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RESEARCH PAPER

A Multiobjective Optimization Approach for Integrated Production Planning, Location-Allocation, and Routing in Three-Echelon Supply Chains

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Abstract

The focus of many researchers in the operations research field has been on supply chain design problems during past decades. Previous works have widely investigated production-inventory planning, vehicle routing, and location-allocation problems. This paper aims to consider these problems simultaneously and present a new integrated production planning-location-routing problem for a three-echelon supply chain, considering several real-world assumptions. The studied supply chain includes multiple production centers, distribution centers, and customers. The distribution centers use a set of non-homogeneous vehicles to deliver the products to the customers. Several features, such as regular and overtime production, production reliability, time-window constraints, and capacity constraints, are incorporated to provide a more realistic problem. The bi-objective model aims to determine the optimal location, allocation, production, and routing decisions to optimize the total cost and servicing time objective functions. Concerning problem's complexity, the non-dominated sorting genetic algorithm-II (NSGA-II) is designed and implemented as a solution approach. The results reveal that this algorithm can solve the model in an acceptable time interval. In addition, the results demonstrate that the NSGA-II algorithm is reliable in finding solutions, and there is no significant difference between the average solution and the best solution of the algorithm in several runs.

Keywords: Supply Chain Planning, Production Planning, Vehicle Routing Problem, Location-Allocation Analysis, Multiobjective Optimization.

Introduction

The manufacturing supply chain is a network consisting of different echelons, including raw material procurement, processing, and delivery of the final item to the buyer. In the past decades, managers believed that independent planning of supply chain components was feasible. However, over time, it was realized that decentralized approaches weaken the performance of supply chains (Pritsker & O'Reilly, 1999). As a result, due to the need for supply chain managers to employ operations management techniques to improve their entire supply chain performance, supply chain planning problems have become one of the most important issues in production and service management that have attracted the attention of many

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operations researchers in recent years. Decision-making in supply chains occurs at three levels: strategic, tactical, and operational. Problems such as facility location, production-inventory planning, vehicle routing, etc., are some examples of the problems that are studied in the three mentioned decision-making levels (Iranmanesh & Kazemi, 2017). Various types of supply chain planning problems in different manufacturing and service sectors have attracted researchers' attention.

For example, Taleizadeh et al. (2011) studied a two-echelon supply chain comprising various suppliers, products, and customers. In this study, the dependency of delivery time on the quantity of products ordered was considered. They used a harmony search algorithm as the solution methodology of the problem. Pasandideh et al. (2015) provided a multiobjective model for optimizing a supply chain's location, flow, and production decisions. However, attention was not paid to distribution and routing decisions. The authors solved the model using metaheuristic algorithms. Sarrafha et al. (2015) suggested a multiobjective planning problem for a four-echelon supply chain including supplier, manufacturer, distributor, and customer. Three metaheuristic algorithms were proposed to minimize objective functions. Supply chain planning problems in non-manufacturing sectors, such as healthcare, energy, and agriculture, have also been studied. For instance, Amani Bani et al. (2022) formulated a mathematical model for managing COIVD-19 vaccine waste flow in a reverse network under stochastic conditions. They used a robust method to manage the uncertainty and proposed the Lagrangian relaxation algorithm as a solution method. Fathollahi-Fard et al. (2019) introduced a multiobjective problem for location-routing decisions optimization in a green homecare network. The authors proposed the epsilon constraint method and multiple metaheuristics as the solution approach. Fallahi, Mousavian Anaraki, et al. (2024) modeled a new plasma supply chain planning problem to respond to the needs of COVID-19 patients. The proposed model simultaneously minimized the economic and environmental impacts of the blood plasma logistic network.

Neiro et al. (2022) presented a three-objective optimization problem for optimizing simultaneous production and distribution decisions in a gas logistic network. They specifically considered an argon supply chain, including multiple production sites and customers, where liquid argon is distributed via refrigerated trucks. Jaigirdar et al. (2023) proposed another multiobjective model for a multi-product perishable fruit and vegetable supply chain. The objectives of this problem were cost, waste, and pollution minimization, which were turned into a single objective using weighted sum method. In another research, Ala et al. (2024) investigated a sustainable stochastic supply chain planning problem for mobile charging stations. They employed a two-stage stochastic programming approach to address uncertainty. Kochakkashani et al. (2023) developed a new mathematical model for planning a COVID-19 four-echelon pharmaceutical supply chain. The uncertainty was taken into account and addressed by the wait-and-see approach. The authors also utilized an unsupervised machine learning approach to cluster pharmaceuticals. Nikoubin et al. (2023) studied a multiobjective vaccine supply chain design problem. They simultaneously formulated economical and social objective functions in their problem. Also, the booster dose injection and mix-and-match strategies were assumed in this research. Fallahi, Pourghazi, et al. (2024) focused on designing a humanitarian supply chain problem considering different types of items. In particular, they assumed the presence of blood, perishable, and non-perishable items in the studied logistic network. Sadeghi and Niaki (2024) addressed a green three-echelon supply chain planning problem under VMI contract. To incorporate the green dimension of sustainability, they considered several aspects such as tax regulation, energy management, water and waste control. Also, ergonomic indicators were used in this problem to establish a more comprehensive framework.

Moreover, the performance of supply chain networks is influenced by the distribution network due to the high transportation costs in supply chains (Kuo, 2013). In such situations, ignoring distribution planning and routing decisions negatively affects the performance of the supply chain. In this regard, Gendreau et al. (1994) designed a tabu search metaheuristic algorithm to solve the vehicle routing problem. Various constraints, such as capacity constraint and route length, were also taken into account. Vidal et al. (2012) proposed and implemented a genetic algorithm-based approach for optimizing three different routing problems. Laganà et al. (2021) developed a customer service differentiated routing problem and solved the model using an adaptive large neighborhood search algorithm. Moreover, in production planning, efforts are made to make optimal production decisions, such as production quantity, inventory holding level, or workforce size, considering constraints such as meeting customer demand completely (Cheraghalikhani et al., 2019). Production planning problems in the supply chain have long been of interest to researchers and scholars. For example, the economic production quantity (EPQ) model was proposed by Taft (1918) over a century ago. This model aimed to determine the economic quantity of production in a production environment to minimize the total production and inventory costs. Taleizadeh et al. (2013) focused on production-inventory management in a single-supplier-single-buyer supply chain. Uncertainty in product demand and delivery time was formulated using probabilistic and fuzzy uncertainty. Mokhtari et al. (2021) modeled the problem of determining optimal production policies in a production-inventory system considering operational constraints such as available space and purchasing budget. Reworking defective items was another assumption of this study. Taleizadeh et al. (2024) investigated a joint pricing and inventory planning problem for an imperfect production-inventory system. They suggested a rework policy for the imperfect items. Also, the backorder shortage was assumed for the system.

Amini and Kianfar (2022) studied a three-echelon supply chain planning problem. In this paper, several types of transportation and production methods with different environmental impacts and costs for the supply chain network were considered. Fatemi Ghomi et al. (2021) introduced a bi-objective model aiming to coordinate production and distribution activities across multiple products and time periods within a green supply chain. They also proposed a multiobjective particle swarm optimization algorithm as the solution methodology. Asadkhani et al. (2022) examined the inventory management problem between a buyer and a vendor in a two-echelon network. They also considered the assumption of producing defective items and developed the problem model under the vendor-managed inventory agreement. More details on previous papers in this field are available in review studies (Braekers et al., 2016; Farahani et al., 2014).

Unlike previous studies that have separately addressed production-inventory planning, vehicle routing, and location-allocation problems, our research integrates these aspects into a single, comprehensive model for a three-echelon supply chain. This model encompasses multiple production centers, distribution centers, and customers, and accounts for several real-world constraints such as regular and overtime production, production reliability, time-window constraints, and vehicle capacity. We propose a bi-objective mathematical model to optimize both total cost and servicing time. From the problem definition point of view, the novelty of our work lies in the simultaneous consideration of these elements in a multi-objective optimization framework, which has not been extensively explored in the literature. From the methodological point of view, we design and implement the well-known and powerful non-dominated sorting genetic algorithm-II (NSGA-II) as an effective solution approach. The algorithm is tailored to the features of the studied problem. This study not only advances the theoretical framework of integrated supply chain planning but also provides practical insights by evaluating the model through various numerical examples. In summary, the novelties of the current paper can be expressed as follows:

- Developing a bi-objective model for planning a three-echelon supply chain comprising multiple production centers, multiple distribution centers, and multiple customer.

- Addressing production planning, vehicle routing, and facility location decisions simultaneously.
- Considering various real-world assumptions such as regular and overtime production, production reliability, time window constraints, vehicle capacity, etc.
- Designing the NSGA-II multiobjective metaheuristic algorithm to solve the model.
- Evaluating the performance of the model and methodology through solving numerical examples.

Finally, Table 1 shows the innovations of the present paper compared to previous research in the literature.

Table 1: A comparison between the current study's contributions and the previous studies in the literature

Research	Year	Supply chain type	Number of echelons	Number of objectives		Decision types				Solution approach	
				Single	Multiple	Production-inventory	Routing	Location	Allocation	Exact	(Meta) heuristic
Gendreau et al. (1994)	1994	Service	2	✓	×	×	✓	×	×	×	✓
Vidal et al. (2012)	2012	Service	2	✓	×	×	✓	×	×	×	✓
Taleizadeh et al. (2013)	2013	Manufacturing	2	✓	×	✓	×	×	×	×	✓
Pasandideh et al. (2015)	2015	Manufacturing	3	×	✓	✓	×	✓	✓	×	✓
Sarrafha et al. (2015)	2015	Manufacturing	4	×	✓	✓	×	×	✓	×	✓
Fathollahi-Fard et al. (2019)	2019	Homecare	3	×	✓	×	✓	✓	✓	✓	✓
Fatemi Ghomi et al. (2021)	2021	Manufacturing	4	×	✓	✓	×	×	✓	✓	✓
Mokhtari et al. (2021)	2021	Manufacturing	1	✓	×	✓	×	×	×	✓	×
Laganà et al. (2021)	2021	Service	2	✓	×	×	✓	×	×	×	✓
Asadkhani et al. (2022)	2022	Manufacturing	2	✓	×	✓	×	×	×	✓	×
Amani Bani et al. (2022)	2022	Vaccine waste	6	×	✓	×	×	✓	✓	✓	×
Amini and Kianfar (2022)	2022	Manufacturing	3	✓	×	✓	×	×	✓	✓	✓
Fallahi, Mousavian Anaraki, et al. (2024)	2022	Blood	4	×	✓	✓	×	✓	✓	✓	×
Nikoubin et al. (2023)	2023	Vaccine	4	×	✓	✓	×	✓	✓	✓	✓
Kochakkashani et al. (2023)	2023	Pharmacy	4	✓	×	✓	×	×	✓	✓	×
Ala et al. (2024)	2023	Charging stations	2	✓	×	×	×	✓	✓	✓	✓
Fallahi, Pourghazi, et al. (2024)	2024	Humanitarian	5	×	✓	×	×	✓	✓	✓	×
Taleizadeh et al. (2024)	2024	Manufacturing	1	✓	×	✓	×	×	×	✓	×
Sadeghi and Niaki (2024)	2024	Manufacturing	3	✓	×	✓	×	×	✓	✓	✓
Current study	2024	Manufacturing	3	×	✓	✓	✓	✓	✓	✓	✓

The continuation of this paper is organized as follows. In Section 2, problem definition and mathematical modeling are presented. In Section 3, the proposed NSGA-II algorithm is explained. In Section 4, the effectiveness of the proposed framework is evaluated through solving numerical examples. In Section 5, the conclusions of the paper are drawn, and suggestions for future research are provided.

Problem description and mathematical formulation

First, a description of the newly investigated problem is provided in this section. Then, the mathematical model is developed by defining notations, parameters, variables, objective functions, and constraints.

Problem description

The problem involves a three-echelon supply chain network consisting of multiple production centers, distributor centers, and customers. At the first level, a set of production centers is responsible for manufacturing products. Each production center has its own production capacity, and production is possible for them during regular and overtime hours. Production during overtime hours incurs additional production costs per unit. Various factors, such as operator fatigue and continuous equipment operation may affect the production capacity of production centers. Therefore, the production reliability coefficient is considered for each production center and for regular and overtime hours. Production centers send the produced goods for distribution to a set of distribution centers. The location of distribution centers should be determined from a set of available candidate locations. Upon receiving the goods, distribution centers utilize vehicles to deliver the goods to customers. These vehicles should be rented based on a predetermined cost. These customers are located in the last layer of the supply chain network. Routing decisions for these vehicles are incorporated into the system. In this study, we assume that vehicles have different transportation costs, speeds, and capacities. One of the constraints in this problem is the presence of a hard time window for delivering goods to customers. More specifically, goods must be delivered to customers before the specified deadline. Two objective functions are considered for the problem to address the needs of managers and customers simultaneously. Total system cost and the total service time to customers minimization are the objective functions. The optimal location, allocation, routing, and production decisions should be determined so that these objectives are optimized. In general, the assumptions can be summarized as follows:

- The investigated supply chain is a three-echelon network consisting of production centers, distribution centers, and customers.
- Product manufacturing by production centers occurs in two timeframes: regular hours and overtime hours.
- There is a reliability level for production centers that impacts the production capacity.
- Optimal distributor locations must be selected from among a set of candidate points.
- Vehicles should be rented by distribution centers for delivering goods to customers.
- Vehicle routing is conducted for delivering goods to customers.
- Vehicles are different in load capacity and speed.
- There is a time window constraint for delivering goods to customers.
- Minimizing the total cost and the total service time to customers are the objectives.

Figure 1 illustrates a schematic view of the examined network.

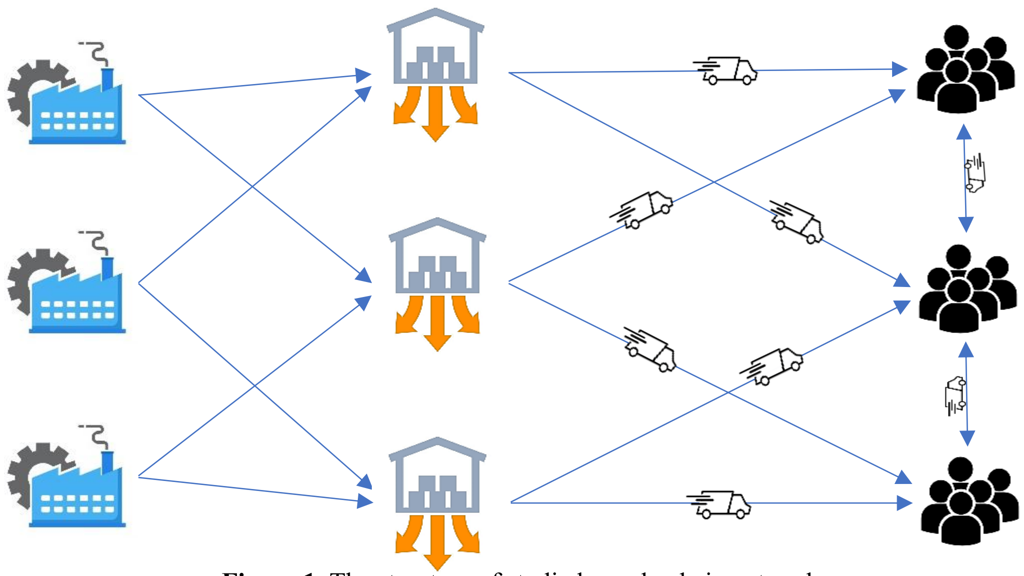

Figure 1. The structure of studied supply chain network

Mathematical formulation

The following notations are used in the modeling of the problem.

Sets

P	The set of production centers
D	The set of distribution centers
C	The set of customers
K	The set of vehicles

Parameters

PCR_p	The unit production cost for production center p during regular hours
PCO_p	The unit production cost for production center p during overtime hours
α_p	The reliability level for production center p during regular hours
β_p	The reliability level for production center p during overtime hours
CPR_p	The production capacity of production center p during regular hours
CPO_p	The production capacity of production center p during overtime hours
LCD_d	The location cost for distribution center d
CPD_d	The capacity of distribution center d
RCV_k	The rental cost for vehicle k
TCC_{ijk}	The transportation cost between customer i and customer j for vehicle k
SPV_k	The speed of vehicle k
CPV_k	The capacity for vehicle k
DIS_{ij}	The distance between customer i and customer j
LST_i	The latest possible service time for customer i
DEM_i	The demand of customer i
M	A very big number

Decision variables

tpr_p	The total production of production center p
pfl_p	The total flow for production center p
dfl_d	The total flow for distribution center d
vlo_k	The total load of vehicle k after leaving distribution center
sts_i	The service starting time of customer i
laf_i	The total load of vehicle after servicing customer i
arp_p	1 if production center p is activated during regular hours, 0 otherwise (A binary decision variable)
aop_p	1 if production center p is activated during overtime hours, 0 otherwise (A binary decision variable)
dlc_d	1 if distribution center d is located, 0 otherwise (A binary decision variable)
apd_{dp}	1 if distribution center d is allocated to production center p, 0 otherwise (A binary decision variable)
vdc_{ijkd}	1 if rented vehicle k by distribution center d has a trip from customer i to customer j, 0 otherwise (A binary decision variable)

Objective functions

The objective functions are as follows:

$$\begin{aligned} Min\ Z_1 = & \sum_{d \in D} dlc_d \times LCD_d + \sum_{k \in K} \sum_{i \in C} \sum_{d \in D} vdc_{ijkd} \times RCV_k \\ & + \sum_{k \in K} \sum_{i \in C \cup D} \sum_{j \in C \cup D} vdc_{ijkd} \times TCC_{ijk} + \sum_{p \in P} arp_p \times PCR_p + \sum_{p \in P} aop_p \times PCO_p \end{aligned} \tag{1}$$

$$Min\ Z_2 = \sum_{i \in C} sts_i \tag{2}$$

The objective function (1) minimizes the total cost, including the location cost of distribution centers, the renting cost of vehicles, the transportation cost of vehicles, the production cost of regular working hours, and the production cost of overtime working hours. In addition, the objective function (2) minimizes the service start time of customers.

Constraints

The objective functions are subjected to the following constraints:

$$\sum_{d \in D} \sum_{k \in K} \sum_{i \in C \cup D} vdc_{ijkd} = 1 \qquad \forall j \in C \tag{3}$$

$$\sum_{j \in C} vdc_{djkd} = \sum_{j \in C} vdc_{jdkd} \qquad \forall d \in D, k \in K \tag{4}$$

$$vdc_{iikd} = 0 \qquad \forall d \in D, k \in K, i \in C \cup D \tag{5}$$

$$\sum_{i \in D \cup C} vdc_{ihkd} = \sum_{j \in D \cup C} vdc_{hjkd} \qquad \forall d \in D, k \in K, h \in C \tag{6}$$

$$\sum_{i \in C} \sum_{d' \in D} vdc_{d'ikd} = 0 \qquad \forall d, d' \in D, k \in K, d \neq d' \tag{7}$$

$$sts_i + \frac{DIS_{ij}}{SPV_k} - M\left(1 - vdc_{ijkd}\right) \leq sts_j \qquad \forall d \in D, k \in K \tag{8}$$

$$sts_d = 0 \qquad \forall d \in D \tag{9}$$

$$sts_i \leq LST_i \qquad i \in C \tag{10}$$

$$vlo_k = \sum_{i \in C} \sum_{j \in C \cup D} \sum_{d \in D} vdc_{ijkd} \times DEM_j \qquad \forall k \in K \tag{11}$$

$$vlo_k \leq CPV_k \qquad \forall k \in K \tag{12}$$

$$laf_j \geq vlo_k - DEM_j - M\left(1 - vdc_{djkd}\right) \qquad \forall d \in D, k \in K, j \in C \tag{13}$$

$$laf_j \geq laf_i - DEM_j - M\left(1 - vdc_{ijkd}\right) \qquad \forall d \in D, k \in K, i, j \in C \tag{14}$$

$$\sum_{p \in P} apd_{dp} = 1 \qquad \forall d \in D \tag{15}$$

$$dfl_d = \sum_{i \in C} \sum_{j \in C \cup D} \sum_{k \in K} vdc_{ijkd} \times DEM_i \qquad \forall d \in D \tag{16}$$

$$dfl_d \leq CPD_d \qquad \forall d \in D \tag{17}$$

$$pfl_p = \sum_{d \in D} dfl_d \times apd_{dp} \qquad \forall p \in P \tag{18}$$

$$tpr_p = arp_p \times CPR_p\left(1 - \alpha_p\right) + aop_p \times CPO_p\left(1 - \beta_p\right) \qquad \forall p \in P \tag{19}$$

$$apd_{dp}, vdc_{ijdk}, arp_p, aop_p \in \{0,1\} \qquad \forall d \in D, k \in K, i, j \in C, p \in P \tag{20}$$

$$tpr_p, vlo_k, laf_j \geq 0 \qquad \forall k \in K, j \in C, p \in P \tag{21}$$

Constraints (3) guarantees that each customer is serviced by only one vehicle. Additionally, each vehicle that departs from a distributor must return to the same distributor after servicing, as ensured by constraints (4). Constraints (5) ensures no route from any node back to itself. Constraints (6) stipulates that each vehicle that visits a customer must exit from the customer node after servicing. Constraints (7) guarantees the allocation of each vehicle to each distributor and prevents a vehicle from one distributor exiting to another distributor. Constraints (8) calculates the service time for each customer. Constraints (9) sets the departure time of the vehicle from the distributor to zero. Constraints (10) represents the strict time window limitation of the problem, ensuring delivery to the customer within the specified timeframe. Constraints (11) calculates the load carried by each vehicle. Constraints (12) accounts for the capacity of vehicles for cargo transportation. Constraints (13) and (14) determine the load of vehicles after leaving the first customer and other customers, respectively. Constraints (15) ensures the allocation of each located distribution center to production centers. Constraints (16) calculates the flow for each distributor. Constraints (17) ensures compliance with the demand of each distributor. Constraints (18) calculates the total flow for each production center. Additionally, Constraints (19) determines the level of product production by each production center during regular and overtime hours considering the reliability level. Finally, Constraints (20) and (21) represent the type of decision variables in the problem.

Linearization

The presence of nonlinear components in the model significantly increases the complexity of solving the problem using standard solvers. Nonlinear models often require more computational resources and time, making them less practical for large-scale instances or real-time applications. To address this issue and enhance the tractability of our model, we linearize the nonlinear constraints (18) in this section. This linearization process simplifies the problem structure, allowing for more efficient and effective solutions using conventional optimization techniques. To this end, zax_d is defined as an auxiliary variable. Using this variable, we can rewrite the constraints (18) as follows:

$$pfl_p = \sum_{d \in D} zax_d \qquad \forall p \in P \qquad (22)$$

Also, the following complementary constraints should be considered in the linear counterpart model:

$$zax_d \leq apd_{dp} \qquad \forall p \in P, d \in D \qquad (23)$$
$$zax_d \leq dfl_d M \qquad \forall d \in D \qquad (24)$$
$$zax_d \geq apd_{dp} - M(1 - dfl_d) \qquad \forall p \in P, d \in D \qquad (25)$$

The solution approach

The supply chain network design problem, which generally encompasses two types of decisions: facility location and flow allocation, is NP-Hard complex (Gourdin et al., 2000). In such circumstances, the solution time exponentially increases with the problem size, making it impractical to solve the problem exactly for large dimensions within a reasonable time frame. Since the current problem is a generalization of the supply chain network design problem, exact methods and commercial solvers are inefficient in solving the problem for large dimensions. Therefore, the NSGA-II algorithm, as one of the most efficient and well-known multiobjective metaheuristics, is proposed as the solution approach. The multi-objective version of the genetic algorithm, known as non-dominated sorting genetic algorithm (NSGA), was provided by Srinivas and Deb (1994). However, some weaknesses of this algorithm, such as the need for high computational power, led to the proposal of the NSGA-II metaheuristic by Deb et al. (2002) as an enhanced version of the previous algorithm.

Similar to other metaheuristic algorithms, determining a set of input parameters is required to execute this algorithm. These parameters include the maximum iteration number, population size, mutation, and crossover probability. After determining the parameters, a population of solutions is randomly generated in the first generation. This initial population is evaluated based on the objective function, serving as a fitness function, and sorted into non-dominated tiers. In this procedure, each solution is compared with all existing solutions in the population to determine whether it is dominated or non-dominated. All non-dominated solutions are placed on the first tier. In the next step, the identified set of solutions for the first tier is temporarily removed from the population, and the aforementioned process is repeated. This process continues until all members of the population are assigned to a single tier.

In subsequent generations, these steps are iteratively repeated. Initially, a set of parents is selected using a tier rank-based selection method and crowding distance. The crowding distance in NSGA-II is an indicator of the density in the space. It quantifies how close a solution is to its neighbors, aiding in maintaining diversity in the population by favoring solutions that are located in less crowded regions of the objective space. Two members of the population are randomly chosen, and if one member has a superior rank, it is selected as the superior member. If both members have equal ranks, the one with the lower crowding distance is chosen. After that, the combination of solutions is performed using the crossover operator, considering the crossover probability parameter. Additionally, the mutation operator is applied according to the mutation probability parameter and a specified number of times. The population resulting from these operators, along with the population from the previous generation, forms a combined population, and the algorithm must select the required number of solutions from this combined population for the next generation. Solution selection is based on their placement in non-dominated tiers and the distribution of solutions along the border. Various criteria, such as iteration number, solution time, number of objective function evaluations, etc., have been considered as stopping criteria in the literature. In this study, the maximum number of iterations is considered as the termination condition for the algorithm. Finally, after the last iteration, the solutions available at the first tier are reported as non-dominated solutions of the algorithm. Figure 2 illustrates the flowchart of the NSGA-II algorithm.

The implementation of metaheuristic algorithms for each particular problem requires a solution encoding scheme. Proper solution encoding has an significant role in the performance of metaheuristic algorithms. In the designed NSGA-II algorithm, each solution consists of three vectors. A random permutation of numbers is assigned to each vector. The first vector includes $n + k - 1$ cells, where n is the number of customers, and k is the number of vehicles. In this vector, numbers 1 to n represent customer numbers, and numbers greater than n specify vehicle numbers. Specifically, cell $n + 1$ represents the first vehicle, and cell $n + i$ represents the i^{th} vehicle. In this case, the numbers that appear before cell $n + i$ are the customers served by the i^{th} vehicle, and their order indicates the sequence in which they are served. All customer numbers that appear after the last number greater than n are assigned to the last vehicle. If there is no customer number before $n + i$, it means that the vehicle is not used. The second vector includes $d + k - 1$ cells, where numbers 1 to k represent vehicle numbers, and number $k + i$ represents the i^{th} distribution center. The vehicle numbers before cell $k + i$ are the vehicles assigned to the i^{th} distribution center. Vehicles after the last number greater than k are served by the last distribution center. If there is no vehicle number before $k + i$, it means that the distributor is not used. The third vector includes $p + d - 1$ cells, where numbers 1 to d represent distribution center numbers, and number $d + i$ represents the i^{th} production center. The distribution center numbers before cell $d + i$ are the distribution center assigned to the i^{th} manufacturer. Distribution centers after the last number greater than d are served by the last production center. If there is no distribution center number before $d + i$, it means that the production center is not used.

To better understand this problem, a numerical example is provided. Suppose there are ten customers, three vehicles, three distribution centers, and three production centers. Figures 3 shows a solution for this problem based on the explained solution encoding scheme. As shown in this figure, vehicle 1 serves customers 1 and 6 in order, vehicle 2 serves customers 9, 3, and 8 in order, and vehicle 3 serves customers 2, 7, 4, 5, and 10. Additionally, vehicles 1 and 2 are assigned to distribution center 1, and vehicle 3 is assigned to distribution center 2, while distribution center 3 is not used. Finally, distribution center 1 is assigned to production center 1, and distribution center 2 is assigned to production center 2 (since distribution center 3 is not located).

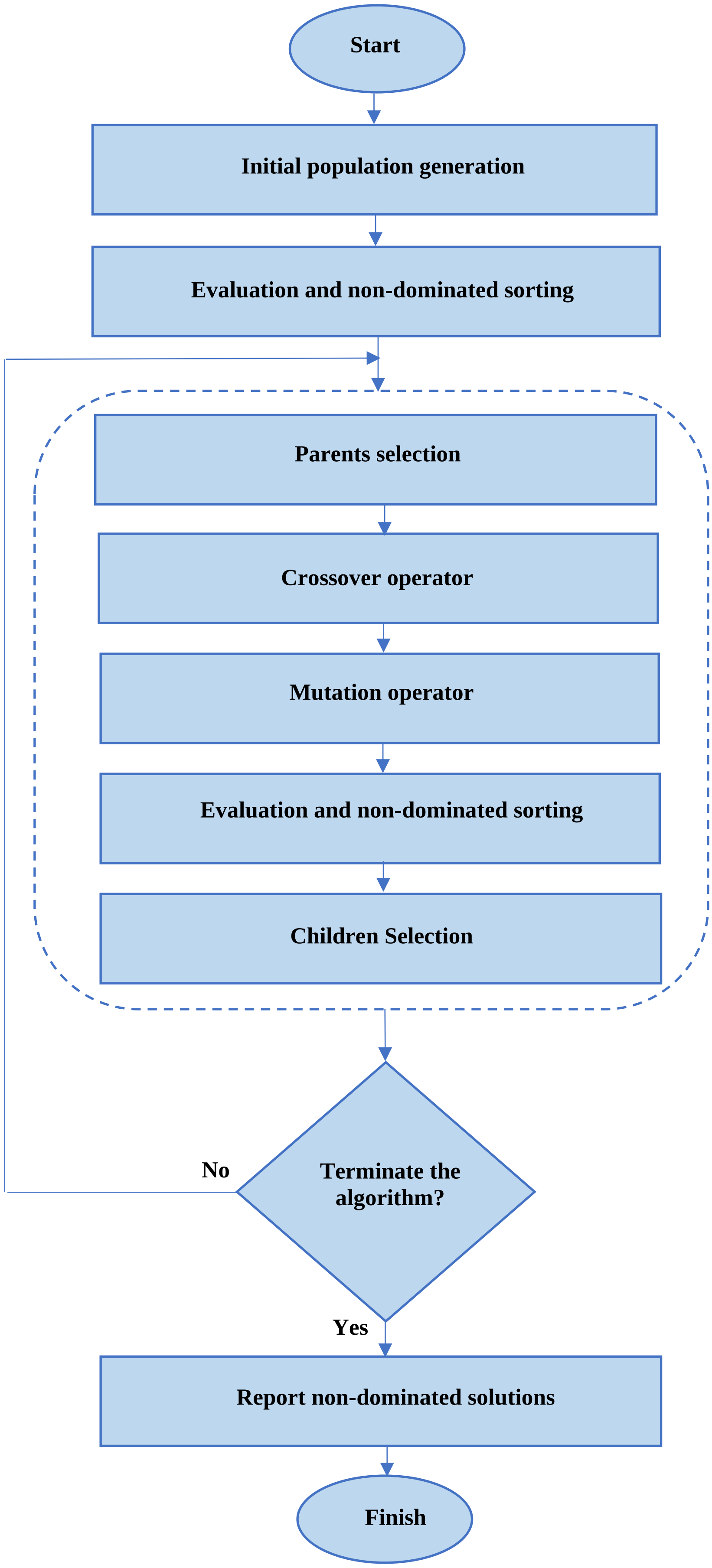

Figure 2. The flowchart of NSGA-II algorithm

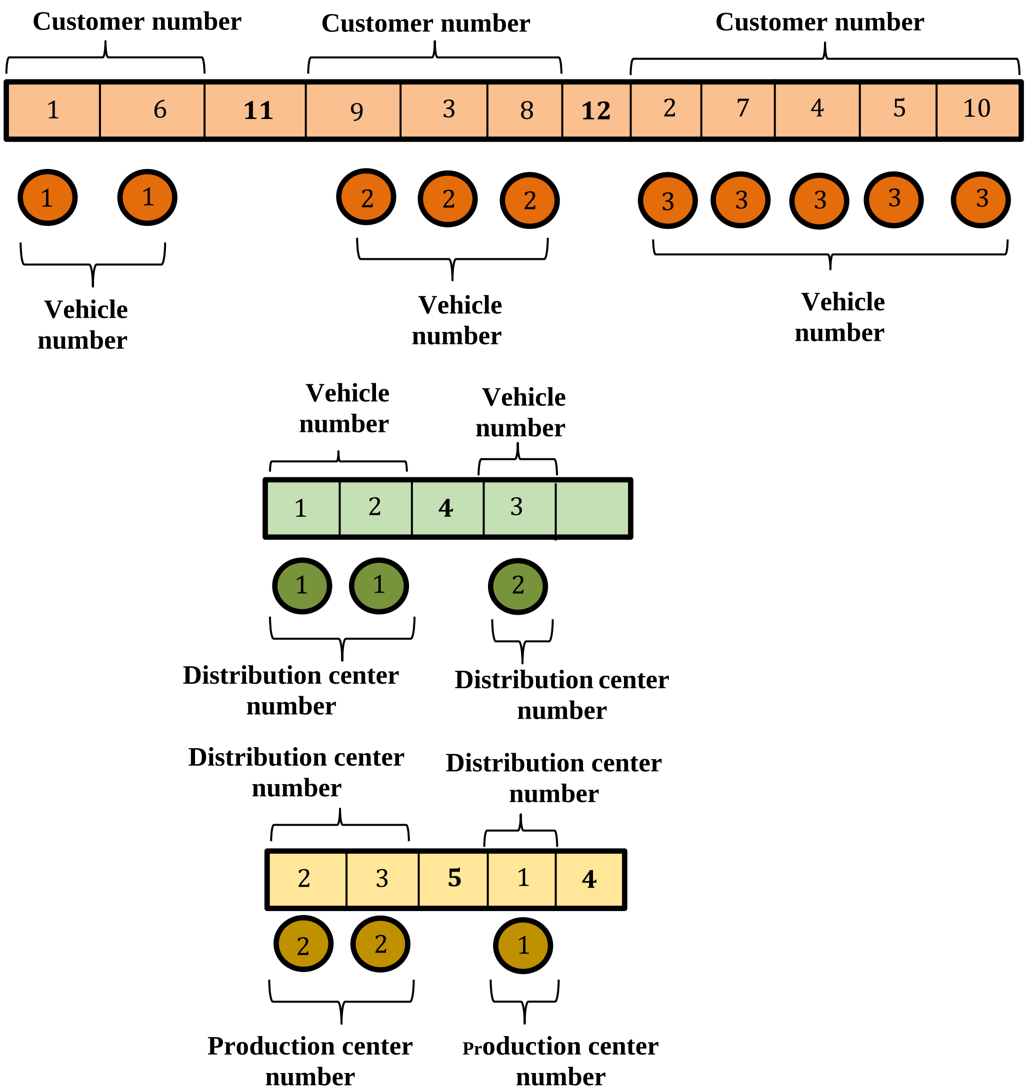

Figure 3. An example of solution encoding for NSGA-II implementation

Computational experiments

In this section, the proposed model and solution methodology performance is evaluated through solving numerical examples. To provide exact solutions, the problem is coded in GAMS programming environment and solved using the commercial CPLEX solver (Heidari-Fathian & Pasandideh, 2018). To this end, the weighted sum method is employed. The weighted sum method is a common approach for managing multiple objective functions in an optimization problem. This method converts the problem into a single-objective model by assigning a weight to each objective function and summing them up to form a composite objective function. By adjusting the weights, different trade-offs among the objectives can be explored, producing a set of Pareto optimal solutions. This enables the decision-maker to select the solution that best aligns with their preferences and priorities. The generated numerical examples consist of 3 production centers, 4 distribution centers, 4 vehicles, and 10 customers. Additionally, the NSGA-II algorithm is implemented and coded in the MATLAB programming environment. Here, the maximum number of iterations, population size, crossover probability, and mutation probability are set 50, 100, 0.8, 0.2. The generation of numerical example parameters is done randomly from a uniform distribution. The considered ranges for parameters in the numerical examples are presented in Table 2. These ranges are determined based on previous studies in the literature and the opinion of experts in the field.

Table 2. The considered intervals for numerical examples generation

Parameter	Range	Parameter	Range
DEM_i	(1,15)	α_p	(0.40,0.80)
PCR_p	(0.1,0.2)	PCO_p	(0.2,0.3)
DIS_{ij}	(1,10)	β_p	(0.40,0.80)
CPV_k	(20,40)	RCV_k	(1,5)
LST_i	(5,25)	LCD_d	(10,30)
CPD_d	(20,35)	CPD_d	(10,40)

All solutions are computed on a personal laptop with 8GB RAM 2.6 GHz intel core i7 CPU. Figure 4 illustrates the output network after solving the example with the CPLEX solver. In this diagram, green pentagons represent producers, yellow rectangles represent distributors, and circles represent customers. The paths taken by the vehicles are depicted by vectors. As mentioned, there is possibility of production during regular and overtime hours for production centers. Table 3 displays the production costs for each production center during regular and overtime hours.

In the second echelon, the distribution centers' location should be determined. The location status of candidate distribution centers and their associated cost are shown in Table 4. As can be seen, three distribution centers should be located for servicing customers.

After location distribution centers, they should be allocated to customers for servicing. Each distribution center rents a vehicle to deliver the products to the customers. The details on the optimal distribution centers-customers allocation and routs of vehicles are summarized in Tables 5 and 6.

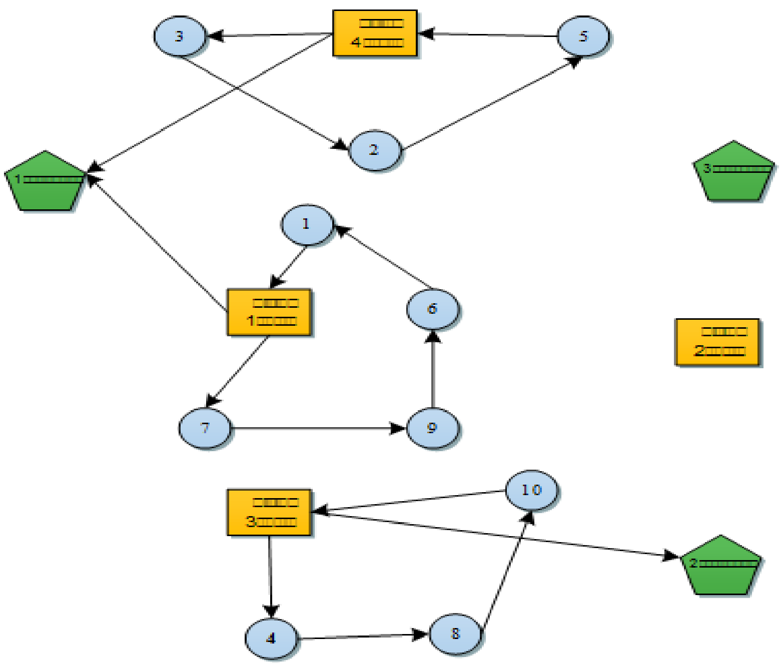

Figure 4. A schematic view of the obtained solution

Table 3. The production cost during regular and overtime hours

Production center	Cost for regular hours (a hundred million Rials)	Cost for overtime hours (a hundred million Rials)
1	7	3
2	4	0

Table 4. The location status of distribution centers

Distribution center	Status	Location cost (a hundred million Rials)
1	Located	20
2	Located	15
3	Not located	0
4	Located	15

Table 5. The optimal allocation and routing decisions for distribution centers and customers

Distribution center	Vehicle number	Service route
4	1	Customer 3-Customer 2-Customer 5
1	2	Customer 7-Customer 9
1	3	Customer 6-Customer 1
3	4	Customer 4-Customer 8-Customer 10

Table 6. The location status of vehicles

Vehicle	Status	Rental cost
1	Rented	4
2	Rented	3
3	Rented	3
4	Rented	2

One of the constraints of the problem is to adhere to the capacity of the vehicles for loading. One of the constraints of the problem is the initial loading amount. The initial loading status of the vehicles relative to their capacity is depicted in Figure 5. As shown in this figure, all vehicles have loaded less than their capacity.

Figure 6 illustrates the comparison between the maximum possible service time allotted for each of the ten customers and the actual service time provided. Adhering to the hard time window constraint, the service time for each customer must fall within the specified maximum service time. As depicted, the service time for all customers remains below their respective maximum service times, ensuring compliance with the imposed constraints.

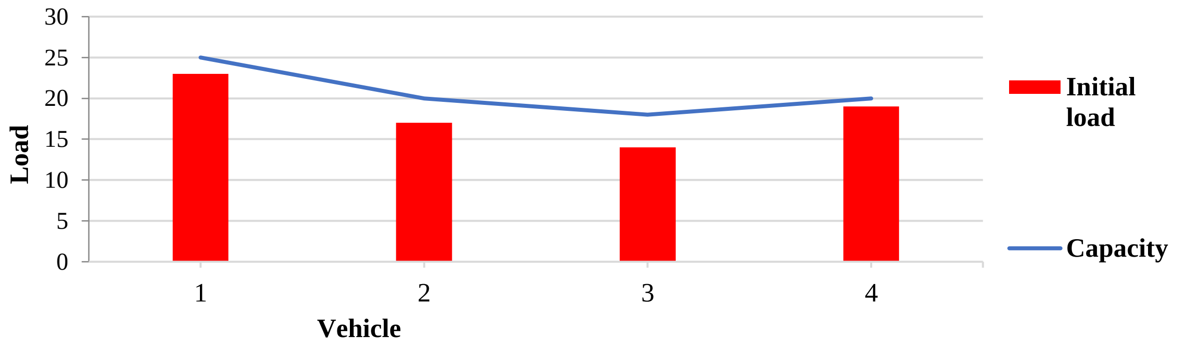

Figure 5. The comparison between the service time and maximum possible service time of customers

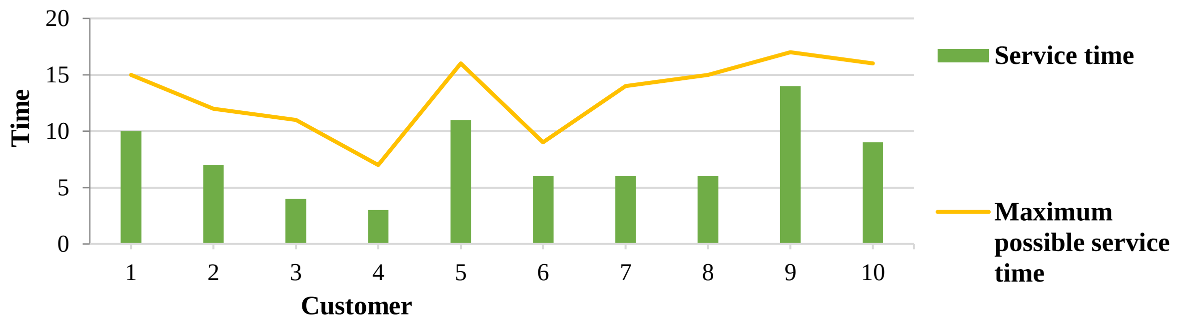

Figure 6. The comparison between the service time and maximum possible service time of customers

Figure 7 delineates the capacity status of the four distribution centers alongside their current load. It is evident that the load at each distribution center remains below its designated capacity. Notably, the figure does not depict the load for distribution center 3, denoting its absence in the network.

In continuing, the sensitivity of the CPLEX solver CPU time with respect to the size of the problem is investigated. 10 numerical examples in different sizes are used for this goal, based on the presented details in Table 7.

Figure 8 illustrates the CPU time the CPLEX solver requires to solve ten numerical examples. A discernible trend emerges, indicating an exponential increase in CPU time as the size of the examples grows. This observation aligns with expectations, given the NP-hard complexity of the problem.

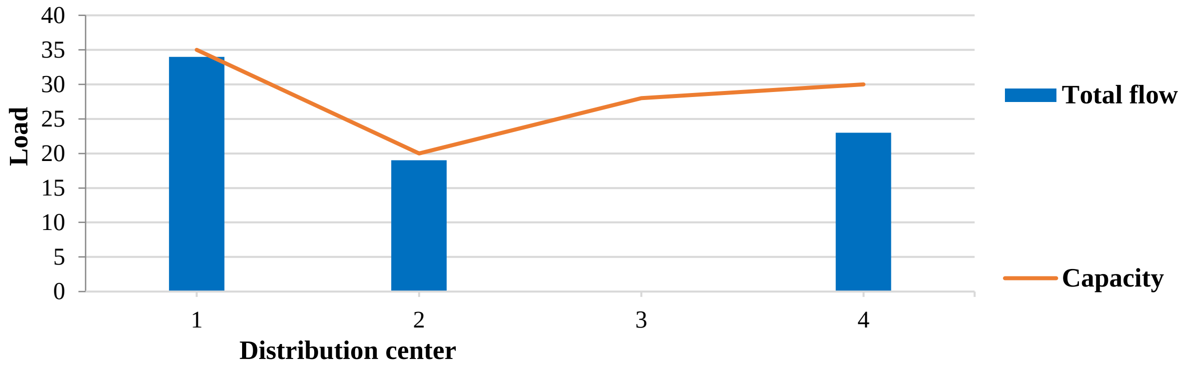

Figure 7. The comparison between the load and capacity of distribution centers

Table 7. The size of studied numerical examples for CPU time evaluation of CPLEX

Example	Production centers	Distribution centers	Vehicles	Customers
1	2	3	2	4
2	2	3	2	6
3	3	4	3	8
4	3	4	3	10
5	3	5	4	12
6	4	5	4	14
7	4	6	5	16
8	4	6	5	18
9	5	7	6	20
10	5	7	6	22

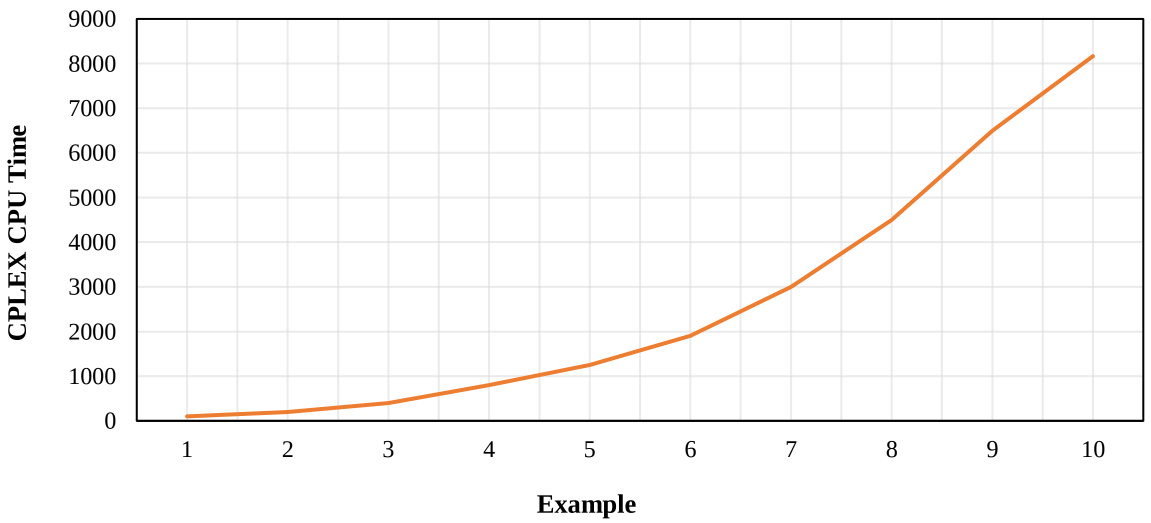

Figure 8: The CPU time of commercial solver in different numerical examples

Now, we are going to investigate the performance of NSGA-II multiobjective metaheuristic algorithm in solving the presented problem. To this end, 5 numerical examples in small and large-size are randomly generated. In small-size examples, we compare the performance of NSGA-II with respect to the best obtained solution for each objective function. The results are presented in Table 8. As obvious, the difference between the solutions are relatively low and acceptable. Since the CPLEX solver cannot reach the solution for large-size examples in a reasonable time interval, we compare the average and best values for each objective function in these examples. The results are summarized in Table 9. Figures 9 and 10 depict the results obtained from NSGA-II for large-size examples. As can be seen, the average and best values exhibit minimal variance, indicating consistent performance of NSGA-II. Overall, this analysis underscores the reliability and stability of NSGA-II across varying problem sizes, validating its effectiveness as a robust solution approach for the studied problem.

Table 8. The performance comparison of CPLEX and NSGA-II in small-size examples

Example	Production centers	Distribution centers	Vehicles	Customers	First objective			Second objective		
					CPLEX	NSGA-II	Variation percentage	CPLEX	NSGA-II	Variation percentage
1	2	2	4	4	59	59.50	0.85 %	34	34.2	0.59 %
2	3	3	7	7	65.2	65.80	0.92 %	25	25.3	1.20 %
3	4	4	10	10	74	74.80	1.08 %	20	20.3	1.50 %
4	5	5	12	12	96	97.30	1.35 %	18	18.3	1.67 %
5	6	6	15	15	112	113.90	1.70 %	13	13.3	2.31 %

Table 9. The performance comparison of CPLEX and NSGA-II in small-size examples

Example	Production centers	Distribution centers	Vehicles	Customers	First objective			Second objective		
					Average	Best	Variation percentage	CPLEX	NSGA-II	Variation percentage
1	5	5	5	20	352	353	0.28%	252	250	0.80 %
2	7	7	7	30	452	456	0.88 %	276	273	1.10 %
3	9	9	9	40	533	539	1.13 %	328	324	1.23 %
4	11	11	11	50	650	659	1.38 %	413	407	1.47 %
5	13	13	13	60	739	756	2.30 %	468	460	1.74 %

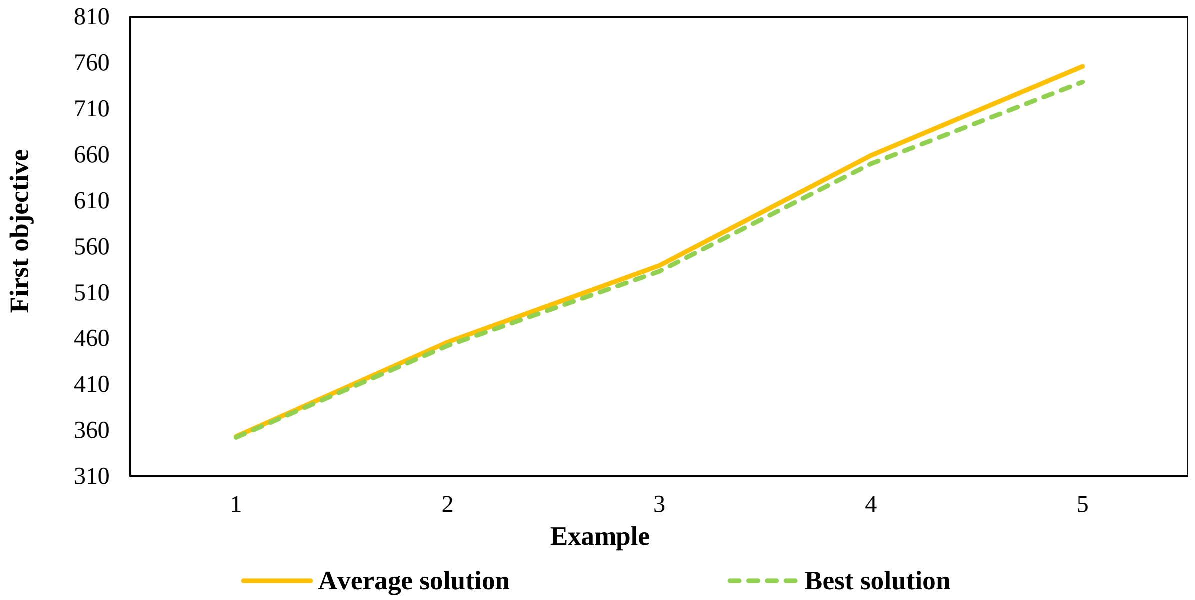

Figure 9. The robustness of NSGA-II algorithm for the first objective function

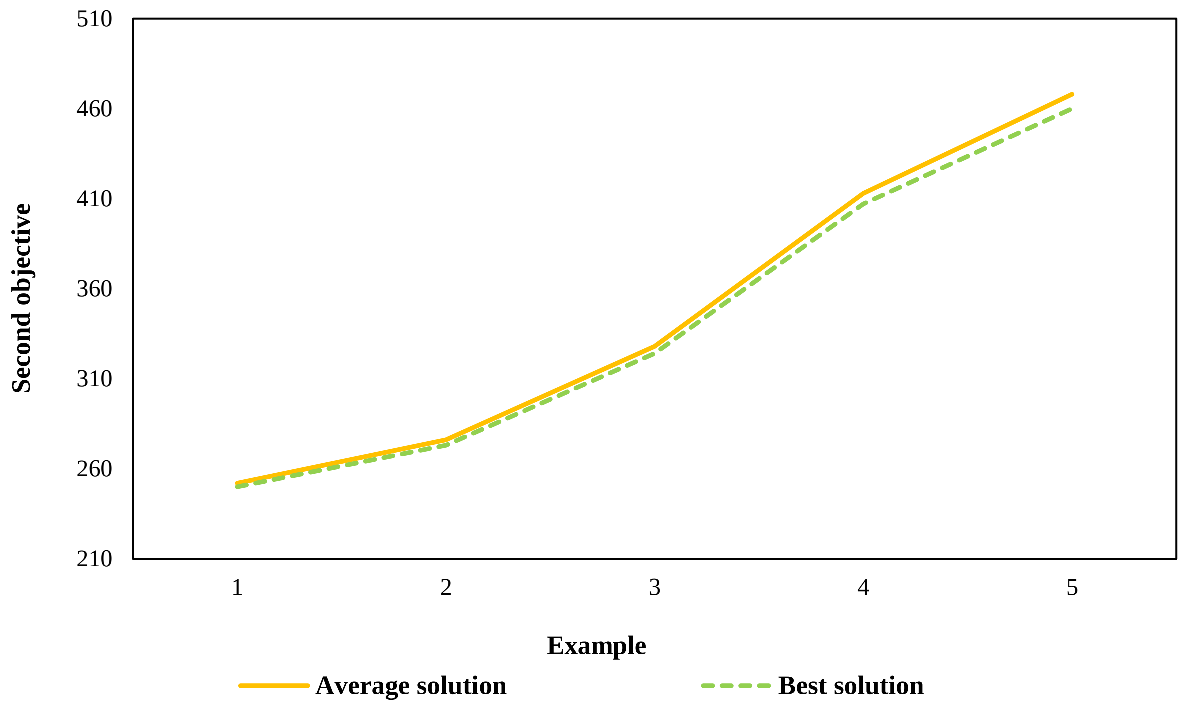

Figure 10. The robustness of NSGA-II algorithm for the second objective function

The performance of NSGA-II and the CPLEX solver in terms of CPU time is compared using the first numerical example from Table 7, where the number of customers ranges from 1 to 8. The results are graphically shown in Figure 11. In the case of CPLEX, a clear exponential trend is observed as the number of customers increases, indicative of significantly escalating computational costs. Conversely, NSGA-II demonstrates a linear trend, with computational costs remaining relatively stable even as the problem size expands. This stark difference underscores the superior scalability and efficiency of NSGA-II, particularly for larger examples, positioning it as a favorable solution approach for tackling complex integrated production planning-location-routing problems within three-echelon supply chains.

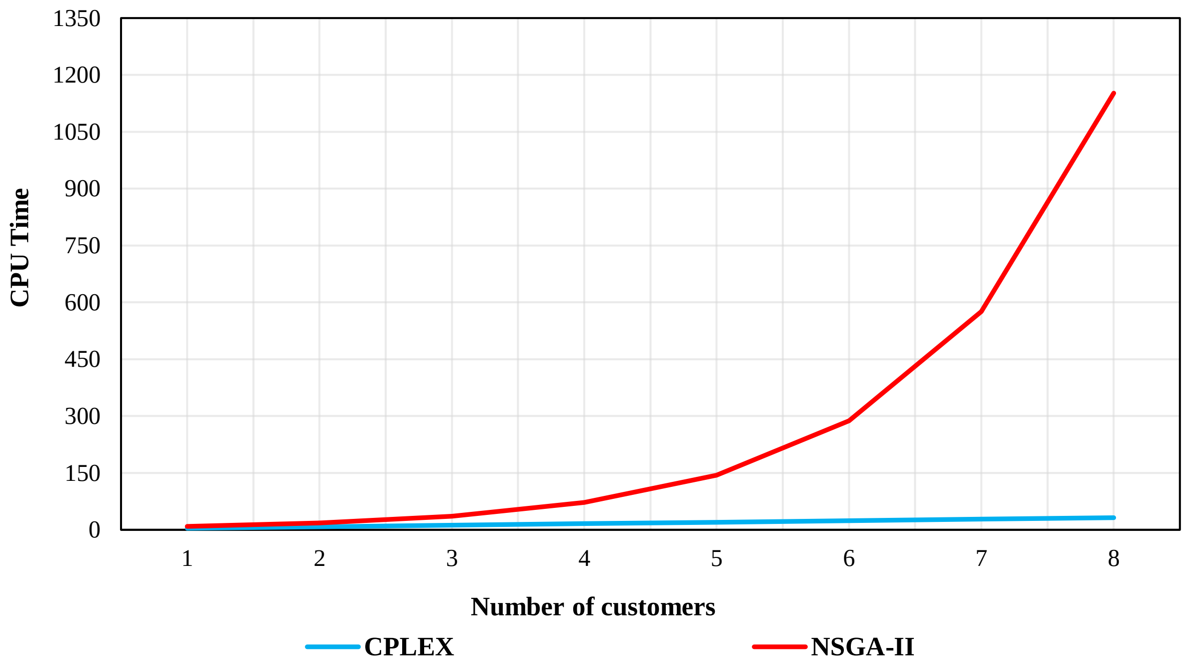

Figure 11. The sensitivity of CPU time to number of customers

Conclusion

The objective of the current paper was to present a location-allocation, routing, and production planning problem for a three-echelon supply chain network, including production centers, distribution centers, and customers. Several realistic assumptions, such as regular and overtime production, reliability, time window constraints, etc., were incorporated into the problem. The problem was modeled with two objective functions. The objectives were minimizing the total costs and the customer service time. Due to the complexity of the problem, the NSGA-II algorithm was designed and implemented as the solution method. Numerical examples of various dimensions were analyzed to evaluate the proposed model and the solution methodology. The validity of the model was demonstrated by detailing the results of solving a problem using the CPLEX commercial solver. The results indicated that as the problem dimension increases, the solution time exponentially increases with the commercial solver. Additionally, the proposed algorithm exhibited satisfactory stability in solving the problem, and the calculated optimal values for each objective function and the average best values do not deviate significantly from each other. The current paper can be further developed in future research in various ways. The current problem modeling was presented assuming certainty in parameter values. Considering uncertainty in parameters such as customer demands and using appropriate approaches such as stochastic, fuzzy, or robust optimization can be a proposal for future research. Also, other important decisions, such as inventory management decisions, can be added to the problem. Designing and implementing other heuristic and metaheuristic algorithms and comparing the quality of the outputs with the proposed algorithm can be another suggestion for future researchers.

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