Advances in Industrial Engineering, June 2024, 58(1): 103-123
DOI: 10.22059/aie.2024.368021.1881
RESEARCH PAPER

Economic Design of Integrated Production Planning Model Based on Adaptive Synthetic EWMA Control Chart and Maintenance Policies

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Received: 12 November 2023, Revised: 19 February 2024, Accepted: 25 February 2024

Abstract

Control charts and maintenance strategies are essential tools in production management. However, despite their inherent connection, these tools are often studied and applied independently. To better reflect real-world scenarios, this paper focuses on the economic design of an integrated production planning model based on a synthetic adaptive Exponentially Weighted Moving Average (EWMA) control chart. To mitigate machine failure rates, two types of maintenance strategies are incorporated: reactive maintenance (RM) and preventive maintenance (PM). The model uses the particle swarm optimization (PSO) metaheuristic algorithm to minimize the total production cycle cost while adhering to statistical quality constraints. A comparative analysis is conducted to evaluate the impact of variable sampling intervals in control charts on overall costs. Sensitivity analysis is performed to examine how model parameters influence optimization policies. Finally, the results are compared with previous studies to demonstrate the effectiveness of the proposed method.

Keywords:
economic control chart design, synthetic adaptive control chart of EWMA, production planning, economic production, maintenance policies

Introduction

One of the main models used in the inventory control is the Economic Production Quantity (EPQ) model. This model focuses mainly on inventory costs, which consist of maintenance and ordering expenses. However, there are some hypotheses/assumptions in the traditional EPQ models that exhibit the need to develop more practical ones.

The first assumption covers the idea that the production process never breaks down (or never becomes out-of-control) while the second assumption focuses on the idea that the production process always produces the corresponding items (production without considering the non-compliant product). In a more realistic situation, the production process may not always be complete and some disturbances may happen over time which may cause quality loss of the products. On the other hand, in the production systems, in addition to quality loss, machine failure may also occur. But in the classic EPQ models, it is assumed that machine failure never occurs in the production cycle. In order to reduce the costs of device failure, researchers have

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stated that production and maintenance planning should be considered simultaneously. Some researchers, such as Chen et al. [1] have investigated this problem**.** Jafari et al. [2] presented a model for joint optimization of economic production (EMQ) and maintenance policy to reduce the total production cost.

While Bouslah et al. [3] investigated the optimization of sampling design and preventive maintenance of a production system subject to the quality loss constraints, Nourelfath et al. [4] studied the integration of production and maintenance for an incomplete process with very large dimensions and ignoring the statistical characteristics of the process. Y. Li et al. [5] designed a joint model of maintenance policy and CUSUM control chart to minimize the total production cost per unit time. In the meanwhile, an integrated problem in terms of production size, quality control and condition-based maintenance for a defective production system exposed to both reliability and quality reduction was investigated by Guo Qing Cheng et al. [6]. Lin Wang et al. [7] developed an integrated model for optimizing the production plan and PM schedule by suggesting an overhaul strategy to minimize the total cost. Li Xue et al. [8] studied the economic design of EWMA chart with variable sampling intervals (VSI) for monitoring mean and standard deviation under preventive maintenance and Taguchi loss functions. Rajesh Saha et al. [9] developed an integrated economic model for the joint optimization of quality control parameters and a preventive maintenance policy using the CUSUM control chart.

The hybrid control charts have attracted researchers' most attention. Spedding et al. [10] combined the Shewhart and Conforming Run Length (CRL) charts for the first time. Some researchers such as Machado et al. [11], Zhang et al. [12], Khoo et al. [13] and Yeong et al. [14] developed the combinatorial charts studies. On the other hand, some researchers have tried to combine the hybrid standard charts with adaptive schemes, for example: Khoo et al. [15] provided a hybrid Double Sampling chart (DS chart). They combined the DS chart and the CRL chart and concluded that the resulting chart was superior to the ordinary chart in terms of detection ability. Lee et al. [16] added the VSI feature to their synthetic multivariate model and showed that the VSI hybrid multivariate control chart is better than the other types of such charts for detecting changes in the covariance matrix. In the following papers researched by Li Xue et al. [17], Fallah Nezhad et al. [18] and Jafarian-Namin et al. [19] an integrated model of economic design, production planning and maintenance policies has been carried out and solved with the help of PSO algorithm. Rasaei et al. [20] investigated the integration of maintenance planning (MP) and statistical process control (SPC) decisions for a two-stage dependent production process. On the other hand, the combination of different maintenance methods and economic design of control charts has been the focus of many researchers in recent years. These maintenance policies increase the reliability of a system. Heydari et al. [21] investigated the economic design of the X-bar control chart under the Burr XII shock model, as an integrated model using preventive maintenance and incomplete maintenance. Shajai et al. [22] created an integrated model of production planning and economic design and maintenance policies in order to minimize total production costs. Based on the information in the following table, the background of the research is as follows:

Table 1. Background research

Number	Authors	Year	Subject	Algorithm
1	A. Salmasniaa, Z. Hajihosseini, M. Namdar, F. Mamashli [23]	2018	Joint determination of production cycle length, maintenance policy and control chart parameters considering time value of money under random variable size	particle swarm optimization algorithm (PSO)
2	M. Namdar [24]	2018	Solid economic-statistical design of adaptive control chart for a secret system under maintenance policies	particle swarm optimization algorithm
3	G. Cheng, B. Zhou, Ling Li [6]	2018	Integrated manufacturing, quality	Simulation-

			control, and condition-based maintenance and repair for defective manufacturing systems	based optimization Monte Carlo simulation
4	M. Pasha, M. Bameni Moghadam [25]	2018	A general version of Ben-Daya-Rahim (2000) and Rahim Banerjee (1993) models in the economic design of $\overline{X}$ control charts in systems with just-in-time replacement (JIT) and preventive maintenance under integrated risk reduction.	Integrated hazard over sampling interval
6	Q. Wan, Y. Wu, W. Zhou, Xiaohong Chen [26]	2018	Integrated economic design of hybrid adaptive chart and maintenance system management	Genetic algorithm
7	A. Salmasnia, F. Soltany, M. Noroozi, B. Abdzadeh [27]	2019	An economic-statistical model for planning production and maintenance and repairs in chi-square non-central comparative diagram	particle swarm optimization algorithm (PSO)
8	A. Farahani, H. Tohidi, A. Shoja [28]	2019	Integrated optimization of quality control chart parameters and preventive maintenance using Markov chain	Modeling with nonlinear regression
9	A. Salmasnia, M. Namdar, B. Abdzadeh [29]	2019	An integrated model of quality and maintenance and repair for a two-unit series production system	particle swarm optimization algorithm (PSO)
10	A. Salmasnia, F. Soltani, E. Heydari, S. Googoonani [30]	2019	An integrated model for joint determination of production length, adaptive control chart parameters and maintenance and repair policy	particle swarm optimization algorithm (PSO)
11	L. Xue and Z. He [8]	2020	Economic Design of EWMA Control Charts with Variable Sampling Intervals for Monitoring the Mean and Standard Deviation under Preventive Maintenance and Taguchi’s Loss Functions.	Taguchi’s Loss Functions
12	S. Jafarian-Namin, M. Saber Fallahnezhad, R. TavakkoliMoghaddam, A. Salmasnia & M. Taghi Fatemi Ghomi [18]	2021	An integrated quality, maintenance and production model based on the delayed monitoring under the ARMA control chart	particle swarm optimization algorithm (PSO)
13	S. Jafarian-Namin, M.S. Fallah Nezhad; R. Tavakkoli Moghaddam, A. Salmasnia, and M.H. Abooie [19]	2021	An integrated model for optimal selection of quality, maintenance, and production parameters with auto correlated data	particle swarm optimization algorithm (PSO)
14	H. Rasay, F. Naderkhani and F. Azizi [20]	2022	Opportunistic maintenance integrated model for a two-stage manufacturing process	genetic algorithm (GA)
15	A. Heydari, M. Tavakoli & A. Rahim [21]	2023	An Integrated Model of Maintenance Policies and Economic Design of X-bar Control Chart Under Burr XII Shock Model	particle swarm optimization algorithm (PSO)
16	M. Shojaee, S. Noori, S. Jafarian-Namin, A. Johannssen [22]	2024	Integration of production–maintenance planning and monitoring simple linear profiles via Hotelling's T2 control chart and particle swarm optimization	particle swarm optimization algorithm (PSO)

The expected total cost (ETC) includes the costs of holding, ordering and sampling of non-conforming product production. The purpose of this article is to model and minimize the expected total cost (ETC) of the production process in the cycle time, according to the statistical limitations caused by using maintenance policies. Therefore, based on the research gap obtained from the above-shown background table, we are focusing on the advantages of the EWMA control chart which are not only using the information of the previous samples, but also its sensitivity to small and medium shifts as well as normality distribution assumption free. All these make the EWMA chart an attractive tool to use in this study. On the other hand, despite the advantages of synthetic adaptive control charts such as lower cost, these types of control charts are used in the literature of common EPQ models. Therefore, we have not only focused on modelling and minimizing the expected total cost (ETC) of the production process, but we have also drawn our attention to an integrated EPQ model based on EWMA synthetic adaptive control chart and maintenance policy, all of which is presented.

The rest of the article is organized as follows. Section 2 describes the problem definition and presents the framework for implementing the proposed adaptive synthetic EWMA chart, while in Section 3, mathematical modeling based on maintenance approaches in the economic production model, is performed. In Section 4, the particle swarm algorithm has been investigated to optimize the presented model, followed by Section 5, in which a numerical example is performed which is then followed by a sensitivity analysis of the optimal policy in the parameters. Section 6 concludes the work.

Problem description

As mentioned in the introduction, in a production system, there are three interrelated problems named as inventory control, quality control and maintenance, which should be considered simultaneously. Therefore, this paper presents an integrated model for the above-mentioned issues by representing the joint optimization of production planning, the parameters of the synthetic adaptive EWMA control chart and the maintenance policy. Control and out-of-control conditions are carefully considered. The process is assumed to start with the control condition, which has a normal distribution with mean μ_0 and standard deviationσ_0 . After a while, due to the mean shift from μ_0 to $\mu_0 + \delta_\mu\sigma_0$ the process goes out-of-control. Therefore, in the out-of-control condition, the quality characteristic follows the normal distribution as $\acute{X} \sim N\left(\mu_0 + \delta_\mu\sigma_0 , \sigma_0\right)$ where δ_μ shows the value of the mean shift which is constant. It should be noted that in this paper, only the positive shifts are considered.

Adaptive EWMA control chart

Salmasnia et al. [30] have shown that traditional control charts with fixed sampling interval are not usually economically optimal and are often inefficient for detecting medium shifts. Whereas adaptive control charts, in which the sampling interval is not fixed (VSI) and changes between the short and long intervals, has the ability of high-speed detection for most of the process shifts. Therefore, in this study, an adaptive EWMA control chart has been used to monitor the mean trend. In (Figure 1), the sampling interval depends on the information obtained from the samples. The adaptive control chart uses the warning limits (LWL/UWL) and the control limits (LCL/UCL). The performance of the control chart and sampling interval varies according to the the place of each sample (Figure 1). If the sample is close to the center line (ie in a safe region), it can be concluded that no shifts in the process parameters has occurred and therefore a longer sampling distance (h_L) should be used for the next sampling attempt. But if the sample is away from the center line (ie in an unsafe region), that should be a signal for an out-of-control state. Therefore, it seems reasonable to use a shorter inspection interval (h_S) in the next sampling [31].

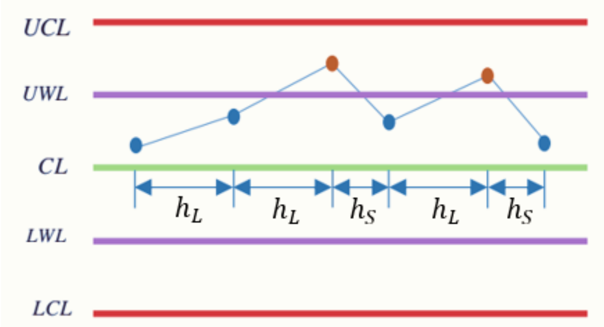

Figure 1. Sampling interval strategy

Therefore, according to the control limits and warning limits [30] , we consider:

$$\begin{cases} h_S \text{ if } UWL < Z_{i-1} \leq UCL \\ h_L \text{ if } LWL \leq Z_{i-1} \leq UWL \\ h_S \text{ if } LCL < Z_{i-1} \leq LWL \end{cases} \tag{1}$$

in Equation 1, the chart statistics is based on the quality characteristic of the synthetic adaptive control chart of EWMA. Due to the above-mentioned conditions for sampling interval, we have two sampling pairs (n, h_S) and (n, h_L) as well as the one when the sample is out-of-control, the process should be checked to determine the assignable cause. Therefore, the probability of placing the sample in the designated areas is as follows [30]:

$$\begin{aligned} P(z_i \in R_1) &= P(LWL \leq z_i \leq UWL) = \varphi(W) - \varphi(-W) = 2\,\varphi(W) - 1 \\ P(z_i \in R_2) &= P(LCL < z_i < LWL) + P(UWL < z_i < UCL) = 2[\varphi(K) - \varphi(W)] \\ P(z_i \in R_3) &= 1 - P(LCL < z_i < UCL) = \varphi(K) - \varphi(-K) = 2\varphi(K) - 1 \end{aligned} \tag{2}$$

in these equations W and K are the warning and control limits coefficients, respectively [30]. In this paper, it is considered, that the time of change follows the Weibull distribution, because the Weibull distribution is suitable to show the failure time of the process.

CRL-EWMA chart

Conforming Run Length (CRL) control chart is defined based on the observed sample number between two nonconforming samples, which includes out of control sample or unit as well. For example, Figure **2** shows that CRL is equal to 4, 6, and 4, respectively [32].

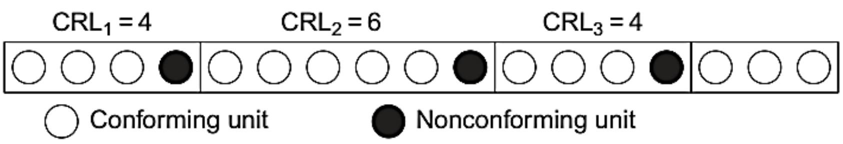

Figure 2. CLR control chart

The CLR control chart statistic has the geometric distribution with the function of $G(CRL) = 1 - (1 - p)^{CRL}$ that the mean is $E(CRL) = 1/p$, where p is nonconforming probability, hence the CRL control chart requires a low control limit (L) and it is calculated as follows [32]:

$$L = \frac{\ln(1\text{-}\,\alpha_{CRL})}{\ln(1\text{ - }p_0)} \qquad \text{where} \qquad \alpha_{CRL} = F_{p_0}(CRL) = 1\text{-}(1 - p_0)^{CRL} \tag{3}$$

where $\alpha_{CRL}$is the first type error of CRL chart, p_0 is the proportion of nonconforming products and L is the lower limit of CRL chart. The value of L must be an integer. In this case, if a CRL sample is less than or equal to L, the probability of a non- conforming product p_0 will increase and give an out-of-control signal [33].

ARL_{CRL} is the average run length of CRL chart. Hence, the in-control ARL for the synthetic adaptive control chart of EWMA, which is represented as $ARL_{CRL-EWMA}$, is calculated from the Equation 4 [34]:

$$\begin{aligned} ARL_{CRL} &= \frac{1}{G(L-1)} = \frac{1}{1-(1-p)^L} \\ ARL_{CRL-EWMA} &= ARL_{EWMA} \times ARL_{CRL} = ARL_{EWMA} \times \frac{1}{1-(1-p)^L} \end{aligned} \tag{4}$$

According to the calculated ARL value, the average time to signal (ATS) can be calculated as follows [34]:

$$ATS_{CRL-EWMA} = ARL_{CRL-EWMA} \times FSI \tag{5}$$

where FSI is the shortest time to receive an out-of-control signal.

Synthetic adaptive control chart of EWMA

The control chart in this article is a combination of EWMA VSI adaptive chart and a Conforming Run Length (CRL) control chart. The basic concept of the VSI feature is that the sampling time interval is determined based on the previous sample place. According to the Equation (6), the Synthetic adaptive control chart of EWMA is divided into three regions, which are defined as the following equations and are shown in Figure 3.

$$\begin{aligned} &R1 \text{ (Central region): } LWL < Z_i < UWL\,; \\ &R2 \text{ (Warning region): } LCL \leq Z_i \leq LWL \text{ or } UWL \leq Z_i \leq UCL\,; \\ &R3 \text{(Signal region): } Z_i < LCL \text{ or } Z_i > UCL; \end{aligned} \tag{6}$$

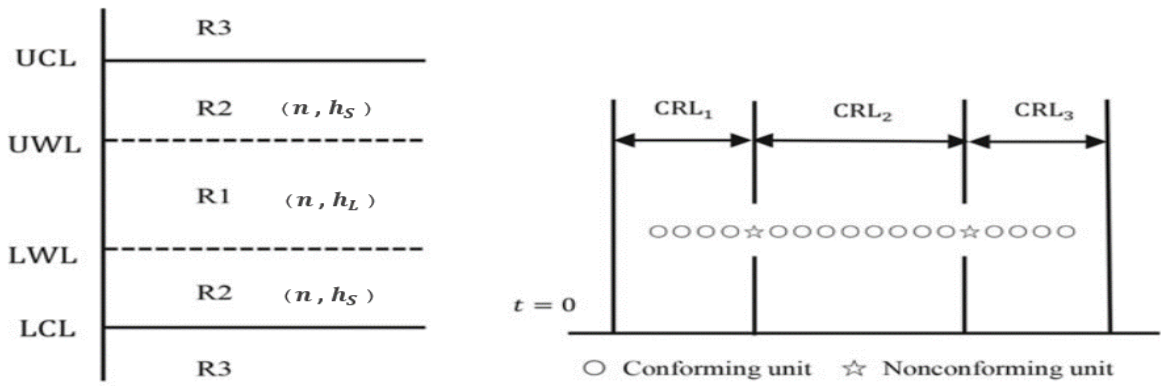

Figure 3. A graphical representation of the synthetic adaptive control chart of EWMA regions

The first region in the EWMA three regions is R1 which is considered as a safe region (in control condition). The second in line is R2 which is known as the unsafe region of which the chart gives an out-of-control signal, and needs to be checked. The last region is R3 which is also defined as the out-of-control region.

In this paper, we consider the variable sampling interval, based on the information in both EWMA VSI chart and the CRL chart to design of the control chart as the following steps:

1. Calculate the optimal values of h_S, h_L, λ, K, W, n, m based on the economic design model to optimize the cost according to the constraints.
2. Set the control limits (UCL , LCL) and the warning limits (UWL , LWL) for the EWMA chart

and specify the lower control limit (L) for the CRL chart.

3. Select a random sample and calculate the sample mean and obtain the Z_i statistic based on that.
4. If the Z_i statistic is in the R3 region, the process is declared out of control. Research and maintenance may have been begun (the conditions are described in the following sections). After that, the control process reaches Step 3 and sampling (n, h_S) is used.
5. If the Z_i statistic is in region R1, the chart is declared in control mode. The sample (n, h_L) is used for the next sampling.
6. If the Z_i statistic is in the R2 region, then the CRL statistic is checked.
   6.1 If the CRL statistic is larger than the lower limit of CRL chart (L), the CRL is in-control, but the sampling scheme (n , h_S) is used in the next sampling.
   6.2 If the CRL statistic is less than the lower limit of CRL chart (L), then the chart is signalled as out-of-control condition, and maintenance may be counted. After that, the control process reaches Step 3 and sampling (n, h_s) is used.

Therefore, according to the research by [34], the ARL_0 for the synthetic adaptive control chart of EWMA is as follows:

$$ARL_0 = 1 + [ARL_{EWMA} - 1](ARL_{CRL}) + ARL_{CRL} - 1 \quad (7)$$

As mentioned in Section 2 the chart parameters should be designed in such a way that the quality cost, production cost and maintenance cost are optimized. Therefore, charts parameters are entered into the optimization model as variables.

Parameters Notation

The variables that are used in the optimization model is shown in Table 2.

Table 2. Table of variables

symbol	Description
K	Coefficient of control limits
w	Coefficient of warning limits
λ	Weight coefficient in EWMA chart
n	Sample size
m	Number of inspection periods until preventive maintenance
h_S	Short-term sampling interval
h_L	Long-term sampling interval
p	nonconforming units
LCL	Lower limit of the synthetic adaptive EWMA chart
UCL	Upper limit of the synthetic adaptive EWMA chart
LWL	Lower warning limit synthetic adaptive EWMA
UWL	Upper warning limit synthetic adaptive EWMA
L	lower control limit of the CRL chart
R_i	the synthetic adaptive EWMA regions
μ_0	Mean of the quality characteristic in normal distribution
σ_0	Variance of the quality characteristic in normal distribution
δ_μ	Shift in the mean parameter (a positive value)
a	Weibull distribution parameter
b	Weibull distribution parameter
d	Daily demand
D	Annual demand
f_1	Proportion of samples in controlled condition with short sampling interval
f_2	Proportion of samples in controlled condition with long sampling distance
c_i	conditions created for maintenance
f(z)	Quality characteristic density function for in-control condition

$f(\acute{z})$	Quality characteristic density function for out-of-control condition
P	Production rate
$P(c_i)$	Probability of i^{th} condition
$P(sig)$	Probability of warning detection by Synthetic EWMA (VSI)
S	Expected number of samples before out-of-control signal
s_1	Expected Number of samples in control condition with short sampling interval
s_2	Expected Number of samples in control condition with long sampling distance
z	Quality characteristic of the process at in-control condition
$\acute{z}$	Quality characteristic of the process at out-of-control condition
A	Setup cost
B	Maintenance cost per unit time
C_F	Fixed cost of sampling
C_P	Preventive maintenance cost
C_R	Reactive maintenance cost
C_V	Variable sampling cost
C_Y	Cost of false signal investigation
C_s	The total cost of the inspection
$E(M)$	Expected maintenance cost
$E(Q)$	Expected quality loss cost
$E(S)$	Expected inspection cost
$E(I)$	Expected inventory maintenance and production start-up costs
ETC	Expected total cost
Q_0	Quality loss cost per unit for in-control condition
Q_1	Quality loss cost per unit for out-of-control condition
ARL_0	Average run length for in-control condition
ARL_1	Average run length for out-of-control condition
ATS_0	Average time to signal for in-control condition
ATS_1	Average time to signal for out-of-control condition
ATS_u	Upper bound of the $\mathbf{ATS_0}$ constraint
ATS_L	Lower bound of the $\mathbf{ATS_1}$ constraint
α	First type of error in EWMA chart
β	Second type of error in EWMA chart
α_{CRL}	First type of error in CRL chart
E	Time needed to record each sample
$E(T_0\|C_i)$	Expected time that the process is in in-control condition for i^{th} condition
$E(T_1\|C_i)$	Expected time that the process is in out-of-control condition for i^{th} condition
P_1	Proportion of time spent at in-control condition using short sampling interval
P_2	Proportion of time spent at in-control condition using long sampling interval
T_l	Time needed to inspection assignable cause
τ_1	Average time between last sample before assignable cause and occurrence of an assignable cause using $\mathbf{h_S}$
τ_2	Average time between last sample before assignable cause and occurrence of an assignable cause using $\mathbf{h_L}$
τ	Average time between last sampling before assignable cause and occurrence of an assignable cause.
v	solution space in PSO optimisation algorithm

Modeling conditions

Regarding on the time that the shift is taking place, three different conditions may occur:

The first condition

If the process is in-control up to M^{th} sample, all preventive maintenance activities are performed in $(M + 1)^{th}$ sample. Figure 4 shows this condition as condition 1.

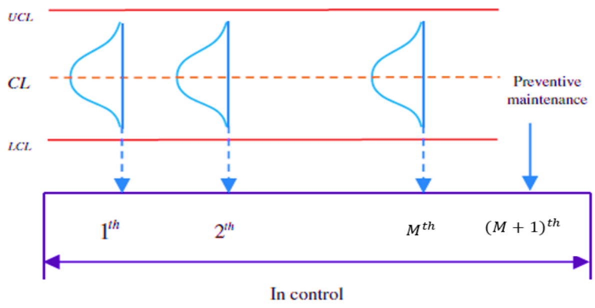

Figure 4. Production cycle with condition 1

In this case, the length of the production period is equal to the duration time under control. If we call this conditionc_1, the predicted time for the process to be in control is [30]:

$$E\,(T_0|c_1) = (m+1)h_s \times P_1 + (m+1)h_L \times P_2 \tag{8}$$

where P_1 and P_2 show the ratio of time spent in control condition using samples with h_s and h_L intervals, respectively, [30]:

$$P_1 = \frac{f_1 h_s}{f_1 h_s + f_2 h_L} \quad P_2 = \frac{f_2 h_L}{f_1 h_s + f_2 h_L} \tag{9}$$

The probability of this condition occurrence is:

$$P(c_1) = 1 - [F[(m+1)h_s] \times f_1 + F[(m+1)h_L] \times f_2] \tag{10}$$

where F(0) is the cumulative function of the Weibull distribution.

The second condition

The process starts in control condition, but between the j^{th} and $(j+1)^{th}$ sample is shifted to out of control condition by an assignable cause. The synthetic adaptive control chart of EWMA cannot detect a change in $(j+1)^{th}$ sample due to the type II error. Finally, in the $(j+i)^{th}$ sample, it releases a signal. Reactive maintenance activities are performed to discover the assigned cause and restore the process to the best possible situation. The production cycle in this condition is shown in Figure 5.

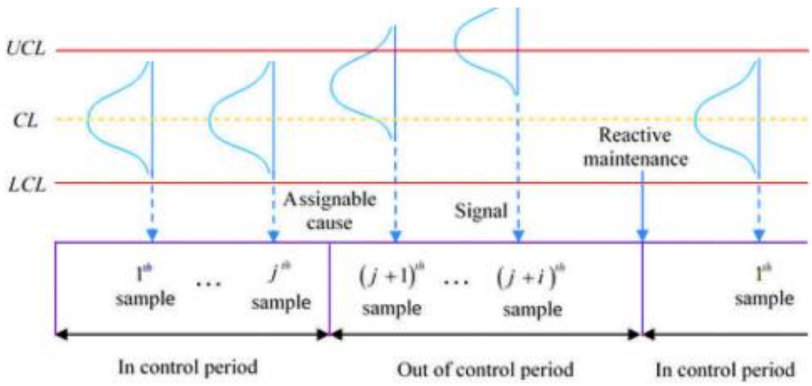

Figure 5. Production cycle with conditions 2

In this condition, the predicted time in control condition has a short Weibull distribution and the distribution function is as follows [30]:

$$f(h|(m+1)h) = \left[\left({}^{b}/_{a}\right)\left({}^{h}/_{a}\right)^{b-1} e^{-\left({}^{h}/_{a}\right)^{b}}\right] \Big/ \left(1 - e^{-\left({}^{h}/_{a}\right)^{(m+1)h}}\right) \tag{11}$$

Therefore, if this condition is denoted byc_2, the expected time under control condition is as follows [30]:

$$E(T_0|c_2) = \left(\int_0^{mh_s} hf(h|(m+1)h_s)dh\right) \times P_1 + \left(\int_0^{mh_L} hf(h|(m+1)h_L)dh\right) \times P_2 \tag{12}$$

The time in out-of-control condition for condition 2 involves three parts. The time of occurrence of an assignable cause, sample review time and result interpretation and the time required to investigate an assignable cause. Expected time for the out-of-control condition and the probability of condition 2 occurrence are calculated as Equations (13-18), [30]:

$$\tau = \tau_1 P_1 + \tau_2 P_2 \tag{13}$$

where this time equals to the time needed for recording each sample size (n):

$$\tau_1 = \int_0^{(m+1)h_s} hf(h|(m+1)h_s)dh - h_s\left(\sum_{j=1}^{m} e^{-\left(\frac{jh_s}{a}\right)^b} - me^{-\left(\frac{(m+1)h_s}{a}\right)^b}\right) \tag{14}$$

$$\tau_2 = \int_0^{(m+1)h_L} hf(h|(m+1)h_L)dh - h_L\left(\sum_{j=1}^{m} e^{-\left(\frac{jh_L}{a}\right)^b} - me^{-\left(\frac{(m+1)h_L}{a}\right)^b}\right) \tag{15}$$

$$E(T_1|c_2) = ATS_1 - \tau + nE + T_1 \tag{16}$$

$$P(c_2) = [F(mh_s) \times f_1 + F(mh_L) \times f_2] \times P(sig) \tag{17}$$

where F is the cumulative function of the Weibull distribution and $P(sig)$ indicates the probability of signal propagation by the control chart. [30]:

$$P(sig) = 1 - \beta^{m-1} \tag{18}$$

The third condition

In condition 3, the process starts in control condition, but between the j^{th} and $(j+1)^{th}$ sample, due to an assignable cause, the process comes to an out-of-control condition. Due to type II error, the control chart cannot show the signal up to the m^{th} sample inspection. Thus, in the $(m+1)^{th}$ sample inspection, planned maintenance is replaced by reactive maintenance activities, as shown in Figure 6.

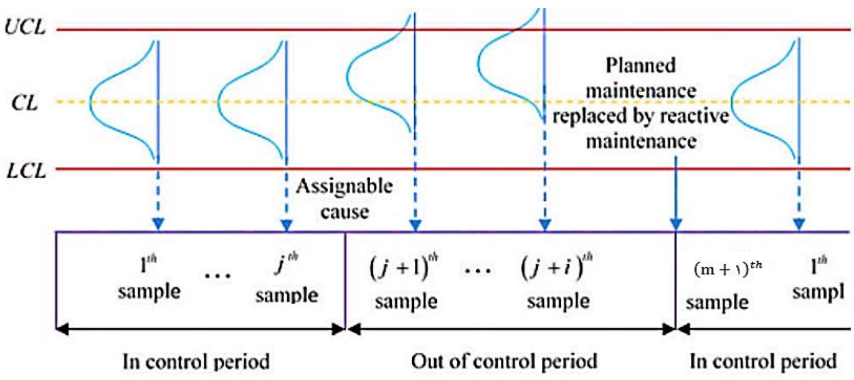

Figure 6. Production cycle with condition 3

This condition is calledc_3. Hence, time in control condition follows the distribution of Weibull and can be calculated as follows, [30]:

$$E(T_0|c_3) = (\int_0^{(m+1)h_s} hf(h|(m+1)h_s)dh \times P_1) + (\int_0^{(m+1)h_L} hf(h|(m+1)h_L)dh \times P_2) \quad (19)$$

Expected time for out-of-control condition and the probability of this situation occurrence are calculated as Equations (20) and (21), [30]:

$$E(T_1|c_3) = [(k+1)h_s \times P_1 + (k+1)h_L \times P_2] - E(T_0|C_3) \quad (20)$$

$$P(c_3) = F[(m+1)h_s] \times f_1 + F[(m+1)h_L] \times f_2 - [F(mh_s) \times f_1 + F(mh_L) \times f_2] \times P(sig) \quad (21)$$

Expected production cycle costs

Production cycle costs are described below to show how each cost is calculated.

Quality cost

In this study, the quality reduction is defined as the distance from the center line in the control chart and being in the unsafe region. The cost of the quality reduction in out of control condition was examined in the previous section and is formulated according to Equation (22):

$$E(Q) = \sum_{i=1}^{3} E(C_Q|c_i)P(c_i) \qquad i = 1،2،3$$
$$\text{when } E(C_Q|c_i) = \begin{cases} Q_0P \times E(T_0|c_i) & \text{for } i = 1 \\ Q_0P \times E(T_0|c_i) + Q_1P \times E(T_1|c_i) & \text{for } i = 2،3 \end{cases} \quad (22)$$

in most of previous studies, Q_0 and Q_1 were considered constant numbers obtained from previous data, but in this study, we use Taguchi quality loss function to determine them.

Inspection cost

Inspection costs include fixed sampling costs and variable sampling costs. The average number of samples in conditions 1 and 3 is equal to m, while the average number in condition 2 is calculated by adding the average number of samples in out of control and in control conditions as follows:

$$E(S) = \sum_{i=1}^{3} E(C_S|c_i)P(c_i) \qquad i = 1،2،3$$
$$E(C_S|c_i) = \begin{cases} (C_F + C_V n)k & \text{for } i = 1،3 \\ (C_F + C_V n)(S + ARL_1) & \text{for } i = 2 \end{cases} \quad (23)$$

when C_F is the fixed sampling cost, C_V is the variable sampling cost, ARL_1 is the average run length for out-of-control state of the synthetic adaptive control chart of EWMA, [30] :

$$S = s_1f_1 + s_2f_2 \quad (24)$$

in Equation (24), the values of s_1 and s_2 are calculated as follows:

$$s_2 = \sum_{j=1}^{m} e^{-(\frac{jh_L}{a})^b} - me^{-(\frac{(m+1)h_L}{a})^b} \qquad s_1 = \sum_{j=1}^{m} e^{-(\frac{jh_s}{a})^b} - me^{-(\frac{(m+1)h_s}{a})^b} \quad (25)$$

Maintenance cost

Maintenance costs include costs of deviation alerts occurred, implementation of preventive maintenance and implementation of reactive maintenance. The costs depend on the situations in which they occur. Since in condition 1 the process is in control, the reactive maintenance cost should be zero and the only cost is preventive maintenance cost. While in conditions 2 and 3, due to the out-of-control conditions, the cost of reactive maintenance replaces the cost of preventive maintenance. Therefore, the maintenance costs can be obtained from the following equations:

$$E(M) = \sum_{i=1}^{3} E(C_M|c_i)P(c_i) \qquad i = 1،2،3$$
$$E(C_M|c_i) = \begin{cases} \frac{mC_Y}{ARL_0} + C_P & \text{for } i = 1 \\ \frac{sC_Y}{ARL_0} + C_R & \text{for } i = 2،3 \end{cases} \tag{26}$$

when C_Y represents the cost of checking for error alerts, C_P represents the cost of preventive maintenance and C_R represents the cost of reactive maintenance.

Setting up machinery and maintaining inventory costs

According to the EPQ model, the cost of setting up and maintaining inventory depends on the production rate and the inventory demand rate. Here, these costs are calculated as follows:

$$E(I) = \frac{DA}{PT} + \frac{B(P-d)T}{2} \tag{27}$$

in Equation (27), the first part shows the expected ordering cost and the second part shows the inventory maintaining cost, where T is the production length, which is calculated as follows:

$$T = E(T_0|c_1) \times P(c_1) + [E(T_0|c_2) + E(T_1|c_2)] \times P(c_2) + [E(T_0|c_3) + E(T_1|c_3)] \times P(c_3) \tag{28}$$

Expected total cost

The expected total cost of production cycle is calculated by adding the costs mentioned in the previous sections to the common costs in the classic EPQ model as Equation (29):

$$ETC = E(I) + E(Q) + E(S) + E(M) \tag{29}$$

Objective function and constraints

The purpose of optimization model is to find the values of control chart parameters such as sample size (n), sampling variable interval (h_s, h_L) and control and warning coefficients (K, w). Also, the parameter of the EWMA chart (λ), the lower control limit (L) of the CRL_EWMA chart and the number of maintenance periods (M). Therefore, the expected cost of the production system is minimized and the statistical indicators (ARL_0, ATS_0, A, β, α_CRL) stay at the desired level. Hence, by adding the statistical constraints to the cost function, the optimization model becomes as:

$$\text{Min } ETC = E(I) + E(Q) + E(S) + E(M) \tag{30}$$
$$\text{Subject to :}$$
$$m(h_s P_1 + h_L P_1) \geq M \tag{30-a}$$
$$1 \leq n \leq n_{max} \tag{30-b}$$
$$ATS_0 > ATS_L \tag{30-c}$$
$$ATS_1 < ATS_u \tag{30-d}$$
$$L_{min} \leq L \leq L_{max} \tag{30-e}$$
$$h_s, h_L, k, w, \lambda, n, m, L > 0 \qquad i = 1.2 \tag{30-f}$$
$$n, m, L \in \text{integer} \tag{30-g}$$

in terms of the economic design of the control charts, the sample size should be less than a predetermined value (it should be noted that the sample size is determined based on the sampling cost), as shown in Equation (30-a). In order to improve the statistical characteristics of the proposed model, the number of error signals should be limited without affecting the control chart performance. Hence, the constraint $ATS_0 > ATS_L$ is added to the model in Equation (30-c) where ATS_L is a predetermined value. In addition, when setting $ATS_1$to less than the present value of ATS_u, the control chart can detect the occurrence of a determinable cause as quickly as possible.

Model solve approach

Particle swarm algorithm (PSO)

We chose this optimization method due to the features which shall be mentioned as follows:

1. PSO is a population-based search algorithm, this feature ensures that it is unlikely to fall into the local optimization trap.
2. Since this algorithm uses probability transfer rules, it has high flexibility and good ability in the compact and indeterminate region.
3. One of the unique features of this algorithm is to create a balance between local and global exploration in search area, which leads to overcoming untimely difficulties and increasing search capabilities.
4. Unlike some other exploratory methods, the quality of PSO solution does not depend on the initial population. The algorithm starts to solve anywhere in the search space, ensuring convergence to the desired solution.

While PSO presents notable advantages in terms of population-based search and flexibility, it is crucial to acknowledge the computational complexity of the underlying optimization problem. Despite the favorable features of PSO outlined earlier, our subject inherently possesses characteristics that align with NP-hard problems:

1. **Computational Complexity:** The nature of the optimization problem under investigation exhibits computational complexity, and the search space involves intricate relationships and dependencies. This complexity contributes to the NP-hard classification, signifying that finding an optimal solution within polynomial time remains a formidable challenge.
2. **Non-Deterministic Polynomial-Hardness:** The reliance on population-based search and probabilistic transfer rules, while advantageous for overcoming local optima and exploring complex regions, introduces non-deterministic aspects. The non-deterministic polynomial-hardness (NP-hardness) of the problem becomes evident, as the algorithm faces challenges in achieving a solution with polynomial time complexity.
3. **Global Exploration and NP-Hardness:** Despite PSO's ability to strike a balance between local and global exploration, the NP-hard nature of our subject introduces inherent difficulties. Achieving an optimal solution in a reasonable amount of time is inherently challenging due to the combinatorial or complex nature of the problem.
4. **Initial Population Independence and NP-Hardness:** Although PSO exhibits independence from the initial population for solution quality, the underlying NP-hardness implies that the algorithm's efficiency in finding an optimal solution is inherently constrained.

It is important to recognize that while PSO serves as a powerful optimization tool, the NP-hard characteristics of our subject pose challenges that extend beyond the capabilities of heuristic algorithms. Acknowledging these complexities for understanding the inherent difficulty associated with solving our optimization problem is quite crucial.

In the PSO algorithm, each of the potential solutions of the optimization problem is considered as a particle with two general characteristics in the solution space. These characteristics are particle location and particle velocity. At first a population of particles with random locations inside the solution space is generated. Then, in order to get closer to the optimal solution, these particles move at a certain speed in subsequent repetitions. In each iteration, the velocity of each particle is updated based on the following three factors:

- Current particle velocity (v_i^t)
- The best place for that particle to repeat this algorithm (pbest)
- The best location between all the particles up to this iteration of the algorithm (gbest)

The steps of PSO algorithm method are shown in Figure 7:

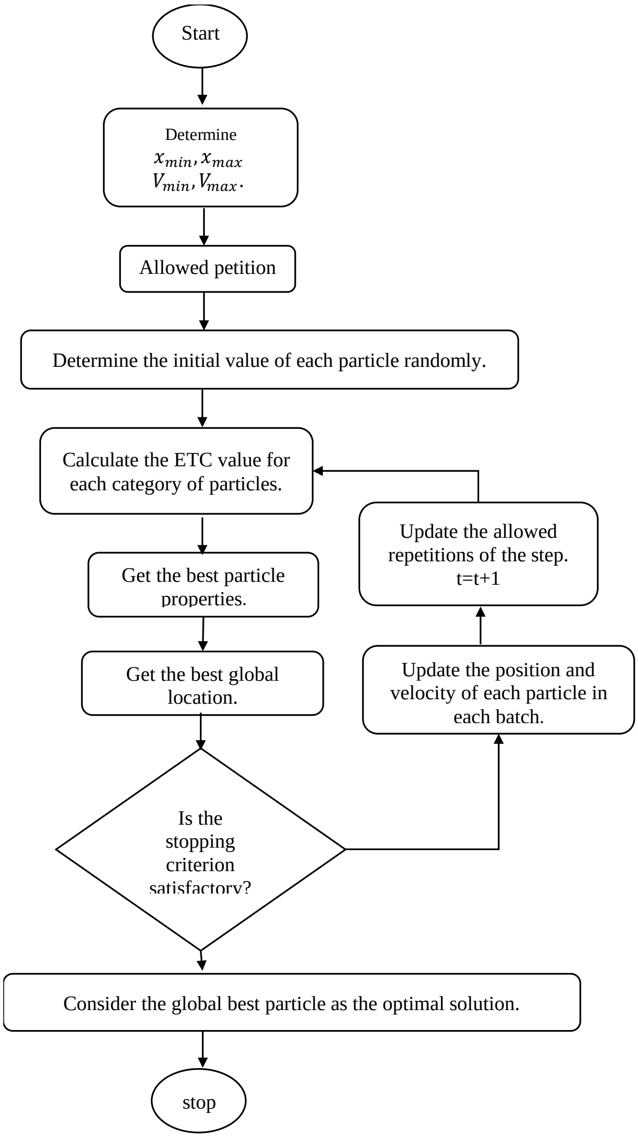

Figure 7. PSO algorithm method

Parameter setting

To solve the model using PSO method, the algorithm parameters must first be estimated. It should be noted that the parameter setting is based on the research review by [24].

In the proposed model, the answer is a nineteenth dimensional vector including $x_i^t = [n, m, L, k, w, \lambda, h_s, h_l, ATS_0, ATS_1, C_F, C_P, C_R, C_V, C_Y, Q_0, Q_1, E, T_1]$.The variables $\{n, L, m\}$ are considered as discrete variables and the rest are considered as continuous decision variables. It is considered a specific interval for discrete variables, which is for n, interval of $[n_{min}, n_{max}]$, L, interval of $[L_{min}, L_{max}]$ and m, interval of $[m_{min}, m_{max}]$ [24]:

$$n \in \{n_{min}, n_{min}+1, n_{min}+2, \dots, n_{max}\} \tag{31}$$

Then, we take into account a continuous variable, for example R, which generates initially random values in the range [0,1]. Then these values are converted into continuous values by Equation (32) [24].

$$n = \mathrm{Min}(n_{min} + \lfloor (n_{max} - n_{min} + 1) \times R \rfloor, n_{max}) \tag{32}$$

This process is also performed for L and m variables. In the case of continuous variables, the values are selected from the range between the upper and lower limits randomly. The process of the algorithm is performed according to Figure 7 to reach the stop criterion. In general, the criterion for stopping PSO depends on the issue under consideration. This stop criterion is, in some instances, the achievement of a pre-determined maximum allowable repetition, while in some other cases, is the achievement of a predetermined error threshold in the g-best value, [24]. To implement PSO, the parameters of the algorithm are set as follows:

- The solution space is equal to 150.
- The number of iterations of the algorithm is considered equal to 300.

It should also be noted that the model was provided and solved by MATLAB software.

Numerical examples

The performance of the proposed model is illustrated by a modified example from [35]. A special food product company operates with a production volume of about 100 units per day. The manufacturer sells this product in the packages with normally distribution weight and mean of 1 kilogram and standard deviation of 0.2 kilogram. In each interval, n-sized samples are taken from the process, with a fixed sampling cost of $ 10 and variable sampling cost of $2. The required time to record each sample is 0.01 unit of time.

When the sample mean is in out-of-control condition, the synthetic EWMA chart shows a signal. The operator should check the accuracy of the out-of-control signal. The required time to validate the signal is about 1 unit of time and the cost of checking for each false alarm is $ 200. If the alert is true, the system pays $ 5,000 reactive maintenance cost. In addition, for a Long-term sampling interval, if the process is in control state, the preventive maintenance repairs are done and the cost is $ 2400. The quality reduction index is about $ 100 for in control condition and $ 300 for out-of-control condition. The annual demand of this product is 10,000 units and the daily demand is 80 units. The start-up cost is about $ 60 and the cost of maintaining of inventory is $ 10 per unit per year. The parameters values are summarized in Table 3.

Table 3. Model parameters

Parameter	**δ μ**	**A**	**b**	**d**	**D**	**P**	**A**	**B**	**C_F**	**C_P**	**C_R**	**C_V**	**C_Y**	**Q_0**	**Q_1**	**E**	**T_1**	**n_{max}**
value	1	0.5	1	80	10000	100	60	10	10	2400	5000	2	200	100	300	0.01	1	**20**

Model solution results

In this paper, the model is solved by PSO algorithm with regarding of the adjusted parameters in Table 3. The solution results are showed in **Error! Reference source not found.**.

Table 4. Model solution results

variable	**ETC**	**E(I)**	**E(Q)**	**E(S)**	**E(M)**	**K**	**w**	**n**	**m**	**h_S**	**h_L**
value	4548.6	1824.5	139.4	184.0	2400.7	1.84	0.86	18	44	0.12	**3.0016**
variable	**ATS_1**	**ATS_0**	**L**	**ARL_0**	**ARL_1**	**λ**	**LCL**	**UCL**		**LWL**	**UWL**
value	8	857	8	1183.8	12.9	0.39	0.957	1.043		0.98	**1.02**

In Table 4, it can be seen that the total cost (ETC) is 4548.6, inventory cost (E(I)) is 1824.5, quality cost (E(Q)) is 139.4, repair and maintenance cost (E(M)) is 2400.7, inspection cost (E(S)) is 184.0. Also, the sample number is equal (n) to 18, the control limit coefficient (λ) of the EWMA chart is equal to 0.39, the weight coefficient (w) of EWMA chart is equal to 0.86 and the control limit of the CRL chart (L) is reported to be equal to 8, for this solution.

Results sensitivity analysis

In order to investigate the effect of model parameters or input variables estimation on the cost function, sensitivity analysis is performed. Many studies have used the sensitivity analysis method with the help of Taguchi test design, for example study of Guo Qing Cheng et al. [6]. It should be noted that the software used to implement this method is MINITAB.

In the context of our study, we use the unique attributes of the Taguchi method to explore and study the system sensitivity to different factors. It is considered three levels for the examined parameters as is presented in Table 5. L27 approach is selected to do the design experiments by Taguchi method, hence as a result, 27 experiments are reported.

Table 5. Factor levels in Taguchi experiment design

Level	Q_0	Q_0	C_F	C_V	C_Y	C_P	C_R	a	b	$\delta\mu$
1	50	200	5	2	200	240	4000	0.1	0.5	0.5
2	100	300	10	5	400	1200	5000	0.5	0.7	1
3	150	400	50	10	1000	2400	6000	1	1	1.5

Taguchi design is one of the known experimental designs to make purposeful changes on the model parameters. The results of L27 Taguchi design are shown in Table 6.

Table 6. Taguchi Experimental Designs

Run	Q_0	Q_1	C_F	C_vV	C_Y	C_P	C_R	a	b	$\delta\mu$
1	50	200	5	2	200	240	4000	0.1	0.5	0.5
2	50	200	5	2	400	1200	5000	0.5	0.7	1.0
3	50	200	5	2	1000	2400	6000	1.0	1.0	1.5
4	50	300	10	5	200	240	4000	0.5	0.7	1.0
5	50	300	10	5	400	1200	5000	1.0	1.0	1.5
6	50	300	10	5	1000	2400	6000	0.1	0.5	0.5
7	50	400	50	10	200	240	4000	1.0	1.0	1.5
8	50	400	50	10	400	1200	5000	0.1	0.5	0.5
9	50	400	50	10	1000	2400	6000	0.5	0.7	1.0
10	100	200	10	10	200	1200	6000	0.1	0.7	1.5
11	100	200	10	10	400	2400	4000	0.5	1.0	0.5
12	100	200	10	10	1000	240	5000	1.0	0.5	1.0
13	100	300	50	2	200	1200	6000	0.5	1.0	0.5
14	100	300	50	2	400	2400	4000	1.0	0.5	1.0
15	100	300	50	2	1000	240	5000	0.1	0.7	1.5
16	100	400	5	5	200	1200	6000	1.0	0.5	1.0
17	100	400	5	5	400	2400	4000	0.1	0.7	1.5
18	100	400	5	5	1000	240	5000	0.5	1.0	0.5
19	150	200	50	5	200	2400	5000	0.1	1.0	1.0
20	150	200	50	5	400	240	6000	0.5	0.5	1.5
21	150	200	50	5	1000	1200	4000	1.0	0.7	0.5
22	150	300	5	10	200	2400	5000	0.5	0.5	1.5
23	150	300	5	10	400	240	6000	1.0	0.7	0.5
24	150	300	5	10	1000	1200	4000	0.1	1.0	1.0
25	150	400	10	2	200	2400	5000	1.0	0.7	0.5
26	150	400	10	2	400	240	6000	0.1	1.0	1.0
27	150	400	10	2	1000	1200	4000	0.5	0.5	1.5

The results for decision variables and statistical properties in 27 determined experiments are given in Table 7-1, 7-2 and 7-3.

Table 7-1. The results for decision variables and statistical properties

Run	n	L	w	M	h_S	h_L
1	14	8	1.86	85	0.60	0.61
2	4	7	3.85	85	0.59	0.68
3	2	8	2.88	83	0.77	0.80
4	4	14	3.66	84	0.70	0.75
5	4	10	0.65	84	0.88	0.97
6	20	9	3.46	70	0.62	0.80
7	5	6	3.30	65	1.35	1.42
8	16	6	4.17	75	0.83	0.90
9	4	19	4.23	60	1.21	1.25
10	3	14	3.63	70	0.95	1.24
11	20	14	2.75	54	1.16	1.19
12	4	7	3.86	20	1.21	1.29
13	20	6	4.79	80	0.58	0.75
14	7	7	1.26	18	1.07	1.11
15	3	19	2.29	52	1.01	1.17
16	5	17	1.49	17	0.28	0.85
17	**18**	**8**	**0.86**	**44**	**0.12**	**3.13**
18	20	6	2.05	87	0.63	0.70
19	3	7	2.34	51	1.16	1.65
20	5	7	3.44	18	1.81	1.87
21	18	7	2.58	43	1.10	1.13
22	4	19	4.67	56	0.71	0.94
23	8	8	3.14	56	1.06	1.09
24	8	9	1.09	75	0.76	0.96
25	19	17	2.98	87	0.48	0.49
26	6	7	2.12	87	0.57	0.70
27	6	7	4.32	60	0.66	0.86

Table 7-2. The results for decision variables and statistical properties

Run	ATS_0	ATS_1	ARL_1	ARL_0
1	518	7	927.5	10.1
2	518	7	826.9	10.8
3	631	7	865.4	8.7
4	523	6	769.4	9.7
5	515	6	630.5	8.8
6	520	6	760.8	7.8
7	749	5	552.2	6.2
8	515	6	654.7	7.04
9	514	6	437.5	5.06
10	520	7	487.8	7.5
11	516	7	452.3	6.004
12	520	5	412.5	5.7
13	522	5	784.01	9.04
14	517	5	504.03	4.9
15	517	7	485.6	9.3
16	521	5	1215.6	12.7
17	**857**	**8**	**1183**	**12.9**
18	520	6	899.2	9.6
19	521	7	340.2	5.9
20	629	5	367.4	5.7
21	521	8	483.4	7.1
22	522	8	619.1	8.8
23	519	7	514.02	5.4
24	517	8	682.8	10.9
25	520	3	1863.9	11.9
26	519	6	954.4	13.1
27	638	7	1051.2	10.3

Table 7-3. The results for decision variables and statistical properties

Run	λ	LCL	UCL	LWL	UWL	ETC
1	0.42	0.413	0.587	0.449	0.551	4998.7
2	0.68	0.764	1.24	0.724	1.28	5161.9
3	0.62	1.22	1.78	1.23	1.77	5235.3
4	0.62	0.783	1.22	0.755	1.25	5579.7
5	0.42	1.364	1.62	1.47	1.53	5036.6
6	0.66	0.409	0.591	0.391	0.609	5621.2
7	0.68	1.33	1.67	1.29	1.71	5006.8
8	0.62	0.417	0.583	0.360	0.640	5553.8
9	0.66	0.782	1.22	0.703	1.30	5394.2
10	0.64	1.26	1.74	1.21	1.79	5679.1
11	0.55	0.431	0.569	0.424	0.576	5615.04
12	0.55	0.807	1.193	0.762	1.24	5803.7
13	0.52	0.417	0.583	0.373	0.627	5790.9
14	0.44	0.863	1.14	0.949	1.05	5234.2
15	0.46	1.31	1.69	1.36	1.64	5598.7
16	0.68	0.772	1.23	0.904	1.09	5097.8
17	**0.39**	**0.957**	**1.043**	**0.988**	**1.05**	**4548.5**
18	0.49	0.419	0.581	0.448	0.552	5644.1
19	0.49	0.772	1.23	0.846	1.15	5400.02
20	0.54	1.33	1.67	1.31	1.69	5379.9
21	0.59	0.399	0.600	0.421	0.579	5804.7
22	0.42	1.36	1.64	1.26	1.74	5509.9
23	0.42	0.376	0.624	0.386	0.614	5517.7
24	0.68	0.881	1.12	0.945	1.06	5759.4
25	0.62	0.394	0.606	0.408	0.592	5705.5
26	0.62	0.812	1.19	0.884	1.12	5476.7
27	0.62	1.34	1.66	1.26	1.74	5082.9

According to Table 7-1, the best values of the parameters are n=18, L=8, W=0.86, M=44, h_S=0.12, h_L=3.13. Also, in Table 7-2 the finest values of the parameters are equal to ATS_0=857, ATS_1=8, ARL_1=1183, ARL_0=12.9. In addition to these, Table 7-3 shows λ=0.39, LCL=0.957, UCL=1.043, LWL=0.988, UWL=1.05 and ETC=4548.5 as the best values of the parameters. Also, to show the effect of parameters values on the objective function, the diagrams from the analysis of Taguchi experiments are drawn in Figure 8.

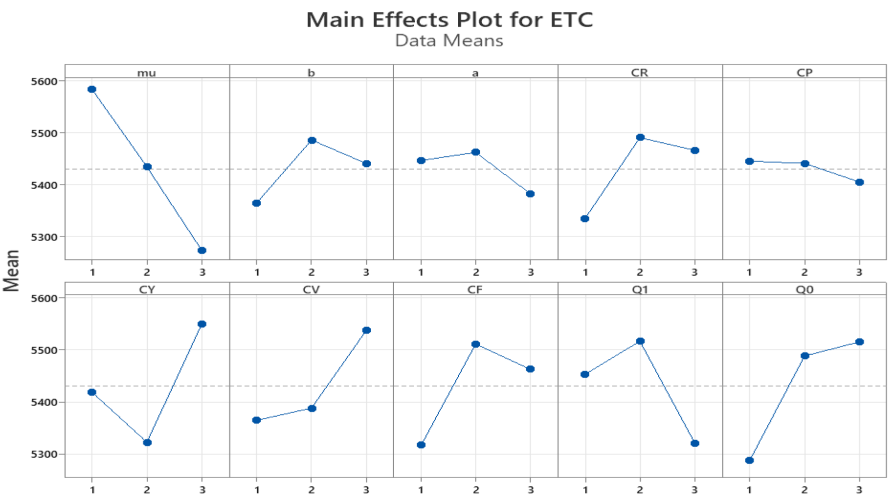

Figure 8. The main effects plot

In sensitivity analysis, a main effect plot is a graphical representation that helps visualize the impact of individual factors (variables) on a chosen outcome or response variable while holding other factors constant. This type of analysis is particularly useful in understanding how changes in each factor influence the overall system. This plot typically shows the average response at each level of a factor. Each line or bar represents a different factor, and the pattern of the lines/bars indicates the influence of that factor on the response. The diagram is drawn from Taguchi's experimental design. Based on the main effect diagram, the effective factors and the effective value of each factor are shown in the Table 8.

Table 8. Proper values of the parameters

Parameter	**δ_μ**	**b**	**a**	**C_R**	**C_P**	**C_Y**	**C_V**	**C_F**	**Q_1**	**Q_0**
Level	1	2	2	2	1	3	3	2	2	**3**
value	0.5	0.7	0.5	5000	240	1000	10	10	300	**150**

Comparison the results

To compare the results of the proposed method and show the performance of it, in the proposed model (Equation 30) for a certain amount of variability ($\delta_\mu = 1$), the optimal values are compared with the research of Salmasnia et al. [30]. It should be noted that in order to make a comparison, first we fixed ATS_0 on the value of 500 and then compared the value of ATS_1 of the two methods. The comparison results are given in Table 9.

Table 9. Comparing the values of the proposed method with Salmasnia et al.

Research	**ETC**	**K**	**w**	**n**	**m**	**h_S**	**h_L**	**ATS_1**	**ATS_0**
Proposed method	5234.2	3.42	1.26	7	18	1.07	1.11	5	**500**
Salmasnia et al. 2019	6995.74	3.24	0.61	4	90	0.58	0.60	5.51	**500**

Table 9, shows adding a synthetic feature to the chart (the lower limit of the CRL control chart), while increases the sample size, reduces the ETC and the number of inspection periods until preventive maintenance. Also, the value of ATS_1 in the current study is less than the method of Salmasnia et al. [30]. In addition, the expected cost values for the L27 Taguchi test designs in the proposed method are compared with Salmasnia et al. [30] in Table 10.

Table 10. Compare expected cost for two models

Run	**Proposed method**	**Salmasnia et al.**
1	4998.7	6480.83
2	5161.9	6860.73
3	5235.3	7739.82
4	5579.7	6114.61
5	5036.6	6880.04
6	5621.2	9214.03
7	5006.8	6281.64
8	5553.8	9350.54
9	5394.2	8609.32
10	5679.1	7878.61
11	5615.04	7402.97
12	5803.7	7181.83
13	5790.9	9071.44
14	5234.2	6280.51
15	5598.7	7094.08
16	5097.8	7869.9
17	4930.5	5900.41
18	5644.1	8440.22
19	5400.02	7212.49
20	5379.9	8029.97
21	5804.7	7190.39
22	5509.9	6925.35
23	5517.7	9885.96
24	5759.4	6236.64
25	5705.5	8018.43
26	5476.7	8084.69
27	5082.9	5891.11

According to Table 10, it can be seen the performance of the presented method is proper than the study by Salmasnia et al. [30].

Conclusion and suggestions

The present study described the economic design of integrated production planning model based on adaptive synthetic EWMA control chart and maintenance policies. In this research, a mathematical model was presented with the aim of minimizing the total cost. Also, particle swarm optimization algorithm was used to solve the model. To show the performance of the proposed method, a numerical example according to the research Stephen M et al. [33] was used. After solving the sample problem, the sensitivity analysis of the model parameters and effect on the objective function was studied. In additions, the comparison of results of the proposed method and Li Xue et al. [17] shows that this method has a superior performance to decrease of total cost and average time to signal index.

In addition, this study provides the valuable inspirations for practical implementation in managerial decision-making. By reflecting on the key findings, several implications for the management can be stand out. The results of this research emphasize the vital importance of taking into account for implementing maintenance strategies. Managers in the industry can benefit from these inspirations to improve operational efficiency, reduce downtime and cost and optimize resource allocation.

Based on the results of this paper, these suggestions are recommended to study for future:

- Economic-statistical design of other control charts should be examined by determining the decision variables and maintenance policies.
- The presented research should be studied to control the variability of the process.
- The results of the presented model in different industries should be examined in conditions of uncertainty.
- Other innovative algorithms such as neural networks, genetic algorithms, etc. can be used and compared to solve the optimization model.

References

[1] D. Y. J. J. E. A. Chen Y, "Integration Of Process-Oriented Tolerancing And Maintenance Planning In Design Of Multistation Manufacturing Processes," IEEE Trans Autom Sci Eng, vol. Vol 4, no. 1, p. 440–453, 2006.

[2] M. V. Jafari L, "Optimal Lot-Sizing And Maintenance Policy For A Partially Observable Production System," Comput Ind Eng, vol. 93, no. 1, p. 88–98, 2016.

[3] G. A. P. R. Bouslah B, "Joint Economic Design Of Production, Continuous Sampling Inspection And Preventive Maintenance Of A Deteriorating Production System," Int J Prod Econ, vol. 173, p. 184–198, 2016.

[4] N. N. B.-D. M. Nourelfath M, "Integrated Preventive Maintenance And Production Decisions For Imperfect Processes," Reliab Eng Syst Saf, vol. 148, p. 21–31, 2016.

[5] L. C. E. P. Z. C. Y. Li, "Jointly Optimal Design Of Perfect Maintenance Policy And CUSUM Control Chart," IEEE International Conference On Industrial Engineering And Engineering Management (IEM), pp. 2157-362, 2017.

[6] B. H. Z. ,. L. L. Guo Qing Cheng, "Integrated production, quality control and condition-based maintenance for imperfect production systems," Reliability Engineering and System Safety, vol. 175, pp. 251-264, 2018.

[7] Z. L. X. H. Lin Wang, "Joint Optimisation Of Production, Maintenance And Quality For Batch Production System Subject To Varying Operational Conditions," International Journal Of Production Research, vol. 57, no. 24, pp. 7552-7566, 2019.

[8] L. X. a. Z. He, "Economic Design of EWMA Control Charts with Variable Sampling Intervals for Monitoring the Mean and Standard Deviation under Preventive Maintenance and Taguchi's Loss Functions," Mathematical Problems in Engineering, vol. 2021, no. 1, p. 14, 2021.

[9] A. A. K. W. H. S. M. A. S. K. P. Rajesh Saha, "Integrated economic design of quality control and maintenance management: Implications for managing manufacturing process," International Journal of System Assurance Engineering and Management, vol. 12, p. 263–280, 2021.

[10] Z. S. T. Wu, "A synthetic control chart for detecting small shifts in the process mean," Journal of Quality Technology, vol. Vol 32, no. 1, p. 32–38., 2000.

[11] M. A. G. A. F. B. C. A. M. A. R. Machado, "The Synthetic Control Chart Based On Two Sample Variances For Monitoring The Covariance Matrix.," Quality And Reliability Engineering Internationa, vol. 25, no. 5, p. 595–606, 2009.

[12] Y. P. C. Z. A. B. C. K. (. .. V. 4. :. Zhang, "The Synthetic .X Chart With Estimated Parameters," IIE Transactions, vol. 43, no. 9, p. 676–87, 2011.
[13] M. B. C. Z. W. A. P. C. Khoo, "Optimal Design Of The Synthetic Chart For The Process Mean Based On Median Run Length," IIE Transactions, vol. 44, no. 9, p. 765–79, 2012.
[14] W. B. C. K. A. C. Yeong, "Economic-Statistical Design Of The Synth.X," Quality And Reliability Engineering International, vol. 31, no. 5, pp. 863-876, 2014.
[15] M. B. C. H. C. L. Z. C.-H. C. A. P. C. Khoo, "A Synthetic Double Sampling Control Chart For The Process Mean," IIE Transactions, vol. 43, no. 1, p. 23–38, 2010.
[16] M. H. A. M. B. C. K. Lee, "Multivariate Synthetic |S| Control Chart With Variable Sampling Interval," Communications In Statistics – Simulation And Computation, vol. 44, no. 4, p. 924–42, 2015.
[17] Z. H. Li Xue, "Economic Design of EWMA Control Charts with Variable Sampling Intervals for Monitoring the Mean and Standard Deviation under Preventive Maintenance and Taguchi's Loss Functions," Mathematical Problems in Engineering, 2021.
[18] M. F. N. e. a. S. Jafarian-Namin, "An integrated model for optimal selection of quality, maintenance, and production parameters with autocorrelated data," Scientia Iranica, vol. 31, no. 3, 2021.
[19] S. J.-N. e. al, "An integrated quality, maintenance and production model based on the delayed monitoring under the ARMA control chart," Journal of Statistical Computation and Simulation, vol. 91, no. 13, 2021.
[20] F. N. &. F. A. Hasan Rasay, "Opportunistic maintenance integrated model for a two-stage manufacturing process," The International Journal of Advanced Manufacturing Technology, vol. 119, no. 1, p. 8173–8191, 2022.
[21] M. T. &. A. R. Ali Akbar Heydari, "An Integrated Model of Maintenance Policies and Economic Design of X-bar Control Chart Under Burr XII Shock Model," Iranian Journal of Science, vol. 47, no. 1, p. 1261–1269, 2023.
[22] S. N. ,. S. J.-N. A. J. Mohsen Shojaee, "Integration of production–maintenance planning and monitoring simple linear profiles via Hotelling's T2 control chart and particle swarm optimization," Computers & Industrial Engineering, vol. 188, 2024.
[23] Z. H. N. F. M. A. Salmasnia, "A joint determination of production cycle length, maintenance policy, and control chart parameters considering time value of money under stochastic shift size," Transactions on Industrial Engineering, vol. 27, no. 1, pp. 427-447, 2018.
[24] N. M. Reza, "Solid economic-statistical design of adaptive control chart for a secret system under maintenance policies," Faculty of Industrial Engineering, Qom University, 2018.
[25] M. B. M. M. A. Pasha, "A generalized version of Ben-Daya-Rahim (2000) and Rahim-Banerjee (1993) cost models in economic design of X-control charts for systems with early replacement and preventive maintenance under decreasing integrated hazard," Communications in Statistics - Simulation and Computation , vol. 49, no. 1, pp. 178-193, 2018.
[26] Y. W. W. Z. &. X. C. Qiang Wan, "Economic design of an integrated adaptive synthetic chart and maintenance management system. Communications in Statistics," Theory and Methods, vol. 0, no. 0, pp. 1-18, 2018.
[27] F. S. M. N. B. A. A. Salmasnia, "An economic-statistical model for production and maintenance planning under adaptive non-central chi-square control chart," vol. 12, no. 1, pp. 35-65 , 2019.
[28] H. T. A. S. A. Farahani, "An integrated optimization of quality control chart parameters and preventive maintenance using Markov chain," Advances in Production Engineering & Management, vol. 14, no. 1, p. 5–14, 2019.
[29] M. N. A. Ali Salmasnia, "An integrated quality and maintenance model for two-unit series systems," Communications in Statistics - Simulation and Computation , vol. 49, no. 4, pp. 886-917 , 2020.
[30] F. S. E. H. &. S. G. Ali Salmasnia, "An integrated model for joint determination of production run length, adaptive control chart parameters and maintenance policy.," Journal of Industrial and Production Engineering, vol. 36, no. 6, pp. 401-417, 2019.
[31] C.-H. C. ,. H.-R. L. CHAO-YU CHOU, "Economic Design of EWMA Charts with Variable Sampling Intervals," Quality & Quantity, vol. 40, no. 1, pp. 879-896, 2006.
[32] A. Haq, "A new nonparametric synthetic EWMA control chart for monitoring process mean. Communications in Statistics," Simulation and Computation, vol. 48, no. 1, pp. 1665-1676, 2018.
[33] M. E. C. Stephen M. Scariano, "The Generalized Synthetic Chart," Sequential Analysis:Design Methods and Applications, vol. vol 28, no. 1, pp. 54-68, 2009.
[34] M. B. C. K. S. Y. T. M. H. L. Lei Yong Lee, "A Variable Sampling Interval Synthetic Xbar Chart for the Process Mean," PLoS ONE, vol. 10, no. 5, 2015.
[35] J. Y. W. S. E. A. Pan E, "An Integrated EPQ Model Based On A Control Chart For An Imperfect Production Process," Int J P Res, vol. 50, no. 1, p. 6999–7011, 2012.