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RESEARCH PAPER

# Material Nonlinear Static Analysis of Axially Functionally Graded Porous Bar Elements

Şeref Doğuşcan AKBAŞ *

*Bursa Technical University, Mimar Sinan Campus, Bursa, 16310, Turkey*

**Abstract**

**This investigation presents material nonlinear analysis of a cantilever bar element made of functionally graded material with porosity properties. The material properties of bar element are considered as changing though axial direction based on the Power-Law distribution and uniform porosity distribution. The stress-strain relation of the material is considered as a nonlinear property according to a Power-Law function. The cantilever bar element is subjected to a point load at the free end. In order to obtain more realistic solution for the nonlinear problem and axially material distribution, nonlinear finite element method is used. In the obtaining of finite element equations, the virtual work principle is used and, after linearization step, the tangent stiffness matrix and residual vector are obtained. In the nonlinear solution process, the incremental force method is implemented and, each load step, the nonlinear equations are solved by using the Newton-Raphson iteration method. In the numerical results, effects of material nonlinearity parameters, porosity coefficients, material distribution parameter and aspect ratios on nonlinear static deflections of the bar are presented and discussed. The obtained results show that the material nonlinear behaviour of the bar element is considerably affected with porosity and material graduation.**

**Keywords:** Porosity; Material Nonlinearity; Functionally Graded Materials; Porosity; Nonlinear Solution; Finite Element Method

## 1. Main text

Structural elements could be exposed to large values loads or extreme conditions. Due to these conditions, large displacements, rotations and strains could be occurred, so linear modelling and analysis are not valid for mechanical behaviour of structures elements. In order to gain more sensitive results about mechanical behaviour of structural elements, nonlinear effects and analysis must be considered within these conditions. Material nonlinearity, one of the nonlinear types, consists of nonlinearity of the relationship between stress and strain. In large strains and inelastic behaviours of structures, stress-strain relations do not perform as linear function, so material nonlinear analysis must be taken into account in the modelling.

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* Corresponding author. Tel.: +90 224 300 34 98;

*E-mail address:* serefda@yahoo.com , seref.akbas@btu.edu.tr

Functionally graded material (FGM) is one of the composite types, in which material properties vary along directions. FGMs are used a lot of engineering projects, for example defence, biomedical, aerospace and aircraft structures, thermal barriers etc. With using FGMs in structural projects, the structural elements could be designed as low weight and high strength and resistance. Over the last 40 years, usage and investigations of the FGMS have been increasing.

In the production stage of FGMs, pores could be occurred due to different production errors. A structural element with porosity could be lost its strength. After a level of porosity, the materials could be exposed to large strains or inelastic stage even if the linear behaviour conditions exist. Therefore, a FGM structural element could be subjected to large strains with porosity. For this reason, material nonlinear analysis gains importance for porous FGMs structures. It is intendent in this paper that learn about both material nonlinearity behaviour and porosity effects of a FGM bar element.

In the literature, a lot of investigations about porosity and nonlinear behaviour FGM structures carried out in last decade. Published papers about porosity and nonlinear behaviour of FGM structures are listed and summarized in the following parts. Wattanasakulpong and Ungbhakorn [1] investigated linear and geometrically nonlinear vibrations of FGM beams with different porosity distributions by using differential transformation method. Ebrahimi and Zia [2] investigated geometrically nonlinear vibration of FGM porous beams based on Timoshenko beam theory by using Galerkin's method and the method of multiple scales. Akbaş [3] investigated free vibration of porous FGM deep beams under temperature effects by using 2D finite element approach. Akbaş [4, 5] presented geometrically nonlinear static analysis of FGM porous beams under mechanical and thermal loadings by using Total Langragian finite element approach. Based on polygonal finite element method, Nguyen et al. [6] studied nonlinear analysis of FGM plates with porosity. Amir and Talha [7] investigated finite element solution of geometrically nonlinear thermo-elastic vibration analysis of FGM curved panels with porosity based on higher shear deformation shell theory. Babaei et al. [8] presented a review study about porous FGM structures. Gao et al. [9] investigated geometrically nonlinear bending analysis of FGM nano beams with porosity by using nonlocal strain gradient theory. Xiao et al. [10], Ahmadi and Foroutan [11], Gao et al. [12], Fouaidi et al. [13], Babaei et al. [8] studied effects of porosity on geometrically nonlinear buckling behaviour of FGM structures. Keleshteri and Jelovica [14] analysed geometrically nonlinear vibration of FGM porous cylindrical panels by using differential quadrature method. Zhu et al. [15] investigated geometrically nonlinear vibration of fluid-FGM porous conveying pipes resting on nonlinear elastic foundation by using variational iteration method and the direct method of multiple scales. Alhaifi et al. [16] presented large deflection analysis of FGM porous plates resting on nonlinear foundation based on shear deformation theory by using generalized differential quadrature method. Fan et al. [17] analysed geometrically nonlinear vibration of FGM porous micro/nano-plates based on nonlocality by using isogeometric analysis. Li et al. [18] presented geometrically nonlinear static analysis of axially FGM microtubes by using modified couple stress theory generalized differential quadrature method. Li et al. [19] used finite element method to investigate nonlinear dynamics of FGM sandwich beams includes porous core. Yapor Genao et al. [20] investigated geometrically nonlinear static analysis of micro scaled FGM plates with porosity based on modified couple stress theory by using nonlinear finite element method. Ansari et al. [21] studied geometrically nonlinear vibration of porous FGM beams resting on foundation under thermal loading based on Timoshenko beam theory by using variational differential quadrature. Hosur Shivaramaiah et al. [22] presented a study about geometrically nonlinear static and vibration of FGM porous plates by using finite element method. Mahesh [23] investigated geometrically nonlinear static analysis of FGM magneto-electro-elastic porous shells subjected to electro-magnetic and mechanical loads based on higher-order shear deformation theory and von-Karman's geometric nonlinearity. Sh et al. [24] examined geometrically nonlinear dynamics of FGM magneto-electro-elastic porous plates by using first-order shear deformation theory, von Karman's nonlinearity and finite element method. Van Long et al. [25] presented geometrically nonlinear static responses of porous FGM beams on elastic foundation by using von Kármán nonlinearity, Timoshenko beam theory. Zargar Ershadi et al. [26] studied geometrically nonlinear dynamic responses of porous circular plates subjected to hygro-thermal loading based on von-Kármán non-linearity by using differential quadrature method. Do and Pham [27] examined nonlinear static behaviour of FGM sandwich plates with porosity resting on foundation based on the von-Kármán non-linearity and first-order shear deformation theory. Joshi and Kar [28] investigated elasto-plastic analysis of FGM porous panels with various distribution direction based on von Karman strain kinematics, von Mises yield criterion using bilinear isotropic strain hardening by using finite element method. Kim et al. [29] studied geometrically nonlinear bending analysis of micro circular/annular plates made of porous FGM subjected to thermal and mechanical loads based on third-order shear deformation theory and by using finite element method. Krysko-jr et al. [30] presented an investigation about material nonlinearity and elasto-plastic analyses of FGM plates with porosity by using different solution techniques. Kumar et al. [31] examined geometrically nonlinear analysis of porous FGM plates based on von Karman's nonlinearity and higher order shear deformation theory by using finite element method. Sobhy and Alsaleh [32] presented an investigation about geometrically nonlinear static analysis of FGM sandwich plate with porosity and metal/graphene materials by using Galerkin and Newton's methods.

It is seen from literature survey that a large number of studies have dealt with geometrically nonlinear analysis in the nonlinear investigations of porous FGM structures. There have been few studies which includes material nonlinearity of FGM porous structures. Also, no previous study has investigated material nonlinearity analysis of FGM porous bars. Previous studies are limited to geometrically nonlinear analysis for nonlinear studies for FGM structures.

The current study aimed to these gaps in the existing literature for material nonlinear analysis of FGM porous bar elements. The current study examined the effects of porosity, material distribution and aspect ratios on the material nonlinear static behavior of the bar elements. In the material nonlinear model of stress-strain relation, a Power-Law function is considered and the material properties of the cantilever bar element are considered as changing though axial direction according to the Power-Law distribution. In solution, finite element method is used and nonlinear equations are solved by using the Newton-Raphson iteration method within the incremental force approach.

## 2. Theory and Formulations

Figure 1 presents a cantilevered prismatic bar element made of porous functionally graded material under a point load (P) at the free and with L length, b width and h height.

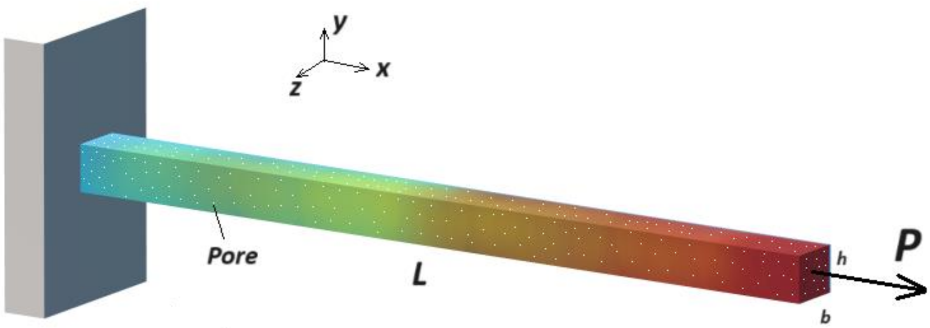


**Fig 1: A Cantilever Axially Functionally Graded Prismatic Porous Bar Element Under Point Load.**

Material properties (*M*) of bar element change though axial direction (*x*) with uniform porosity model according to following power-law function:

$$\mathrm{M(x)} = (\mathrm{M_L} - \mathrm{M_R})\left(1 - \frac{\mathrm{x}}{\mathrm{L}}\right)^{\mathrm{n}} + \mathrm{M_R} - \frac{(\mathrm{M_L}+\mathrm{M_R})}{2} a_{pr} \tag{1}$$

In equation 1, $M_L$ and $M_R$ are material properties of left and the right faces of bar and *n* is graduation coefficient, $a_{pr}$ is volume fraction of porosity. In the uniform porosity model, the pores spread uniformly in the material. In equation 1, $M=M_L$ if x=0, $M=M_R$, material of bar gets homogenous full material of left face if $n$=0, material of bar gets homogenous full material of right face if $n=\infty$ and the material gets fully FGM without pores if $a_{pr}=0$. In order to show how change the material properties along axial direction *x* according to the power-law function in equation 1, change of nonlinear material Modulus (*C*) is plotted for different *n* parameters for values $C_L = 500\ MPa$, $C_R = 0.5E_L$, $a_{pr}$=0 and *L*=5 m in figure 2. The nonlinear elastic stress-strain relation is considered as a power-law function as following function:

$$\sigma_x = C(x, a_{pr})(\varepsilon_x)^k \tag{2}$$

where $\sigma_x$ and $\varepsilon_x$ are normal stress and axial strain, respectively. $C$ is and nonlinear material modulus which depends x coordinate and porosity according to equation 1. In equation 2, $k$ indicates the material constant which defines material nonlinearity and ranges 0-1. If $k$=1, stress-strain relation yields to linear elastic property, otherwise stress-strain relation has a nonlinear property. Equation 2 is plotted for different values of material nonlinear for $C$=500 MPa in figure 3.

Based on finite element method, considered nonlinear problem is solved. Finite elements with 2 nodes are used with one freedom in each node as shown in figure 4. The displacement field ($u$) is written in terms of nodal displacements ($u_i$ and $u_j$) as follows:

$$u = \left[\left(1-\frac{x}{L}\right) \quad \frac{x}{L}\right]\begin{Bmatrix} u_i \\ u_j \end{Bmatrix} = [N]\{u\},$$
$$[N] = \left[\left(1-\frac{x}{L}\right) \quad \frac{x}{L}\right], \{u\} = \begin{Bmatrix} u_i \\ u_j \end{Bmatrix} \qquad (3)$$

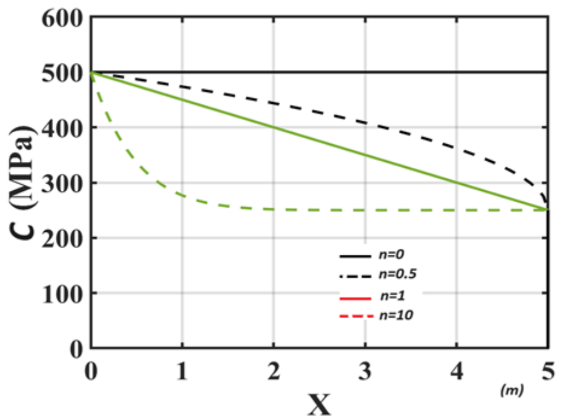


**Fig 2: Change of Nonlinear Material Modulus though axial direction for power-law function.**

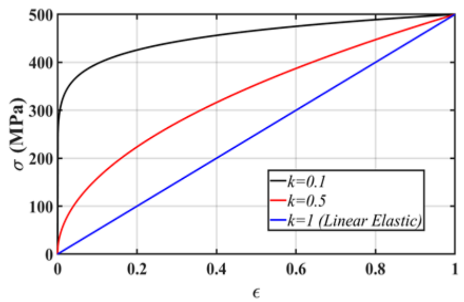


**Fig. 3. Nonlinear Elastic Stress-Strain relation for different k parameters according to a power-law function.**

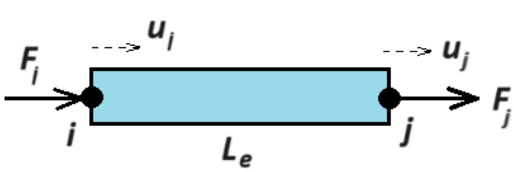


**Fig. 4. Finite element with 2 nodes.**

In equation 3, $[N]$ and $\{u\}$ indicate matrix of interpolation functions and displacement vector for a finite element. The axial strain ($\varepsilon_x$) –displacement ($u$) relation and its expression in terms of nodal displacements ($u_i$ and $u_j$) and interpolation function are presented as follows:

$$\varepsilon_x = \frac{\partial u}{\partial x} = \begin{bmatrix} -\frac{1}{L} & \frac{1}{L} \end{bmatrix} \begin{Bmatrix} u_i \\ u_j \end{Bmatrix} = [B]\{u\} \tag{4}$$

Where [B] represents a matrix for the derivatives of interpolation functions as follows:

$$[B] = \begin{bmatrix} -\frac{1}{L} & \frac{1}{L} \end{bmatrix} \tag{5}$$

In the obtaining in equilibrium equations the virtual work principle is implemented. The virtual work expressions of internal forces ($\delta W_I$) and external forces ($\delta W_e$) are presented for a finite element model as follows:

$$\delta W_e = \{\delta u_i \quad \delta u_j\}^T \begin{Bmatrix} F_i \\ F_j \end{Bmatrix} = \{\delta u\}^T\{F\} \tag{6}$$

$$\delta W_I = \int_V \sigma_x \, \delta\varepsilon_x \, dV = \int_V \sigma_x \, [B]\{\delta u\} dV = \{\delta u\}^T \int_V [B]^T \sigma_x \, dV \tag{7}$$

Where $F_i$ and $F_j$ are node forces, $\delta u_i$ and $\delta u_i$ are virtual displacements of nodes, $V$ indicates the volume. Substituting Eqs (2) into Eq. (7), The virtual work expressions of internal forces ($\delta W_I$) can be obtained following equation:

$$\delta W_I = \{\delta u\}^T \int_V [B]^T (x, a_{pr}) (\varepsilon_x)^k \, dV$$
$$= \{\delta u\}^T \int_V [B]^T C(x, a_{pr}) ([B]\{u\})^k \, dV = \{\delta u\}^T \int_0^{L_e} [B]^T C(x, a_{pr}) A ([B]\{u\})^k \, dx \tag{8}$$

Where $A$ indicates cross sectional area. With virtual work principle $\delta W_I = \delta W_e$, the equilibrium equation of a finite element can be obtained as follows:

$$\{P(u)\} = \{F\} \tag{9}$$

Where $\{P\}$ and $\{F\}$ are internal and external forces vectors and theirs expressions are presented as follows:

$$\{P(u)\} = \int_0^{L_e} [B]^T C(x, a) A ([B]\{u\})^k \, dx \tag{10}$$

$$\{F\} = \begin{Bmatrix} F_i \\ F_j \end{Bmatrix} \tag{11}$$

After obtaining finite element equations for an element, the total finite element equations can be derived by assembling of the finite element matrixes and vectors. As seen from equation (10), the internal force is a nonlinear function respect to $\{u\}$ because of using material nonlinearity. In solution of nonlinear equation (9), Newton-Raphson method is used. In addition, the incremental force method is implemented in order to obtain rapid convergence for iteration and easy estimation of the initial displacement vector for the Newton-Raphson iteration process. In this approach, the applied load is divided by a number, and the Newton Raphson process is implemented at each load step. With linearization process on the internal force vector based on the Newton-Raphson method, the linearized equation is presented for $r$th load step as follows:

$$[K_T]^p \{\Delta u\}^p = \{R\}^p \tag{12}$$

Where $[K_T]^p$, $\{\Delta u\}^p$ and $\{R\}^p$ are tangent stiffness matrix, incremental displacement vector and residual vector for $i$th iteration and $j$th load step, respectively. The tangent stiffness matrix of this problem for $i$th iteration is obtained as follows:

$$[K_T]^p = \frac{\partial\{P(u)^p\}}{\partial\{u\}} = \int_0^{L_e} k \, C(x, a_{pr}) A [B]^T [B] ([B]\{u\}^p)^{k-1} \, dx \tag{13}$$

The residual vector $\{R\}^p$ for $p$th iteration and $r$th load step is expressed as follows:

$$\{R\}^p = \{F\}^r - \{P(u)^p\} \tag{14}$$

Where $\{P(u)^p\}$ is the internal force vector for $p$th iteration . With the initial estimation of displacement vector $\{u\}^1$ the iteration starts in first iteration and incremental displacement vector is calculated at every iteration. After solution of incremental displacement vector for $p$th in equation (12), the next displacement vector $\{u\}^{p+1}$ for $p$+1th is expressed as follow:

$$\{u\}^{p+1} = \{u\}^p + \{\Delta u\}^p \tag{15}$$

The iteration continue until a termination criterion reaches a value which show convergence of the iteration. In nonlinear solution of this problem, following termination criterion which based on tolerance of the residual vector is used as follows (Kim [33]):

$$conv = \frac{\sum_{j=1}^{n}\left(R_j^{p+1}\right)^2}{1+\sum_{j=1}^{n}\left(F_j^r\right)^2} \tag{16}$$

If convergence value in equation 16 is less than a tolerance value, the iteration is stopped. The flowchart of solution process is presented in figure 5.

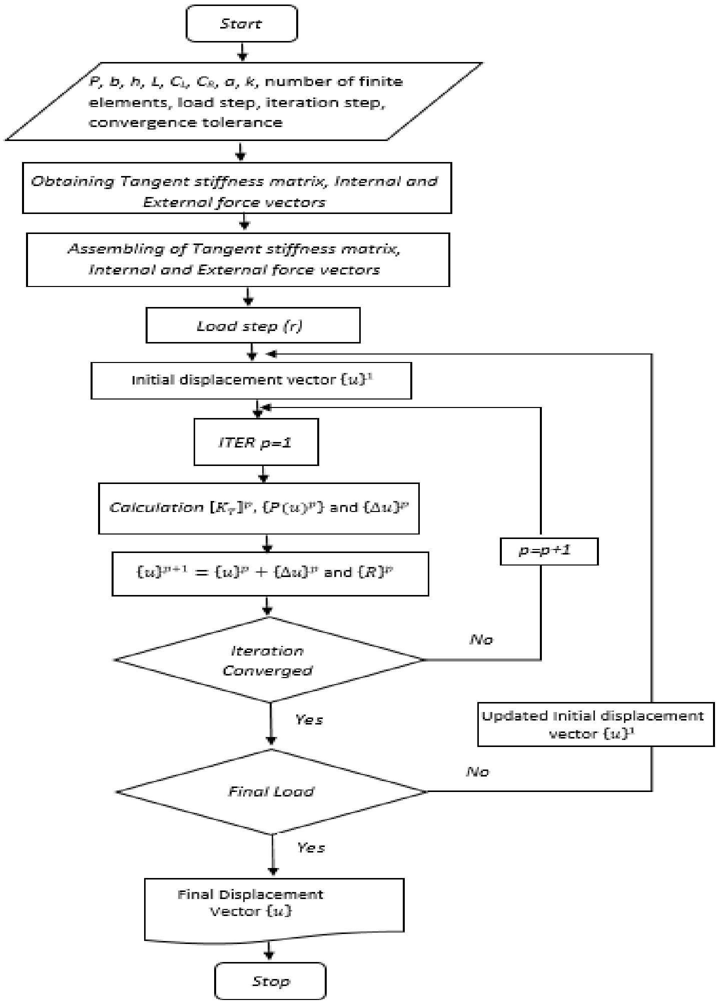


**Fig. 5. The flowchart of solution process.**

### 3. Analysis Results and Discussion

In this section, a lot numerical studies are carried out in order to investigated effects of material nonlinearity parameters, porosity coefficients, material distribution indexes and aspect ratios on the nonlinear static deflections of the porous cantilever bar. In the solution of formulation and obtaining results, an algorithm is developed according to the flowchart which is shown in figure 5 in MATLAB.

The nonlinear deflections are obtained by using finite elements and the nonlinear finite element equation is solved by using the Newton-Raphson iteration method via the Newton-Raphson iteration. In order to obtain more precise results for nonlinear solution, the applied load is divided by large numbers and the Newton-Raphson iteration method is implemented in each load step. The tolerance value of the iteration is selected as 10-5 in the solution of nonlinear process.

Unless otherwise stated, the height and width of cross-section are taken as b=h=0.01 m in the numerical studies. The length of the bar is selected according to different aspect ratios (L/h) for each analysis. The nonlinear material modulus at left face of the bar element is taken as CL=500 MPa. The right face of the bar element is considered as CR=0.5CL MPa. This material property varies along axial direction (x) from left face to right face according to equation (1).

In order to determine the number of finite element for sensitivity of results in the analysis, a convergence study is presented for relation between finite elements and nonlinear displacements at free end in figure 6. In this convergence study is obtained for the following parameters: L/h=50, apr =0.3, n=0.5, k=0.8, P=1000 N. As seen from figure 6, it is converged when the finite element is as 80 and this value is used in the numerical studies.

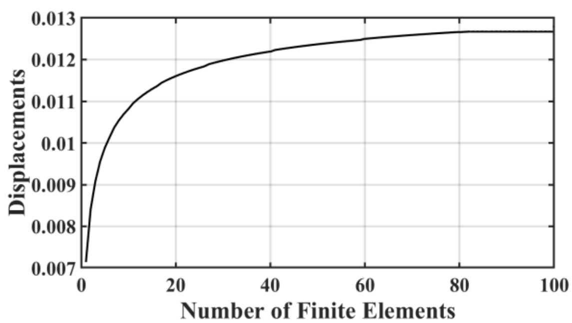


**Fig. 6. Convergence analysis of numbers of Finite Elements for nonlinear displacements at free end.**

In order to verify the used formulations and method, special results of this study is obtained and compared with analytical results of cantilever bar element made of homogenous material without pores under axial point load at free end in figure 7. Analytical solution of this comparison study can be obtained as $v = (P/CA)^{(1/k)}L$ at free end. In this comparison study, nonlinear displacements at free end versus load are compared with analytical solution. This comparison study is obtained for following parameters: *L*/*h*=20, $a_{pr}$ =0, *n*=0, *k*=0.5, *P*=1000 N. Figure 7 shows that there are good harmony in between present results and the analytical solution.

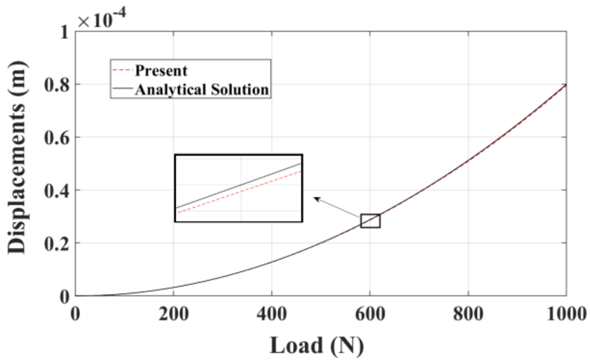


**Fig. 7. Comparison study: nonlinear displacements at free end versus load in both present results and analytical solution for *k*=0.5.**

In the obtaining numerical results, firstly, effects of graduation coefficient of FGM ($n$), material nonlinearity coefficient ($k$) and the volume fraction of porosity ($a$) on the nonlinear responses of the porous cantilever bar element are examined separately in figures 8-10. One of these parameters is taken as a variable and other is kept constant in figures 8-10. Secondly, combined effects of these parameters are examined with different values of aspect ratio ($L/h$) in figure 11.

In figure 8, influences of graduation coefficient of FGM ($n$) on material nonlinear deflections of axially graded porous bar are plotted. In this figure, relations between load-nonlinear displacements at free end of the bar for different $n$ parameter for aspect ratio $L/h$=50, volume fraction of porosity $a_{pr}$=0.1, material nonlinearity coefficient $k$=0.8. As can be seen from figure 8, increasing graduation coefficient ($n$) yields to increasing nonlinear displacements. This increasing can be explained from figure 2 and equation 1 that increasing $n$ the material properties yields to right face properties. In the used parameters for this study, material constant of the left face is bigger than those of the right face. The rigidity of the bar element decrease by increasing $n$, so the nonlinear displacements increase naturally.

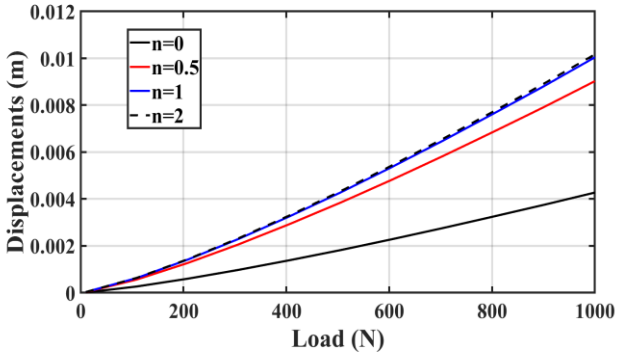


**Fig. 8. Nonlinear displacements at free end of the porous bar element versus load for different values of the graduation coefficient (*n*).**

Figure 9 displays influences of material nonlinearity coefficient ($k$) on nonlinear displacements of the axially graded porous bar element at free end versus load. This figure is plotted for used following parameters: $L/h$=50, $n$=0.5, $a_{pr}$=0.1. It is mentioned in section 2 that $k$ represents a constant which nonlinearity in stress-strain relation, namely material nonlinearity. Looking at figure 9, it is clear that the displacements change considerably with the change of $k$ parameter. When $k$ parameter approaches to 1, displacements increase significantly. The reason of this situation can be explained from figure 3 and equation 2 that the rigidity of material decreases when $k$ increases. It shows that material nonlinear property for a material is very effective on the mechanical behavior. As figure 8 shows, there is a considerably difference among of results of different $k$ parameter on the nonlinear behavior of bar element.

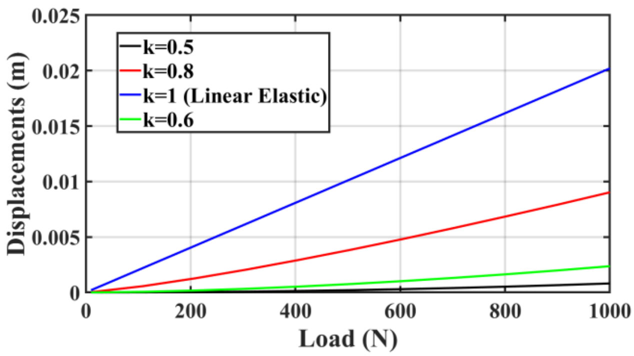


**Fig. 9. Nonlinear displacements at free end of the porous bar element versus load for various values of material nonlinearity coefficient (*k*).**

In figure 10, nonlinear displacement-load relation is presented for different values of the volume fraction of porosity ($a_{pr}$) for $L/h$=50, $n$=0.5, $k$=0.8. It is seen from figure 10 that displacements increase significantly when porosity coefficient increase. Increasing the volume fraction of porosity yields to stiffness of the material according to equation 1, so displacements increase naturally.

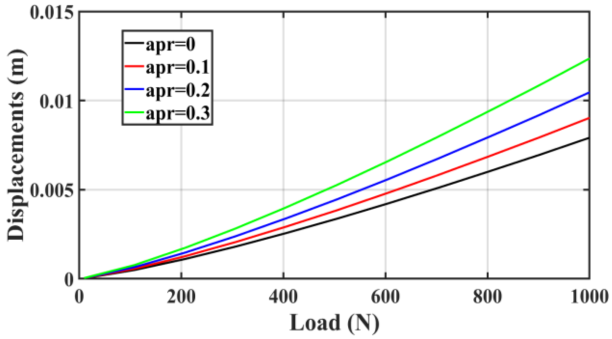


**Fig. 10. Nonlinear displacements at free end of the porous bar element versus load for different values of the volume fraction of porosity ($a_{pr}$).**

In figure 11, combined effects of graduation coefficient of FGM ($n$), material nonlinearity coefficient ($k$) and the volume fraction of porosity ($a_{pr}$) on the nonlinear displacements of the axially graded bar element are presented with different values of aspect ratio ($L/h$) for $P$=1000 N. As can be seen from figure 11, $L/h$ ratio plays important role on the nonlinear behavior of bar elements. In higher values of $L/h$ ratio, effects of porosity and material nonlinearity coefficient on the static behavior of the axially graded porous bar element increase even more.

A significant difference is found among of results of porosity parameter in lower values of material nonlinearity parameter ($k$) in figure 11. Especially, this difference is more visible for higher value of material graduation parameter ($n$). In case of linear elastic case ($k$=1), the displacements are the biggest values and porosity is more effective on the static behavior of the structure. In higher value of $n$ parameter and lower values of $k$ parameter, porosity gain important role on the static deflections of the bar element. In lower values of $k$ parameter, there is no significant change in the displacements when porosity and material graduation parameters increase or decrease. It shows that constitute behavior of material is very effective on the influences of porosity and graduation of material on the mechanical behavior of structures.

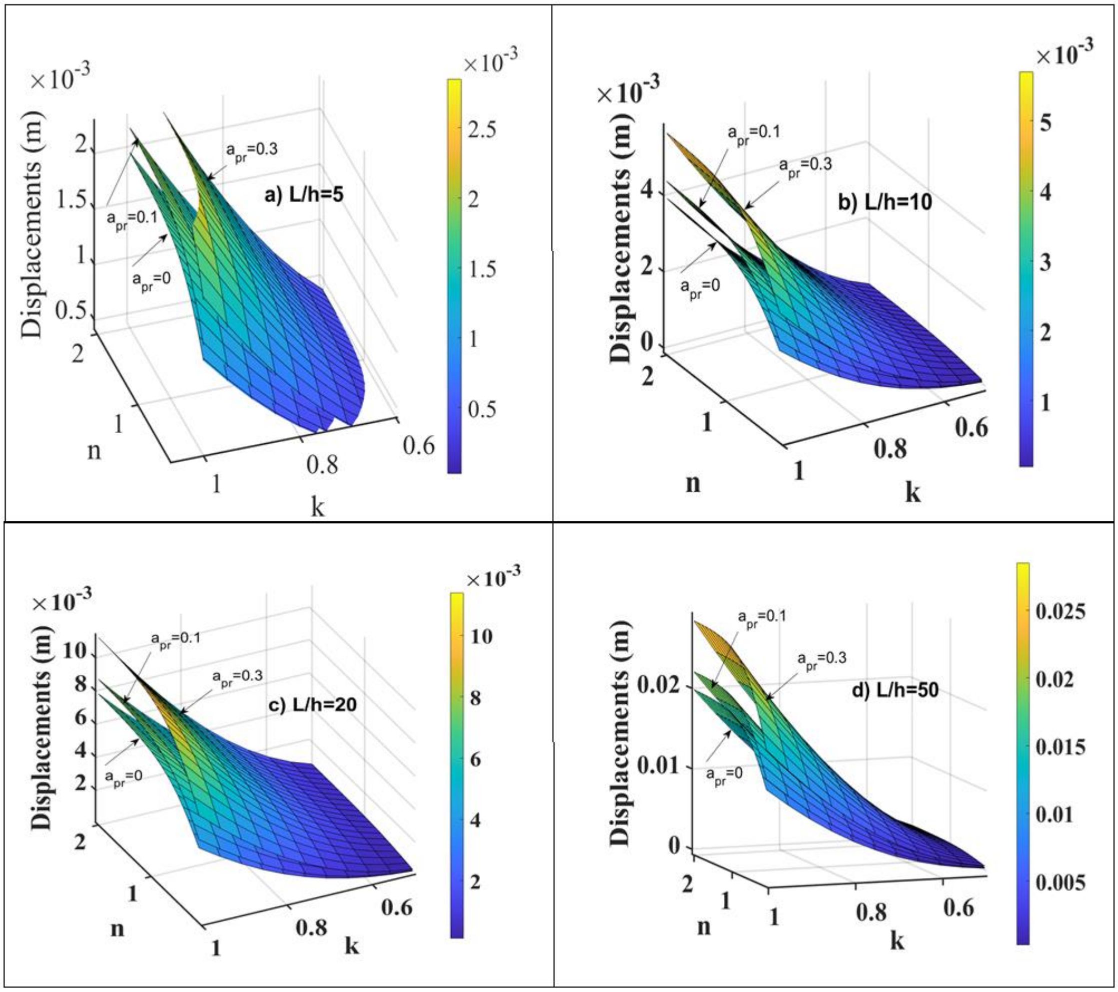


**Fig. 11 Nonlinear displacements of the porous bar at free end with different values of *n*, *k*, $a_{pr}$ and *L/h* for a) for *L/h*=5, b) for *L/h*=10, c) for *L/h*=20 and d) for *L/h*=50.**

## 4. Conclusions

In this paper, material nonlinear static analysis of a prismatic porous bar element made of functionally graded material is investigated with uniform porosity distribution model in numerically. In the material nonlinearity, the stress-strain relation is considered to change according to a Power-Law function. Material distribution of bar element change along axial direction according to a Power-Law function. Considered problem is modelled by using one dimensional finite element. The nonlinear finite element equations are obtained from the virtual work principle and solved by using he Newton-Raphson iteration method based on the incremental force approach. Influences of material nonlinearity parameters, porosity coefficients, material distribution parameters and aspect ratio on the nonlinear displacements of the bar element are investigation.

This investigation has shown that material nonlinearity coefficients play important role on nonlinear static responses of axially functionally graded porous bar elements. In lower values of material nonlinearity coefficient, influences of porosity and material graduation parameters on the nonlinear deflection are very small. When the material of bar element gets to linear elastic property, namely increasing in k parameter, the bar element gains more sensitive to porosity and graduation of material. Also, aspect ratio is very effective on the nonlinear behaviour of axially graded porous bar elements.

In the literature, no previous study has investigated material nonlinearity analysis of functionally graded porous bars and material nonlinearity studies about functionally graded structures are limited. In this study, it is aimed to investigate these gaps in the existing literature. It is believed that used formulations and material nonlinear model could be useful for researches. A limitation of this study is to consider with geometrically nonlinear of this problem. In future, a study could be investigated and developed with considering geometrically nonlinearity by researches.

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