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Fate and Transport of Solute with Temporally Varying Pulse Type Input Source under Sorption in Heterogeneous Porous Formation

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ABSTRACT

A mathematical model is developed to describe the conservative solute migration under sorption in a groundwater reservoir. For the complexity of the aquifer, it is assumed as heterogeneous and semi-infinite. Dispersion is considered as a varying power of seepage velocity. For the sake of real scenario of the aquifer, the seepage velocity, first-order decay (FOD), zero-order production (ZOP), and retardation factor are taken as spatio-temporal dependent parameters. Initially, the aquifer is assumed as polluted by a background source throughout the domain. Also, a temporally dependent pulse type sinusoidal input source is taken at origin of the aquifer. The other end of the aquifer is assumed as flux free. The retardation factor considered with a special form due to regional and complication of the porous medium. The transient velocity is considered as sinusoidal, exponential, algebraic sigmoid and asymptotic forms to study the solute transport behavior under different velocity patterns. The analytical solution of the proposed model is obtained by Laplace and inverse Laplace transform techniques. All the graphical plots are obtained by MATLAB software. The present study may be helpful for scientists, geologists to determine the time and position of harmless concentration level and can be treated as preliminary tool for solute migration for the future researchers.

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INTRODUCTION

In recent decades, the pollution of Earth sub-surface natural water has been a major problem throughout the world. It is polluted either naturally or by anthropogenic activities of human beings. Day by day quality of groundwater is cautiously deteriorating; as a result it became unfit for human beings. Once the groundwater become polluted, it is very difficult, time consuming and expensive to clean it up. Mathematical model plays important role to study about the solute or pollutant transport in aquifer.

To study groundwater pollution, groundwater modelling is very helpful as it enables engineers or researchers to better understand complex systems, predict their behaviour, optimize designs and strategies, assess risks, and develop innovative solutions to address environmental and geotechnical challenges. In grigoundwater system, the solute dispersion and groundwater seepage velocity play a major role in solute transportation. Many researchers and scientists analyzed the relationship between these two crucial processes in the classical model of advection-diffusion equation (ADE). Freeze & Cherry (1979) found that dispersion correlates with the n^{th} power of the seepage velocity with the exponent typically falling between

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1 and 2. Also, according to the Indian geological formation, Ghosh & Sharma (2006) presented that the dispersion is linked to a power n of the seepage velocity with the exponent varying between 1 and 1.2. Chen et al. (2008) derived an analytical solution (AS) taking dispersion as hyperbolic asymptotic space-dependent. Applying Laplace transform technique (LTT), Gao et al. (2010) obtained an AS for the reactive solute in mobile-immobile zone subject to scale dependent dispersion and first-order decay (FOD) in a heterogeneous porous medium. Guerrero & Skaggs (2010) derived an AS by solving a one-dimensional (1D) ADE with space dependence dispersion in a finite heterogeneous porous media. Applying the integral transform for prediction of mixing of contaminate into groundwater, dispersion coefficient has significant impact in hydrology. Ziskind et al. (2011) derived an AS of the classical 1D ADE in a finite domain subject to pulse type boundary condition. Also, Chang & Yeh (2012) derived an AS to investigate the non-reactive solute in non-stationary unsaturated flow fields. Using LTT, Singh et al. (2012) examined solute distribution by solving a 1D ADE subject to pulse type varying point source at origin in a medium of linear heterogeneity. Using finite difference method (FDM), Savovic & Djordjevich (2012) discussed the solute migration with steady and unsteady flow by solving a classical 1D ADE with constant and oscillating boundary condition in a semi-infinite homogeneous and inhomogeneous media.

Cvetkovic et al. (2014) developed a unique three-dimensional multi-indicator model to discuss the solute migration behaviour in a spatially dependent heterogeneous formation. Wu et al. (2014) explored analytical and experimental studies of solute dispersion in a non-uniform porous media. The distance dependent dispersion was taken as linear, parabolic, asymptotic, hyperbolic or exponential functions. To analyse solute migration in a heterogeneous medium, Abgaze & Sharma (2015) applied the hybrid finite volume method and represented with the breakthrough curve and found that solute dispersion increases with the variation of hydraulic conductivity. Singh et al. (2015) investigated the migration of solute in both parallel and opposing directions to sinusoidally varying flows, utilizing a point source with a pulse-type release. Using LTT, Kumar & Yadav (2015) obtained an analytical solution of a 1D ADE with uniform and pulse-type input source in a non-uniform porous media subject to space varying linear dispersion and seepage velocity. Applying integral transform, Suk (2016) derived a semi-analytical solution under the FOD and subject to temporally and spatially dependent dispersion with flow velocity. Moreover, Zhao et al. (2017) developed an experimental study of non-reactive solute transport in non-uniform aquifers. The authors explained the relationship between dispersivity of pollutant and hydraulic conductivity.

Yadav et al. (2018) analytically solved a two-dimensional (2D) conservative ADE subject to temporal and scale-varying dispersion in a heterogeneous aquifer. Thakur et al. (2019) explored the transport mechanism of contaminated groundwater in a porous formation. The study derived the solution of a 2D ADE for semi-infinite domain with varying flow velocities. Canuto et al. (2019) employed the decomposition method of a time dependent ADE to analyse the conservative solute movement in a heterogeneous porous formation under various time steps in different sub-domains. Applying FDM, Savovic & Djordjevich (2020) obtained solution for solute migration associated with constant and oscillatory type concentration conditions. Chaudhary et al. (2020) analysed the 1D solute distribution by taking solute dispersion as square of space dependent groundwater velocity in a semi-infinite aquifer. Also, Chaudhary & Singh (2020) employed the homotopy analysis method to develop the series solutions of a 1D multispecies convection-dispersion equation.

Khuzhayorov et al. (2020) discussed the solute migration under non-equilibrium sorption in the elements of fractured porous medium with a heterogeneous porous block. Applying LTT, Rajput & Singh (2021) investigated a 2D solute migration in presence of off-diagonal dispersion effect in a heterogeneous porous medium. The authors considered transverse, longitudinal and off-diagonal dispersion, all varies with space. With the help of homotopy analysis method, Kumar et al. (2022) investigated 1D solute fate in the study domain by semi-analytical approach subject to time dependent dispersion and seepage velocity. Mehmood et al. (2023) analysed the fluid flow and pollutant transport in a uniform porous formation with solid plate stacks. Also, the pollutant transport equation is solved using FDM. Moreover, Singh et al. (2023) presented the 2D mathematical model under non-linear sorption and solved the governing equation using FDM subject to axial input sources. Liang & Isa (2024) discussed the problem of heavy metal contamination in Earth porous formation and its impact on environment. The mathematical model is presented for the transport of heavy metals through soil under the consideration of adsorption, desorption, emission, and retardation factors.

Numerous studies have been reported previously in which solute transport along groundwater flow in porous media is addressed with input source at origin for a certain period of time. However, the pulse type input source may occur naturally or human activities in the porous medium. This concept inspired our investigation into solute transport behaviour with a pulse-type point source in an aquifer. Analytical technique is utilised to solve the ADE under the sorption condition. The retardation factor, FOD and zero-order production (ZOP) rates are also incorporated in the governing equation to study about their effects on the solute transport.

MATHEMATICAL FORMULATION

By conservative mass law and Fick's law of diffusion, governing solute transport model is formulated by considering only mechanical dispersion. The proposed governing solute transport equation together with the concentration conditions are modelled as follows:

$$\frac{\partial c}{\partial t}+\frac{\rho}{\phi}\frac{\partial s}{\partial t}=\frac{\partial}{\partial x}\left(D\frac{\partial c}{\partial x}-uc\right)-\lambda c+\gamma \tag{1}$$

$$c(x,t)=c_1+\frac{\gamma}{u}x;\ \ x\geq 0,\ t=0 \tag{2}$$

$$c(x,t)=\begin{cases} c_0\left[1+\sin\left(kt\right)\right] & 0<t\leq t_0 \\ 0 & t>t_0 \end{cases};\ \ x=0 \tag{3}$$

$$\frac{\partial c}{\partial x}=0;\ \ x\to\infty,\ t>0 \tag{4}$$

NOMENCLATURE

$c\left[\mathrm{ML^{-3}}\right]$: Liquid phase concentration

$c_1\left[\mathrm{ML^{-3}}\right]$: Constant background solute concentration

$c_0\left[\mathrm{ML^{-3}}\right]$: Constant solute concentration at the origin

$s\left[\mathrm{MM^{-1}}\right]$: Solid phase concentration

$D\left[\mathrm{L^{2}T^{-1}}\right]$: Longitudinal dispersion

$u\left[\mathrm{LT^{-1}}\right]$: Pore-water velocity

$k\left[\mathrm{T^{-1}}\right]$: Decay parameter

$\lambda\left[\mathrm{T}^{-1}\right]$: FOD rate

$\gamma\left[\mathrm{ML}^{-3}\mathrm{T}^{-1}\right]$: ZOP rate

$\rho\left[\mathrm{ML}^{-3}\right]$: Bulk density of porous formation

$\phi[-]$: Porosity of the porous medium

$x[\mathrm{L}]$: Space variable

$t[\mathrm{T}]$: Time variable

The initial condition (2) interprets here that initially the domain is supposed as not solute free. It is linearly combined space dependent function associated with ZOP and initial concentration c_1. The change in solute mass happens as a result of chemical gradients in groundwater, radioactive decay, and the effect of bacterial activities. Due to some anthropogenic activities, the temporally dependent sinusoidal boundary condition (3) is considered in the splitting time domain. The boundary condition (4) shows that contaminant mass flux is considered as no flow at the final boundary of the semi-infinite aquifer.

The diagrammatic representation of the solute transport with concentration condition is shown in Figure 1.

The linear relationship between the concentration of dissolved mass in the liquid phase (c) and the concentration of sorbed (s) mass in the solid phase is defined as follows (Zheng & Bennett, 2002):

$$s = k_d c \tag{5}$$

where, $k_d\left[\mathrm{M}^{-1}\mathrm{L}^{3}\right]$ is distribution coefficient.

Now from Eq. (5), we get

$$\frac{\partial s}{\partial t} = k_d \frac{\partial c}{\partial t} \tag{6}$$

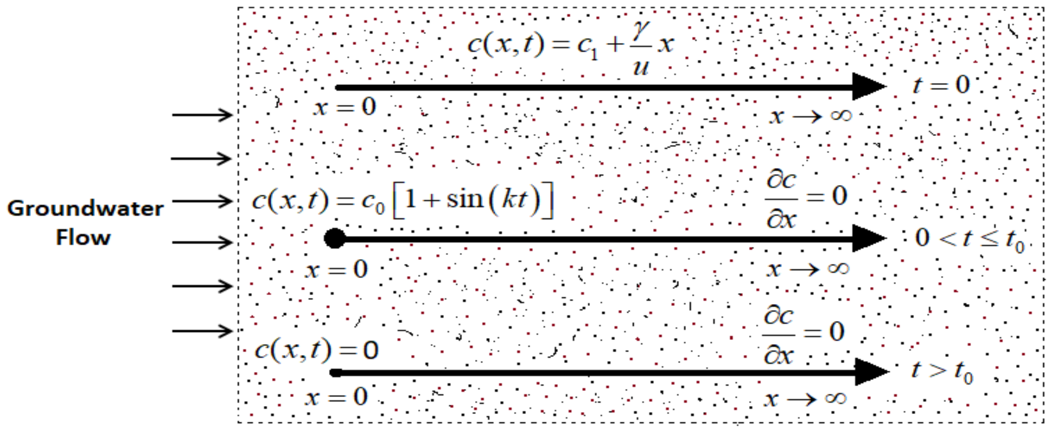

Fig. 1. Diagrammatic representation of the model problem

Using Eq. (6) in Eq. (1) gives

$$R^*\frac{\partial c}{\partial t}=\frac{\partial}{\partial x}\left(D\frac{\partial c}{\partial x}-uc\right)-\lambda c+\gamma\,,\ \text{where}\ R^*=1+\frac{\rho}{\phi}k_d\ \text{is the retardation factor.} \tag{7}$$

Case-I:

According to Freeze & Cherry (1979), the dispersion coefficient shows a direct proportionality to a some power b of pore-water velocity i.e., $D=Au^b$, where A is proportionality constant, which depends on the geometry of the porous formation and range of b is $1\le b\le 2$. Based on this theory, the advection, dispersion, and the other parameters are generalised as follows:

$$\left.\begin{aligned}&u(x,t)=u_0(1+ax)^b\,v(mt);\ D(x,t)=D_0(1+ax)^{b+1}\,v(mt);\ R^*(x,t)=R_0(1+ax)^{b-1}\,v(mt);\\&\lambda(x,t)=\lambda_0(1+ax)^{b-1}\,v(mt);\ \gamma(x,t)=\gamma_0(1+ax)^{b-1}\,v(mt)\end{aligned}\right\} \tag{8}$$

where, $0<a<1$ is the heterogeneity factor having dimension as the inverse of distance, $m[\mathrm{T}^{-1}]$ is constant coefficient, u_0, D_0, R_0, λ_0, and γ_0 are their corresponding initial state values.

In the present work, the range of b is considered as $0\le b\le 1$. Using the above generalised definitions of D, R^*, λ, γ and u in Eq. (7), we get

$$R_0\frac{\partial c}{\partial t}=D_0(1+ax)^2\frac{\partial^2 c}{\partial x^2}-\left(u_0-aD_0(b+1)\right)(1+ax)\frac{\partial c}{\partial x}-\left(abu_0+\lambda_0\right)c+\gamma_0 \tag{9}$$

Case-II:

Consider the retardation factor as space dependent only i.e., $R^*(x,t)=R_0(1+ax)^{b-1}$.

Now, applying the transformation $T=\int_0^t v(mt)dt$ (Crank, 1975) in Eq. (7), we get

$$R^*v(mt)\frac{\partial c}{\partial T}=\frac{d}{dx}\left(D\frac{\partial c}{\partial x}-uc\right)-\lambda c+\gamma \tag{10}$$

To make the variable coefficient of Eq. (9) constant, the following transformation is introduced:

$$\frac{1}{a}\log(1+ax)=X\ ,\ \text{ i.e., } x=\frac{1}{a}\left(e^{aX}-1\right) \tag{11}$$

The Eq. (9) takes the following form on using Eq. (11):

$$R_0\frac{\partial c}{\partial t}=D_0\frac{\partial^2 c}{\partial X^2}-U_0\frac{\partial c}{\partial X}-\mu_0 c+\gamma_0\,,\ \text{where}\ U_0=u_0-abD_0\ \text{and}\ \mu_0=abu_0+\lambda_0 \tag{12}$$

The respective concentration conditions become

$$c(X,t)=c_1+\frac{\gamma_0}{u_0}\frac{1}{a}\left(1-e^{-aX}\right);\ \ X\ge 0,\ t=0 \tag{13}$$

$$c\left(X,t\right)=\begin{cases} c_0\left[1+\sin(kt)\right] & 0<t\le t_0 \\ 0 & t>t_0 \end{cases}\ ;\ X=0 \tag{14}$$

$$\frac{\partial c}{\partial X}=0,\quad X\to\infty,\ t>0 \tag{15}$$

Again, Eq. (12) is simplified by applying the transformation introduced as follows:

$$c(X,t)=K(X,t)\,\mathrm{e}^{\frac{X}{2}\frac{U_0}{D_0}-\frac{1}{R_0}\left(\frac{U_0^2}{4D_0}+\mu_0\right)t}+\frac{\gamma_0}{\mu_0} \tag{16}$$

Using Eq. (16) in Eq. (12), we get

$$\frac{\partial^2 K}{\partial X^2}-\frac{R_0}{D_0}\frac{\partial K}{\partial t}=0 \tag{17}$$

Also, the concentration conditions are transformed as follows:

$$K\left(X,t\right)=\left(c_1-\frac{\gamma_0}{\mu_0}+\frac{\gamma_0}{au_0}\right)e^{-\frac{U_0}{2D_0}X}-\frac{\gamma_0}{au_0}e^{-\left(\frac{U_0}{2D_0}+a\right)X};\quad X\ge 0,\ t=0 \tag{18}$$

$$K(X,t)=\begin{cases} c_0\left[1+\sin(kt)-\dfrac{\gamma_0}{\mu_0}\right]e^{\frac{1}{R_0}\left(\frac{U_0^2}{4D_0}+\mu_0\right)t}, & 0<t\le t_0 \\ & \qquad X=0 \\ \left[-\dfrac{\gamma_0}{\mu_0}\right]e^{\frac{1}{R_0}\left(\frac{U_0^2}{4D_0}+\mu_0\right)t}, & t>t_0 \end{cases} \tag{19}$$

$$\frac{\partial K}{\partial X}=-\frac{U_0}{2D_0}K;\ X\to\infty,\ t>0 \tag{20}$$

Applying Laplace transformation in Eq. (17), we get

$$\frac{d^2\overline{K}}{dX^2}-RS\overline{K}(X,S)=-R\left(c_1-\frac{\gamma_0}{\mu_0}+\frac{\gamma_0}{au_0}\right)\mathrm{e}^{-\frac{U_0}{2D_0}X}+R\frac{\gamma_0}{au_0}\,\mathrm{e}^{-\left(\frac{U_0}{2D_0}+a\right)X} \tag{21}$$

where, $R=\dfrac{R_0}{D_0}$ and S is the transformed real parameter.

The general solution of Eq. (21) is obtained in Laplacian domain as

$$\overline{K}(X,S)=A\,\mathrm{e}^{X\sqrt{RS}}+B\,\mathrm{e}^{-X\sqrt{RS}}+\left(c_1-\frac{\gamma_0}{\mu_0}+\frac{\gamma_0}{au_0}\right)\frac{e^{-\frac{U_0}{2D_0}X}}{S-\frac{1}{R}\left(\frac{U_0}{2D_0}\right)^2}-\frac{\gamma_0}{au_0}\frac{e^{-\left(\frac{U_0}{2D_0}+a\right)X}}{S-\frac{1}{R}\left(\frac{U_0}{2D_0}+a\right)^2} \tag{22}$$

where, the arbitrary constants A and B can be obtained using initial and boundary conditions.

Applying the Laplace transformation in equations (18), (19) and (20), and using in (22), we get the arbitrary constants as follows:

$$A=0 \tag{23}$$

$$B=\left[\begin{array}{l}\dfrac{1-e^{-(S-\theta)t_0}}{(S-\theta)}+k\dfrac{1-e^{-(S-\theta)t_0}}{(S-\theta)^2}-k^3\dfrac{1-e^{-(S-\theta)t_0}}{(S-\theta)^4}-kt_0\dfrac{e^{-(S-\theta)t_0}}{(S-\theta)}+\dfrac{k^3t_0^3}{6}\dfrac{e^{-(S-\theta)t_0}}{(S-\theta)}+\dfrac{k^3t_0^2}{2}\dfrac{e^{-(S-\theta)t_0}}{(S-\theta)^2}\\ +\dfrac{k^3t_0}{1}\dfrac{e^{-(S-\theta)t_0}}{(S-\theta)^3}-\dfrac{\gamma_0}{\mu_0}\dfrac{1}{(S-\theta)}-\left(c_1-\dfrac{\gamma_0}{\mu_0}+\dfrac{\gamma_0}{au_0}\right)\dfrac{1}{S-\dfrac{1}{R}\left(\dfrac{U_0}{2D_0}\right)^2}+\dfrac{\gamma_0}{au_0}\dfrac{1}{S-\dfrac{1}{R}\left(\dfrac{U_0}{2D_0}+a\right)^2}\end{array}\right] \tag{24}$$

where, $\theta=\dfrac{1}{R_0}\left(\dfrac{U_0{}^2}{4D_0}+\mu_0\right)$.

Using the values of A and B in Eq. (22), we get

$$\overline{K}(X,S)=\left[\begin{array}{l}c_0\left\{\begin{array}{l}\dfrac{1-e^{-(S-\theta)t_0}}{(S-\theta)}+k\dfrac{1-e^{-(S-\theta)t_0}}{(S-\theta)^2}-k^3\dfrac{1-e^{-(S-\theta)t_0}}{(S-\theta)^4}-kt_0\dfrac{e^{-(S-\theta)t_0}}{(S-\theta)}\\ +\dfrac{k^3t_0^3}{6}\dfrac{e^{-(S-\theta)t_0}}{(S-\theta)}+\dfrac{k^3t_0^2}{2}\dfrac{e^{-(S-\theta)t_0}}{(S-\theta)^2}+\dfrac{k^3t_0}{1}\dfrac{e^{-(S-\theta)t_0}}{(S-\theta)^3}\end{array}\right\}\\ -\dfrac{z}{\mu_0(S-\theta)}-\left(c_1-\dfrac{\gamma_0}{\mu_0}+\dfrac{\gamma_0}{au_0}\right)\dfrac{1}{S-\dfrac{1}{R}\left(\dfrac{U_0}{2D_0}\right)^2}+\dfrac{\gamma_0}{au_0}\dfrac{1}{S-\dfrac{1}{R}\left(\dfrac{U_0}{2D_0}+a\right)^2}\end{array}\right]\mathrm{e}^{-\sqrt{RS}X}$$
$$+\left(c_1-\dfrac{\gamma_0}{\mu_0}+\dfrac{\gamma_0}{au_0}\right)\dfrac{1}{S-\dfrac{1}{R}\left(\dfrac{U_0}{2D_0}\right)^2}e^{-\frac{U_0}{2D_0}X}-\dfrac{\gamma_0}{au_0}\dfrac{1}{S-\dfrac{1}{R}\left(\dfrac{U_0}{2D_0}+a\right)^2}e^{-\left(\frac{U_0}{2D_0}+a\right)X} \tag{25}$$

Now, using the inverse Laplace transform in Eq. (25), we get the required solution as follows:

$$K(X,T)=\begin{cases}\left.\begin{array}{l}c_0\left[F(X,T)+k\{H(X,T)\}-k^3\{G(X,T)\}\right]-\dfrac{\gamma_0}{\mu_0}F(X,T)+\dfrac{\gamma_0M(X,T)}{au_0}\\ -\left(c_1-\dfrac{\gamma_0}{\mu_0}+\dfrac{\gamma_0}{au_0}\right)J(X,T)+\left(c_1-\dfrac{\gamma_0}{\mu_0}+\dfrac{\gamma_0}{au_0}\right)U(X,T)-\dfrac{\gamma_0V(X,T)}{au_0}\end{array}\right\},\ 0<T\le T_0\\ \left.\begin{array}{l}c_0\left[\begin{array}{l}F(X,T)-F(X,T-T_0)+k\{H(X,T)-H(X,T-T_0)\}\\ -k^3\{G(X,T)-G(X,T-T_0)\}\end{array}\right]-kTF(X,T)\\ +kT_0F(X,T-T_0)+\dfrac{k^3T_0^3}{6}\{F(X,T)-F(X,T-T_0)\}+\dfrac{k^3T_0^2}{2}H(X,T)+\dfrac{k^3T_0^2}{2}\\ \times H(X,T-T_0)+\dfrac{k^3T_0}{1}\{I(X,T)-I(X,T-T_0)\}-\dfrac{\gamma_0F(X,T)}{\mu_0}-\dfrac{\gamma_0V(X,T)}{au_0}\\ -\left(c_1-\dfrac{\gamma_0}{\mu_0}+\dfrac{\gamma_0}{au_0}\right)J(X,T)+\dfrac{\gamma_0M(X,T)}{au_0}+\left(c_1-\dfrac{\gamma_0}{\mu_0}+\dfrac{\gamma_0}{au_0}\right)U(X,T)\end{array}\right\},\ T\ge T_0\end{cases} \tag{26}$$

where;

$$\mathrm{F}(X,T)=\frac{1}{2}\left[\exp\left(\theta T-X\sqrt{\theta R}\right)erfc\left(\frac{X\sqrt{R}}{2\sqrt{T}}-\sqrt{\theta T}\right)+\exp\left(\theta T+X\sqrt{R\theta}\right)erfc\left(\frac{X\sqrt{R}}{2\sqrt{T}}+\sqrt{\theta T}\right)\right],$$

$$H(X,T)=\begin{bmatrix}\frac{1}{4\sqrt{\theta}}\left(2\sqrt{\theta}T-\sqrt{R}X\right)\exp\left(\theta T-\sqrt{\theta R}X\right)erfc\left(\frac{X\sqrt{R}}{2\sqrt{T}}-\sqrt{\theta}\sqrt{T}\right)\\+\frac{1}{4\sqrt{\theta}}\left(2\sqrt{\theta}T+\sqrt{R}X\right)\exp\left(\theta T+\sqrt{\theta R}X\right)erfc\left(\frac{X\sqrt{R}}{2\sqrt{T}}+\sqrt{\theta}\sqrt{T}\right)\end{bmatrix},$$

$$J(X,T)=\frac{1}{2}\left[\exp\left(\frac{T}{4R}-\frac{X\sqrt{R}}{\sqrt{4R}}\right)erfc\left(\frac{X\sqrt{R}}{2\sqrt{T}}-\sqrt{\frac{T}{4R}}\right)+\exp\left(\frac{T}{4R}+\frac{X\sqrt{R}}{\sqrt{4R}}\right)erfc\left(\frac{X\sqrt{R}}{2\sqrt{T}}+\sqrt{\frac{T}{4R}}\right)\right],$$

$$M(X,T)=\frac{1}{2}\begin{bmatrix}\exp\left(\frac{T}{4R_0}\frac{u_0^2}{D_0^2}-\frac{X}{2}\frac{u_0}{D_0}\right)erfc\left(\frac{X\sqrt{R}}{2\sqrt{T}}-\frac{u_0}{2D_0}\sqrt{\frac{T}{R_0}}\right)\\+\exp\left(\frac{T}{4R_0}\frac{u_0^2}{D_0^2}+\frac{X}{2}\frac{u_0}{D_0}\right)erfc\left(\frac{X\sqrt{R}}{2\sqrt{T}}+\frac{u_0}{2D_0}\sqrt{\frac{T}{R_0}}\right)\end{bmatrix},$$

$$G(X,T)=-\frac{I}{3\theta}+\frac{1}{96\theta^{3/2}}\begin{bmatrix}-\frac{X\sqrt{R}}{\theta}\exp\left(\theta T-\sqrt{\theta R}X\right)erfc\left(\frac{X\sqrt{R}}{2\sqrt{T}}-\sqrt{\theta T}\right)\\+\frac{X\sqrt{R}}{\theta}\exp\left(\theta T+\sqrt{\theta R}X\right)erfc\left(\frac{X\sqrt{R}}{2\sqrt{T}}+\sqrt{\theta T}\right)\\+\left(2T+\frac{\sqrt{R}X}{\theta}\right)\begin{Bmatrix}\left(2\sqrt{\theta}T-\sqrt{R}X\right)\exp\left(\theta T-\sqrt{\theta R}X\right)erfc\left(\frac{X\sqrt{R}}{2\sqrt{T}}-\sqrt{\theta T}\right)\\+2\sqrt{\frac{T}{\pi}}\exp\left(\theta T-\sqrt{\theta R}X\right)erfc\left(-\left(\frac{X\sqrt{R}}{2\sqrt{T}}-\sqrt{\theta T}\right)^2\right)\end{Bmatrix}\\+\left(2T-\frac{\sqrt{R}X}{\theta}\right)\begin{Bmatrix}\left(2\sqrt{\theta}T+\sqrt{R}X\right)\exp\left(\theta T+\sqrt{\theta R}X\right)erfc\left(\frac{X\sqrt{R}}{2\sqrt{T}}+\sqrt{\theta T}\right)\\+2\sqrt{\frac{T}{\pi}}\exp\left(\theta T+\sqrt{\theta R}X\right)erfc\left(-\left(\frac{X\sqrt{R}}{2\sqrt{T}}+\sqrt{\theta T}\right)^2\right)\end{Bmatrix}\end{bmatrix},$$

$$U(X,T)=\exp\left(\frac{T}{4R_0}\frac{u_0^2}{D_0^2}-\frac{X}{2}\frac{u_0}{D_0}\right),\ V(X,T)=\exp\left(\frac{T}{R_0}\left(a+\frac{u_0}{2D_0}\right)^2-X\left(a+\frac{u_0}{2D_0}\right)\right),$$

$$N(X,T)=\frac{1}{2}\begin{bmatrix}\exp\left(\frac{T}{R_0}\left(a+\frac{u_0}{2D_0}\right)^2-X\left(a+\frac{u_0}{2D_0}\right)\right)erfc\left(\frac{X}{2}\sqrt{\frac{R}{T}}-\left(a+\frac{u_0}{2D_0}\right)\sqrt{\frac{T}{R_0}}\right)\\+\exp\left(\frac{T}{R_0}\left(a+\frac{u_0}{2D_0}\right)^2+X\left(a+\frac{u_0}{2D_0}\right)\right)erfc\left(\frac{X}{2}\sqrt{\frac{R}{T}}+\left(a+\frac{u_0}{2D_0}\right)\sqrt{\frac{T}{R_0}}\right)\end{bmatrix},$$

$$I(X,T)=\frac{1}{16\theta}\left[\begin{array}{l} X\sqrt{\frac{R}{\theta}}\exp\left(\theta T-\sqrt{\theta R}X\right)erfc\left(\frac{X\sqrt{R}}{2\sqrt{T}}-\sqrt{\theta T}\right)-X\sqrt{\frac{R}{\theta}}\exp\left(\theta T+\sqrt{\theta R}X\right) \\ \times erfc\left(\frac{X\sqrt{R}}{2\sqrt{T}}+\sqrt{\theta T}\right)+\left(2\sqrt{\theta}T-\sqrt{R}X\right)^{2}\exp\left(\theta T-\sqrt{\theta R}X\right)erfc\left(\frac{X\sqrt{R}}{2\sqrt{T}}-\sqrt{\theta T}\right) \\ +\left(2\sqrt{\theta}T-\sqrt{R}X\right)\exp\left(\theta T-\sqrt{\theta R}X\right)\exp\left\{-\left(\frac{X\sqrt{R}}{2\sqrt{T}}-\sqrt{\theta T}\right)^{2}\right\}2\sqrt{\frac{T}{\pi}} \\ +\left(2\sqrt{\theta}T+\sqrt{R}X\right)^{2}\exp\left(\theta T+\sqrt{\theta R}X\right)erfc\left(\frac{X\sqrt{R}}{2\sqrt{T}}+\sqrt{\theta T}\right) \\ -\left(2\sqrt{\theta}T+\sqrt{R}X\right)\exp\left(\theta T+\sqrt{\theta R}X\right)\exp\left\{-\left(\frac{X\sqrt{R}}{2\sqrt{T}}+\sqrt{\theta T}\right)^{2}\right\}2\sqrt{\frac{T}{\pi}} \end{array}\right]$$

RESULTS AND DISCUSSION

In the present study, the presence of input source at the origin is considered for the period of 0.2 years and then it will be disappeared after that. In tropical regions, like India and other sub-continental countries, the groundwater follows sinusoidal form of seepage velocity. On the other hand, mountain hill area, like as Himalayan basin, the seepage velocity is of exponential form. For the case $b=1$, it demonstrated that dispersion varies as the proportional to pore-water velocity with power 2, which is the degenerate form linear space and temporal function, whereas FOD, ZOP and retardation factor all are temporal dependent. On the other hand, for $b=0$, it gives dispersion varies proportional to seepage velocity, which is only temporal dependent and whereas FOD, ZOP, and retardation factor all are temporal dependent and inversely proportional to space function.

Table 1. Four different forms of groundwater velocity are considered to analyse the result

Sl No	Velocity pattern	Mathematical expression	New time formation
1.	Sinusoidal form	$v(mt)=1-\sin\left(pmt\right)$	$T=\frac{1}{mp}\left[pmt - \left\{1-\cos\left(mpt\right)\right\}\right]$
2.	Exponentially decaying form	$v(mt)=\exp\left(-mpt\right),\ mpt<1$	$T=\frac{1}{mp}\left[1-\exp\left(-mpt\right)\right],\ mpt<1$
3.	Asymptotic form	$v(mt)=\frac{mt}{mt+p}$	$T=\frac{1}{m}\left[mt-p\log\left(\frac{mt+p}{p}\right)\right]$
4.	Algebraic sigmoid form	$v(mt)=\frac{mt}{\sqrt{\left(\left(mt\right)^{2}+p^{2}\right)}}$	$T=\frac{1}{m}\left[\sqrt{\left(\left(mt\right)^{2}+p^{2}\right)}-p\right]$

The input data which are used for graphical presentation are as follow:

$c_0=1$, $c_1=0.1$, $u_0=0.1$, $D_0=0.4$, $\gamma_0=0.01$, $\lambda_0=0.5$, $m=0.3$, $k_d=0.15$, $\rho=2.49$, $\phi=0.3$, $t=0.1$, $a=0.5$, $p=1$, $k=0.3$.

Figure 2 depicts the solute level decreasing with space throughout the medium that reaches to its harmless level around 0.7 km of the domain. In general, after solute transport, it will not completely disappear immediately from the aquifer. As continuous background source always exists throughout the domain and already existing transport solute gradually increases with space and attained its maximum level at a particular space, near the source end and then decreases continuously up to its harmless level.

In Figure 3, it is observed that concentration level is highest around 0.4 km in clay medium and beyond that the solute tendency is reversed. From Figures 2 and 3, the solute level comparatively lesser in shale medium than sandstone and clay media. This indicates that solute transport is more controllable in shale medium than the other two media. Also, near the source end of the medium, the solute level is higher for $b=0$ compare to $b=1$, after that this tendency is reversed and maximum difference occurred at the final boundary of the domain.

Figures 4 and 5 depict solute distribution for different values of a. It indicates that on increasing the heterogeneity parameter, the solute level increases throughout the domain. The influence of a on solute transport is more relevant. This comparison is more significant for actual scenario of the aquifer. For both the cases of presence and absence of input source, after

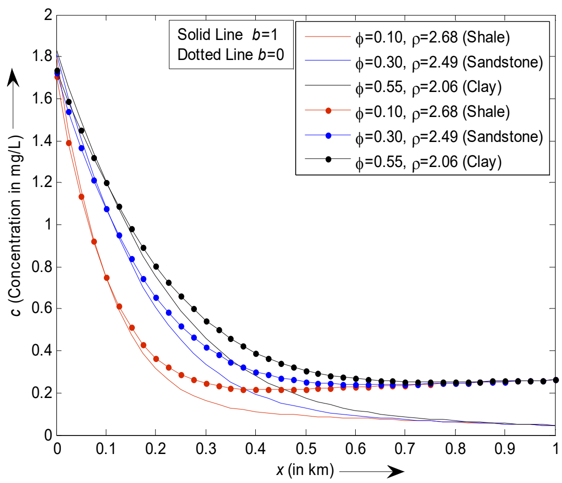

Fig. 2. Comparison of solute distribution in various geological formations in presence of input source

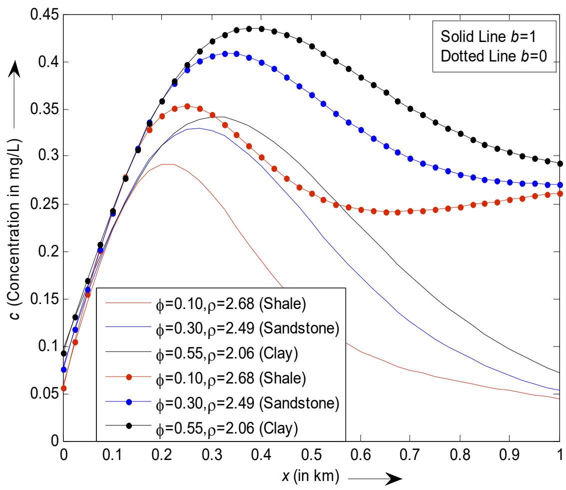

Fig. 3. Comparison of solute distribution in various geological formations in absence of input source

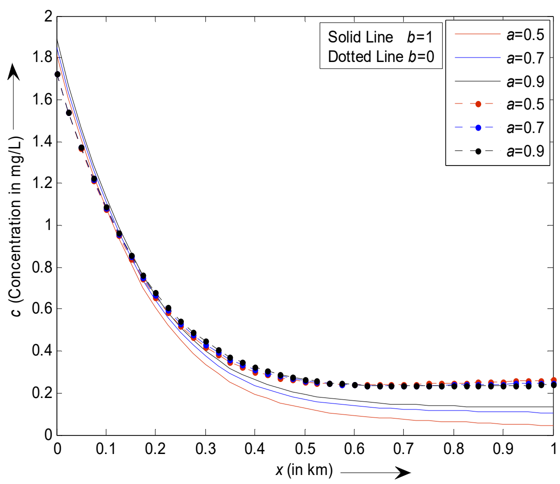

Fig. 4. Solute distribution profiles for different values of a in presence of input source

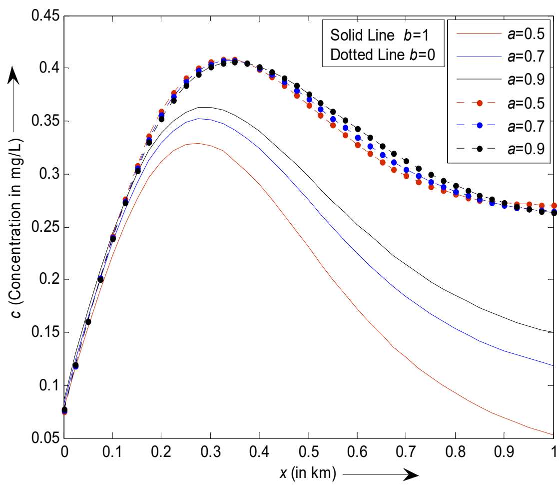

Fig. 5. Solute distribution profiles for different values of a in absence of input source

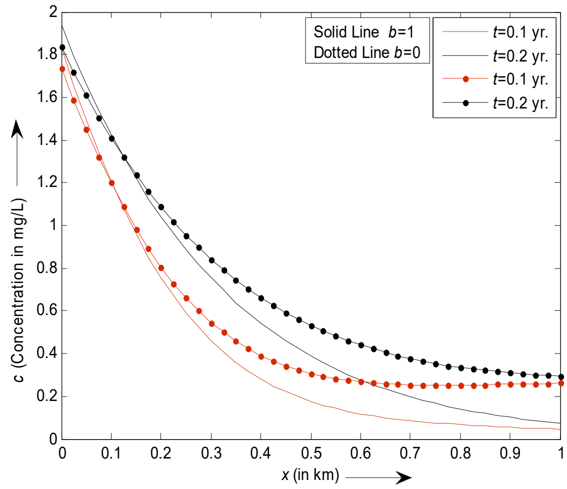

Fig. 6. Solute distribution profiles at different time periods in presence of input source in clay medium

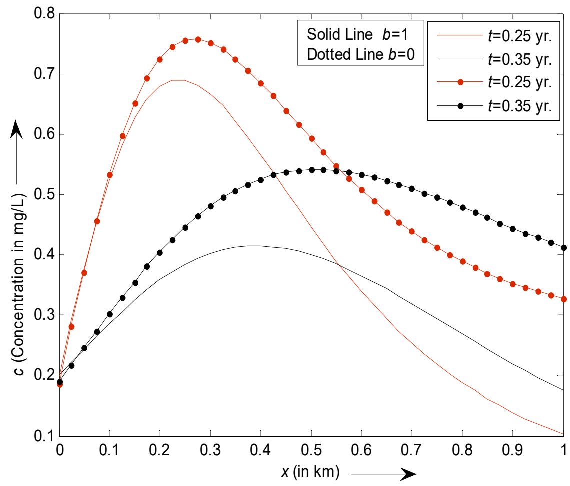

Fig. 7. Solute distribution profiles at different time periods in absence of input source in clay medium

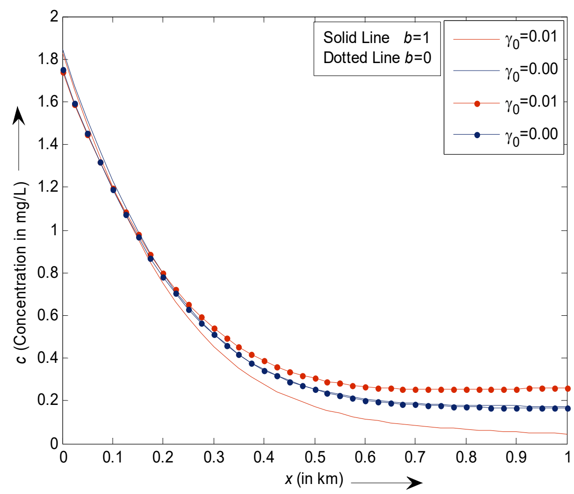

Fig. 8. Solute distribution profiles for uniform and variable background sources in presence source

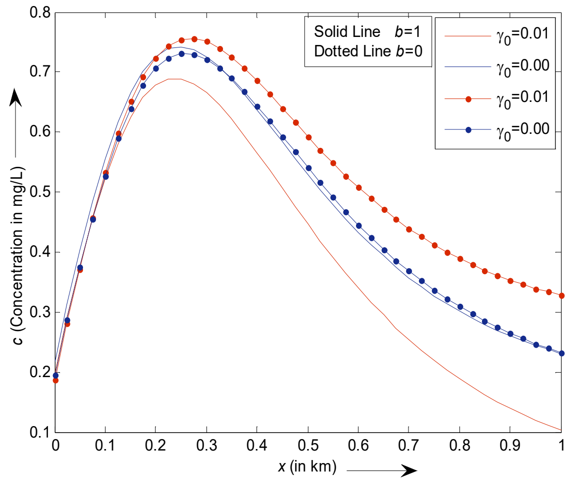

Fig. 9. Solute distribution profiles for uniform and variable background sources in absence source

a certain distance of 0.1 km from the origin, solute level is lower for $b=1$ than $b=0$.

Figures 6 and 7 depict the solute profiles in presence and absence of input source. It is considered that the source exists for 0.2 year and then it will be disappeared forever. The patterns of solute distribution for both the cases of b are similar but some variance occurred due time variation. At a specific location, the solute level is higher for higher value of t, as time dependent input source gives the increasing nature of pollutant. Also, the rehabilitation process is faster for the case of $b=1$ than $b=0$. Figure 7 shows that at each time period, the solute attains maximum level near the source end and then starts decreasing up to the harmless level. For both the cases (i.e., $b=1$, $b=0$) as time elapse, the level gap of the concentration increase with space. Also for any time, maximum peak attained for $b=0$.

Figures 8 and 9 show the solute concentration profile for increasing values of ZOP. The concentration level taking up throughout the domain with the increasing values of ZOP when $b=0$, where as it decreases when $b=1$.

Figures 10 and 11 depict solute transport profiles with four different forms of velocities. The concentration transport pattern under the sinusoidal and exponential forms of velocity follows

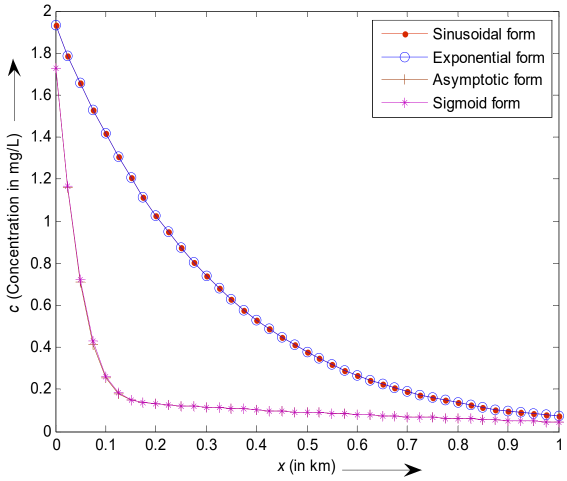

Fig.10. Solute distribution profile with different form of seepage velocity in presence of input source

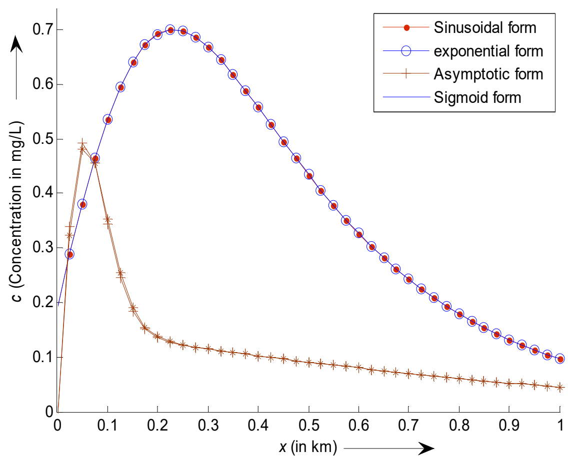

Fig. 11. Solute distribution profile with different form of seepage velocity in absence of input source

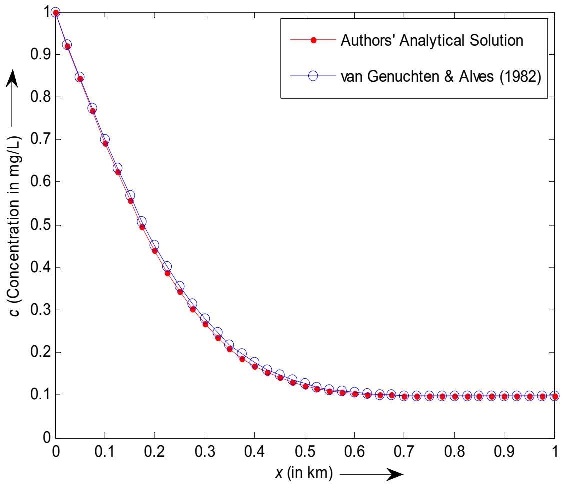

Fig. 12. Solute distribution profile in presence of input source

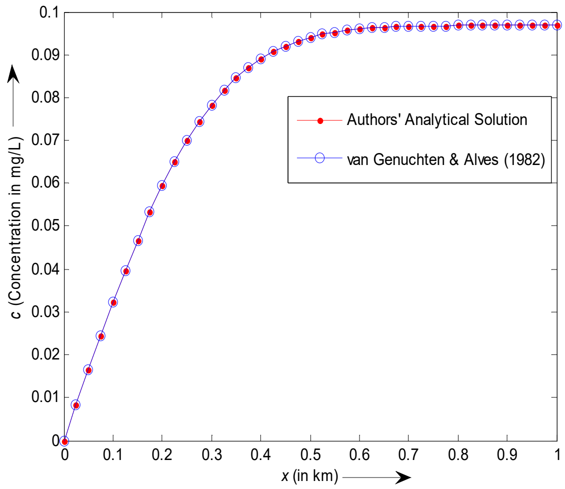

Fig. 13. Solute distribution profile in absence of input source

the similar path in presence and absence of the point source. Also, the sigmoid and algebraic forms follow similar path together. However, the transport profiles of sinusoidal and exponential forms are different from the sigmoid and algebraic forms. In figure 10, the attenuation rate of solute distribution is very fast for sigmoid and asymptotic form of velocities than sinusoidal and exponential form of velocities.

Figures 12 and 13 depict the comparison between present analytical solution and the existing solution (van Genuchten & Alves, 1982) of the ADE under special case in presence and absence of input source, respectively. For the comparison purpose, the present model problem is simplified by considering the initial background source and the input source at the origin both as constant. Also, the advection and dispersion coefficients are taken as constant. In the figures, it can be observed that both the graphical solutions in presence and absence of input source are almost similar to each other.

CONCLUSIONS

A solute transport equation under sorption associated with FOD and ZOP is solved analytically in a heterogeneous porous formation subject to sinusoidally varying pulse type input source.

The research outcomes of the present work are observed as follows:

1. The solute concentration attained higher level in clay formation compared to sandstone and shale formations in absence and presence of input source.
2. The Solute concentration attained higher level in the medium under sinusoidal or exponential form of velocities as compared to sigmoid or asymptotic form of velocities.
3. The solute concentration level takes up in the porous formation when the heterogeneity parameter value is increasing, whereas it decreases in case of seepage velocity power is taken as zero.
4. The solute concentration level increases in the medium with time in presence of input source, whereas it decreases in absence of input source.
5. The solute concentration level increases throughout the domain on increasing the values of ZOP when the power of seepage velocity is taken zero, whereas it decreases when the power is taken one.
6. The present study may be helpful for scientists or geologists to determine the harmless concentration level in a porous medium and can be treated as preliminary tool for solute migration for the future researcher.

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CONFLICT OF INTEREST

The authors declare that there is not any conflict of interests regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/ or falsification, double publication and/or submission, and redundancy has been completely observed by the authors.

LIFE SCIENCE REPORTING

No life science threat was practiced in this research.

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