-RESEARCH PAPER-

International Journal of Mining and Geo-Engineering

IJMGE

IJMGE 58-4 (2024) 401-410

DOI: 10.22059/IJMGE.2024.374224.595160

Evaluating the relationship of vertical drilling rate with rock properties in the Marble quarry mining

Mohammad Rezaei [a, *], Navid Nyazyan [a], Mostafa Asadizadeh [b]

[a] *Department of Mining Engineering, Faculty of Engineering, University of Kurdistan, Sanandaj, Iran.*

[b] *Lead consultant, Mine stability department, WSP Global Inc., Colorado, USA.*

Article History:

Received: 21 March 2024.

Revised: 19 August 2024.

Accepted: 29 October 2024.

ABSTRACT

In this research, the relationship of vertical drilling rate (*VDR*) with rock properties was studied in the marble quarry mining based on the combination of field surveys and laboratory experiments. To achieve this, the *VDR* was initially measured while drilling at the Malawi marble quarry mine. Then, the physical and mechanics tests were conducted on the core specimens prepared from the collected minor blocks corresponding to the under-drilling mine benches. The parametric study revealed that natural density, dry density, slake durability index, Schmidt hammer rebound, compression wave velocity, point load index, uniaxial compressive strength and modulus of elasticity exhibit an inverse relationship with *VDR*. Conversely, VDR was found to have a direct relationship with porosity, water content, Los Angeles abrasion, and Poisson's ratio. Moreover, it was confirmed that *the VDR* is more correlated with the rock mechanical properties rather than the rock physical features, considering the obtained mean coefficient of determination (*COD*) values of 0.8948 and 0.9206, respectively. Besides, sensitivity analysis showed that modulus of elasticity and water content are the most and least effective variables on *VDR* with influence values of 1.152 and 0.8865, respectively. Additionally, statistical analyses were conducted and optimum empirical quadratic, power, and exponential relationships with high accuracy (*COD* values from 0.861 to 0.987) were proposed to determine the *VDR* based on each rock property. Finally, comparative analysis was conducted for further verification of the current study. Accordingly, the values of lower and upper limits of *COD*, and mean relative error were obtained 0.36, 0.9576, and 0.359 for previous researches, compared to 0.861, 0.987, and 0.114% for this study. These results confirmed the superiority of the current research compared to similar previous studies.

Keywords: *Marble quarry mining, Parametric study, Rock properties, Statistical analysis, Vertical drilling rate.*

1. Introduction

Decorative stones, building facades and aggregate materials are widely used in the different stages of buildings and road constructions. These materials are commonly extracted from the quarry mining. Any optimization during their extraction, production and preparation of these materials leads to enhance efficiency and productivity in the construction system of the road and building structures. Marble stones are the most commonly used for building facades and other constructions. For this reason, the optimization of different processes in the marble quarry industry has been investigated by different researchers [1,2].

In the marble quarry mining, drilling operation is one of the most important processes which involves high costs for bit wear and casing consumptions. A comprehensive understanding of the drilling environment and rock mass characteristics can help with the optimum selection of drilling system, precise determination of the type and number of drilling tools, and accurate prediction of the drilling rate. In conclusion, the above optimizations lead to enhanced drilling efficiency and mining productivity [3]. In rock materials, some variables, including controllable and uncontrollable parameters can generally affect the drilling rate (*DR*). The characteristics of drilling machine and its related tools are the uncontrollable parameters. However, the physical, mechanical and structural characteristics of rock are categorized as the controllable variables. Estimating the equipment's drilling rate has a significant effect on the cost assessment stage during the planning and execution of drilling projects. For successful planning and designing a drilling process, recognition of the effective parameters on the *DR* is essential. The drillability index of different rocks has been studied by numerous researchers who evaluated the relationship between rock characteristics and *DR* for accurate estimation. In addition, effect of rock properties on drilling resistance and drilling tools requirements, i.e., machine type, bit, wear, and casing were investigated by some researchers [4]. Available studies in the field of research issue prove its importance and practicality.

Drilling process plays an important role in the quarry and open-pit mining operations that can affect the final productivity of mine. One of the main tasks for decreasing operational costs and optimizing mining operations is enhancing drilling efficiency and reducing related costs in order to minimize the drilling process [5]. Principally, drilling operation is a time-consuming process in quarry mines and the downstream processes, i.e., cutting operations, depend on the completion of this process. Accordingly, optimizing the drilling rate is extremely significant in quarry mining, as the downstream processes are dependent on it. Overall drilling costs can be estimated using the predictive empirical equations. Also, similar equations can be used for predicting the efficiency of other related drilling tools, depending on the specific studied case [6]. For this purpose, understanding the drilling

---

* Corresponding author. *Tel*: +988733660073, *Fax*: +988733668513, *E-mail address:* m.rezaei@uok.ac.ir (M.Rezaei).

environments and rock mass characteristics can help the optimum prediction of *DR* and penetration rate (*PR*), the optimum selection of drilling system and the desired determination of type and number of drilling machineries. The above optimum process leads to an accurate evaluation of drilling efficiency and precise estimation of production capacity in the mining operation. In addition, accurate estimation of drilling performance in quarry mines is further important for cost estimation and planning objectives. According to the above-mentioned points, the evaluation of the effect of physical and mechanical properties of different rocks on the *DR* was investigated by several researchers, as briefly outlined here. Relationships between rock properties and penetration rate, and drilling rate were previously investigated by different researchers and indirect/inverse relationships between *PR* and various rock characteristics were acquired [2,3,7–23]. Yenice et al. [2] evaluated the effects of natural density (ρ_n), shore hardness (*SH*), uniaxial compressive strength (*UCS*) and tensile strength (*TS*) on the *DR* and proposed some linear and quadratic relationships in this regard. Hoseinie et al. [5] conducted a physical modelling to study the impact of grain size (*GS*), Mohs hardness (*MH*), UCS, and joint characteristics, i.e., joint spacing (J_c), aperture (J_{ap}), and joint dipping (J_p) on the rock mass drillability. Kahraman et al. [7] studied the impact of *UCS*, Brazilian tensile strength (*BTS*), point load strength (*PLI*), Schmidt hammer rebound (*SHR*), impact strength index (ISI), modulus of elasticity (*E*), ρ_n and P-wave velocity (V_p) on the *PR* in percussive drilling operations. The relationships of *UCS*, *SHR* and ρ_n of limestone with *PR* investigated by Okewale and Olaleye [9] based on the field and laboratory data. Demirdag et al. [10] evaluated the effects of different rock properties on the both horizontal and vertical drilling rates and concluded that porosity and unit volume weight are the most effective parameters on *DR*. Ataei et al. [12] utilized the rock mass drillability index to forecast *DR* in open-pit mining operations, considering different mechanical and structural characteristics. index in rotary surface drilling operations. Kahraman et al. [14] proposed a new drillability index for *PR* prediction and proved that the model can be applicable for carbonaceous rocks with unconfined compressive strength greater than 40 *MPa*. A novel brittleness index was suggested by Altindag [15] to evaluate its correlation with the drillability Stavropoulou [16] investigated the rotary drilling using numerical modelling based on laboratory records from marble samples. He found that the cohesion and internal friction angle parameters have the most effect on the rock drillability. Yarali and Soyer [17] surveyed the relationship of *DR* with the mechanical characteristics of different rocks based on the statistical models and proved the linear inverse relationships. Saeidi et al. [18] proposed a statistical model to predict the *PR* during rotary drilling. Munoz et al. [19] presented a novel rock brittleness index to assess the rock behaviour under rotary drilling using the energy dissipation concept. Capik et al. [20] investigated the relationship of *DR* index with the physical and mechanical characteristics of rock and specified their inherent direct or inverse relationships. Derdour et al. [21] used the Taguchi method and response surface methodology to optimize the *PR* in rotary drilling operations. Feng et al. [22] examined the impacts of drilling variables on the consumed energy and *DR*. They proved the important role of rock strength characteristics on the relationship of the *PR* with the applied thrust. Finally, Kolapo [23] studied the impact of rock mechanical characteristics on *PR* and proposed some empirical relationships with high correlation.

According to the above-reviewed publications, an inverse relationship of *PR* or *DR* with the UCS [2,3,7,9,10,12,17,20,23], tensile strength [2,7–10,12,17,20,23], impact strength [7,10], modulus of elasticity [3,7,11,12], hole diameter [13], point load index [3,7,17,20] shore hardness [2,17], Schmidt hammer rebound [3,7,9,10,12,17,20], brittleness index [10,19], compressional wave velocity [3,7,8,10,12], shear wave velocity [12] and natural density [2,3,7,9,10] has been proven. On the other hand, it was found that *PR* or *DR* has a direct relationship with porosity [10,12,20] and Los Angeles and Bohme abrasions [3,10]. The summary of the literature review and used physical and mechanical properties of rock applied in *PR* modelling is briefly presented in Table 1.

Considering the comprehensive literature review conducted in this study, it is concluded that the previous studies were based on field and laboratory measurements, as well as numerical and analytical modelling. Also, the availability of many research studies in this field proved the importance of the studied issue and the influence of drilling efficiency on the mine productivity especially in the marble quarry mines. However, not all possible physico-mechanical properties of a specific rock were simultaneously considered in the modelling of *DR*. Moreover, there are limited studies on the drilling efficiency in quarries especially in the marble quarry mines which are very important materials in buildings and road constructions. Accordingly, the current study is conducted to overcome the above-mentioned shortcomings for minimizing the marble production costs and improving the economic aspects of buildings and roads. The main purpose of this research was to determine the inherent relationships between rock characteristics and *VDR*. In addition, the determination of the most effective physical and mechanical properties of rock on *VDR* has been conducted based on statistical and sensitivity analyses. For this purpose, the vertical drilling information (length and time of the drilling) was recorded in the case study and the corresponding block samples were collected to perform the laboratory tests. Then, standard core specimens were prepared and various physical and mechanical tests were carried out. Therefore, a valid database was prepared, and parametric study and statistical analyses were performed on the datasets.

Table 1. A summary of literature review along with the used parameters in PR modelling and suggested optimum relationship types.

Used rock properties	Optimum relationship type	Reference
ρ_n, *SH, UCS* and *TS* versus *DRI*	Quadratic and Linear	[2]
ρ_n, ρ_d, *n*, w_a, *PLI, SHR*, V_p, *LAA, PLI, UCS, E* and ν versus *HDR*	Quadratic and exponential	[3]
GS, MH, UCS, J_c, J_{ap}, J_p	Linear and logarithmic	[5]
UCS, BTS, PLI, V_p, ρ_n, *ISI, SHR* and *E* versus *PR*	Linear	[7]
UCS, *SHR* and ρ_n versus *PR*	Linear	[9]
UCS, BTS, ISI, BAS, V_p, *n*, ρ_n, *SHR* and *BI* versus *DR*	Power, logarithmic and exponential	[10]
UCS, TS, V_p, V_s, *n*, E_{dyn}, *RDi* and *SHR* versus *DR*	Linear, logarithmic and exponential	[12]
UCS, PLI, BTS, SHR and *SH* versus *DRI*	Linear	[17]
BI versus *PR*	Power	[19]
UCS, PLI, BTS, SHR, e and *n* versus *DRI*	Linear	[20]
UCS and *TS* versus *PR*	Linear	[23]

Note: *PR* and ν both are Poisson ratio, ρ_n is dry density, w_a is water content, *LAA* is Los Angeles abrasion, *n* is porosity, *DRI* is drilling rate index, *HDR* is horizontal drilling rate, *ABS* is Bohme abrasion strength, V_s is shear wave velocity, E_{dyn} is dynamic Young's modulus, *DRi* is rock mass drillability index, *BI* is brittleness index, and *e* is void ratio. The other parameters were described in the text.

2. Case study

In this research, the Malawi marble quarry mine is considered as the case study which is located in the Islamabad-e-Gharb city, west of Iran. This mine contains a measured reserve of 1250000 tons with an annual extraction of 8000 tons. The studied mine is located at the longitude of 46° 36′ 35.033″ and latitude" of 34° 3′ 53.41″ in the southeast of Islamabad-e-Gharb city, on the left side of the Islamabad-e-Gharb-Homeil way. The geographic situation along with the access routes to the case study are shown in Fig. 1. Also, the view of mine area is represented in Fig. 2 and an under working bench face of the mine is demonstrated in Fig. 3.

From the structural viewpoint, the study area is located in the folded Zagros zone. This zone has a northwest-southeast trend approximately. The common available rocks are sedimentary and the sediments are mainly chemical with marl and shale components. There are chemical sediments in this area and no magmatism activity is seen in this basin. The geological age of this area is related to the upper Eocene and its containing rock is associated with the Shahbazan type. From the lithological view, the Malawi marble quarry mine is categorized as the dolomite type.

The extraction method of the Malawi marble quarry mine is the open-pit quarrying. The drilling device used in this mine is pneumatic RASOL which can drill vertical and horizontal holes with diameter variations of 70 to 152 *mm*. The commonly used hole diameter in the Malawi mine is 76 *mm*. Also, a diamond wire cutting machine was applied to cut and separate the rock blocks from the workface and to product the suitable cubs and slates. Finally, water flow was also used to cool the diamond wires during the cutting operations. An example of remaining traces of vertical drilled holes after the block cutting is represented in Fig. 4.

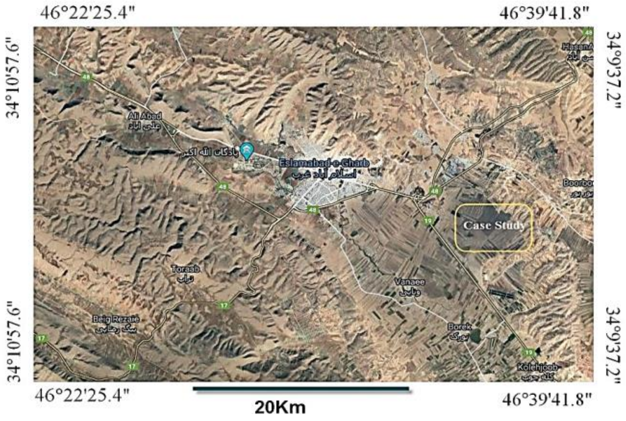


Fig. 1. The geographic situation of Malawi marble quarry mine.

Fig. 2. The view of Malawi marble quarry mine.

Fig. 3. An under working bench face of the mine.

Fig. 4. An example of remaining traces of vertical drilled holes after the block cutting.

3. Data preparation

3.1. Field measurement

For *VDR* measurement in the mine field, the required data, including the drilling depth and the drilling time was recorded for 20 under-drilling blocks and their related *VDR* vales were measured. A part of measured field data for *VDR* calculation is given in Table 2. In the next step, field sampling was conducted and sufficient block samples were prepared to perform the required laboratory tests. For this purpose, a conventional sampling procedure was used to collect the required minor blocks samples. In this approach, minor block samples with approximate dimensions of 20×20×25 cm were separated from the corresponding under-drilling major blocks using the cutting machine. From each major block, one minor block was separated in line with the direction of vertical drilling to obtain the effect of rock properties on the drilling direction. These collected minor block samples were then transferred to the laboratory for conducting the coring operation and preparing the required core specimens to perform the desired experiments.

Table 2. A part of field measurement data for VDR calculation.

No.	Drilling Length (m)	Drilling Time (h)	Geological conditions
1	15	3	Pure rock
2	14	5	With soil
3	13	2.45	Pure rock
4	12	3	With some soil
5	9	3	With some soil

3.2. Laboratory measurement

Coring operation upon the block samples was first conducted in the laboratory to prepare the required suitable cylindrical specimens. The coring device used during the core capturing and some of the prepared core specimens are shown in Figs. 5 and 6, respectively. It should be noted that the core specimens were prepared with a 76 *mm* diameter to match the used drilling diameter in the mine. In the next stage, the samples were cut using the cutting machine at the standard ratio of height to diameter and polished by a core cross-sectional polisher. After the cores were prepared, different tests were performed to measure the physical and mechanical properties of marble specimens. The key physical and mechanical characteristics of marble specimens, including natural (ρ_n) and dry (ρ_d) densities, porosity (n), water content (w_a), slake durability index (I_d), Schmidt hammer rebound (*SHR*), compression wave velocity (V_p), Los Angeles abrasion (*LAA*), point load index (*PLI*), uniaxial compressive strength (*UCS*), modulus of elasticity (E) and Poisson ratio (ν) were measured in accordance with the proposed standards by the International Society for Rock Mechanics (*ISRM*) [24–26]. More details about the implementation procedures of these tests are given in the literature [24–29]. Therefore, no further explanations are provided here. The devices used for implementing the required laboratory tests are given in Fig. 7

Fig. 5. Coring device while the core capturing.

Fig. 6. Some of the prepared core specimens.

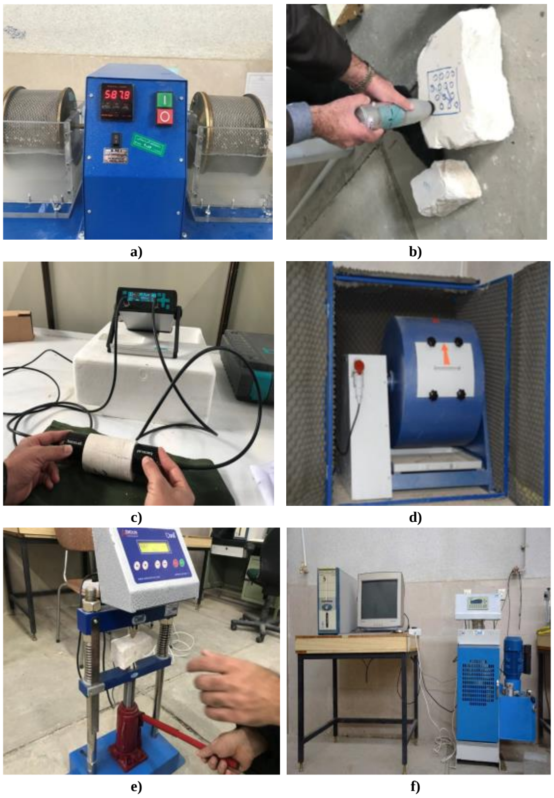


Fig. 7. Implementation of the laboratory tests: a) Id measurement devise b) Proceq device to determine SHR; c) PUNDIT device to measure Vp; d) The LAA test tool; e) Used machine to determine PLI; f) Uniaxial compression device to measure UCS, E and ν.

3.3. Summary of acquired data

According to the field and laboratory measurements, a total of 20 reliable data series were prepared for further related analyses. Five samples of the measured values of the physical and mechanical properties of rock are given in Table 3. Also, the statistical characteristics of the prepared datasets are demonstrated in Table 4.

4. Results and discussion

4.1. Parametric study

The parametric study is commonly used to evaluate the effect of input variables on a specific output. Here, the parametric study was applied to assess the influence of rock properties on the *VDR*. Indeed, the relationship type (direct or inverse) of the rock properties with the *VDR* is now studied which can help improve knowledge of rock properties' relationships with the *VDR*. Practically, this process can assist in the optimum selection of drilling device tools related to a specific rock type. This process enhances the efficiency of drilling operations and mining economics.

Based on the parametric study, the inherent relationships of rock properties with *VDR* are demonstrated in Fig. 8. As it can be seen from this figure, ρ_n, ρ_d, I_d, *SHR*, V_p, *PLI*, *UCS* and E parameters had an inverse relationship with the *VDR* and a negative effect on it. On the other hand, n, W_a, *LAA* and ν parameters had a positive effect on the *VDR* and a direct relationship with it. Quantitatively, the obtained values of the coefficient of determination (COD or R-Square) for ρ_n, ρ_d, n, W_a, I_d, *SHR*, V_p, *LAA*, *PLI*, *UCS*, ν and E were 0.9205, 0.8372, 0.9079, 0.9086, 0.8582, 0.9327, 0.9445, 0.8493, 0.9367, 0.9339, 0.9225, and 0.9247, respectively. These results confirmed that *PLI*, *UCS*, ν and E parameters

Table 3. The measured values of physical and mechanical properties of five rock samples.

ρ_n (g/cm^3)	P_d (g/cm^3)	n (%)	W_a (%)	I_d (%)	SHR (-)	V_p (m/s)	LAA (%)	PLI (MPa)	UCS (MPa)	ν (-)	E (MPa)
2.25	2.246	1.4	0.56	0.96	46	3651	68.74	1.09	12.96	0.39	10.11
2.256	2.25	1.39	0.613	0.978	59	3784	43.86	1.75	17.56	0.37	18.89
2.3338	2.268	1.2	0.56	0.98	59	3830	41.6	2.31	29.98	0.351	19.42
2.407	2.402	0.89	0.363	0.991	70	4211	30.19	3.21	42.32	0.283	30.43
2.517	2.485	0.60	0.263	0.993	73	4321	28.23	4.6	65.12	0.2	35.1

Table 4. Statistical characteristics of prepared datasets.

Parameter	Symbol	Max	Min	Var.	Std. dev.
Natural density (*g/cm^3*)	ρ_n	2.517	2.25	0.00574	0.07579
Dry density (*g/cm^3*)	ρ_d	2.485	2.246	0.00508	0.07131
Porosity (%)	n	1.4	0.60	0.056	0.2370
Water content (%)	w_a	0.613	0.263	0.1244	0.11155
Slake durability index (%)	I_d	0.993	0.96	7.91E-5	0.008893
Schmidt hammer rebound (-)	SHR	73	46	45.7493	6.7838
Compression wave velocity (*m/s*)	V_p	4321	3651	38411.08	195.987
Los Angeles abrasion (%)	LAA	68.74	28.23	114.5243	110.7016
Point load index (*MPa*)	PLI	4.6	1.09	0.90528	0.95146
Uniaxial compressive strength (*MPa*)	UCS	65.12	12.96	197.7189	14.0612
Poisson ratio (-)	ν	0.39	0.2	0.002652	0.0515
Modulus of elasticity (*GPa*)	E	35.1	10.11	38.2837	6.1873
Vertical drilling rate (*cm/min*)	VDR	9.12	3.84	2.4526	1.56608

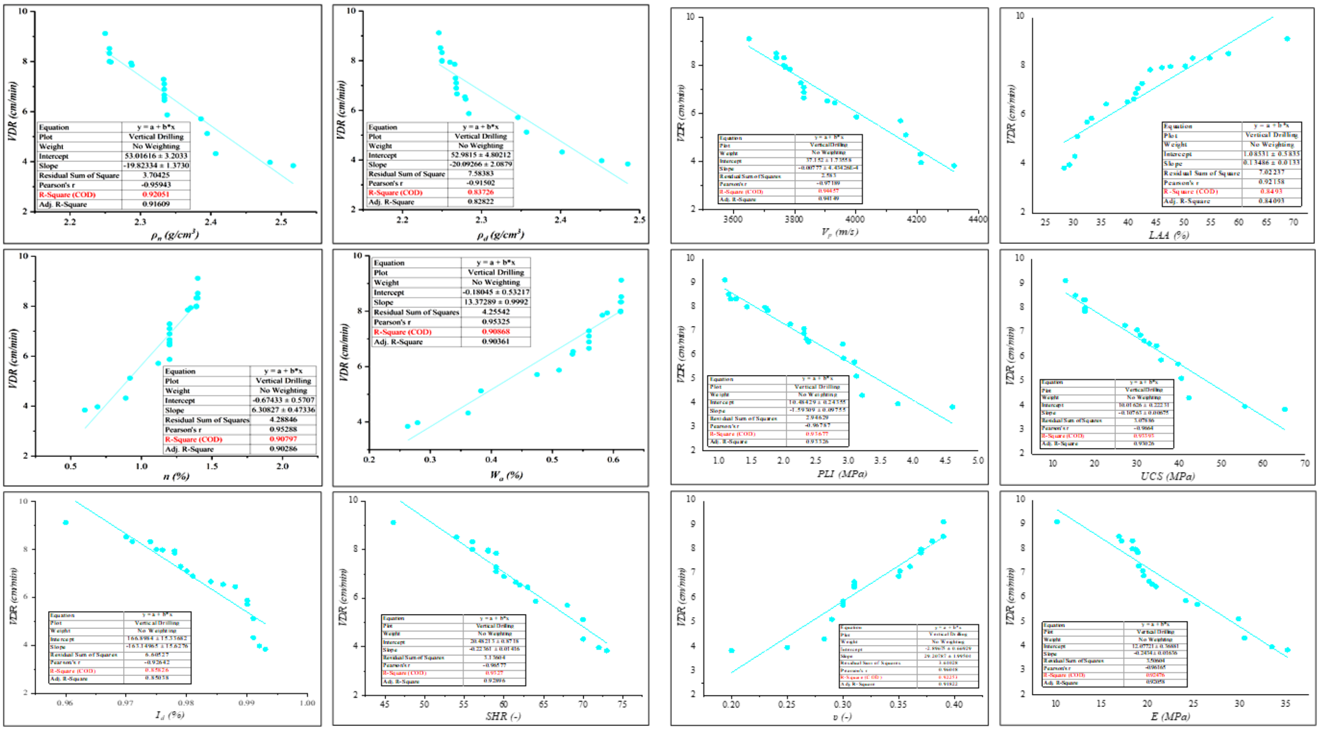


Fig. 8. Relationships of different rock properties with the VDR.

were more correlated with *VDR* compared to the other input variables. According to the above *COD* results, it is found the average *COD* of physical and mechanical properties of rock were 0.8948 and 0.9294, respectively. Therefore, it is generally concluded that *VDR* is more correlated with the mechanical properties of rock rather than the rock's physical properties.

The result obtained from this section can be significant, because it showed the greater importance of mechanical properties in vertical drilling compared to the physical properties. This may due to the considerable resistance of rock mechanical properties against the vertical forces that are available in vertical drilling. This point can be especially noticeable in vertical drilling within marble quarry mines. However, more research on this issue is required for further validation and to evaluate other aspects of this output.

4.2. Sensitivity analysis

Sensitivity analysis is normally carried out to estimate the most and least effective variables on the defined objective variable(s) in a specific problem. According the sensitivity analysis result, the effective parameters can receive more attention in the optimum selection of drilling machine. The sensitivity analysis can be generally performed using two different methods, including new and custom procedures. In the custom approach, the effect of a possible input variable on an output variable is determined based on their variations [30–32]. Nevertheless, new approaches, such as the cosine amplitude method (*CAM*) were utilized for the sensitivity analysis based on the theoretical and statistical relationships [33–35]. In this study, multivariate sensitivity analysis is used to determine the effective parameters on the *VDR*. For this purpose, interval variations of the input variables (12 considered rock properties) and one output parameter (*VDR*) were first calculated and normalized between 0 and 100%. Then, input variations versus their corresponding output changes were separately plotted as linear lines. The relationships between inputs variations *and* the *VDR* variations are demonstrated in Fig. 9. The presented lines in this figure specify the effects of rock properties on the *VDR* based on their variation percent values. Indeed, each line determines the relationship between the input and output variables, and a higher line slope shows a greater effect on the output (*VDR*). The slope values of the plotted lines related to each parameter in Fig. 9 are given in Table 5 in order to identify the effective rock properties on the *VDR*. In Table 5, the positive and negative signs indicate a direct or inverse effect, while the absolute value of the number indicates the effect of the parameter on the *VDR*. It is concluded from Fig. 9 and Table 5 that *E* and *SHR* had the highest effect on the *VDR* with the influence values of 1.152 and 1.1435, respectively. On the other hand, w_a and ρ_d had the lowest effect on this parameter with the influence amounts of 0.8865 and 0.9095, respectively. However, the effects of *E*, *PLI* and *UCS* parameters on *VDR* were also considerable and cannot be ignored.

The main output of this sensitivity analysis is acquiring the importance values of each rock properties on the *VDR* and determining their relationship type with the *VDR* (inverse or direct). Indeed, the importance order of rock properties on the *VDR*, and the most and least effective ones are specified. As a general result of this sensitivity analysis, it can be concluded that the most effective parameters can receive more attention in the drilling operations of the marble quarry mines for the optimum selection of drilling tools requirements, i.e., machine type, bit, wear, casing and etc. This can help minimize drilling costs, optimize the drilling operations, and enhance the mining productivity.

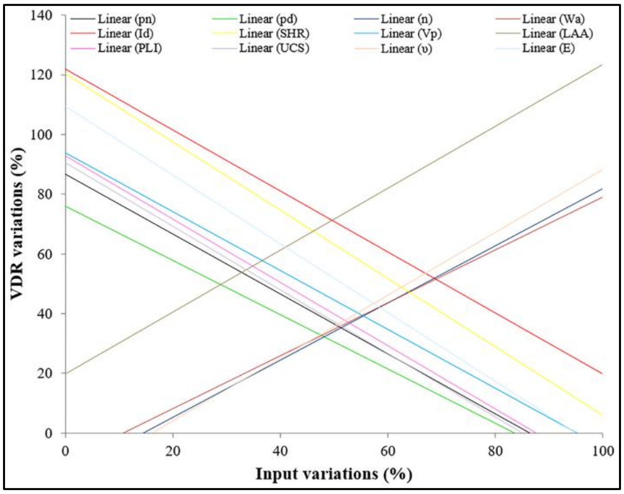


Fig. 9. Sensitivity analysis of the input's variations and the VDR variations.

4.3. Regression modelling

In this section, multiple regression analysis between *VDR* and different rock properties was carried out. Indeed, constant and coefficients between rock characteristics and output (*VDR*), and the statistical indices, including the variance inflation factor (*VIF*), coefficient of determination (*COD*), Fisher-test coefficient (*F*), statistical significance (*Sig.*) and standard error of the estimation values were achieved in the multiple regression analysis. The key conclusions of this analysis for *VDR* prediction are given in Table 6. According to Hair et al. [36], *VIF* is commonly greater than 1 and there is no considerable correlation for variables with *VIF* values less than 5. As it can be seen in Table 6, the obtained values of *VIF* for all inputs are less than 4. Therefore, it can be concluded that there is no considerable correlation between the input parameters. However, the *VIF* values for n, w_a, V_p, and ν were somewhat high which showed a minor correlation among these parameters, but they remained in regression modelling according to Hair et al. [36] standard. The *VIF* results indicated a relatively higher correlation between the *VDR* and the mechanical properties, as opposed to the physical characteristics. This output was in agreement with the parametric study results in section 4.1. Besides, the obtained *Sig.* value was less than 0.05 which proved the statistical significance of the current multiple regression analysis. Also, the achieved *COD* and *F* values from the *VDR* regression modelling were very high which proved the accuracy of this multiple regression analysis between rock properties and *VDR*. Finally, the standard error of the estimate achieved from the *VDR* statistical modelling was very low which confirmed the capability of conducted regression model.

The results of Table 5 can lead to a new multivariate regression equation to predict the *VDR* based on different rock properties. One of the main advantages of this acquired multivariate regression model is that it is not a one-dimensional equation and the *VDR* is simultaneously estimated based on 12 different rock properties. Therefore, it is a comprehensive and multi-dimensional estimate that can be more effective in practical drilling at marble quarry mines.

4.4. Development of empirical equations to predict VDR

To develop the predictive empirical equations between rock properties and *VDR* in this research, statistical modelling was utilized based on the analysis of variance (*ANOVA*). For this purpose, five specific equations, i.e., linear, quadratic polynomial, logarithmic, exponential and power types were separately developed between the rock characteristics and *VDR*. The *COD*, *F* and *Sig.* indices were applied for selecting the optimum relationship among the above-mentioned equations. The *ANOVA* results to find the optimum relationships between each rock property and *VDR* are given in Table 7. Based on three criteria (*COD*, *F* and *Sig.*) mentioned above, the optimum relationships to predict *VDR* were selected and presented in Table 7. As it can be seen from this Table, quadratic, power, and exponential equations were specified as the optimum relationships between different rock properties and *VDR*. Indeed, an optimum power type was acquired only for the relationship of dry density with *VDR*, whereas the quadratic polynomial and exponential types were achieved as the optimum relationships between other rock properties with *VDR*. Accordingly, these optimum equations were proposed in this research to forecast the *VDR* that can be used as the predictive tools for other cases with similar conditions.

Using the obtained equations in this section, different values of *VDR* can be achieved in a specific drilling operation depending on any type of rock property. Therefore, the lower and upper limits of *VDR* can be determined for conducting engineering decisions regarding the optimum selection of the drilling tools and determining the drilling efficiency to choose the requirements of the desired product. In cases of cost and time limitations or when high accuracy (especially in the early stages of mining operations), the *VDR* value can be estimated based on one parameter or a limited number of rock properties, minimizing felid and laboratory measurement costs.

4.5. Comparative analyses based on the actual data

To validate and prove the superiority of the current research, the obtained results were compared with the results achieved from previous similar studies [2, 7, 9, 12, 17, 19, 20, 23]. For this comparison, four new datasets were separately measured. These datasets were not utilized in developing the proposed equations. Based on these new datasets, *VDR*

Table 5. Influence values of the resulted lines slope from the sensitivity analysis in order to specify the effective rock properties on the VDR.

Parameter	Line slope value in sensitivity analysis	Parameter	Line slope value in sensitivity analysis
ρn	-1.0094	Vp	-0.9855
ρd	-0.9095	LAA	+1.0347
n	+0.9558	PLI	-1.059
wa	+0.8865	UCS	-1.0633
Id	-1.0197	ν	+1.051
SHR	-1.1435	E	-1.152

Table 6. Key outputs of multiple regression analysis for the VDR prediction.

Dependent variable	Predictor	Coefficient	*VIF*	Summary of model results			
				COD	*F*	*Sig.*	Std. Error of the Estimate
VDR	Constant	119.323	----	0.995	119.21	0.0	0.20092
	ρ_n	-2.04	2.275				
	ρ_d	-11.813	2.988				
	n	5.92	3.648				
	w_a	13.35	3.409				
	I_d	-95.87	1.133				
	SHR	-0.011	1.140				
	V_p	-0.004	3.94				
	LAA	0.001	1.284				
	PLI	-0.93	1.371				
	UCS	-0.005	1.578				
	ν	1.015	3.968				
	E	-0.023	1.648				

Table 7. Summary of the ANOVA results to find the optimum relationship between rock properties and VDR.

Input parameter	Optimum equation type	Equation	COD	F	Sig.
ρn	Exponential	VDR=63981e-0.04ρn	0.937	267.812	0.00
ρd	Power	VDR=21746ρd-9.79	0.919	204.541	0.00
n	Exponential	VDR=1.423e1.258n	0.924	217.898	0.00
wa	Exponential	VDR=1.549e2.694Wa	0.943	296.228	0.00
Id	Quadratic	VDR=-83.450Id2+87.164	0.861	111.457	0.00
SHR	Quadratic	VDR=-0.004SHR2+0.322SHR+3.940	0.987	212.886	0.00
Vp	Quadratic	VDR=5.325E-6Vp2-0.05Vp+121.563	0.955	179.844	0.00
LAA	Quadratic	VDR=-0.004LAA2+0.478LAA-6.323	0.960	203.111	0.00
PLI	Quadratic	VDR=0.076PLI2-1.989PLI+10.927	0.939	131.765	0.00
UCS	Quadratic	VDR=0.001UCS 2-0.168UCS+10.912	0.950	159.979	0.00
ν	Quadratic	VDR=70.121ν 2-14.208ν+3.613	0.941	135.443	0.00
E	Exponential	VDR=15.866e-0.04E	0.940	283.387	0.00

values were first estimated from the proposed equations in the current research and other previous literature. Then, the error of each dataset for all the used comparable equations was calculated. Subsequently, the mean relative error (*MRE*) of each comparable research was computed in percent. Commonly, an equation with a lower *MRE* is superior compared to the other equations. In addition to the above comparison, the lower and upper limits of *COD* achieved from the optimum proposed equations were compared with the similarly acquired *COD* values from previous studies. Indeed, the lower and upper limits of *COD* related to the proposed *VDR* relationships in this study (Table 7) were compared with the similar *COD* values obtained from the previous studies. In general, *COD* criterion was applied as an efficient tool to evaluate the model's ability to calculate the outputs. Commonly, an equation or model with a greater value of *COD* is more precise and can forecast the accurate amounts of a specific target variable.

Results of the comparative analysis are given in Table 8 and also demonstrated in Figs. 10 and 11 in terms of lower and upper limits of *COD* and *MRE* (based on four actual datasets). As seen from Table 8, similar tendencies were generally detected from all compared research in the form of the achieved equations types. On the other hand, it is generally observed that there is a good agreement between the overall results of this research and prior studies. However, it is clear from Table 8 and Fig. 10 that the acquired values of lower limit of *COD* were 0.861 and 0.36 for this research and previous studies, respectively. Likewise, the obtained amounts of upper limit of *COD* were 0.987 and 0.9576. Accordingly, it is concluded that both the upper and lower limits of *COD* resulted from this study were higher than the similar values obtained from previous studies. Additionally, it can be observed from Table 8 and Fig. 11 that the values of obtained *MRE* from previous research and the current study were 0.359% and 0.114%, respectively. This output showed that the achieved *MRE* from the current study was also much lower than the acquired *MRE* from the previous comparable research. Considering the above results, it is concluded that the current research results were more accurate than those of the similar previous studies in determining *VDR* based on different rock properties.

4.6. Benefits, restrictions, and future research

In this paper, the relationships of 12 rock properties with the *VDR* were investigated based on the combination of field and experimental measurements. One output of this study was specified the relationships of each rock property (direct or inverse) with the *VDR* which can be effective in the productivity of the drilling operations. In this evaluation, a greater correlation of rock mechanical properties with the *VDR* was found compared to the physical properties of rock which should be noted in the vertical drilling operations. Another beneficial output of this research was the sensitivity analysis of the impact of rock properties on the *VDR* and the determination of the most and least effective ones. This result can be generally useful in selecting suitable drilling tools for drilling operations, resulting in the quarry mining with low cost and high productivity. The third result of this paper was the introduction of a new multivariate regression equation to predict the *VDR* considering 12 rock properties simultaneously. This multi-dimensional *VDR* estimation can be effectively used in practical marble quarry drilling. Finally, 12 optimum empirical equations with high accuracy were proposed for the indirect determination of the *VDR* based on 12 different rock properties. In the case of the cost and time limitations or in the initial phase of the drilling operation, the *VDR* value can be estimated based on the one or more proposed *VDR* equations. This indirect estimation of *VDR* involves minimum cost and time, enhancing the efficiency of drilling operations in quarry marble mining.

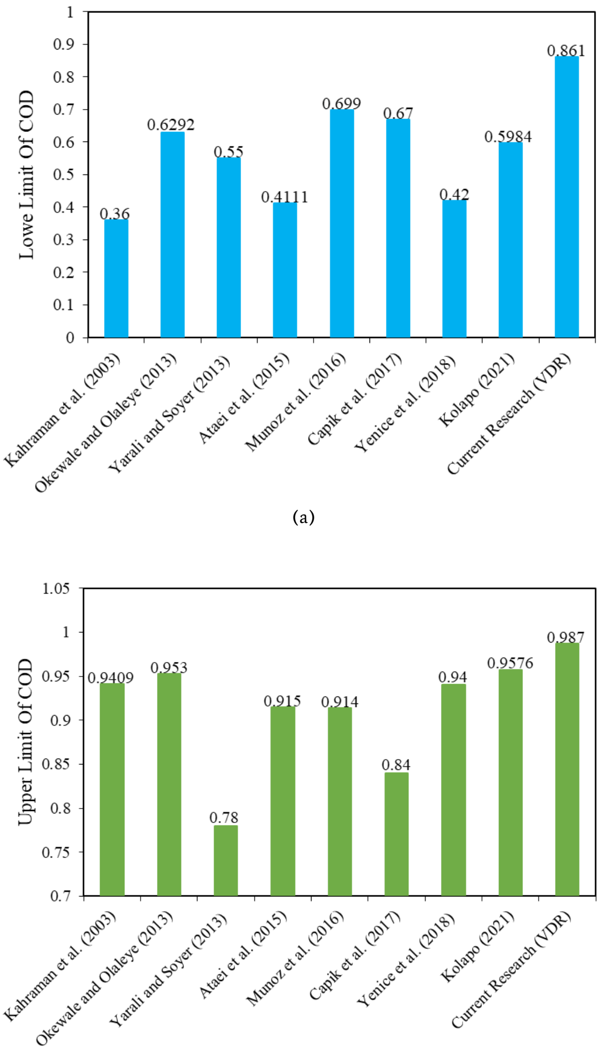
The research outputs were verified by comparing their results with recent research results using the obtained quantitative values of error and accuracy indices. In addition to the above-proved quantitative dominance, another superiority of the current study is that all the possible rock properties were simultaneously considered for *VDR* prediction compared to previous research. Considering the above comparative/verification analyses, it can be concluded that the suggested empirical-experimental relationships in this paper are convincing tools with acceptable abilities for *VDR* determination based on different rock properties. Therefore, they could be applied in the mine drilling operations, especially in marble quarry mines with similar rock properties. However, further investigations will be needed to evaluate the effect of rock varieties on the relationships of rock properties with *VDR*. This can be examined in different mines other than marble quarry mines. Besides, the effect of the possible structural characteristics of rock (i.e., joints, anisotropy, lamination and bedding) on the *VDR* can be considered in the future research. Furthermore, it is better to evaluate the effect of drilling machine parameters and loading direction on the *VDR* in marble quarry mining. As a final recommendation to cover the current research limitations, investigating the impact of rock properties on directional drilling can be an attractive research issue, since horizontal and inclined drilling is usually done in quarries and other mines.

Table 8. Comparing the acquired values of COD and MRE from the current study results with previous similar researches.

Lower and upper limits of achieved COD	Mean relative error (MRE)	Reference
0.42-0.94	32.56 %	[2]
0.36-0.9409	33.24 %	[7]
0.6292-0.953	28.12 %	[9]
0.4411-0.915	30.41 %	[12]
0.55-0.78	39.54 %	[17]
0.699-0.914	No data on BI	[19]
0.67-0.84	36.97 %	[20]
0.5984-0.9576	51.09 %	[23]
0.861-0.987	1.114 %	Current research

5. Conclusions

The influence of physical and mechanical properties of rock on the *VDR* was studied in this study. For this purpose, the *VDR* was first measured in the Malawi marble quarry mine, located in the Islamabad-e-Gharb, Iran. In the next step, core specimens were prepared from the minor blocks related to the mine under drilling of major blocks. Then,



(a)

(b)

Fig. 10. Comparing the resulted COD values from current study with similar previous studies based on the actual data: a) Lower limit of COD, and b) Upper limit of COD.

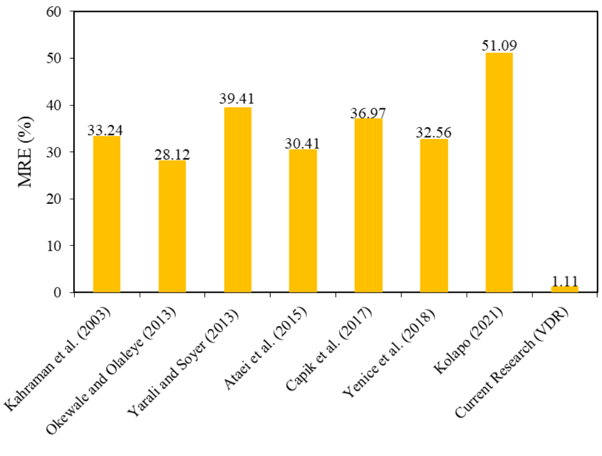


Fig. 11. Comparing the resulted MRE value from current study with similar previous studies based on the actual data.

required physical and mechanical tests were implemented in the laboratory to measure the rock properties. Lastly, the obtained datasets were analysed and the following results were achieved:

- The parametric study confirmed that *VDR* was inversely affected by ρ_n, ρ_d, I_d, *SHR*, V_p, *PLI*, *UCS* and E characteristics. Conversely, it was directly influenced by n, w_a, *LAA* and ν variables.
- According to the results of the parametric study and *VIF* analysis, *VDR* was more closely correlated with rock mechanical properties than with physical characteristics.
- The multivariate sensitivity analysis showed that E and w_a had the highest and lowest effects on *VDR* with the influence values of 1.152 and 0.8865, respectively.
- A multiple regression model with a high value of *COD* (0.995) and a low amount of error (0.20092) was proposed for *VDR* estimation based on 12 rock properties.
- According to the *ANOVA* results, it was observed that quadratic, power, and exponential equations were the optimum relationships to determine *VDR* based on the rock properties.
- Comparative analyses confirmed that there was a reasonable trend agreement between the results of the current study and previous ones in the field of the proposed relationships type. However, it was proved that the results of the current study were more accurate than the results of prior investigations.

Considering the above results, it is concluded that the outputs of this study are satisfied and confident and can be practically applied to estimate *VDR* with reliable accuracy, resulting in cost reduction and productivity enhancement in the drilling operations of quarry mining. Even though the results of this paper are rather superior compared to the previous researches, further studies are required to assess the effect of different rock types, rock structural properties and drilling machine characteristics on the *VDR* and other directional drillings.

Acknowledgment

The authors gratefully acknowledge the support of the Malawi marble quarry mine management and mine technical team for their assistance in data preparation.

References

[1] Yavuz, A. B., Turk, N., & Koca, M.Y. (2005). Geological parameters affecting the marble production in the quarries along the southern flank of the Menderes Massif, in SW Turkey. *Engeering Geology, 80*(3–4), 214–241. doi: https://doi.org/10.1016/j.enggeo.2005.05.003

[2] Yenice, H., Özdoğan, M. V., Özfırat, M. K. (2018). A sampling study on rock properties affecting drilling rate index (DRI). *Journal of African Earth Sciences, 141*, 1–8. doi: https://doi.org/10.1016/j.jafrearsci.2018.01.015

[3] Rezaei, M., & Nyazyan, N. (2023). Assessment of effect of rock properties on horizontal drilling rate in marble quarry mining: field and experimental studies. *Journal of Minining and Environment, 14*(1), 321–339. doi: https://doi.org/10.22044/jme.2023.12595.2287

[4] Yasar, E., Ranjith, P. G., & Viete, D.R. (2011). An experimental investigation into the drilling and physico-mechanical properties of a rock-like brittle material. *Journal of Petroleum Science and Engineering, 76*(3–4), 185–193. doi: https://doi.org/10.1016/j.petrol.2011.01.011

[5] Hoseinie, S. H., Aghababaei, H., & Pourrahimian Y. (2008). Development of a new classification system for assessing of rock mass drillability index (RDi). *International Journal of Rock Mechanics and Mining Sciences, 45*(1), 1–10. doi: https://doi.org/10.1016/j.ijrmms.2007.04.001

[6] Kahraman, S. (1999). Rotary and percussive drilling prediction using regression analysis. *International Journal of Rock Mechanics and Mining Sciences, 36*(7) 981–989. doi: https://doi.org/10.1016/S0148-9062(99)00050-9

[7] Kahraman, S., Bilgin, N., & Feridunoglu, C. (2003). Dominant rock properties affecting the penetration rate of percussive drills. *International Journal of Rock Mechanics and Mining Sciences, 40*(5), 711–723. doi: https://doi.org/10.1016/S1365-1609(03)00063-7

[8] Bilim, N. (2011). Determination of drillability of some natural stones and their association with rock properties. *Scientific Research and Essays, 6*(2), 382–387. doi: https://doi.org/10.5897/SRE10.878

[9] Okewale, I. A., & Olaleye, B. M. (2013). Correlation of strength properties of limestone deposit in Ogun state, Nigeria with penetration rate using linear regression analysis for engineering applications. *International Journal of Engineering Science, 2*(7), 18–24.

[10] Demirdag, S., Ugur, N., Efe, T., Akbay, D., & Altindag R. (2014). Variation of vertical and horizontal drilling rates depending on some rock properties in the marble quarries. International *Journal of Mining Science and Technology, 24*(2), 269–273. doi: https://doi.org/10.1016/j.ijmst.2014.01.020

[11] Hoseinie, S. H., Ataei, M., & Aghababaie A. (2014). A laboratory study of rock properties affecting the penetration rate of pneumatic top hammer drills. *Journal of Minining and Environment, 5*(1), 25–34. doi: https://doi.org/10.22044/JME.2014.216

[12] Ataei, M., KaKaie, R., Ghavidel, M., & Saeidi O. (2015). Drilling rate prediction of an open pit mine using the rock mass drillability index. *International Journal of Rock Mechanics and Mining Sciences, 73*, 130–138. doi: https://doi.org/10.1016/j.ijrmms.2014.08.006

[13] Kivade,, S., Murthy, C. H., & Vardhan H. (2015). Experimental investigations on penetration rate of percussive drill. *Procedia Earth and Planetary Science, 11*, 89–99. doi: https://doi.org/10.1016/j.proeps.2015.06.012

[14] Kahraman, S., Balcı, C., Yazıcı, S., & Bilgin N. (2000). Prediction of the penetration rate of rotary blast hole drills using a new drillability index. *International Journal of Rock Mechanics and Mining Sciences, 37*(5), 729–743. doi: https://doi.org/10.1016/S1365-1609(00)00007-1

[15] Altindag, R. (2002). The evaluation of rock brittleness concept on rotary blast hole drills. *Journal of the Southern African Institute of Mining and Metallurgy, 102*(1), 61–66. doi: https://hdl.handle.net/10520/AJA0038223X_2763

[16] Stavropoulou, M. (2006). Modeling of small-diameter rotary drilling tests on marbles. *International Journal of Rock Mechanics and Mining Sciences, 43*(7), 1034–1051. doi: https://doi.org/10.1016/j.ijrmms.2006.03.008

[17] Yarali, O., & Soyer E. (2013). Assessment of relationships between drilling rate index and mechanical properties of rocks. *Tunnelling and Underground Space Technology, 33*, 46–53. doi: https://doi.org/10.1016/j.tust.2012.08.010

[18] Saeidi, O., Torabi, S. R., Ataei, M., & Rostami, J. (2014). A stochastic penetration rate model for rotary drilling in surface mines. *International Journal of Rock Mechanics and Mining Sciences, 68*, 55–65. doi: https://doi.org/10.1016/j.ijrmms.2014.02.007

[19] Munoz, H., Taheri, A., & Chanda, E. K. (2016). Rock drilling performance evaluation by an energy dissipation-based rock brittleness index. *Rock Mechanics and Rock Engeering, 49*(8), 3343–3355. doi: https://doi.org/10.1007/s00603-016-0986-0

[20] Capik, M., Yilmaz, A. O., & Yasar, S. (2017). Relationships between the drilling rate index and physicomechanical rock properties. *Bulletin of Engineering Geology and the Environment, 76*(1), 253–261. doi: https://doi.org/10.1007/s10064-016-0991-2

[21] Derdour, F. Z., Kezzar, M., & Khochemane L. (2018). Optimization of penetration rate in rotary percussive drilling using two techniques: Taguchi analysis and response surface methodology (RMS). *Powder Technology, 339*(9), 846–853. doi: https://doi.org/10.1016/j.powtec.2018.08.030

[22] Feng, S., Wang, Y., Zhang, G., Zhao, Y., Wang, S., Cao, R., & Xiao E. (2020). Estimation of optimal drilling efficiency and rock strength by using controllable drilling parameters in rotary non-percussive drilling. *Journal of Petroleum Science and Engineering, 193*, 107376. doi: https://doi.org/10.1016/j.petrol.2020.107376

[23] Kolapo, P. (2021). Investigating the effects of mechanical properties of rocks on specific energy and penetration rate of borehole drilling. *Geotechnical and Geological Engineering, 39*(11), 1715–1726. doi: https://doi.org/10.1007/s10706-020-01577-y

[24] ISRM. (1978). Suggested method for determining sound velocity. *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts, 15*(2), 53–58. doi: https://doi.org/10.1016/0148-9062(78)91678-9

[25] ISRM. (1979). Suggested method for determining water content, porosity, density, absorption and related properties and swelling and slake durability index properties. *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts, 16*(2), 141–156. doi: https://doi.org/10.1016/0148-9062(79)90287-0

[26] ISRM. (2007). The complete ISRM suggested methods for rock characterization, testing and monitoring: 1974–2006. Kozan Ofset Matbaacılık, Ankara. Doi: https://doi.org/10.2113/gseegeosci.15.1.47

[27] Rezaei, M., & Koureh Davoodi, P. (2021). Determining the relationship between shear wave velocity and physicomechanical properties of rocks. *International Journal of Mining and Geo-Engineering, 55*(1), 63–70. doi: https://doi.org/10.22059/IJMGE.2019.275851.594782

[28] Seyed Mousavi, S.Z., & Rezaei, M. (2022). Correlation assessment between degradation ratios of UCS and non-destructive properties of rock under freezing-thawing cycles. *Geoderma, 428*, 116209. doi: https://doi.org/10.1016/j.geoderma.2022.116209

[29] Seyed Mousavi, S.Z., & Rezaei, M. (2023). Assessing the long-term durability and degradation of rocks under freezing-thawing cycles. *Geomechanics and Engineering, 34*(1), 51–67. doi: https://doi.org/10.12989/gae.2023.34.1.051

[30] Rezaei, M. (2018). Long-term stability analysis of goaf area in longwall mining using minimum potential energy theory. *Journal of Minining and Environment, 9*(1), 169–182. doi: https://doi.org/10.22044/JME.2017.5950.1408

[31] Rezaei, M., Majdi, A., Hossaini, M. F., & Najmoddini, I. (2018). Study the roof behavior over the longwall gob in long-term condition. *Journal of Geology and Mining Research, 10*(2), 15–27. doi: https://doi.org/10.5897/JGMR2017.0284

[32] Rezaei, M. (2020). Feasibility of novel techniques to predict the elastic modulus of rocks based on the laboratory data. *International Journal of Geotechnical Engineering, 14*(1), 25–34. doi: https://doi.org/10.1080/19386362.2017.1397873

[33] Rezaei, M., & Rajabi, M. (2021). Assessment of plastic zones surrounding the power station cavern using numerical, fuzzy and statistical models. *Engineering with Computers, 37*(2), 1499–1518. doi: https://doi.org/10.1007/s00366-019-00900-3

[34] Asadizadeh, M., & Rezaei, M. (2021). Surveying the mechanical response of non-persistent jointed slabs subjected to compressive axial loading utilising GEP approach. *International Journal of Geotechnical Engineering, 15*(10), 1312–1324. doi: https://doi.org/10.1080/19386362.2019.1596610

[35] Sayadi, A.R., Tavassoli, S.M.M., Monjezi, M., & Rezaei, M. (2014). Application of neural networks to predict net present value in mining projects. *Arabian Journal of Geosciences*, 7(3), 1067–1072. https://doi.org/10.1007/s12517-012-0750-z

[36] Hair, J.F., Black, W.C., Babin, B.J., Anderson, R.E., & Tatham, R.L. (2010). Multivariate Data Analysis. 7th ed. New York: Pearson. doi: https://doi.org/10.1080/19386362.2019.1596610