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RESEARCH PAPER

Analytical Modeling and Parametric Study of Equivalent Stiffness for Auxetic Structures

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Abstract

Auxetic materials have a negative Poisson's ratio, which is different from most engineering materials. Auxetic materials are used in various fields such as medicine, sports science, sensors and actuators, etc. An auxetic structure is made of several cells in parallel and series. In this paper, the equivalent stiffness of an auxetic cell and structure are analytically extracted. The effect of the geometrical parameters, such as the angle and beam length of the auxetic cell, on auxetic cell and structure equivalent stiffness, are investigated. The extracted equations are verified using the simulation of the auxetic structure in the Abaqus software. In this study, numerical simulation is used in order to study the effect of the parameters of the auxetic cell on its equivalent mass. The results of this study show that changing the geometrical parameters of the auxetic cell, affect the vibration behavior of the auxetic structure. Also, the effect of geometrical parameters of the auxetic structure on the Poisson's ratio is investigated.

Keywords: Auxetic structure, Poisson's ratio, Equivalent stiffness, Equivalent mass;

1. Introduction

When a material is subjected to tension, the ratio of lateral contraction strain to longitudinal tensile strain is Poisson's ratio, and most materials have a positive Poisson's ratio[1]. Auxetic materials have a negative Poisson's ratio, which is different from most engineering materials[2]. Auxetic materials can be used in various fields such as medicine, sports, science, sensors and actuators, etc.[3-6]. These materials are used in various fields. For this reason, the researchers investigated the behavior of auxetic materials. Iyer et al.[7] investigated the electromechanical properties of auxetic piezoelectric cellular solids using finite element method. Their study showed that the auxetic piezoelectric has a higher efficiency than the ordinary non- auxetic piezoelectric solid. Zhang and Yang [8] proposed a lightweight vibration isolator based on auxetic materials. They investigated the dynamic response of the presented design numerically and experimentally. Their study shows that the auxetic vibration isolator has less weight and less vibration transmission than the traditional isolator. Zhang and Yang [9] parametrically analysed the effect of Poisson's ratio and relative density of honeycombs on bearing capacity and dynamics using the finite element method and experimental models. Their analysis shows that the ultimate bearing capacity of auxetic materials is independent of scale by keeping Poisson's ratio and relative density constant. Imbalzano et al. [10] proposed a sandwich panel with an auxetic core for impact resistance. They investigated the behavior of their proposed structure numerically. The results of their study show that the panel with an auxetic core has the same energy absorption as the monolithic panels. The similarity in energy absorption is that plastic deformation is reduced

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by 56% with an auxetic core. Duc et al. [11] investigated the vibration behaviour and dynamic response of the sandwich panel with an auxetic core using an analytical approach. They derived the governing equations using Reddy's first-order shear deformation theory and Airy stress function, the Galerkin method, as well as fourth-order Runge–Kutta. They are able to design a suitable auxetic composite structures under the blast and other mechanical loads. Peng et al. [12] investigated the dynamic behaviour of the auxetic structure under shock using finite element. According to their study, auxetic materials are suitable for suppressing shock and reducing peak values of longitudinal acceleration response. Also, in addition to shock absorption, these materials perform well in vibration isolation. Lee et al. [13] investigated the behaviour of auxetic materials in collision using experiment and the finite element method. In their review, they measured speed reduction and specific energy absorption. The results of their investigation show that the auxetic tube has better damping in low impact. Zhang et al.[14] studied the nonlinear transient responses of an auxetic honeycomb sandwich plate under impact loading. They derived the equations using Hamilton's principle and Reddy's higher shear deformation theory. By numerical simulation, they investigated the effect of different parameters such as core thickness, Poisson's ratio, and cell angle. Their study showed that a honeycomb sandwich plate with negative Poisson's ratio performs better in dynamic loading in some structures. Li et al. [15] investigated the nonlinear bending vibration behavior of sandwich beams in thermal environments using 3D finite element simulation. The core of sandwich beam Poisson's ratio is negative. The results reveal that temperature changes can significantly affect the effective Poisson's ratio (EPR)-deflection curves. Eipakchi and Nasrekani [16] presented a mathematical method to study the vibration behavior of a three-layer cylindrical shell with internal pressure. The core of the cylindrical shell is made of auxetic material. They used Hamilton's principle to derive the equation of motion. Their study shows that using an auxetic core reduces the weight of the structure, while its natural frequency remains almost constant. Dutta et al. [17] calculated Poisson's ratio using numerical analysis and Abaqus software for different auxetic structures. They investigated the configuration of structures such as trusses under four-point bending. The results of their investigation show that cell angle is one of the main factors determining Poisson's ratio. Kamthe and Walame [18], by changing the geometrical parameters of auxetic materials such as aspect ratio, thickness, number of layers and angle, investigated the energy absorption capacity of the auxetic re-entrant Hexagon structure. They simulated their model in Ansys software and used a uniaxial tensile device to test it. Their study shows that increasing the absorption thickness and decreasing the angle of the auxetic materials increases energy absorption. Ebrahimian et al. [19] used auxetic materials to increase energy harvesting and reduce the resonance frequency of an energy harvester. They used finite element modeling to evaluate structures and extract optimal parameters. The results of their study show that the use of auxetic materials in the energy harvester increases the output of electrical power and reduces the resonant frequency by 30%. Pham et al. [20] presented a finite element method for vibration analysis of sandwich nanoplates with an auxetic core. The size effect is applied in their study using nonlocal elasticity theory. They investigated the effect of geometrical parameters of auxetic materials on the free vibration behavior of sandwich nanoplates. Pham et al. [21] investigated the vibrational behaviour of carbon fiber-reinforced polymer three-dimensional auxetic materials. They investigated the effect of negative Poisson's ratio of auxetic materials analytically. The results of their study show that the configuration of the auxetic core affects the fundamental frequency. Jing Li and Ji Zhou [22] investigated the vibrational behavior of a sandwich plate with an auxetic core. They derived the nonlinear equations of motion based on the nonlinear von Karman theory and the third-order shear deformation theory. The results of their study show that there will be no periodic vibration without transverse excitation.

In this study, the equivalent stiffness of an auxetic cell and an auxetic structure made of several cells placed together in parallel and in series, are analytically extracted. In this work, the effect of the geometrical parameters of the auxetic cell and the auxetic structure on the equivalent stiffness is investigated. Also, using equivalent stiffness and natural frequency of the auxetic cell, which was extracted using numerical simulation, the effect of the angle of the auxetic cell on the equivalent mass was also investigated.

2. Modeling

Fig 1 shows the shape of an auxetic cell under tension. In this figure, l_1 is the length of the auxetic cell beam, and alpha (α) is the angle of the auxetic cell.

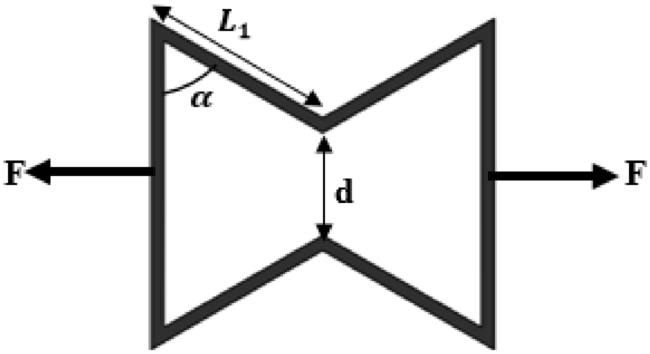

Fig 1: Auxetic cell under tension

The displacement of the auxetic cell can be calculated by using figures 2 and 3.

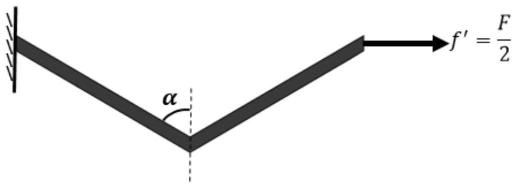

Fig 2: Auxetic half-cell under tension

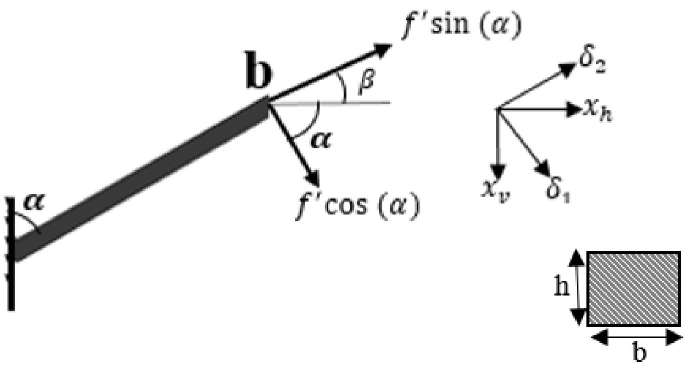

Fig 3: Displacement of the system

According to Fig 3, the displacement along the beam and perpendicular to the beam is derived as

$$\delta_1 = \frac{\left(f'\cos(\alpha)\right)l_1^3}{12EI} \tag{1}$$

$$\delta_2 = \frac{\left(f'\sin(\alpha)\right)l_1}{EA} \tag{2}$$

where, I is the moment of inertia ($I=bh^3/12$) and A is the area of cross-section ($A=bh$). Also, the modulus of elasticity is shown by E. Displacement of the beam in Fig 3 in the direction of x_h can be calculated as

$$x_1 = \delta_1 \cos(\alpha) + \delta_2 \sin(\alpha) \tag{3}$$

$$x_1 = \frac{f' \, l_1^3}{12EI} \cos^2(\alpha) + \frac{f' \, l_1}{EA} \sin^2(\alpha) \tag{4}$$

Using equations 3 and 4 can be written as

$$x_h = 2x_1 \tag{5}$$

where x_h is the horizontal displacement of the auxetic half-cell under tension. From the equation 5 displacement of the auxetic cell equal x_h. Displacement of the beam in Fig 3 in the direction of x_v can be calculated as

$$x_v = \delta_1 \sin(\alpha) - \delta_2 \cos(\alpha) \tag{6}$$

Therefore, Poisson's ratio of auxetic cell is:

$$\upsilon_{cell} = \frac{-2x_v}{x_h} = 2\frac{\frac{l_1}{EA}\sin(\alpha)\cos(\alpha) - \frac{l_1^3}{12EI}\cos(\alpha)\sin(\alpha)}{\frac{l_1}{EA}\sin^2(\alpha) + \frac{l_1^3}{12EI}\cos^2(\alpha)} \tag{7}$$

Also, Poisson's ratio is calculated for the auxetic structure with i cells in series and j cells in parallel (isjp) as the following equation:

$$\upsilon_{axc.str.} = \frac{\upsilon_{cell}}{i} \tag{8}$$

The equivalent stiffness of auxetic cell can be calculated as

$$k_{eq.\,cell} = \frac{F}{x_h} = \frac{1}{\frac{l_1}{EA}\sin^2(\alpha) + \frac{l_1^3}{12EI}\cos^2(\alpha)} \tag{9}$$

$$\omega_n = \sqrt{\frac{k_{eq.\,cell}}{M_{eq.\,cell}}} \tag{10}$$

where ω_n and M_{eqcell} represent the natural frequency and the equivalent mass of the auxetic cell, respectively. Numerical simulation is used to calculate the natural frequency of the auxetic cell. The ratio of auxetic cell's equivalent mass to the auxetic cell's mass is a function of the auxetic cell angle (α). Therefore, it can be written as

$$C(\alpha) = \frac{M_{eq.\,cell}}{M_{cell}} \tag{11}$$

where M_{eqcell} is the mass of the auxetic cell. The auxetic structure consisting of i series and j parallel auxetic cell (isjp) can be calculated the equivalent stiffness of the auxetic structure. Auxetic structure in Fig 4 consists of 3 series cells and 3 parallel cells (3s3p).

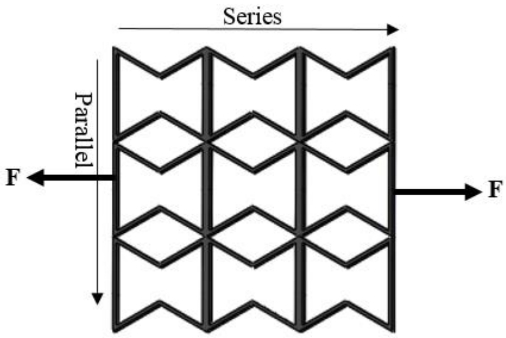

Fig 4: Auxetic structure 3s3p

For *m* series auxetic cell, the equivalent stiffness is:

$$\frac{1}{k_{eq.m\ series\ axc.\ cell}} = \underbrace{\frac{1}{k_{eq.\ cell}} + \frac{1}{k_{eq.\ cell}} + \ldots + \frac{1}{k_{eq.\ cell}}}_{m\ series\ axc.\ cell} \tag{12}$$

$$k_{eq.\ m\ series\ axc.\ cell} = \frac{k_{eq.cell}}{m} \tag{13}$$

For n parallel auxetic cell, equivalent stiffness is:

$$k_{eq.n\ parallel\ axc.\ cell} = \underbrace{k_{eq.cell} + k_{eq.cell} + \ldots + k_{eq.cell}}_{n\ parallel\ axc.\ cell} \tag{14}$$

Use of equations 13 and 14, the equivalent stiffness of auxetic structure can be calculated as

$$k_{axc.\ str.} = \frac{nk_{eq.cell}}{m} \tag{15}$$

When the elastic structure has a length *X* and width *Y*, so that the sum of series cells is equal to *X* and the sum of parallel cells is equal to *Y*, the equivalent stiffness can be obtained as

$$X = 2ml_1 \sin(\alpha) \quad , \quad Y = n\left(d + 2l_1 \cos(\alpha)\right) \tag{16}$$

By inserting equation 16 in equation 15, the equivalent stiffness of auxetic structure can be derived as

$$k_{eq.\ axc.\ str.} = \frac{2l_1 \sin(\alpha) Y k_{eq.cell}}{X\left(d + 2l_1 \cos(\alpha)\right)} \tag{17}$$

with $d = ql_1$, equation 17 can be written as

$$k_{eq.\ axc.\ str.} = \frac{2\sin(\alpha) Y k_{eq.\ cell}}{X\left(q + 2\cos(\alpha)\right)} \tag{18}$$

In this case, Poisson's ratio obtained as

$$\upsilon_{axc.\ str.} = \frac{2l_1 \sin(\alpha)\upsilon_{cell}}{X} \tag{19}$$

3. Result and discussion

The equation 4 is employed to obtain displacement and stiffness of the auxetic structure. Therefore, this equation is very important. The accuracy of this equation has been compared with the results obtained from the simulation of the auxetic structure in Abaqus. Fig 5 shows an example of modelling. The comparison results are presented in Table 1.

Table 1: Comparison of analytical with simulation results

Number of cells	Cross section (A) (mm^2)	Length (l_1) (mm)	Angle (α)	Simulation (mm)	Analytical (mm)	Error (%)
single cell	1.5*0.1	1.3	π/3	1.802e-2	1.7709e-2	1.7
single cell	1.5*0.1	1.3	π/6	5.324e-2	5.2303e-2	1.7
3s2p	1.5*0.1	1.3	π/6	7.985e-2	7.8454e-2	1.6
3s2p	1.5*0.1	1.3	π/3	2.703e-2	2.6563e-2	1.7
3s2p	0.01 π	1.3	π/6	1.318e-1	1.2555e-1	4

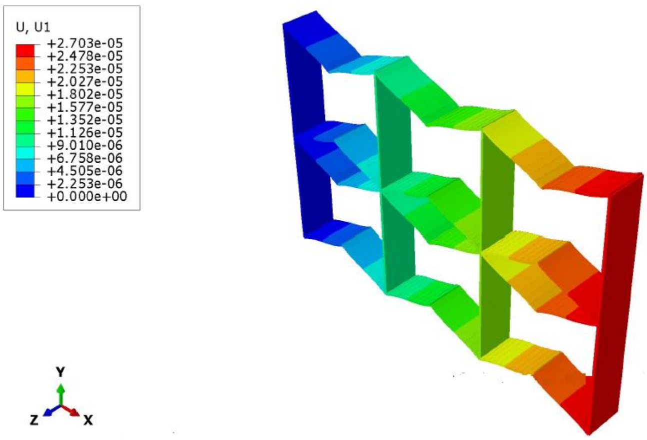

Fig 5: Simulation example in Abaqus

The negative Poisson's ratio is one of the reasons for using an auxetic structure. Fig 6 depicts the influence of the cell angle (α) and the cell beam length on the Poisson's ratio of the auxetic structure. Fig 6 shows that the absolute value of Poisson's ratio increases from 0° to 90°. At an angle of 90 degrees, Poisson's ratio becomes zero. Obtained results in Fig 6 show that increasing l1 increases the absolute magnitude of Poisson's ratio.

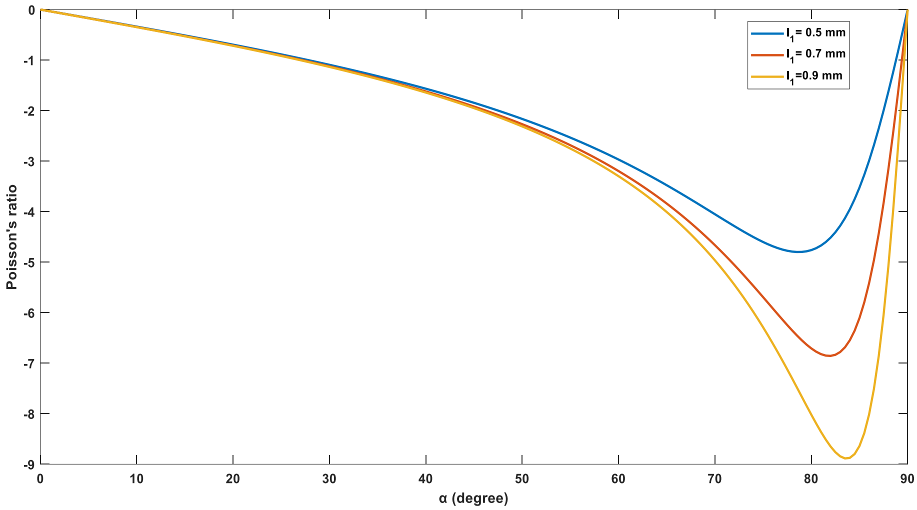

Fig 6: Poisson's ratio of the auxetic cell for α= 0 - 90 degree

The effect of α and l_1 on the equivalent stiffness of an auxetic cell is presented in Fig 7. As illustrated, the equivalent stiffness increases with the increase of the angle from 0 to 90 degrees. Also, the stiffness decreases with the increase in the beam length of the auxetic cell. Comparison of the Fig 6 and Fig 7 shows that the effect of beam length on equivalent stiffness and Poisson's ratio is opposite. The opposite behaviour indicates that the Poisson's ratio decreases as the cell's stiffness increases.

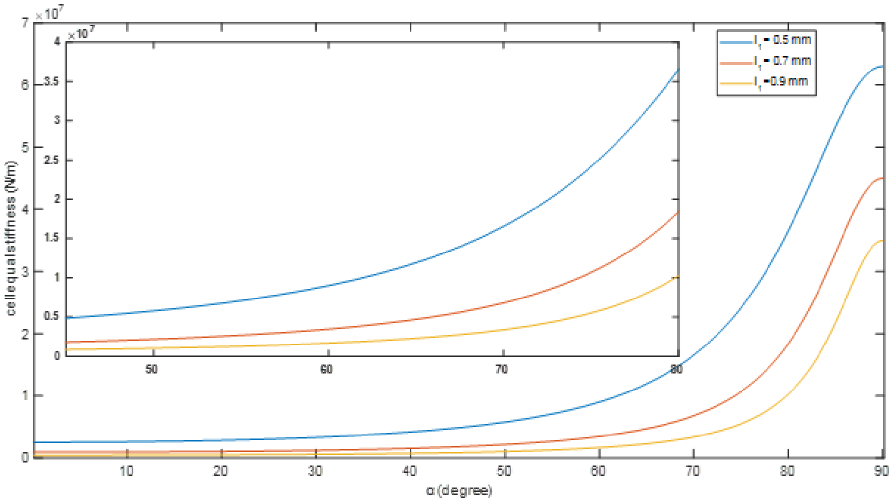

Fig 7: Equivalent stiffness of the auxetic cell for α= 0 - 90 degree

The auxetic structure length is 100 mm. The Poisson’s ratio for this auxetic structure is presented in Fig 8. The behaviour of this diagram similar to the behaviour of the Fig 7, which is for an auxetic cell, but the Poisson's ratio has decreased.

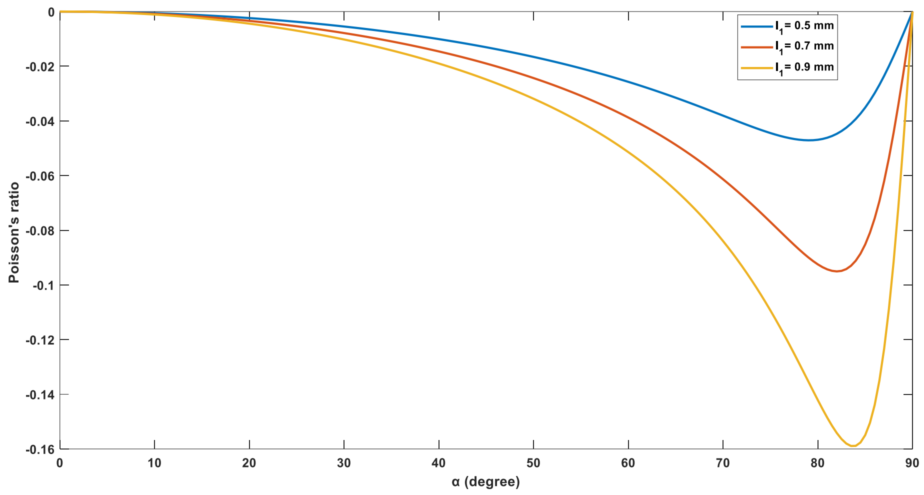

Fig 8: Poisson's ratio of the auxetic structure for α= 0 – 90 degree

The auxetic structure length and width is 100 mm. The equivalent stiffness of this auxetic structure is presented in Fig 9. The general behaviour of an auxetic structure is similar to an auxetic cell.

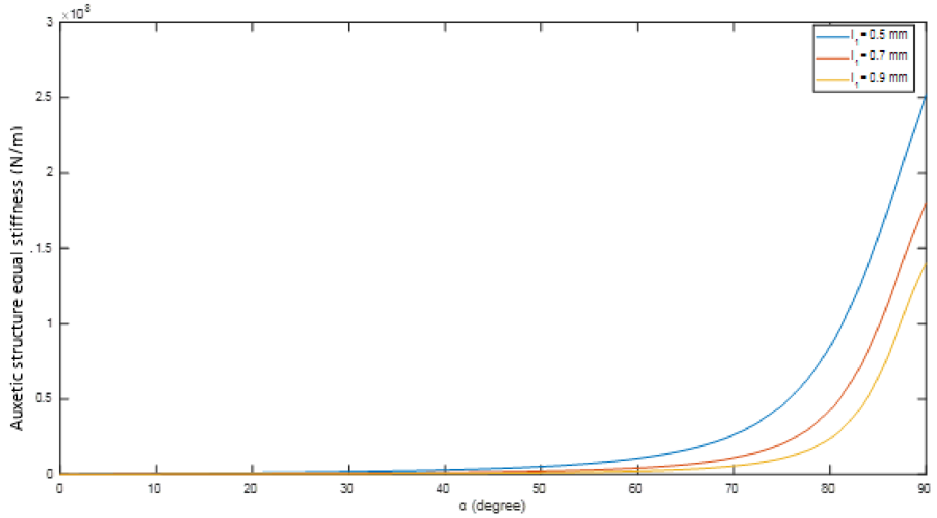

Fig 9: Equivalent stiffness of the auxetic structure for α= 0 - 90 degree

Fig 9 depicts the equivalent stiffness of the auxetic structure for the angle from 0 to 60 degree is smaller than the equivalent stiffness of the auxetic structure for the angle from 60 to 90 degree. For this reason, the equivalent stiffness difference in 3 different lengths of the auxetic structure beam at 0 to 60 degree is unclear. Fig 10 depicts the equivalent stiffness for the angle from 0 to 60 degree.

The auxetic cell angle effect on the equivalent mass is demonstrated in Fig 12. The equivalent mass increases with the increase of the angle of the auxetic cell. Unlike normal engineering materials with a positive Poisson's ratio, the equivalent mass in system's vibration is less than the beam's mass. In these materials, according to the angle of the auxetic cell, the equivalent mass is greater than the mass of the auxetic cell. According to the diagram below, from an angle of 70 degrees and above, the equivalent mass is greater than the mass of the auxetic cell. Using curve fitting in MATLAB, an equation is extracted for the ratio of equivalent mass to cell mass at angle from 5 to 80 degrees.

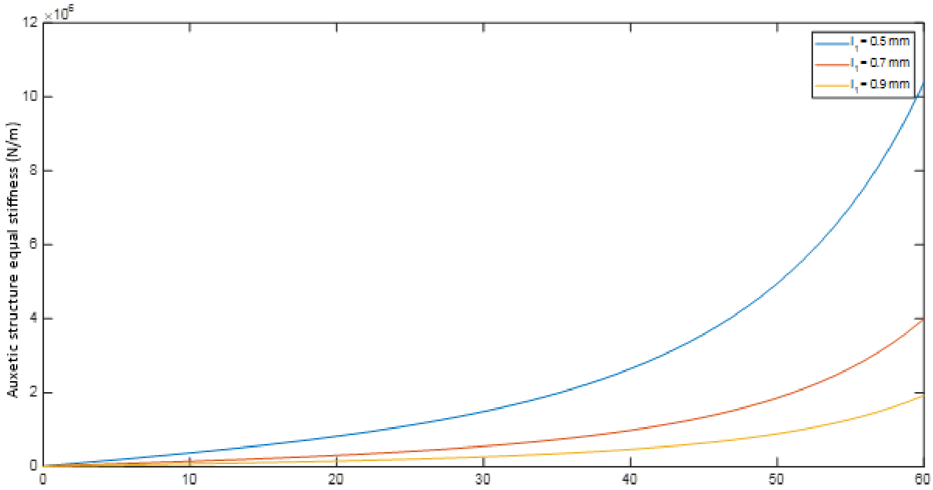

Fig 10: Equivalent stiffness of the auxetic structure for *α*= 0 - 60 degree

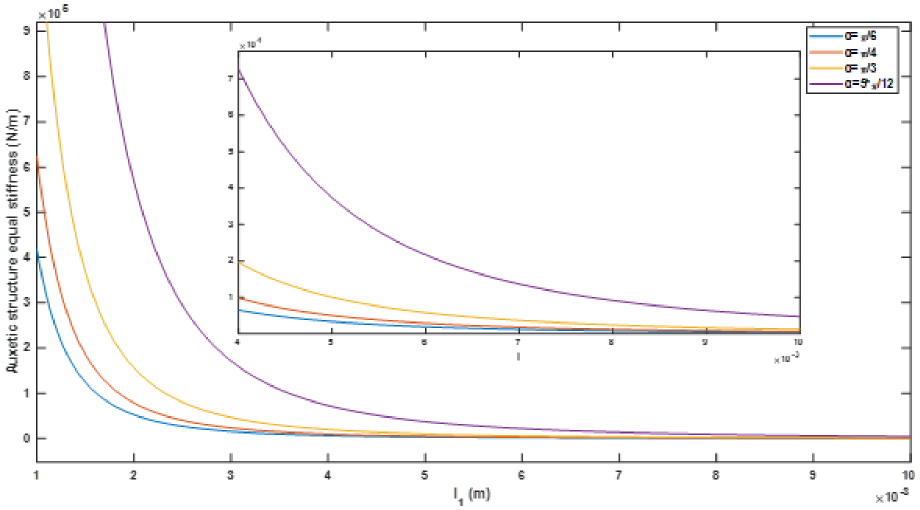

Fig 11: Equivalent stiffness of the auxetic structure for l_1= 1 mm to 10 mm

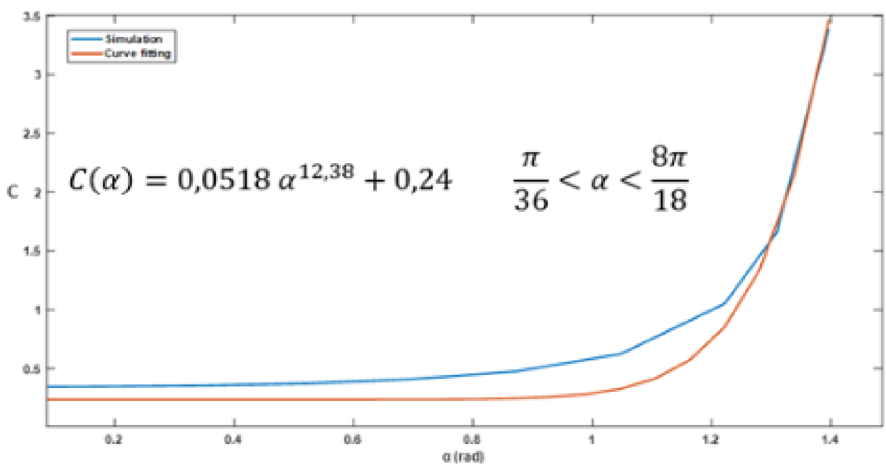

Fig 12: The ratio of the Equivalent mass to the mass of the cell

Fig 13 shows that the natural frequency increases with an increase in the angle of the auxetic cell. According to Figs 7 and 10, increasing the angle of the auxetic cell increases the equivalent stiffness and equivalent mass of the cell. According to equation 10 and the behaviour of Fig 13, the stiffness increase rate is greater than the equivalent mass increase rate.

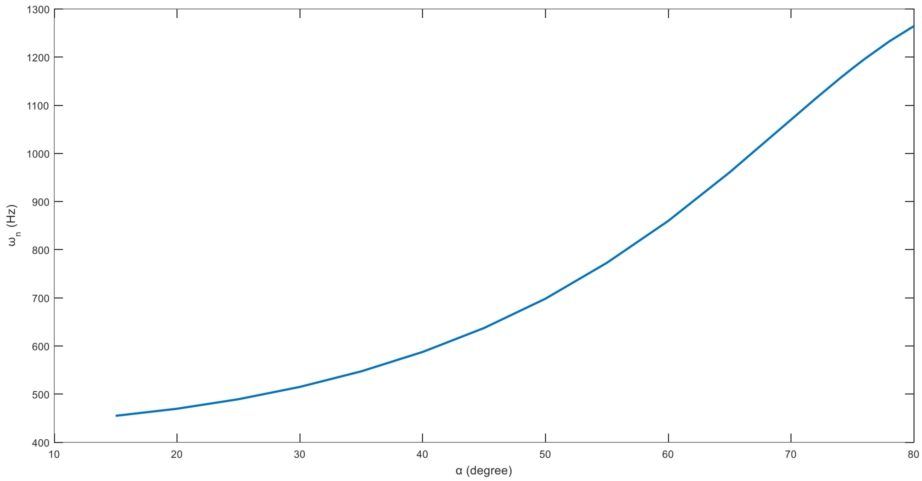

Fig 13: First natural frequency of the auxetic cell

4. Conclusion

In this study, the equivalent stiffness and equivalent mass of a type of auxetic material were analysed. The purpose of this investigation is to determine equivalent stiffness and equivalent mass of the auxetic cell and structures. The geometrical parameters of the cell and the auxetic structure, such as cell length and cell angle were investigated. The influences of the number of cells that are placed next to each other in parallel and series in a structure were examined. The obtained results reveal that

- Increasing the angle of the cell increases the Poisson's ratio of the auxetic cell.
- Increasing the angle of the auxetic cell increases the equivalent stiffness of the cell.
- By increasing the number of series cells in a fixed length, the behaviour of Poisson's ratio in different angles of the auxetic cell is constant, but the Poisson's ratio decreases.
- By increasing the number of series cells in a fixed length of the auxetic beam structure, the equivalent stiffness increases due to the reduction of the length of the auxetic cell beam. In other words, increasing the length of the beam in the auxetic cell reduces the equivalent stiffness of the auxetic structure.
- Increasing the angle of the auxetic cell increases the equivalent mass of the auxetic cell.

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