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RESEARCH PAPER

A New Contributory Pension Model – Evidence from Chile

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Abstract

We develop a model of individual account (IA) pension systems that considers the wage distribution of the economy and models the density of contributions or so-called "Lagunas" (gaps) as a function of wage inequality. People with lower wages are forced to work in the informal market where they do not contribute to their individual pension accounts. This model allows us to find the entire distribution of pensions; with the possibility to evaluate the effects of policies on average pensions as well as changes in distribution. We applied the model to the Chilean pension system and found that for workers who have contributed less than 18 years an increase in contribution rates has a greater effect than an increase in the interest rate. The people most affected by labor informality benefit more from a marginal increase in the contribution rate than from a marginal increase in the interest rate. Along the same lines, we find that the marginal effect of increasing the contribution rate is greater than the marginal effect of increasing the retirement age. From our study, we find a recommendable policy due to its positive effects in increasing the level of pensions as well as reducing the level of inequality in the contribution rate in the account of each individual at the time of birth.

Keywords: Chile, Contributory Pension**,** Individual Account (IA), Inequality, Pension Distribution.

JEL Classification: H55, H75, J32.

1. Introduction

In the last 50 years, countries have been migrating from Pay-as-you-go (PAYG) pension systems to individual account pension systems (IA). The latter represents a fiscally sustainable alternative (Cerda, 2008) for increasingly deficient pension systems, overburdened due to different factors among which the increase in life expectancy stands out (Montenegro et al., 2019). However, the transition from PAYG systems raises concerns about redistributive effects (Brown, 2007), mainly changes to IA systems where pensions depend directly on people's wages. Therefore, wage inequality in the labor market is transferred to wage inequality in pensions. In developing countries, the problem is even more significant due to the low density of contributions (also called Lagunas) mainly from people with lower wage levels who work in the informal part of the labor market (Arenas de Mesa et al., 2004).

Recently Chile went through tremendous changes in its socio-political and economic dynamics due to many factors like the social movement of September 2019, the global pandemic, and the election of the constituent assembly to draft a new constitution due to the social movement of September 2019 to name a few. There have been many Constitutional amendments made to withdraw the pension amount partially due to the pandemic crisis and the need to reform the existing pension system can be found in the public discourse. In fact, in a public opinion poll conducted by the Centre for Public Studies (Leitch et al., 2019) in December 2019, the "low pension" was the second most important reason behind the 2019 social demonstrations in Chile. Besides, the global coronavirus pandemic has made pension reform an urgent global imperative (Mitchell, 2020), as there is an increase in unemployment with the fall in economic growth and a decline in financial investment returns. In this context, the present study aims to analyze and review the Chilean pension system in its current form through the construction analytical model of the Chilean pension system that allows us to evaluate the effects of different public policy alternatives from both the analytical and empirical points of view.

In this study, a model of individual account pension systems (IA) is developed that takes into account the wage distribution of the economy and models the density of contributions or so-called Lagunas as a function of wage inequality, so that people with lower wages are forced to work in the informal market where they do not contribute to their individual pension account. This model allows finding the entire distribution of pensions; which gives us the possibility to evaluate the effects of policies on average pensions as well as changes in distribution.

This work is closely related to papers that develop computational models for analyzing public policies in the pension system and simulation of policy recommendations such as Nicholas and Diamond (2016), and Benavides Salazar and Valdés (2018).

Among the results to be highlighted, we find that for people with a higher probability of finding formal employment than the net contribution rate, a policy that generates a marginal increase in the contribution rate generates a higher increase in the total expected saving accumulated, and therefore pensions than a policy that marginally reduces the Lagunas.

In the same direction, we found that for the Chilean case, for workers who have contributed less than 18 periods[1] an increase in contribution rates has a greater effect than an increase in the interest rate. The people most affected by labor informality benefit more from a marginal increase in the contribution rate than from a marginal increase in the interest rate. Along the same lines, we find that the marginal effect of increasing the contribution rate is greater than the marginal effect of increasing the retirement age (i.e., the working life).

The previous policies were aimed at increasing average pensions; however, the model allows recovering the full distribution of pensions, so that policies aimed at reducing inequality can be analyzed. In this case, we find that a very recommendable policy due to its positive effects in increasing the level of pensions as well as reducing the level of inequality is the contribution in the account of each individual at the time of birth.

The remainder of the paper is organized as follows. Section 2 characterizes and explains the functioning of the Chilean pension system along with the relevant literature review. In section 3, an analytical model is developed while section 4 deduces results from the point of view of the expected pension. In section 5 we develop a Monte Carlo experiment aimed at simulating the effects of different policies in terms of changes in inequality levels and the replacement rate, and finally, section 6 offers conclusions.

2. Chilean Pension System

2.1 Characterization of the Chilean Pension System

Until 1980, the Chilean pension system was a Pay-As-You-Go (PAYG) system, with contributions from employers, workers, and the State, which were administered by a series of pension funds, each of which had different requirements and characteristics that determined the amount of the pension. In 1981 in Chile, the Pinochet regime reformed the state-led PAYG pension system into the private pension system. The Chilean experiment attracted the attention of both politicians and experts around the world and laid the foundations for the new pension orthodoxy.

According to the superintendence of pensions at the end of 2018 (before the social outbreak and the COVID-19 crisis) there were 10.6 million members of the pension system, of which 1.3 million were pensioners and 5.4 contributors, that is, of all possible contributors only 58.1% were contributing to the system. During the same period, the system accumulated funds of $200 billion, representing 74% of Chile's GDP.

Monthly, contributors contribute 12.68% of their income to the system, of which 10% goes directly to the contributor's pension savings account, 1.15% to the

[1]. Period here refer to one – year.

AFP as a fee for managing the funds, and 1.53% is directed to Disability and Survivorship Insurance (SIS). This last amount is not important for our case, so we assume that the average contributor contributes 11.15% to the pension system.

Since 2002, the system has gone from offering a single fund to a multi-fund system with five alternatives (A, B, C, D, and E) which vary in their portfolio composition between fixed income and variable income assets, and therefore vary in the level of risk to which they expose contributors. Fund A is the riskiest, distributing its portfolio with 79.3% of assets in equities and 20.7% in fixed income. Fund E is the least risky and distributes its portfolio in 5.0% of assets in equities and 95.0% in fixed income.

One of the main problems of the system is the periods in which the contributor cannot contribute to his individual capitalization account, either because he is unemployed or because he is working in the informal sector. According to the Superintendence of Pensions (2018), men on average contribute 58.5% of the total periods in their working life, while women contribute 47%, for an overall rate for the entire workforce of 53.5%.

2.2 Previous Studies on the Pension System

OECD (2019) in their report titled *Pensions at a Glance 2019: OECD and G20 Indicators*, state that average contributions per member in Chile (5.8% of average wages) were lower than the mandatory 10% of earnings. Theoretically, this reflects irregular contributions from plan members to pension plans. Research in the field of the pension system in Chile has been carried out from various approaches. Berstein and Morales (2021) used Chile as a case study to look at the longevity insurance for defined-contribution (DC) systems to examine its importance and potential. Biehl et al. (2021) in their research study highlighted the existing gender inequality in retirement in Chile. Madero-Cabib et al. (2019) analyzed a longitudinal dataset employing sequence analysis from Chile and examined the pension contribution histories of a group of individuals and their formal employment pathways since the early 1980s when this group started working with the introduction of the Chilean private pension system. In their study, they highlighted the importance of the growing diversity and precariousness of labor-force trajectories, which should be considered by policymakers when evaluating the performance and sustainability of pension systems. Bentancor (2020) analyzed the evolution of the Chilean pension system between 2000 and 2017. In the analysis, the focus is on three important aspects of the pension system, which are the coverage achieved, the observed gap between benefits and adequacy levels, and the trajectory of public spending. She concluded that for increasing the benefits of the population and their replacement rates, there is a need for improvement and reform of the pension system. In another study, Vallejos et al. (2019) analyzed the comovement among Pension Fund Administrators (AFPs) in the Chilean private pension system and found evidence of strong comovement among the returns of all AFPs. Undurraga and Becker (2019) in their research explored the perception of the pension system amongst young Chilean women between 20 and 35 years of age (both professional and nonprofessional). The results highlighted the negative perception of the system and a suspicion that the pension could provide them with a good standard of living after retirement. Edwards et al. (2021) with a sample of 500 thousand workers in their study underlined the significant difference in the life expectancy at the age of retirement by gender and income level in the Chilean context. Gómez-Rubio et al. (2016) carried out a qualitative research study to know the experiences and perceptions of 16 Chilean retired women about their pension amount. They found three emerging themes in the study, which are the perception of fraud and regret in the transfer to the AFP; gender discrimination in the workplace impacting their retirement; and impoverishment and suffering in daily life. They concluded that it would be difficult to aspire to be a gender equity society if the pension system is configured as discriminatory and lacking in solidarity. Leitch et al. (2020) in their study highlighted a few relevant points regarding the Chilean pension system while comparing the same with other countries like Australia, Sweden, etc. Some of them are:

- The informal sector poses a relevant problem in Chile because it hinders the coverage of the systems, reduces density, and, therefore, the final pensions. The density of contributions in Chile (53%) is very low compared to other better-placed countries like Sweden (90%)
- The Chilean contribution rate is substantially lower than most of the countries above it in the ranking, which also has an impact on the replacement rate.
- Retirement ages in Chile, especially for women, are among the lowest on the list, surpassed only by Colombia, Turkey, and Thailand. This indicator is even more important considering that our country's post-65 life expectancy is very similar to that of the most developed OECD countries, which are in the process of increasing their retirement ages to combat population aging.

3. Model

3.1 Before Retirement

We assume that individuals have no utility for leisure, so they always prefer to work rather than not work. We also assume that income levels in the formal sector are always sufficiently higher than in the informal sector so that the incentives to move from a job in the formal sector to a job in the informal sector due to tax payments, retirement rates, etc. do not exist. That is, the individual always prefers working to not working, as well as working in the formal sector to working in the informal sector.

We assume that there exist N individuals who are in the labor market, all born at time $t = 0$. Individuals are categorized as female ($j = 1$) and male ($j = 0$). They enter the market at $y_{t,b}^{j}$ years of age, and their age of retirement is $y_{t,e}^{j}$. These individuals are heterogeneous in the sense that when they enter the labor market their initial salary is different and denoted by $w_{0,j}^{i}$. The initial salary is assigned randomly from a log-normal distribution[1] shown below. It is assumed that due to inherent discrimination in the labor market, women's initial salary distribution is different from that of men.

$$f_{w_{0,j}^{i}}\left(w_{0,j}^{i};\mu_j,\sigma_j\right) = \frac{1}{w_{0,j}^{i}\sigma_j\sqrt{2\pi}}\exp\left\{-\frac{\left(\log w_{0,j}^{i}-\mu_j\right)^2}{2\sigma_j^2}\right\}, \quad w_{0,j}^{i} > 0 \qquad (1)$$

After entering the labor market, the constant annual growth rate of wages of individuals with gender j is g_j. That is, we have that $w_{t,j}^{i} = w_{0,j}^{i}\left(1+g_j\right)^{t}$. All individuals are exposed to idiosyncratic risk[2] in the labor market. At the beginning of each period, the worker i is assigned to one of the two possible states. The first one is where the individual is either working in the informal sector or is unemployed ($d_{t,j}^{i} = 0$) and therefore does not contribute to the capitalization account and the second one is the state where she has a job in the formal sector ($d_{t,j}^{i} = 1$) and she contributes to her capitalization account. That happens according to the following Bernoulli process shown in equation (2).

$$d_{t,j}^{i} = \begin{cases} 0, & \pi_j^{i} \geq \dfrac{\log(w_{0,j}^{i})}{Max\{\log(Max\,\mathbf{w}_{0,0}),\log(Max\,\mathbf{w}_{0,1})\}} \\ 1, & \pi_j^{i} < \dfrac{\log(w_{0,j}^{i})}{Max\{\log(Max\,\mathbf{w}_{0,0}),\log(Max\,\mathbf{w}_{0,1})\}} \end{cases} \qquad (2)$$

where $\mathrm{w}_{0,j}$ is the column vector of N rows containing the initial salaries of all individuals with genders j. The probability of being assigned to a job in the formal sector (π_j^{i}) depends on wage inequality. The poorer the contributor, the more likely the individual is working in the informal sector or is unemployed and does not contribute to the individual capitalization account. The opposite is true of higher-paid workers. We define the initial contributable salary as $z_{0,j}^{i} = w_{0,j}^{i}d_{0,j}^{i}$, given that $z_{t,j}^{i}$ is the contributable salary because only from this salary it is possible to deduct pension contributions. As shown in Result 6 in Appendix 1, the initial contributable salary has a Bernoulli-log normal distribution.

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[1]. Gibrat's (1931) takes log-normal distribution to explain the income levels, which is multiplicative in nature.

[2]. Risk pertaining from an individual's tax policy, inflation factor, customer demands and changes in interest rates.

We also assume that $d_{t,j}^{i}$ is independent over time, so for a given individual with gender j we know that the π_{j}^{i} will be the proportion of time that he/she has a formal job. We define the "*Laguna*" as the proportion of time that the individual doesn´t contribute to their individual capital accounts in the pension system. Thus, for individual i with gender j the Laguna is nothing more than the complement of π_{j}^{i}, i.e., $L_{j}^{i} = 1 - \pi_{j}^{i}$.

3.2 From the Time of Retirement

After retirement, the amount of pension that the individual i will receive depends on parameters given by the Superintendence of Pensions of a given country and the discount rate applied. Since these parameters are established exogenously by the pension regulator, these parameters are the same for all individuals in the same age group and gender. The survival rate, $p_{t,j}^{i}$, is defined as the complement of the mortality rate $q_{t,j}^{i}$.

$$p_{t,j}^{i} = 1 - q_{t,j}^{i} \quad (3)$$

These survival rates are assumed to be independent, so the probability that individual i with gender j will still be alive in period t, after the time of retirement $y_{t,e}^{j}$, is given by the multiplication of the survival rates from the time of retirement to period t. As shown in Equation (4):

$$\widetilde{xp}_{t,j}^{i} = \prod_{y_{t,e}^{j}}^{t} p_{t,j}^{i} \quad (4)$$

In Equation (5) we define the annuities, which is nothing more than the probability of remaining alive in period t, discounted by the nominal interest rate r, so that it can be evaluated in the period $y_{t,e}^{j}$.

$$A_{t,j}^{i} = \widetilde{xp}_{t}^{i} \left(\frac{1}{1+r}\right)^{t-(y_{t,e}^{j}-1)} \quad (5)$$

Since this model is of interest to individuals who are alive at least until retirement, we define the probability of being alive one period before retirement as 1 for both genders, i.e., $\widetilde{xp}_{y_{t,e}^{j}-1,j}^{i} = 1$ and therefore $A_{y_{t,e}^{j}-1,j}^{i} = 1$ for $j \in \{0,1\}$.

The total annuity factor for each year during the individual retirement period and when receiving pensions is given by equation (6). It depends on life expectancy, which is a parameter $y_{d,j}$ established by the Superintendence of Pensions to guarantee a very low probability that the pensioner will not consume all of his savings before she dies.

$$TF_{t,j}^{i} = \sum_{t=(y_{t,e}^{j}-1)}^{y_{d,j}} A_{t,j}^{i} \quad (6)$$

Finally, the level of annual pensions ($LAP_{i,j}$) is assumed by the total level of savings accumulated by the pensioner up to the time of the individuals' retirement divided by the total Annuities factor.

$$LAP_{i,j} = TA^{i}_{t,y^{j}_{t,e}} / TF^{i}_{t,j} \tag{7}$$

whereas the level of monthly pensions is given by the level of annual pensions divided by 12 months of the year.

$$LPM_{i,j} = \frac{LAP_{i,j}}{12} \tag{8}$$

Once an individual has retired, there are very few possibilities to affect the pensions of individuals. Below, we incorporate in the model the most important public policy on pensions implemented by the Chilean government.

3.3 Solidarity Pension Contribution Policy

Up to this point, the model shows the case of an IA pension system, however, since we are interested in the Chilean case, we now model the behavior of the government. In Chile, there is a social program called Aporte Previsional Solidario (Solidarity Pension Contribution) aimed at helping people with monthly pensions below a threshold "$PMAS$" called Pensión Máxima con Aporte Solidario (Maximum Pension with Solidarity Contribution). The government transfers an amount of money called Complemento Solidario (Solidarity Complement) "CS" to the individuals, this amount of money can be at most "PBS", that is, the subsidy works according to the following rule.

$$\mathrm{CS}_{i,j} = \begin{cases} PBS - \left(\frac{PBS}{PMAS}\right) LPM_{i,j}, & LPM_{i,j} \leq PMAS \\ 0, & LPM_{i,j} > PMAS \end{cases} \tag{9}$$

That is to say, with this social program, individuals will have pensions after subsidy equivalent to the sum of their pensions plus the Solidarity Contribution, which guarantees that no individual will have pensions lower than the so-called Basic Solidarity Pillar "PBS".

$$LPMAS_{i,j} = LPM_{i,j} + \mathrm{CS}_{i,j} \tag{10}$$

Since we are working with N heterogeneous individuals, the final result of the model shows the distribution of monthly pensions for the different socio-economic groups in the country, based on their income, gender, and their condition of informality in the labor market. This model can help us to simulate and analyze a wide variety of possible policy alternatives.

4. Results

Result 1. The expected number of periods that the individual will contribute to the pension system is given by $E[n(p)] = (y_{t,e}^{j} - y_{t,b}^{j})\pi_j^i$ *and the variance by* $V[n(\pi)] = (y_{t,e}^{j} - y_{t,b}^{j})\pi_j^i(1-\pi_j^i)$.

(Proof in Appendix 1).

when the individual has a job in the informal sector, he/she does not contribute to the pension system. If the same individual is working in the formal sector, he/she would contribute an $c\%$ of his/her salary. The contributions made by the individuals are managed by a private firm that charges a $\alpha\%$ of salary as a fee for managing these funds. We denote the annual contribution at time t for individual i with gender j by $PC_{t,j}^{i}$ shown in Equation (11),

$$PC_{t,j}^{i} = (c-\alpha)w_{t,j}^{i}d_{t,j}^{i} = z_{t,j}^{i}(c-\alpha) \quad (11)$$

Note that the annual contribution can be zero if the individual does not have a job in the formal sector. We assume that at all times the period t in which we are is known.

When $d_{t,j}^{i} = 0$, two cases can happen, we define the realization of the random variable $m_{t,j}^{i} \in \{0,1\}$, when $m_{t,j}^{i} = 1$, the individual has a job in the informal sector, in which case we assume that his income is $w_{t,j,m}^{i} = (1-\delta)w_{t,j}^{i}$ with $\delta > c-\alpha$, or it could happen that $m_{t,j}^{i} = 0$, in which case the individual is unemployed, in this case, he does not receive any wages. We do not delve further into the random variable $m_{t,j}^{i}$ in order not to increase the complexity of the analysis.

Result 2. The annual contribution has the following mean and variance

$$E\left[PC_{t,j}^{i}\right] = (c-\alpha)(1+g_j)^{t}e^{\mu_j+\frac{1}{2}\sigma_j{}^{2}}\pi_j^i \quad (12)$$

(Proof in Appendix 1).

In each period t the firm invests the money and gets the nominal interest rate of return r that allows the contributor's annual contribution at time t to be transformed into $PA_{t,y_{t,e}^{j}}$ at the time of retirement, $y_{t,e}^{j}$,i.e.,

$$PA_{t,y_{t,e}^{j}}^{i} = PC_{t,j}^{i}(1+r)^{y_{t,e}{}^{j}-t} \quad (13)$$

At the end of the last year of contribution, the total saving accumulated is given by the sum of all the contributions made during his lifetime, transferred to the period $y_{t,e}^{j}$, as shown in Equation (14).

$$TA_{t,y_{t,e}^{j}}^{i} = \sum_{t=y_{t,b}^{j}}^{y_{t,e}^{j}} PA_{t,y_{t,e}^{j}}^{i} \quad (14)$$

Result 3 shows how the expected value of total savings accumulated depends directly on the most commonly used variables in public policy, viz: the rate of contribution rate c, the mean income distribution $e^{\mu_j+\frac{1}{2}\sigma_j{}^2}$, the laguna 1-$\pi_j^i$, the contribution years $y_{t,e}^j - y_{t,b}^j$, the rate of return achieved by the AFP companies r and the wage growth rate g_j.

Result 3. The total saving accumulated *has the following mean and variance*

$$E\left[TA^i_{t,y^j_{t,e}}\right] = (c-\alpha)e^{\mu_j+\frac{1}{2}\sigma_j{}^2}\pi_j^i \sum_{t=0}^{y^j_{t,e}-y^j_{t,b}}(1+r)^t(1+g_j)^t \quad (15)$$

(Proof in Appendix 1).

Result 4. If $\pi_j^i \geq (c-\alpha)$ *the marginal effects on the expected total saving accumulated from increases in the contribution rate are larger than the marginal reduction of the Lagunas.*

(Proof in Appendix 1).

We can deduce some interesting conclusions from Result 4, for people with a higher probability of finding formal employment, π_j^i than the net contribution rate, $(c-\alpha)$, a policy that generates a marginal increase in the contribution rate generates a higher increase in the total expected saving accumulated than a policy that marginally reduces the Lagunas. In the Chilean case, the average probability of finding a formal job according to Superintendencia de Pensiones (2018) is $\bar{\pi} = 0.535$ while the net contribution rate is $c-\alpha = 0.1$, so clearly most Chilean workers fall into this case.

To take into account which policy would generate the greatest net benefit, it is necessary to quantify the costs. The expected costs of increasing the contribution rate are covered directly by the workers. It is equal to the increase in the contribution rate times the number of times the individual will contribute multiplied by the expected income,

$$E[Cost_1(\Delta c)] = \Delta \mathrm{c} E\left[n\left(\pi_j^i\right)w_{t,j}^i\right]. \quad (16)$$

However, in this case, the model does not consider the possibility that the individual migrates to the informal sector because the wage may be higher (the model assumes that the wage in the formal sector is always sufficiently higher than in the informal sector for this case to occur). In the case that it is the firms that cover this cost, the model does not consider the possibility of their closing and therefore, the possible increase in the Lagunas or the possible reduction of taxes collected.

The expected cost in the case of a policy that reduces Lagunas is directly linked to the increase in the worker's contributions but is offset by an increase in income both in the case where he goes from having an informal job to a formal one and in which it goes from unemployment to formal employment,

$$E\left[Cost_2\left(\Delta\pi_j^i\right)\right] = (c-\alpha)\Delta E\left[n\left(\pi_j^i\right)\right]E\left[w_{t,j}^i\right] - \mathrm{E}\left[w_{t,j}^i - w_{t,j,m}^i \middle| m_{t,j}^i = 1\right] - \mathrm{E}\left[w_{t,j}^i \middle| m_{t,j}^i = 0\right]. \quad (17)$$

In this case, the model does not include other possible costs and benefits, for example, the cost to the government of the public policy that reduces labor informality and the benefits that would accrue to the government in the form of taxes.

Result 5. If the number of periods contributed to social security is greater than or equal to 18, then for the existing parameter values in Chile, we have that the marginal effects on the expected total saving accumulated from increases in the contribution rate are the largest with respect to the increase in the interest rate. (Proof in Appendix 1).

For workers who have contributed less than 18 periods an increase in contribution rates has a greater effect than an increase in the interest rate. The people most affected by labour informality benefit more from a marginal increase in the contribution rate than from a marginal increase in the interest rate.

From Result 1 we know that the expected number of periods that workers will contribute is given by the expression, $E[n(\pi)] = (y_{t,e}^{j} - y_{t,b}^{j})\pi_{j}^{i}$, so the most benefited individuals correspond to those for whom the probability of finding a formal job in each period is $\pi_{j}^{i} \leq E[n(\pi)]/(y_{t,e}^{j} - y_{t,b}^{j})$. With the values for the Chilean case provided in calibration section, we find that this probability corresponds to is $\pi_{0}^{i} \leq 0.439$ for men and $\pi_{1}^{i} \leq 0.5$ for women. Another way to read it is that the most benefited would be men with lagunas greater than 56% of their working life and women with lagunas greater than 50%. In Chile according to the Superintendencia de Pensiones (2018) the average rate of labor gaps for men is $\bar{L}_0 = 1 - \bar{\pi}_0 = 0.415$ and that of women is $\bar{L}_1 = 1 - \bar{\pi}_1 = 0.521$. That is, most women benefit more from a marginal increase in the contribution rate than in the interest rate, while the opposite is true for men.

It is important to note that policies aimed at increasing the competitiveness of AFP companies would generate a reduction in fees (α) or increases in interest rates (i). A reduction in commissions has the same effect as an increase in the contribution rate (c) but its reduction margin is limited because its percentage level is low in absolute terms (1.15%). An increase in the interest rate is not limited in the same way and given that the interest rate obtained by the AFPs for the riskiest fund is well below the market, there is much potential for improvement. It is also important to note that unlike the policies of increasing the net contribution rate, the policies aimed at increasing the interest rate by increasing competition in the market do not generate costs for workers or the government, but simply reduce the profits of the AFP market, bringing them closer to those of a competitive market.

Another variable that affects the levels of total expected saving accumulated is the growth rate of salaries, which from Equation (15) we see that it has the same effects as the interest rate, so the same results of Result 5 apply to it.

Finally, we can prove that the marginal effect of increasing the contribution rate is greater than the marginal effect of increasing the retirement age (i.e., the working life), the demonstration would follow the same steps as Results 4 and 5. But there is more intuitive way of looking at it. Given that in the Chilean case the interest rate is higher than the rate of salary increase, also, given the probability of finding a formal job, on average men will contribute to social security for 24 years and women for 17 years. Therefore, it is more likely that contributing $\Delta c = 0.01$ of 24 periods salaries and 17 periods salaries generates more benefits than contributing $c = 0.1$ of the salary in the last period with a probability of 0.585 for men and 0.479 for women (even though the latter salary is the greatest in the person's working life).

In the following section, we perform montecarlo exercises to show the effects of most frequently mentioned public policies on the pension levels for each gender, replacement rate and the most commonly used inequality indicator function, the Gini coefficient of pension distribution.

5. Monte Carlo Experiment

5.1 Calibration

We apply this model to the Chilean economy. We begin finding the parameters of the income density distribution $\{\mu_j, \sigma_j\}$. We use the data of income from the National Socioeconomic Characterization Survey[1] (2017) and estimate the parameter using the maximum likelihood method[2]. Pension contribution rate parameters in Chile is 11.5% of the salary. Each contributor pays 10% of his/her income to his/her individual capitalization in the pension system and the remaining 1.15% is paid to AFP as a fee charged for managing the funds (SP, 2018).

The average interest rate offered by the different AFP from 2000 to 2017 according to the Superintendence of Pensions for fund c, which is the most representative fund, is 5.14%. According to the Chilean AFP Association (2019), the age of entering the labour market and making the first corresponding contribution has increased over time and in 2019 had already reached 29 years on average while in the 1980s it was 20 years. In this case we set the age of the first contribution at 24 years for both genders. On the other hand, the retirement age is currently 60 for women and 65 for men.

For mortality and life expectancy rates, the most current values (2016) established by the Superintendence of Pensions (SP) and the Superintendence of

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[1] Encuesta de Caracterización Socioeconómica Nacional (CASEN by its acronym Spanish).

[2]. In order to find the average wages upon entry into the labour market of people who retire in 2017, the mean income found is multiplied by the factor $\frac{1}{(1+g)^{y^m_{t,e}-y^m_{t,b}}}$.

Securities and Insurance (SVS) are used (see Appendix). As per the mortality table of Chilean pension system, the final age is considered to be 110 years for both men and women. In the same direction, the value of the Basic Solidarity Pillar (PBS) and the Maximum Solidarity Pension (PMAS) for 2017 are obtained from the Superintendence of Pensions (SP). Finally, for this model, we have considered a wage growth rate of 1.85% as in Benavides and Valdés (2018).

Table 1. Model Parameters

Parameter	symbol	Men ($\boldsymbol{j=0}$)	Women ($\boldsymbol{j=1}$)
Mean of income density	μ_j	12.54203	12.30205
Standard deviation of income density	σ_j	0.756749	0.741032
Average annual wage growth rate	g	0.0185	0.0185
Pension contribution rate	c	0.1115	0.1115
fee rate	α	0.0115	0.0115
rate of return	r	0.0514	0.0514
Age of entry into the labour market.	$y_{t,b}^{j}$	24	24
Age of retirement of the contributor.	$y_{t,e}^{j}$	65	60
Life expectancy	$y_{d,j}$	110	110
Pilar Básico Solidario (US$)	PBS	165.25	165.25
Pensión Máxima Solidaria (US$)	$PMAS$	488.33	488.33

Source: Research finding.

Table 2 shows the average pension after subside and the replacement rate that results from the model and compares it with the real data of the Chilean economy in 2017. In both cases, the differences are not statistically significant, so the model correctly replicates the Chilean case. In a way, this also shows that the overall fit of the model is very good.

The results shown are from a population of 20,000 simulated individuals, 10,000 men and 10,000 women. Figure 1 shows the complete distribution of pensions generated by the model. Since the model makes it possible to recover the entire distribution, it is possible to analyze the effects of public policies on its different moments, so that it is possible to observe increases in the average pension, replacement rate and variations in the levels of inequality in the distribution of pensions. This, added to the fact that the model differentiates between genders, makes it possible to analyze various policy effects that are not usually considered. In this way, policy makers can decide on those policies that generate the greatest effect in the areas of interest.

Table 2. Model Output and Fits

Variable	Model: Means of Sample Distribution (Standard Deviations)	Chile Economy: Sample Values at 2017	Difference[1]
Pension Mean (US$)	429.42	425.47	3.85
Replacement Rate	0.45	0.44	-0.02
Women's Pension Mean (US$)	398.21	389.83	8.38
Women's Replacement Rate	0.35	0.38	0.03
Men's Pension Mean (US$)	456.43	456.53	-0.10
Men's Replacement Rate	0.54	0.56	-0.02

Source: Research finding and Superintendence of Pensions (SP).
Note: To find the total value of the population, a weighted average is used, according to SP in 2017 53% of pensioners were men and 47% were women.

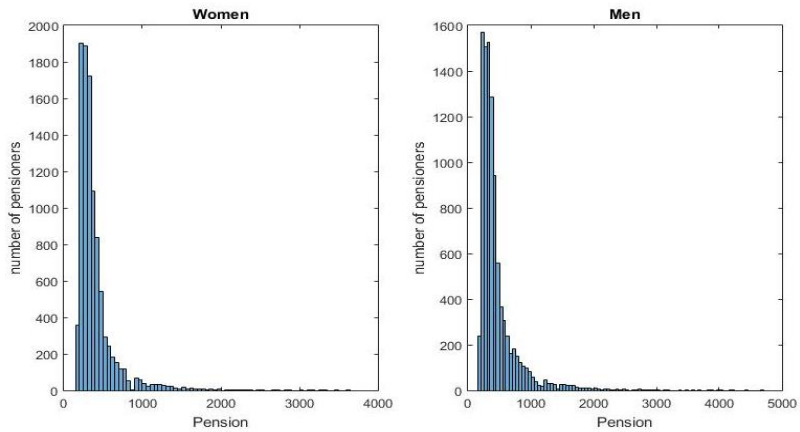

Figure 1. Distribution of pensions (US$ dollars)
Source: Research finding.

5.2 Public Policy Simulations

Finally, in this section we proceed to analyze the most frequently mentioned public policies. We proceeded to evaluate by means of simulations three frequently proposed pension policies. First, a 4% increase in the contribution rate, from 10% to 14%; second, a 5-year increase in the retirement age, so that women retire at 65 and men at 70; third, at birth, deposit US$ 3 thousand of 2017 in the individual account. It should be noted that the model is very general and can evaluate many other possible public policies.

As can be seen in the result tables below, all the proposed policies generate increases in the average pension levels, although the increase in the contribution rate and years of contribution generate an increase in the average pension close to

[1]. The comparison in means of the average pension is made on the logarithmic transformation of the results.

30% and the contribution at birth generates an increase of 10%. It is important to note that only this last proposal generates a drop in the levels of inequality in the distribution of pensions both within each gender (there is less inequality measured by the Gini coefficient between individuals of the same gender) and between genders (there is less inequality, i.e., a lower Gini coefficient in the population as a whole). This can also be seen in the male-female pension ratio.

Table 3. Model Total Population Output and Fits

Simulations	Variable	Benchmark	Model (Means of Sample Distribution)	Difference	Percentage Change
	Pension Mean (US$)	429.07	552.84	123.77	28.85
$\alpha = 0.14$	Replacement Rate	0.45	0.58	0.13	
	Gini coefficient	0.43	0.43	-	
	Pension Mean (US$)	429.07	547.06	117.99	27.50
$\Delta {y_{t,e}}^{m} = 5$	Replacement Rate	0.45	0.56	0.11	
	Gini coefficient	0.43	0.44	0.01	
	Pension Mean (US$)	429.07	472.42	43.35	10.10
Contribution at birth	Replacement Rate	0.45	0.52	0.06	
	Gini coefficient	0.43	0.37	-0.06	-14.04

Source: Research finding.

Table 4. Model Output and Fits for First Policy

Simulations	Variable	Benchmark	Model (Means of Sample Distribution)	Difference	Percentage Change
	Women's Pension Mean (US$)	398.21	507.10	108.89	27.34
$\alpha = 0.14$	Women's Replacement Rate	0.35	0.46	0.11	
	Women's Gini coefficient	0.274	0.324	0.05	
	Men's Pension Mean (US$)	456.43	593.40	136.97	30.01
$\alpha = 0.14$	Men's Replacement Rate	0.54	0.69	0.15	
	Men's Gini coefficient	0.312	0.36	0.048	

Source: Research finding.

Table 5. Model Output and Fits for Second Policy

Simulations	Variable	Benchmark	Model (Means of Sample Distribution)	Difference	Percentage Change
	Women's Pension Mean (US$)	398.21	510.38	112.17	28.17
$\Delta y_{t,e}{}^{m} = 5$	Women's Replacement Rate	0.35	0.45	0.1	
	Women's Gini coefficient	0.274	0.327	0.053	
	Men's Pension Mean (US$)	456.43	579.58	123.15	26.98
$\Delta y_{t,e}{}^{m} = 5$	Men's Replacement Rate	0.54	0.6508	0.1108	
	Men's Gini coefficient	0.312	0.361	0.049	

Source: Research finding.

Table 6. Model Output and Fits for Third Policy

Simulations	Variable	Benchmark	Model (Means of Sample Distribution)	Difference	Percentage Change
	Women's Pension Mean (US$)	398.21	436.62	38.41	9.65
Contribution at birth	Women's Replacement Rate	0.35	0.42	0.07	
	Women's Gini coefficient	0.274	0.256	-0.018	
	Men's Pension Mean (US$)	456.43	504.17	47.74	10.46
Contribution at birth	Men's Replacement Rate	0.54	0.60	0.06	
	Men's Gini coefficient	0.312	0.291	-0.021	

Source: Research finding.

This last public policy has this characteristic: the benefits it generates in terms of inequality reduction are difficult to replicate by policies applied before the individual has retired. Therefore, it is very different from policies that transfer resources to pensioners such as the one presented in section 3.3 (Solidarity Pension Contribution) to generate reductions in inequality levels. Because of this, it is a policy with a low implementation cost and high potential benefit, since it benefits from compound interest over the lifetime of the individual.

So depending on the objectives of the policy maker different policies may be the best, for increases in the average pension, as shown in result section and quantified in this section, increasing the contribution rate may be the most beneficial, while seeking not only to increase pensions but to reduce inequality, contributions at birth is a preferable policy. However, the latter policy generates longer-term effects than the former.

6. Conclusion

The Chilean pension system is under significant criticism and pressure to reform as it is growing. There is concern about the low level of pensions it provides and the high levels of inequality it produces. There are three main characteristics of the Chilean pension system: first, it is a private individual account pension system (IA); second, it is a country with high levels of inequality; and third, it shows a high rate of informality in the labor market. Under these circumstances, any policy analysis must consider these underlying conditions when analyzing possible policy proposals.

In this paper we develop an analytical model of the Chilean pension system, it is assumed that individuals are heterogeneous and that at birth they are assigned an income level, individuals assigned to higher income groups will have a higher probability of finding formal employment when entering the labor market and will be less likely to be affected by idiosyncratic shocks that leads to unemployment or informal employment, so they have a higher probability of remaining in their socioeconomic stratum throughout their lives. The probability of finding a formal job at the beginning of the labor market is lower as the individual is assigned a lower income level at birth, so that the poorer the individual, the greater the probability of being affected by idiosyncratic shocks that send him/her to unemployment or informal employment. This model allows us to analyze inequality within and between genders, because it assumes that the underlying income distribution is different between genders.

Pension fund management companies manage the funds, achieving a return in exchange for a fee. After the person retires from the labor market, if his or her income levels are low, he/she will benefit from a social program implemented with the government.

We find that, for people with a higher probability of finding formal employment, π_j^i than the net contribution rate, $(c - \alpha)$, a policy that generates a marginal increase in the contribution rate generates a higher increase in the total expected saving accumulated than a policy that marginally reduces the Lagunas. In the Chilean case the average probability of finding a formal job according to Superintendencia de Pensiones (2018) is $\bar{\pi} = 0.535$ while the net contribution rate $c - \alpha = 0.1$, so clearly most Chilean workers fall into this case.

For workers who have contributed less than 18 periods an increase in contribution rates has a greater effect than an increase in the interest rate. The people most affected by labor informality benefit more from a marginal increase in the contribution rate than from a marginal increase in the interest rate. The most benefited would be men with Lagunas greater than 56% of their working life and women with Lagunas greater than 50%. In Chile according to the Superintendencia de Pensiones (2018) the average rate of labor gaps for men is $\bar{L}_0 = 1 - \bar{\pi}_0 = 0.415$ and that of women is $\bar{L}_1 = 1 - \bar{\pi}_1 = 0.521$. That is, most women benefit more from a marginal increase in the contribution rate than in the interest rate, while the opposite is true for men.

Policies aimed at increasing the competitiveness of AFP companies would generate a reduction in fees (α) or increases in interest rates (i). A reduction in commissions has the same effect as an increase in the contribution rate (c) but its reduction margin is limited because its percentage level is low in absolute terms (1.15%). An increase in the interest rate is not limited in the same way and given that the interest rate obtained by the AFPs for the riskiest fund is well below the market, there is much potential for improvement. It is also important to note that unlike the policies of increasing the net contribution rate, the policies aimed at increasing the interest rate by increasing competition in the market do not generate costs for workers or the government, but simply reduce the profits of the AFP market, bringing them closer to those of a competitive market.

Another variable that affects the levels of total expected saving accumulated is the growth rate of wages, we find that it has the same effects as the interest rate, so the same results apply to it. We also find, the marginal effect of increasing the contribution rate is greater than the marginal effect of increasing the retirement age (i.e., the working life).

Finally, we proceeded to evaluate by means of simulations three frequently proposed pension policies. First, a 4% increase in the contribution rate, from 10% to 14%; second, a 5-year increase in the retirement age, so that women retire at 65 and men at 70; third, at birth, deposit US$ 3 thousand of 2017 in the individual account. It should be noted that the model is very general and can evaluate many other possible public policies.

All the proposed policies generate increases in the average pension levels, although the increase in the contribution rate and years of contribution generate an increase in the average pension of close to 30% and the contribution at birth generates an increase of 10%. It is important to note that only this last proposal generates a drop in the levels of inequality in the distribution of pensions both within each gender (there is less inequality measured by the Gini coefficient between individuals of the same gender) and between genders (there is less inequality, i.e., a lower Gini coefficient in the population as a whole). This can also be seen in the male-female pension ratio. This last public policy has this characteristic: the benefits it generates in terms of inequality reduction are difficult to replicate by policies applied before the individual has retired.

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Appendix 1

Result 1. The expected number of periods that the individual will contribute to the pension system is given by $E[n(\pi)] = (y^j_{t,e} - y^j_{t,b})\pi^i_j$ and the variance by $V[n(\pi)] = (y^j_{t,e} - y^j_{t,b})\pi^i_j(1 - \pi^i_j)$.

Proof: The proof follows directly from the assumption that $d^i_{t,j}$ is a Bernoulli process and independent over time, and that the total number of trials is given by the total number of periods that the worker is active in the labor market $(y^j_{t,e} - y^j_{t,b})$.

Result 2. The annual contribution has the following mean and variance

$$E[PC^i_{t,j}] = (c - \alpha)(1 + g_j)^t e^{\mu_j + \frac{1}{2}\sigma_j{}^2}\pi^i_j$$

$$Var[PC^i_{t,j}] = (c - \alpha)^2(1 + g_j)^{2t} e^{2\mu_j + \sigma_j{}^2}\pi^i_j(e^{\sigma_j{}^2} - \pi^i_j)$$

Proof: Remember that $w^i_{t,j} = (1 + g_j)^t w^i_{0,j}$ so we have that

$$PC^i_{t,j} = (c - \alpha)w^i_{t,j}d^i_{t,j} = (c - \alpha)(1 + g_j)^t w^i_{0,j}d^i_{t,j}$$

Although it is not mentioned in the text, we assume that g is a random variable that is normally distributed with mean γ and ρ variance, this random variable is independent of $d^i_{t,j}$ and $w^i_{0,j}$. Then applying the conditional expectation operator $E\left[PC^i_{t,j}|g_j\right]$ and for simplicity throughout all remaining proofs, we will omit showing the conditional, but unless otherwise noted, we are conditionalizing on g_j. Now using that $w^i_{0,j}$ and $d^i_{t,j}$ are independent for $t > 0$ we have that $E\left[PC^i_{t,j}\right] = (c-\alpha)\left(1+g_j\right)^t E\left[w^i_{0,j}\right]E\left[d^i_{t,j}\right]$. Given that $w^i_{0,j}$ is a log-normal random variable and $d^i_{t,j}$ we know that their mean are $E\left[w^i_{0,j}\right] = e^{\mu_j+\frac{1}{2}\sigma_j{}^2}$and $E\left[d^i_{t,j}\right] = p_j{}^i$ so we have $E\left[PC^i_{t,j}\right] = (c-\alpha)\left(1+g_j\right)^t e^{\mu_j+\frac{1}{2}\sigma_j{}^2}\pi_j{}^i$.

For the variance, applying the expectation operator we have:

$$\begin{aligned}
\mathrm{Var}\left(PC^i_{t,j}\right) &= (c-\alpha)^2\left(1+g_j\right)^{2t}\mathrm{Var}\left(w^i_{0,j}d^i_{t,j}\right)\\
&= (c-\alpha)^2\left(1+g_j\right)^{2t}\{\left[\mathrm{Var}\left(w^i_{0,j}\right)+E\left(w^i_{0,j}\right)^2\right]\left[\mathrm{Var}\left(d^i_{t,j}\right)\right.\\
&\left.+E\left(d^i_{t,j}\right)^2\right]-E\left(w^i_{0,j}\right)^2E\left(d^i_j\right)^2\}\\
&= (c-\alpha)^2\left(1+g_j\right)^{2t}\left\{\left[\left(e^{\sigma_j{}^2}-1\right)e^{2\mu_j+\sigma_j{}^2}\right.\right.\\
&\left.\left.+e^{2\mu_j+\sigma_j{}^2}\right]\left[\pi^i_j\left(1-\pi^i_j\right)+\pi^i_j{}^2\right]-e^{2\mu_j+\sigma_j{}^2}\pi^i_j{}^2\right\}\\
&= (c-\alpha)^2\left(1+g_j\right)^{2t}\left[e^{2\mu_j+2\sigma_j{}^2}\pi^i_j-e^{2\mu_j+\sigma_j{}^2}\pi^i_j{}^2\right]\\
&= (c-\alpha)^2\left(1+g_j\right)^{2t}e^{2\mu_j+\sigma_j{}^2}\pi^i_j\left(e^{\sigma_j{}^2}-\pi^i_j\right)
\end{aligned}$$

Result 3. The total saving accumulated has the following mean and variance:

$$E\left[TA^i_{t,y^j_{t,e}}\right] = (c-\alpha)e^{\mu_j+\frac{1}{2}\sigma_j{}^2}\pi^i_j\sum_{t=0}^{y^j_{t,e}-y^j_{t,b}}(1+r)^t(1+g_j)^t$$

$$\mathrm{Var}\left[TA^i_{t,y^j_{t,e}}\right] = (c-\alpha)^2e^{2\mu_j+\sigma_j{}^2}\pi^i_j\left(e^{\sigma_j{}^2}-\pi^i_j\right)\sum_{t=0}^{y^j_{t,e}-y^j_{t,b}}(1+r)^{2t}(1+g_j)^{2t}$$

Proof: First we rewrite the total saving accumulated in the following way:

$$TA^i_{t,y^j_{t,e}} = \sum_{t=y^j_{t,b}}^{y^j_{t,e}} PA^i_{t,y^j_{t,e}} = \sum_{t=0}^{y^j_{t,e}-y^j_{t,b}} PA^i_{t,y^j_{t,e}} = \sum_{t=0}^{y^j_{t,e}-y^j_{t,b}}(1+r)^t PC^i_{t,j}$$

Now we can applied expectation operator and the results of Result 1:

$$E[TA^i_{t,y^j_{t,e}}] = \sum_{t=0}^{y^j_{t,e}-y^j_{t,b}} (1+r)^t E[PC^i_{t,j}]$$
$$= \sum_{t=0}^{y^j_{t,e}-y^j_{t,b}} (1+r)^t (c-\alpha)(1+g_j)^t e^{\mu_j+\frac{1}{2}\sigma_j^2} \pi_j{}^i$$
$$= (c-\alpha) e^{\mu_j+\frac{1}{2}\sigma_j^2} \pi_j{}^i \sum_{t=0}^{y^j_{t,e}-y^j_{t,b}} (1+r)^t (1+g_j)^t$$

For the variance we have:

$$Var[TA^i_{t,y^j_{t,e}}] = \sum_{t=0}^{y^j_{t,e}-y^j_{t,b}} (1+r)^{2t} Var[PC^i_{t,j}]$$
$$= \sum_{t=0}^{y^j_{t,e}-y^j_{t,b}} (1+r)^{2t} (c-\alpha)^2 (1+g_j)^{2t} e^{2\mu_j+\sigma_j{}^2} \pi^i_j (e^{\sigma_j{}^2} - \pi^i_j)$$
$$= (c-\alpha)^2 e^{2\mu_j+\sigma_j{}^2} \pi^i_j (e^{\sigma_j{}^2} - \pi^i_j) \sum_{t=0}^{y^j_{t,e}-y^j_{t,b}} (1+r)^{2t} (1+g_j)^{2t}$$

Result 4. If $\pi^i_j \geq (c-\alpha)$ the marginal effects on the total expected saving accumulated from increases in the contribution rate are larger than a marginal reduce of the Lagunas.

Proof: Taking first derivative of the expected total saving accumulated we have

$$\frac{\partial E\left[TA^i_{t,y^j_{t,e}}\right]}{\partial c} = e^{\mu_j+\frac{1}{2}\sigma_j{}^2} \pi^i_j \sum_{t=0}^{y^j_{t,e}-y^j_{t,b}} (1+r)^t (1+g_j)^t$$
$$\frac{\partial E\left[TA^i_{t,y^j_{t,e}}\right]}{\partial \pi^i_j} = (c-\alpha) e^{\mu_j+\frac{1}{2}\sigma_j{}^2} \sum_{t=0}^{y^j_{t,e}-y^j_{t,b}} (1+r)^t (1+g_j)^t$$

We first compare the marginal effects of increasing the contribution rate with reducing Lagunas (increasing the probability of obtaining formal employment).

$$\frac{\partial E\left[TA^i_{t,y^j_{t,e}}\right]}{\partial c} \geq \frac{\partial E\left[TA^i_{t,y^j_{t,e}}\right]}{\partial \tilde{p}^i_j}$$

$$\left[e^{\mu_j+\frac{1}{2}\sigma_j{}^2}\sum_{t=0}^{y_{t,e}^j-y_{t,b}^j}(1+i)^t(1+g_j)^t\right]\left(\pi_j^i-(c-\alpha)\right)\geq 0$$

The left-hand term in square brackets is composed of positive elements, so this inequality is defined by the second term.

$$\left(\pi_j^i-(c-\alpha)\right)\geq 0$$

That is, for people with a higher probability of finding formal employment than the net contribution rate, a marginal increase in the contribution rate generates a higher total expected saving accumulated than a policy that marginally reduces the Lagunas.

Result 5. If the number of periods contributed to social security is greater than or equal to 18, then for the existing parameter values in Chile, we have that the marginal effects on the expected total saving accumulated from increases in the contribution rate are the largest with respect to the increase in the interest rate.

Proof: Again taking first derivative of the expected total saving accumulated we have:

$$\frac{\partial E\left[TA_{t,y_{t,e}^j}^i\right]}{\partial c}=e^{\mu_j+\frac{1}{2}\sigma_j{}^2}\pi_j^i\sum_{t=0}^{y_{t,e}^j-y_{t,b}^j}(1+r)^t(1+g_j)^t$$

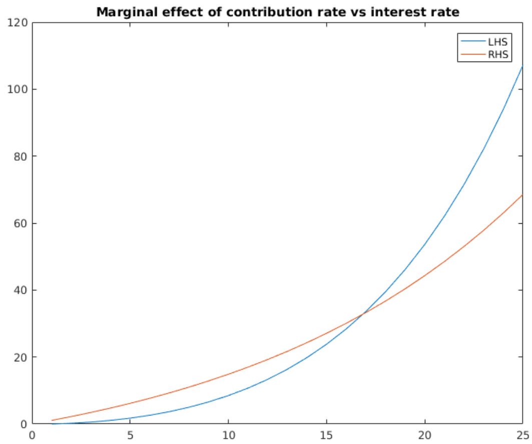
$$\frac{\partial E\left[TA_{t,y_{t,e}^j}^i\right]}{\partial i}=(c-\alpha)e^{\mu_j+\frac{1}{2}\sigma_j{}^2}\pi_j^i\sum_{t=0}^{y_{t,e}^j-y_{t,b}^j}t(1+r)^{t-1}(1+g_j)^t$$

Now compare the marginal effects of increasing the contribution rate with increasing the interest rate.

$$\frac{\partial E\left[TA_{t,y_{t,e}^j}^i\right]}{\partial c}\geq\frac{\partial E\left[TA_{t,y_{t,e}^j}^i\right]}{\partial i}$$

$$\sum_{t=0}^{y_{t,e}^j-y_{t,b}^j}(1+r)^t t\left(1+g_j\right)^{t-1}(c-\alpha)\leq\sum_{t=0}^{y_{t,e}^j-y_{t,b}^j}(1+r)^t(1+g_j)^t$$

Section 4.1 presents values for the Chilean case, where $r=0.0514$, $g=0.0185$, $c=0.1115$, $\alpha=0.0115$. With these values the inequality holds only if people contributed less than 18 periods.



In other words, for people who have contributed less than 18 periods, an increase in contribution rates has a greater effect than an increase in the interest rate. The people most affected by labor informality benefit more from a marginal increase in the contribution rate than from a marginal increase in the interest rate.

Result 6. For each individual and gender the initial contributable salary follow a Bernoulli-log normal distribution with parameters $\tilde{p}_j^i, \mu_j$ and σ_j.

$$F_{Z_{0,j}^i}\left(z_{0,j}^i; \pi_j^i, \mu_j, \sigma_j\right) = 1 - \pi_j^i + \pi_j^i \int_0^z \frac{1}{w_{0,j}^i \sigma_j \sqrt{2\pi}} \exp\left\{-\frac{\left(\log w_{0,j}^i - \mu_j\right)^2}{2\sigma_j^2}\right\} dw_{0,j}^i,$$

Proof: The random variable $Z_{0,j}^i$ doesn't take values in $(-\infty, 0)$. Then, we just have to take in account $Z_{0,j}^i \in [0, +\infty)$. In that case, we have:

$$\begin{aligned} F_{Z_{0,j}^i}\left(z_{0,j}^i\right) &= \Pr\left(d_{0,j}^i = 1\right) \Pr\left(w_{0,j}^i d_{0,j}^i \leq z_{0,j}^i | d_{0,j}^i = 1\right) \\ &\quad + \Pr\left(d_{0,j}^i = 0\right) \Pr\left(w_{0,j}^i d_{0,j}^i \leq z_{0,j}^i | d_{0,j}^i = 0\right) \\ &= \Pr\left(d_{0,j}^i = 1\right) \Pr\left(w_{0,j}^i \leq z_{0,j}^i\right) + \Pr\left(d_{0,j}^i = 0\right) \Pr\left(0 \leq z_{0,j}^i\right) \\ &= \pi_j^i \Pr\left(w_{0,j}^i \leq z_{0,j}^i\right) + (1 - \pi_j^i) \\ &= 1 - \pi_j^i + \pi_j^i \int_0^z \frac{1}{w_{0,j}^i \sigma_j \sqrt{2\pi}} \exp\left\{-\frac{\left(\log w_{0,j}^i - \mu_j\right)^2}{2\sigma_j^2}\right\} dw_{0,j}^i \end{aligned}$$

In addition, the density is given by:

$$f_{Z_{0,j}^i}\left(z_{0,j}^i\right) = \frac{dF_{Z_{0,j}^i}}{dw_{0,j}^i} = \pi_j^i \frac{1}{w_{0,j}^i \sigma_j \sqrt{2\pi}} \exp\left\{-\frac{\left(\log w_{0,j}^i - \mu_j\right)^2}{2\sigma_j^2}\right\}$$

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