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RESEARCH PAPER

Pull-in behaviour of a micro switch actuated by the electrostatic under a uniform longitudinal magnetic field based on nonlocal couple stress theory

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Abstract

Today the micro electromechanical systems industry is widely developed. This article aims to study static pull-in instability of a clamped micro-switch which is exerted by an electric potential difference in presence of a longitudinal magnetic field. The size dependent nonlocal couple stress theory in framework of Bernoulli-Euler beam hypothesis is utilized to model a clamped micro-switch. The equilibrium equation of micro-beam in micro-switch is derived using the principle of virtual work. To obtain the dimensionless pull-in voltage of micro-switch, the equilibrium equation is solved by Galerkin method. The effect of longitudinal magnetic field and some geometric parameter of micro-beam on the pull-in voltage is studied, taking into account the effects of a set of size dependent factors with and without considering the fringing field. The results from developed model are validated by comparing them with benchmark results.

Keywords: Pull-in, micro switch, nonlocal couple stress, magnetic field, Bernoulli-Euler;

1. Main text

In recent years, micro electromechanical systems (MEMS) have been widely used in many scientific and industrial fields such as medical, automotive, communication and aviation [1-5]. So that many researches are devoted to design and fabricate of MEMS devices. MEMS is a set of techniques and processes for design and fabricate of microstructures. The factor that distinguishes these modern techniques from conventional solutions is ability to operate more accurately and stability[6]. MEMS has become popular duo to their various advantages such as small size, high performance, low cost of the batch fabrication, reduction in power consumption, high accuracy and reliability [2, 4, 6-8]. MEMS devices are generally divided into two categories: sensors and actuators. Sensor devices gather information from their surrounding and actuators execute given commands from electrical elements. About actuators, some type of actuations can be exerted. Microswitches are the actuators that can operate based on the electrostatic, electrothermal, electromagnetic, and piezoelectric actuations. Among those, electrostatic actuation acts based on the attraction forces duo to an electrical potential deference induced between two conductive electrodes or elements[9], and magnetic actuation operates based on Lorentz forces. Microbeam is one of the most common mechanical components in microswitches. Microswitches are usually comprise of a conductive flexible microbeam,

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a fixed conductive substrate and a dielectric layer in which microbeam is suspended above the substrate by the dielectric layer[10]. Microbeam will be at rest as long as no voltage is applied. If a constant voltage is applied across the microbeam and substrate, an attractive force forms between them, and movable electrode deforms toward the substrate. The electrostatic force associated with voltage is nonlinear[11], so at a critical voltage which is known as pull-in voltage, microbeam collapse on the substrate. In recent years many researchers investigate the pull-in instability of microbeam structures. Tavakolian et al. [12] considered small scale effects to study pull-in instability of the microswitch using Eringen's nonlocal elasticity theory in the presence of thermal and residual stress effects. Chowdhury et al. [13] developed a linearized, uniform approximate model conjunction with the load deflection model of a MEMS cantilever beam to calculate the pull-in voltage with high accuracy. A simple methodology is presented to determine small deflection of diaphragm in MEMS structures with pressure by Sharma and George [14]. Mobki et al. [15] used a modified non-linear mass–spring model to investigate the behavior of a microswitch composed of a microbeam suspended between two conductive stationary plates. Fakhrabadi et al. [16] studied the vibration, natural frequencies and dynamic pull-in characteristics of the carbon nanotubes in detail using modified couple stress theory. Torabi et al. [17] analyzed the dynamic and static pull-in of rectangular nanoplates made of functionally graded materials based on the nonlocal strain gradient theory and considering disparate boundary conditions. Hosseini et al. [18] study the pull-in voltage and the effects of electrostatic forces, fringing field, and initial gap in detail for different boundary conditions in nanobeams based on the nonlocal strain gradient theory. Rahaeifard et al. [19] investigates the deflection and static pull-in voltage of microcantilevers based on the modified couple stress theory and they show that the couple stress theory can remove the gap between the experimental observations and the classical theory.

The accurate determination of pull-in voltage is critical in microswitches design. When the size of structure is close to micro scale, the small-scale effect on the mechanical behaviour becomes impressive. All experimental studies and molecular simulations conform the size-effect in micro and nano scale. Experimental methods are very expensive and difficult, and molecular simulations is computationally very expensive[20], on the other hand classical continuum mechanic is not able to predict size-scale effect on the mechanical behaviour, so to remove this enormous barrier, some nonclassical (high-order) continuum theories containing length scale parameters have been presented by scientists such as: non-local elasticity, modified couple stress, strain gradient and micropolar.

In this paper, the static pull-in instability of an electrostatically actuated clamped microswitch under a uniform longitudinal magnetic field is investigated using size-dependent nonlocal couple stress theory.

2. Equilibrium equation

To illustrate the structure of double-clamed microswitch under electrostatic and magnetic actuation, a conceptual schematic view of it, is depicted in Fig. 1.

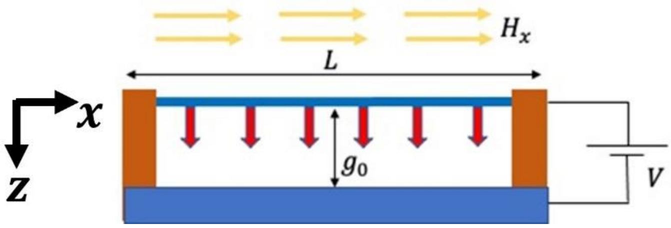

Fig 1: Schematic of double-clamped microswitch

The microswitch consists of a conductive microbeam of length L with rectangular cross section of width b and thickness h, which is suspended above the substrate at the distance of g_0. According to Euler-Bernoulli beam hypothesis, the displacement field of microbeam defined in the form below:

$$u_1 = -z\frac{dw}{dx}, \quad u_2 = 0, \quad u_3 = w \tag{1}$$

where, u_1, u_2 and u_3 denote the infinitesimal displacement components of microbeam in x, y and z direction respectively, and w is the transverse deformation of microbeam. According to the nonlocal couple stress theory, by considering the local rotational degree of freedom of a specific particle, the components of the strain and the symmetric curvature tensor for microbeam are respectively defined as follow[21]:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial u_j} + \frac{\partial u_j}{\partial u_i}\right) \tag{2}$$

$$\chi_{ij} = \frac{1}{2}\left(\frac{\partial \theta_i}{\partial x_j} + \frac{\partial \theta_j}{\partial x_i}\right), \qquad \vec{\theta} = \frac{1}{2}curl\left(\vec{u}\right) \tag{3}$$

where θ_i are the components of the rotation vector So, the nonzero components of the strain and the symmetric curvature tensor based on the nonlocal couple stress theory respectively achieved as below:

$$\varepsilon_{xx} = -z\frac{d^2w}{dx^2} \tag{4}$$

$$\chi_{xy} = \chi_{yx} = \frac{-1}{2}\frac{d^2w}{dx^2} \tag{5}$$

According to the nonlocal couple stress theory, the local rotation, at a point of the continuum induces an additional couple stress, so the variation of strain energy is defined as below:

$$\delta U = \int_0^L \int_A \left(\sigma_{ij}\delta\varepsilon_{ij} + m_{ij}\delta\chi_{ij}\right) dAdx \tag{6}$$

Where, σ_{ij} is components of the Cauchy stress and m_{ij} is components of the deviatoric part of the couple stress tensor[22].

$$\sigma_{ij} = \lambda\ \varepsilon_{kk}\delta_{ij} + 2G\varepsilon_{ij} \tag{7}$$

$$m_{ij} = 2\ell^2 G\chi_{ij} \tag{8}$$

Where λ and G are the Lame's constants in classical elasticity theory and δ_{ij} is the Kronecker delta. on the other hand, stress at a point in the continuum is a function not only of the strains at that point, but also of the strains at all other points of the continuum, so by ignoring the Poisson's ratio υ, the constitutive relations for microbeam are given by[23]:

$$\left(1-\mu^2\nabla^2\right)\sigma_{xx} = E\varepsilon_{xx} \tag{9}$$

$$\left(1-\mu^2\nabla^2\right)m_{xy} = 2\ell^2 G\chi_{xy} \tag{10}$$

Where, ℓ denotes the material length scale parameter that measures the effect of couple stress, and μ is the nonlocality parameter. Moreover, ∇ and E are Nabla operator and Yang modulus respectively. Using Eq. (6), the strain energy relation is simplified as follow:

$$\delta U = \int_0^L -\left(M+Y\right)\frac{d^2 w}{dx^2}\left(\delta w\right)dx \tag{11}$$

Where, M and Y are defined below:

$$M = \int_A z\sigma_{xx}\,dA \tag{12}$$

$$Y = \int_A m_{xy}\,dA \tag{13}$$

So, using integral by parts, the strain energy relation and related boundary conditions are obtained:

$$\delta U = \int_0^L -\frac{d^2}{dx^2}\left(M+Y\right)\left(\delta w\right)dx - \left(M+Y\right)\delta\left(\frac{dw}{dx}\right)\Bigg|_0^L + \frac{d}{dx}\left(M+Y\right)\left(\delta w\right)\Bigg|_0^L \tag{14}$$

The variation of the external work contains of electrostatic field and magnetic forces including fringing field is given by:

$$\delta W^{ext} = \int_0^L \left(F_e + F_m\right)\delta w\, dx = \int_0^L \frac{\varepsilon_0 b V^2}{2\left(g_0 - w\right)^2}\left(1 + 0.65\frac{\left(g_0 - w\right)}{b}\right)\delta w dx + \int_0^L \eta H_x^2 A \frac{d^2 w}{dx^2}\delta w dx \tag{15}$$

where $\varepsilon_0 = 8.854\times10^{-12} C^2 N^{-1} m^{-2}$ denotes vacuum permittivity, $\eta = 4\pi\times10^{-7}$ is magnetic permeability and H_x, V and A are longitudinal magnetic field, applied voltage and cross section area of microbeam. By multiplying Eqs. (9) and (10) by z and integrating over cross section area of the microbeam then second order derivation respect x yields:

$$\left(1 - \mu^2 \frac{d^2}{dx^2}\right)\frac{d^2 M}{dx^2} = -EI\frac{d^4 w}{dx^4} \tag{16}$$

$$\left(1 - \mu^2 \frac{d^2}{dx^2}\right)\frac{d^2 Y}{dx^2} = -\ell^2 GA\frac{d^4 w}{dx^4} \tag{17}$$

Consequently, constitutive equation will be obtained from the summation of Eqs. (16) and (17):

$$\left(1 - \mu^2 \frac{d^2}{dx^2}\right)\frac{d^2\left(M + Y\right)}{dx^2} = -\left(EI + \ell^2 GA\right)\frac{d^4 w}{dx^4} \tag{18}$$

To derive equilibrium equation of transverse deformation of microbeam, principle of virtual displacement is applied.

$$\delta U - \delta W^{ext} = 0 \tag{19}$$

Substitution of Eqs. (14) and (15) into Eq. (19), we get

$$\begin{aligned}
&\left[EI + \mu^2 \eta H_x^2 A + \ell^2 GA\right]\left(g_0 - w\right)^4 \frac{d^4 w}{dx^4} \\
&+\left[\frac{\mu^2 \varepsilon_0 b V^2}{2}\left(2\left(g_0 - w\right) + \frac{0.65}{b}\left(g_0 - w\right)^2\right) - \eta H_x^2 A\left(g_0 - w\right)^4\right]\frac{d^2 w}{dx^2} \\
&+\left[\frac{\mu^2 \varepsilon_0 b V^2}{2}\left(6 + \frac{1.3}{b}\left(g_0 - w\right)\right)\right]\left(\frac{dw}{dx}\right)^2 \\
&-\frac{\varepsilon_0 b V^2}{2}\left(\left(g_0 - w\right)^2 + \frac{0.65}{b}\left(g_0 - w\right)^3\right) = 0
\end{aligned} \tag{20}$$

To derive equilibrium equation of microbeam in nondimensional form, these dimensionless quantities are employed:

$$\bar{w}=\frac{w}{g_0},\ \bar{x}=\frac{x}{L},\ \bar{H}=\frac{\eta H_x^2}{E},\ \bar{V}=\frac{\varepsilon_0 L^2 V^2}{Eh g_0^3},\ G=\frac{E}{2(1+\upsilon)} \tag{21}$$

consequently, the dimensionless equilibrium equation of microswitch actuated by electrostatic and magnetic field based on the nonlocal couple stress theory is presented as below:

$$\begin{aligned}
&\left[1+\frac{6}{1+\upsilon}\frac{1}{(\ell)^2}+\bar{H}\frac{12}{(h/\mu)^2}\right](1-\bar{w})^4\frac{d^4\bar{w}}{d\bar{x}^4}\\
&+\left[\frac{6}{(h/\mu)^2}\bar{V}\left(2(1-\bar{w})+\frac{0.65g_0}{b}(1-\bar{w})^2\right)-12\bar{H}\frac{1}{(h/L)^2}(1-\bar{w})^4\right]\frac{d^2\bar{w}}{d\bar{x}^2}\\
&+\left[\frac{6}{(h/\mu)^2}\bar{V}\left(6+1.3\frac{g_0}{b}(1-\bar{w})\right)\right]\left(\frac{d^2\bar{w}}{d\bar{x}^2}\right)^2\\
&-\left[6\bar{V}\frac{1}{(h/L)^2}\left((1-\bar{w})^2+0.65\frac{g_0}{b}(1-\bar{w})^3\right)\right]=0
\end{aligned} \tag{22}$$

And boundary conditions related to clamped microbeam can be expressed as below:

$$\bar{w}=0,\ \frac{d\bar{w}}{d\bar{x}}=0,\quad @\ \bar{x}=0,1 \tag{23}$$

3. Solution method

To determine the pull-in voltage in microswitch, the equilibrium equation of microbeam should be solved. Duo to nonlinearity of the system, the Galerkin weighted residual method is employed to achieve displacement of middle of microbeam in clamped microswitch. Applying an electrical voltage between microbeam and substrate causes an attractive electrostatic force between them. At a critical voltage called pull-in voltage, the displacement of middle of microbeam suddenly increases duo to an infinitesimal increase in applied voltage. According to Galerkin weighted residual method an approximate solution is selected as below:

$$\bar{w}=\sum_{i=0}^{N}a_i\varphi_i(\bar{x})=a_1\varphi_1(\bar{x})+a_2\varphi_2(\bar{x})+\ldots a_N\varphi_N(\bar{x}) \tag{24}$$

where $\varphi_i(\bar{x})$ are trial functions which must satisfy the geometry boundary conditions of the equilibrium equation, so the first term of the approximate solution is considered, and the nondimensional first mode shape function of the classical Bernoulli-Euler beam model is selected because it satisfies the clamped boundary condition of microbeam as below.

$$\begin{aligned}
&\varphi(\bar{x})=\cosh(\Omega\bar{x})-\cos(\Omega\bar{x})-(\alpha)\left(\sinh(\Omega\bar{x})-\sin(\Omega\bar{x})\right)\\
&\alpha=0.9825022145,\quad \Omega=4.730040745
\end{aligned} \tag{25}$$

and a_i is the unknown coefficient which must be calculated by setting the integral below over domain of the deferential equation to zero.

$$\int_0^1 R(\bar{x})\varphi_1(\bar{x})d\bar{x} = 0 \tag{26}$$

where $R(\bar{x})$ called residual which is obtained by substituting approximate solution into equilibrium equation.

4. Results

Geometric and material properties of microswitch in Table 1 are presented to determine nondimensional pull in voltage. It should be noted that in this study, the effective modulus of elasticity $E/(1-\upsilon^2)$ is utilized.

Table 1: Material and geometric properties utilized to calculate pull-in voltage in this paper.

Quantities	**values**
Length (L)	$100\mu m$
Width (b)	$10\mu m$
thickness (h)	$1\mu m$
Young's modulus (E)	$169Gpa$
Poisson's ratio (υ)	0.3
Initial Gap (g_0)	$1\mu m$

In addition to valid the results of this paper, pull-in voltages for cantilever microbeam are compared with results of Rahaeifard et al[19]. in table 2.

Table 2: comparison pull-in voltages between results of this paper and results given by Rahaeifard in cantilever microbeam model for $b = 50\mu m$, $h = 2.94\mu m$, $g_0 = 1.05\mu m$, $G = 65.8Gpa$, $E = 169.2Gpa$, $\mu = 0$.

$L(cantilever\ length)$	$V_p(classic)$		$V_p(nonclassic)$			
	Present study	Rahaeifard	Present study		Rahaeifard	
			$h/l = 4$	$h/l = 8$	$h/l = 4$	$h/l = 8$
100	39.43	39.69	44.76	40.83	45.11	41.11
150	17.53	17.64	19.9	18.15	20.05	18.27
200	9.86	9.92	11.19	10.21	11.27	10.28

Fig. 2 is presented to investigate the effect of the ratio h/l on the dimensionless pull-in voltage of clamped microswitch for various magnetic field intensity. In all diagrams, in small value of h/l the pull-in voltage is size dependent. in the other hand, as the ratio h/l increases, the size effect on the pull-in voltage becomes negligible. It is also observed that not considering the fringing field on the electrostatic force relation predicts higher pull-in voltage. In addition, in all cases increasing the intensity of magnetic field increases the pull-in voltage of clamped microswitch.

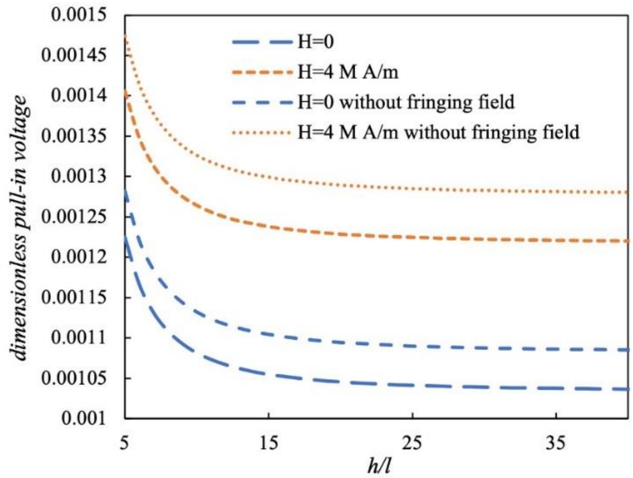

Fig. 2 the effect of ratio of h/l on the dimensionless pull-in voltage for various magnetic field intensity with and without considering fringing field with $L = 100\mu m$ and $l = 0$.

In the other hand the effect of ratio h/μ on the dimensionless pull-in voltage is depicted in Fig. 3. As inferred from this figure increasing the ratio h/μ reduces the dimensionless pull-in voltage for various magnetic field intensity. In small values of the ratio h/μ nonlocal size effect is more significant than higher values of them. In this case, increasing the magnetic field intensity reduces the dimensionless pull-in voltage in clamped microswitch.

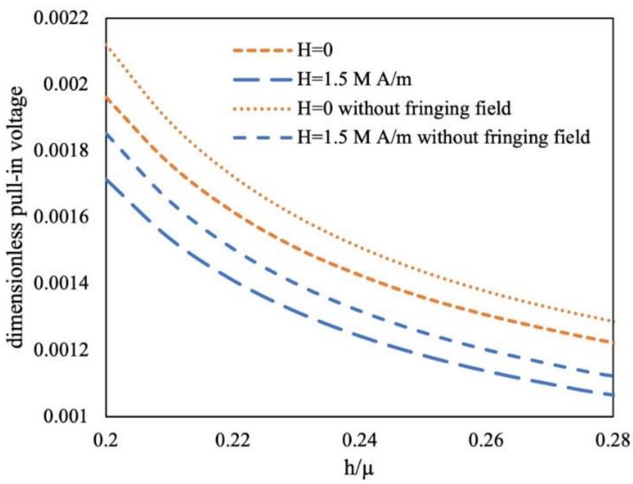

Fig. 3 the effect of ratio of h/μ on the dimensionless pull-in voltage for various magnetic field intensity with and without considering fringing field.

Fig. 4 shows the effect of dimensionless magnetic field versus dimensionless pull-in voltage for various b/g_0 taking into account to fringing field. It can be seen increasing the dimensionless magnetic field increases dimensionless pull-in voltage. As the ratio b/g_0 increases, the results approach to condition that the fringing field is not considered. It is worth noting when the fringing field effect is ignored, the pull-in voltage in the microbeam with a rectangular cross section is independent of the microbeam width.

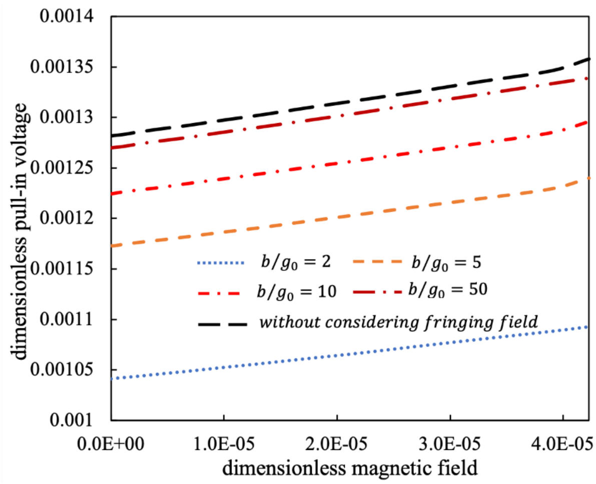

Fig. 4 dimensionless pull-in voltage versus dimensionless magnetic field for various b/g_0 with $L=100\mu m$, $\mu=0$ and $h/l=5$.

As shown in Fig. 5 increasing the dimensionless magnetic field causes an increase in dimensionless pull-in voltage. As the ratio L/h increases the dimensionless pull-in voltage approaches to the results obtained from classical elasticity theory. It should be noted that influence of longitudinal magnetic field intensity is same in both classical and nonclassical theory.

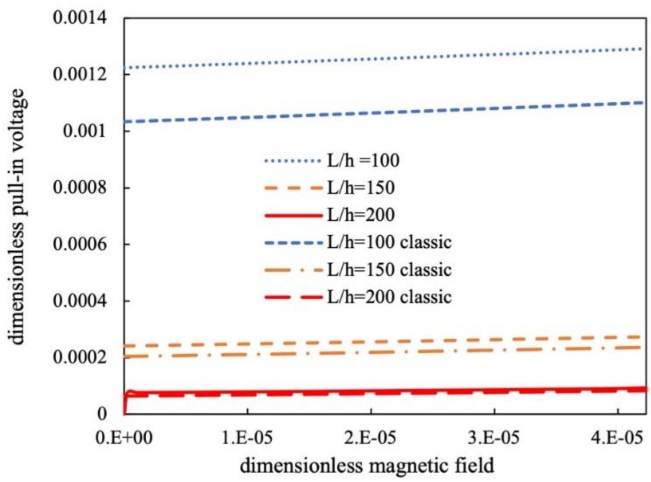

Fig. 5 dimensionless pull-in voltage versus dimensionless magnetic field for various L/h with $\mu=0$, $h/l=5$ and $b/g_0=10$.

5. Conclusion

In this paper a study is performed to investigate the influence of several geometric and material properties on the static pull-in voltage of a clamped microswitch considering induced Lorentz force duo to a longitudinal magnetic field. Using the Galerkin weighted residual method, the results are computationally solved taking into account tow size dependent factors of nonlocal couple stress theory which compared with results obtained from classical elasticity theory. The accuracy of the results is validated by comparing those with literatures. As a result, the static pull-in instability of microswitch is extremely size-dependent, so ignoring the size dependent effects cause a significant difference in prediction of pull-in voltages. By setting the $\mu = 0$, increasing of the magnetic field intensity increase the dimensionless pull-in voltage while by considering $l = 0$ increasing of the magnetic field intensity decrease the dimensionless pull-in voltage. Consequently, it can be concluded that nonlocal couple stress theory could predict both hardening and softening effects on the pull-in instability of microswitch. As the length of the microswitch increases, the electromechanical behaviour of the microswitch approaches to the classical state. In addition, the fringing field effect decreases with increasing beam width.

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