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RESEARCH PAPER

Stability analysis of conveying-nanofluid functionally graded nanotube under based on nonlocal couple stress theory

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Abstract

The dynamics behavior and stability of axially functionally graded fluid-conveying nanotube is investigated, in this paper. The simultaneous influence of both fluid flow and variation of modulus of elasticity on the behavior of simply–simply supported (S-S) and clamp-clamp (C-C) boundary conditions conveying fluid were studied. Small-scale effects are considered using nonlocal couple stress theory in the solid part and in the fluid part. Based on the nonlocal couple stress theory, Bernoulli-Euler beam theory, and Hamilton's principle, the governing equation of motion, and associated boundary conditions were derived to explain fluid-structure interaction (FSI). These equations were solved using Galerkin numerical method and temporal differential equation analysis method. The effects of some parameters such as Knudsen number, density, size parameter, and … were investigated. According to the results, it can be seen that the present method has created an equilibrium for the effect of the size parameters (μ, l) on the critical velocity. The higher value of the Knudsen number caused sooner divergence and flutter instabilities to happen. The results show that if the parameters of the size effect are not considered, it causes errors in the calculations. The obtained results confirm the crucial effects of size.

Keywords: size-dependent solid-fluid interaction; nonlocal couple stress theory; functionally graded materials; nanotube.

1. Introduction

Much research has been done in the field of nanomechanics in the last two decades [1-43]. Nanobiotechnology is an important and widely used branch of nanotechnology [44-56]. In the meantime, nanotubes have found wide applications in modern medicine, and they are widely used in drug delivery and biosensors. Since 2004, nanotubes have been widely investigated as drug carriers for intracellular delivery of chemotherapy drugs, proteins, and genes. In vivo, cancer treatment using carbon nanotubes has been proven in animal experiments by various research groups. Applications of nanotechnology in medical science include the use of self-exploding microcapsules in drug delivery, the role of carbon nanotubes in human bone repair, stop bleeding in less than 15 seconds, nanotechnology and cancer treatment, etc. The use of carbon nanotubes containing fluid as nanoscale fluid transfer units as well as their application in targeted drug delivery to a specific cell area in the tissue of living organisms has caused many researchers from different fields of science to engage in research in the advancement of related research. Carbon nanotubes are known to be the principal member of most nano-scale material transfer and transfer tools in the future. Many studies have been carried out in the field of carbon nanotubes conveying fluid, some of which will be reviewed below.

In the frameworks of nonlocal strain gradient theory, Zhu *et al*. [1] investigated the vibration behavior of magnetically embedded spinning axially functionally graded nanotubes conveying fluid under axial loads. A new

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model for fluid-conveying nanotubes made of bi-directional functionally graded was proposed by Tang and Yang [2]. In the frameworks of nonlocal strain gradient theory, Nematollahi *et al*. [3] investigated fluttering and divergence instability of functionally graded viscoelastic nanotubes conveying fluid. Hu *et al*. [4] studied the nonlinear instability of a fluid-conveying radially functionally graded nanopipe based on the Euler-Bernoulli beam theory and nonlocal strain gradient theory.

Shaat et *al*. [5] studied wettability and confinement size effects on the vibration and stability of water-conveying nanotubes. Gorbani et *al*. [6] investigated and studied carbon nanotubes (CNTs) using molecular dynamics simulation and nonlocal strain gradient continuum model. Longitudinal vibration behaviors of CNTs conveying viscous fluids was investigated by Oveissi et *al*. [7]. Rashidi et *al*. [8] presented an innovative model for coupled vibrations of nanotubes conveying fluid by considering the small-size effects on the flow field. Based on the molecular dynamics simulation, nonlocal strain gradient shell model for buckling analysis of nanotubes was calibrated by Mehralian et *al*. [9]. Jabbari et *al*. [10] investigated water-solid interaction and thermal resistance using molecular dynamics simulation. Ghanbari et *al*. [11] reappraised the vibration analysis of a double-walled carbon nanotube (DWCNT) conveying viscous flow based on the modified strain gradient. Nonlocal strain gradient shell model was calibrated for vibration analysis of a CNT conveying viscous fluid using molecular dynamics simulation by Mohammadi et *al*. [12]. Bidgoli et *al*. [13] used orthotropic Mindlin shell theory to study nonlinear vibration and instability of embedded temperature-dependent uniform and FG cylindrical shell conveying viscous fluid resting on temperature-dependent orthotropic Pasternak medium are investigated. Based on the Eringen and Euler–Bernoulli beam theory, Bahaadini and Hosseini [14] Nonlocal divergence and flutter instability analysis of embedded fluid‑conveying carbon nanotube in a Winkler and Pasternak foundation under magnetic field. Mahinzare et *al*. [15] introduced a new model for vibration and instability analysis of a single-walled carbon nanotube (SWCNT) conveying viscous fluid flow based on the first-order shear deformation shell model and nonlocal strain gradient theory. In the frameworks of nonlocal strain gradient theory, Lee and Chang [16] investigated coupled vibration of fluid-conveying double-walled carbon nanotubes and the influences of nonlocal effect, aspect ratio and van der Waals interaction on the fundamental frequency. In the frameworks of strain gradient elasticity theory combined with inertia gradients, Wang [17] introduce a model for the vibration of fluid-conveying nanotubes using Euler-Bernoulli or Timoshenko assumptions. Hashemnia et *al*. [18] investigate vibrational single-layered graphene sheets and single-wall carbon nanotubes based on the molecular structural mechanics approach. In addition to these, other valuable research have also been carried out [19-24].

Functionally graded materials (FGMs) are composite materials in which mechanical properties vary smoothly and continuously from one surface to the other [25, 26]. Functional grading of material can be used to achieve a variety of goals, including alleviation of residual stresses, reducing stresses during the lifetime of the structure, improvement of stability and dynamic response, preventing fracture and fatigue, etc [27]. Several papers considering various aspects of FGM have been published in recent years [28-33]

Based on the nonlocal couple stress and Bernouli-Euler beam theory, the effects of some parameters were investigated in a conveying-fluid nanotube made of functionally graded materials. In this framework and Hamilton's principle, the governing equation of motion was derived.

2. Theory and formulation

If u_1, u_2 and u_3 are components of the displacement vector for nano-beams based on Euler-Bernoulli beam theories, then they are defined by:

$$u_1 = u_0(x) - z\frac{\partial w}{\partial x}$$

$$u_2 = 0 \tag{1}$$

$$u_3 = w(x.t)$$

where, $w(x,t)$ is the transverse displacement of any point of the beam. The non-zero component of strain tensor and curvature tensor based on the displacement field is defined by:

$$\varepsilon_{xx} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w}{\partial x^2}$$
$$\chi_{xy} = -\frac{1}{2}\left[\frac{\partial^2 w}{\partial x^2}\right] \tag{2}$$

Based on the nonlocal couple stress theory, the constitutive equation is expressed as follows:

$$\sigma_{xx} - \mu^2\frac{\partial^2 \sigma_{xx}}{\partial x^2} = \mathrm{E}\varepsilon_{xx}$$
$$m_{xy} - \mu^2\frac{\partial^2 m_{xy}}{\partial x^2} = 2Gl^2.\chi_{xy} \tag{3}$$

where the material properties (modulus of elasticity and density) are defined as;

$$E = E_0 X_1$$
$$\rho = \rho_0 X_2 \tag{4}$$

and

$$G = \frac{E}{2(1+\nu)} \tag{5}$$

Employing Hamilton's principle, the governing equation of a nanotube conveying nanofluid is obtained. So;

$$\int_0^t (\delta U + \delta W - \delta K)dt = 0 \tag{6}$$

In which, δU, δW and δK present variations of the strain energy, work and kinetic energy, respectively. According to the nonlocal couple stress theory, variation of the strain energy density function of an isotropic linear elastic material with volume V experiencing an infinitesimal displacement is obtained by:

$$\delta U = \int_V (\sigma_{xx}\delta\varepsilon_{xx} + 2m_{xy}\delta\chi_{xy})dV$$
$$= \int_0^l ((N_{xx}\frac{\partial \delta u_0}{\partial x}) - (M_{xx}\frac{\partial^2 \delta w}{\partial x^2}) - (Y_{xy}\frac{\partial^2 \delta w}{\partial x^2}))dx \tag{7}$$

where, N, M and Y are resultant stresses defined by:

$$N_{xx} = \int_A \sigma_{xx} dA$$
$$M_{xx} = \int_A z\sigma_{xx} dA \tag{8}$$
$$Y_{xy} = \int_A m_{xy} dA$$

The variation of kinetic energy can be written as:

$$\int_0^t \delta K dt = \int_0^t \int_V \rho\left[\dot{u}_1\delta\dot{u}_1 + \dot{u}_2\delta\dot{u}_2 + \dot{u}_3\delta\dot{u}_3\right]dVdt$$
$$= \int_0^t \int_0^L \int_A \rho_0 X_2\left[z^2\frac{\partial^2 w}{\partial x\partial t}\frac{\partial^2 \delta w}{\partial x\partial t} + \frac{\partial w}{\partial t}\frac{\partial \delta w}{\partial t}\right]dAdxdt \tag{9}$$
$$= \int_0^t \int_0^L \rho_0 X_2\left[I\frac{\partial^2 w}{\partial x\partial t}\frac{\partial^2 \delta w}{\partial x\partial t} + A\frac{\partial w}{\partial t}\frac{\partial \delta w}{\partial t}\right]$$

where, ρ is the nanotube density, and;

$$I=\int_A z^2 dA \\ A=\int_A dA \tag{10}$$

The variation of work done by the external force, F_{ext}, is as follows:

$$\delta W=\int_V F_{ext}\delta w dV \tag{11}$$

By substituting Eqs. (7), (9), (11) into Eq. (6) and through mathematical analysis, the governing equations yield as follows:

$$\frac{\partial N_{xx}}{\partial x}=0 \\ -\frac{\partial^2}{\partial x^2}\left(M_{xx}+Y_{xy}\right)-\frac{\partial}{\partial x}\left(\rho_0 X_2 I\frac{\partial^3 w}{\partial x\partial t^2}\right)+\rho_0 X_2 A\frac{\partial^2 w}{\partial t^2}-F_{ext}=0 \tag{12}$$

Therefore;

$$\frac{\partial^4\left(M_{xx}+Y_{xy}\right)}{\partial x^4}=-\frac{\partial^2 F_{ext}}{\partial x^2}-\rho_0\frac{d^3X_2}{dx^3}I\frac{\partial^3 w}{\partial x\partial t^2}-3\rho_0\frac{d^2X_2}{dx^2}I\frac{\partial^4 w}{\partial x^2\partial t^2}-3\rho_0\frac{dX_2}{dx}I\frac{\partial^5 w}{\partial x^3\partial t^2} \\ -\rho_0 X_2 I\frac{\partial^6 w}{\partial x^4\partial t^2}+\rho_0\frac{d^2X_2}{dx^2}A\frac{\partial^2 w}{\partial t^2}+2\rho_0\frac{dX_2}{dx}A\frac{\partial^3 w}{\partial x\partial t^2}+\rho_0 X_2 A\frac{\partial^4 w}{\partial x^2\partial t^2} \tag{13}$$

The surface integrals of Eq. (3) simplify the constitutive equation according to the resultant stress tensors as follows:

$$M_{xx}-\mu^2\frac{\partial^2 M_{xx}}{\partial x^2}=-E_0X_1I\frac{\partial^2 w}{\partial x^2} \\ Y_{xy}-\mu^2\frac{\partial^2 Y_{xy}}{\partial x^2}=-\frac{E_0X_1A}{2(1+\nu)}l^2\frac{\partial^2 w}{\partial x^2} \\ N_{xx}-\mu^2\frac{d^2N_{xx}}{dx^2}=E_0X_1A\frac{du_0}{dx} \tag{14}$$

Therefore;

$$\frac{\partial^2\left(M_{xx}+Y_{xy}\right)}{\partial x^2}-\mu^2\frac{\partial^4(M_{xx}+Y_{xy})}{\partial x^4}= \\ -E_0\left(I+\frac{Al^2}{2\left(1+\nu\right)}\right)\left[\frac{d^2X_1}{dx^2}\frac{\partial^2 w}{\partial x^2}+2\frac{dX_1}{dx}\frac{\partial^3 w}{\partial x^3}+X_1\frac{\partial^4 w}{\partial x^4}\right] \tag{15}$$

To derive the effect of flow on the vibration and instability of the nanotube, the momentum balance equation for the fluid flow is developed. The well-known Navier–Stokes equation for a fluid is stated as follows [34]:

$$\rho\frac{Dv}{Dt}=-\nabla p+\mu\nabla^2 v \tag{16}$$

To obtain the force exerted on the nanotube by the fluid flow, the following assumptions are made

(a) The fluid flow is fully developed in the flow direction, i.e:.

$$v = v_x(r)\hat{e}_x + v_r(x,t)\hat{e}_r \tag{17}$$

(b) The slender body theory is applied to the nanotube with a high aspect ratio Wang and Ni [35].

$$v_r(x,t) = \frac{Dw}{Dt} \tag{18}$$

where, v_r is the average flow velocity in the direction of fluid flow. The force exerted on the nanotube induced by the fluid flow is directly related to the fluid pressure gradient. The Navier–Stokes equation in the r direction is developed as follows:

$$\frac{\partial p}{\partial r} = \mu\left[\frac{\partial^3 w}{\partial x^2 \partial t} + \bar{v}_x \frac{\partial^3 w}{\partial x^3}\right] - \rho\left[\frac{\partial^2 w}{\partial t^2} + 2\bar{v}_x \frac{\partial^2 w}{\partial x \partial t} + \bar{v}_x^2 \frac{\partial^2 w}{\partial x^2}\right] \tag{19}$$

where, the terms multiplied by ρ and μ on the right-hand side of Eq. (19) are associated with the fluid inertia and the fluid viscosity, respectively. Since the first term is small, it can be dropped according to Wang and Ni [35]. Using Eq. (19), an equation for the forces induced by the fluid flow F_{ext} is obtained as:

$$F_{ext} = -m_f\left[\frac{\partial^2 w}{\partial t^2} + 2\bar{v}_x \frac{\partial^2 w}{\partial x \partial t} + \bar{v}_x^2 \frac{\partial^2 w}{\partial x^2}\right] \tag{20}$$

where, m_f is the fluid mass per unit length.

The flow behavior at the nano-scale is differ substantially from those at large scales, especially at low Reynolds number [36]. For this problem, it is assumed there is a slip boundary condition between the fluid flow and the nanotube walls. Therefore, we use the velocity correction factor (VCF) proposed by Rashidi et *al*. [8] to consider the slip boundary condition and the small size effects on the fluid flow:

$$VCF = \frac{\bar{V}_X}{U} = (1+\alpha' Kn)(4(\frac{2-\sigma_V}{\sigma_V})(\frac{Kn}{1+Kn})+1) \tag{21}$$

where, $\bar{V}_x$ and U are the average flow velocities through the nanotube with and without considering the slip boundary condition, respectively. Kn is the Knudsen number, σ_V is the tangential momentum accommodation coefficient and is considered to be equal to 0.7 [8], and α' is a coefficient, which can be defined as a function of Kn using Eq. (22) [8]

$$\alpha' = \alpha_0 \frac{2}{\pi}\left(\tan^{-1}(\alpha_1 Kn^B\right) \tag{22}$$

where, α_1 and B are empirical parameters and are equal to 4 and 0.4, respectively [8]. α_0 is given by:

$$\alpha_0 = \frac{64}{3\pi(1-\frac{4}{b})} \tag{23}$$

For the second-order term of slip boundary condition, b is set at one [37]. Therefore, the external force is equal to;

$$F_{ext} = -m_f\left[\frac{\partial^2 w}{\partial t^2} + 2(VCF)U \frac{\partial^2 w}{\partial x \partial t} + (VCF)^2 U^2 \frac{\partial^2 w}{\partial x^2}\right] \tag{24}$$

By substituting the external forces given by Eq. (24) into Eq. (13), and then applying the nonlocal elasticity and the couple stress theories, the fluid–structure interaction (FSI) governing equation for the nanotube can be written as follows:

$$\begin{aligned} &\left(-\rho_0 X_2 I + 3\mu^2 \rho_0 \frac{d^2 X_2}{dx^2} I - \mu^2 \rho_0 X_2 A - \mu^2 m_f\right)\frac{d^4 w}{dx^2 dt^2} \\ &\left(-\rho_0 \frac{dX_2}{dx} I + \mu^2 \rho_0 \frac{d^3 X_2}{dx^3} I - 2\mu^2 \rho_0 \frac{dX_2}{dx} A\right)\frac{d^3 w}{dx dt^2} \end{aligned} \tag{25}$$

$$+\left(m_f (VCF)^2U^2 + E_0\left(I + l^2As\right)\frac{d^2X_1}{dx^2}\right)\frac{d^2w}{dx^2}$$

$$+2m_f (VCF)U\frac{d^2w}{dxdt} + \left(m_f - \rho_0\mu^2\frac{d^2X_2}{dx^2}A + \rho_0X_2A\right)\frac{d^2w}{dt^2}$$

$$+\mu^2\rho_0X_2I\frac{d^6w}{dx^4dt^2} + 3\mu^2\rho_0\frac{dX_2}{dx}I\frac{d^5w}{dx^3dt^2}$$

$$-2\mu^2m_f (VCF)U\frac{d^4w}{dx^3dt} + 2E_0\left(I + l^2As\right)\frac{dX_1}{dx}\frac{d^3w}{dx^3}$$

$$+\left(-\mu^2m_f (VCF)^2U^2 + E_0\left(I + l^2As\right)X_1\right)\frac{\partial^4w}{\partial x^4} = 0$$

where

$$s = \frac{1}{2\left(1+\nu\right)} \tag{26}$$

Using dimensionless variables and parameters, Eq.(25) can be rewritten in a dimensionless form as:

$$\begin{aligned}
&\left(-\bar{\mu}^2(VCF)^2\bar{U}^2 + \left(1+\frac{\bar{l}^2s}{\bar{I}}\right)X_1\right)\frac{\partial^4\bar{w}}{\partial\bar{x}^4} \\
&+2\left(1+\frac{\bar{l}^2s}{\bar{I}}\right)\frac{dX_1}{d\bar{x}}\frac{d^3\bar{w}}{d\bar{x}^3} \\
&+\left((VCF)^2\bar{U}^2 + \left(1+\frac{\bar{l}^2s}{\bar{I}}\right)\frac{d^2X_1}{d\bar{x}^2}\right)\frac{d^2\bar{w}}{d\bar{x}^2} \\
&\left(-X_2\bar{I} + 3\bar{\mu}^2\frac{d^2X_2}{d\bar{x}^2}\bar{I} - \bar{\mu}^2X_2 - \bar{\mu}^2\beta\right)\frac{d^4\bar{w}}{d\bar{x}^2d\tau^2} \\
&\left(-\frac{dX_2}{d\bar{x}}\bar{I} + \bar{\mu}^2\frac{d^3X_2}{d\bar{x}^3}\bar{I} - 2\bar{\mu}^2\frac{dX_2}{d\bar{x}}\right)\frac{d^3\bar{w}}{d\bar{x}d\tau^2} \\
&+\left(\beta - \bar{\mu}^2\frac{d^2X_2}{d\bar{x}^2} + X_2\right)\frac{d^2\bar{w}}{d\tau^2} + 2\beta^{\frac{1}{2}}(VCF)\bar{U}\frac{d^2\bar{w}}{d\bar{x}d\tau}
\end{aligned} \tag{27}$$

$$+3\bar{\mu}^2 \frac{dX_2}{d\bar{x}} \bar{I} \frac{d^5\bar{w}}{d\bar{x}^3 d\tau^2} + \bar{\mu}^2 X_2 \bar{I} \frac{d^6\bar{w}}{d\bar{x}^4 d\tau^2} +$$

$$-2\bar{\mu}^2 \beta^{\frac{1}{2}} (VCF) \bar{U} \frac{d^4\bar{w}}{d\bar{x}^3 d\tau} = 0$$

where

$$\bar{\mu} = \frac{\mu}{L}$$

$$\bar{l} = \frac{l}{L}$$

$$\bar{U} = U \left(\frac{E_0 \mathrm{I}}{m_f} \right)^{-\frac{1}{2}} L$$

$$\tau = \left(\frac{E_0 I}{\rho_0 A} \right)^{\frac{1}{2}} \frac{t}{L^2} \tag{28}$$

$$\bar{I} = \frac{I}{AL^2}$$

$$\beta = \frac{m_f}{\rho_0 A}$$

Therefore

$$R = \left(-\bar{\mu}^2 (VCF)^2 \bar{U}^2 + \left(1 + \frac{\bar{l}^2 s}{\bar{I}} \right) X_1 \right) \frac{\partial^4 \bar{w}}{\partial \bar{x}^4}$$

$$+2 \left(1 + \frac{\bar{l}^2 s}{\bar{I}} \right) \frac{dX_1}{d\bar{x}} \frac{d^3\bar{w}}{d\bar{x}^3}$$

$$+ \left((VCF)^2 \bar{U}^2 + \left(1 + \frac{\bar{l}^2 s}{\bar{I}} \right) \frac{d^2 X_1}{d\bar{x}^2} \right) \frac{d^2\bar{w}}{d\bar{x}^2} \tag{29}$$

$$\left(-X_2 \bar{I} + 3\bar{\mu}^2 \frac{d^2 X_2}{d\bar{x}^2} \bar{I} - \bar{\mu}^2 X_2 - \bar{\mu}^2 \beta \right) \frac{d^4\bar{w}}{d\bar{x}^2 d\tau^2}$$

$$\left(-\frac{dX_2}{d\bar{x}} \bar{I} + \bar{\mu}^2 \frac{d^3 X_2}{d\bar{x}^3} \bar{I} - 2\bar{\mu}^2 \frac{dX_2}{d\bar{x}} \right) \frac{d^3\bar{w}}{d\bar{x} d\tau^2}$$

$$+\left(\beta-\bar{\mu}^2\frac{d^2X_2}{d\bar{x}^2}+X_2\right)\frac{d^2\bar{w}}{d\tau^2}+2\beta^{\frac{1}{2}}(VCF)\bar{U}\frac{d^2\bar{w}}{d\bar{x}d\tau}$$

$$+3\bar{\mu}^2\frac{dX_2}{d\bar{x}}\bar{I}\frac{d^5\bar{w}}{d\bar{x}^3d\tau^2}+\bar{\mu}^2X_2\bar{I}\frac{d^6\bar{w}}{d\bar{x}^4d\tau^2}+$$

$$-2\bar{\mu}^2\beta^{\frac{1}{2}}(VCF)\bar{U}\frac{d^4\bar{w}}{d\bar{x}^3d\tau}$$

To compute the complex eigenvalues of the nanotube vibrations, we employ the extended Galerkin method to solve the partial differential equation of motion, and the associated boundary conditions, by a finite dimensional system of coupled ordinary differential equations. Accordingly, the flexural displacement of the nanotube can be represented in the form of a series given by:

$$\tilde{w}=\sum_{i=1}^{n}\phi_i\left(\bar{x}\right).T_i\left(\tau\right) \tag{30}$$

where, n is the number of vibrational modes, $T_i(\tau)$ represents the n^{th} modal coordinate and $\phi_i(\bar{x})$ denotes the basis function for the n^{th} eigenmode. For a simply supported nanotube, ϕ_I can be written as:

$$\begin{aligned}&\phi_i=\sin\frac{i\pi x}{L}\\&\phi_i=1-\cos 2i\pi x\\&\int_0^1\phi\left(\bar{x}\right)Rd\bar{x}=0\end{aligned} \tag{31}$$

3. Validation

Verification of the results of this research with those available in other research. Comparing the results shows that the presented method is very accurate

Table 1: Verification of the results of this research with those available in other research.

$[K_n,\bar{\mu}]$	$[0,0]$	$[0.001,0]$	$[0.001,0.2]$
Present results	3.142	3.119	2.641
Sadeghi-Goughari et al. [37]	3.142	3.118	2.64
Ni et al. [38]	3.142	3.118	2.64
Mirramezani and Mirdamadi [39]	3.142	3.118	2.64

4. Results and discussions

The effect of different parameters on the dynamics of functionally graded nanotubes is studied, and numerical results are presented in this section. The effect of β changes on the modulus of elasticity is visible in fig. 1. It can be seen in figs. 1 and 2, nanotubes can be used for higher velocities, and instability will occur at higher fluid velocities. Therefore, increasing the modulus of elasticity has a positive effect on the structure's efficiency.

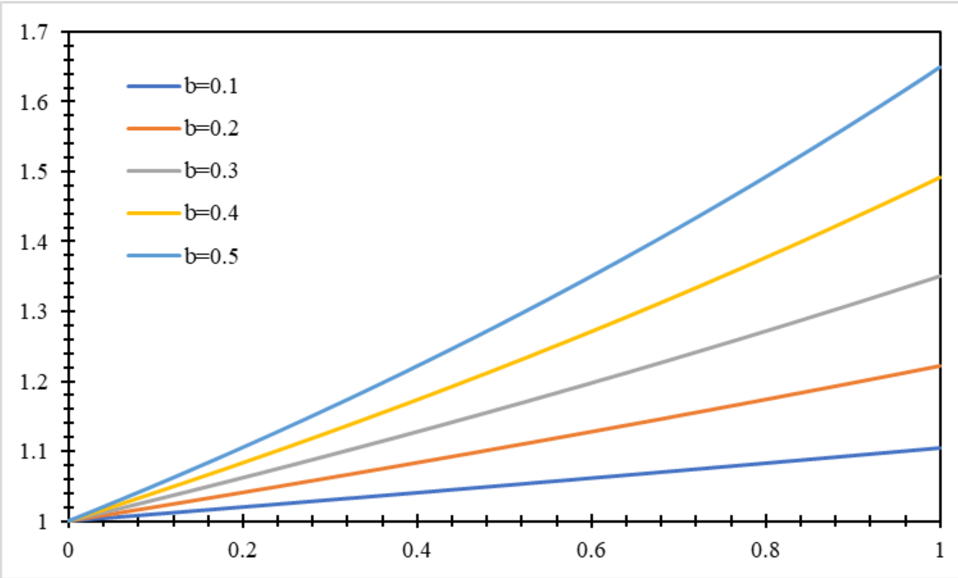
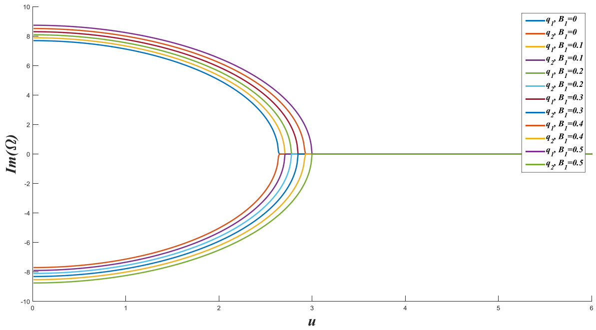
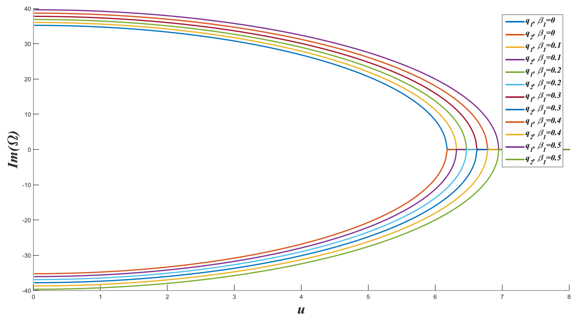


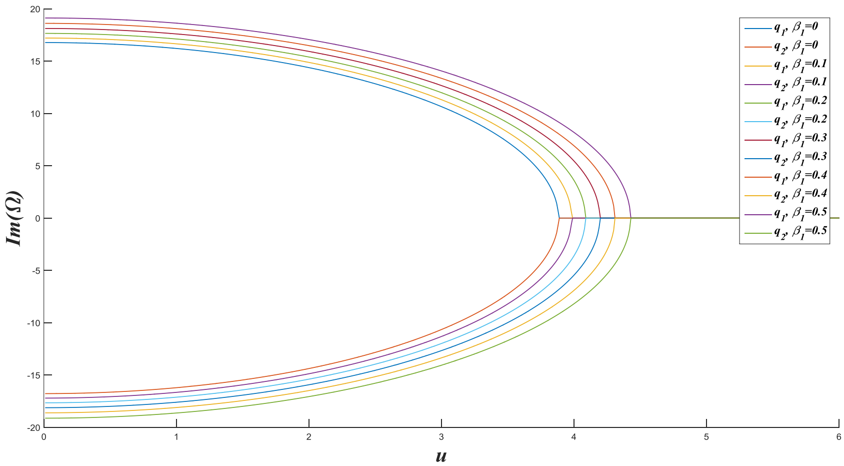
Fig. 1: The effect of β on the modulus of elasticity.



(a) simply–simply supported (S-S) boundary conditions



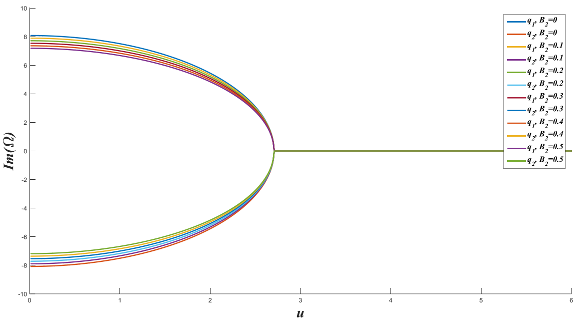
(b) simply–simply supported (S-S) boundary conditions



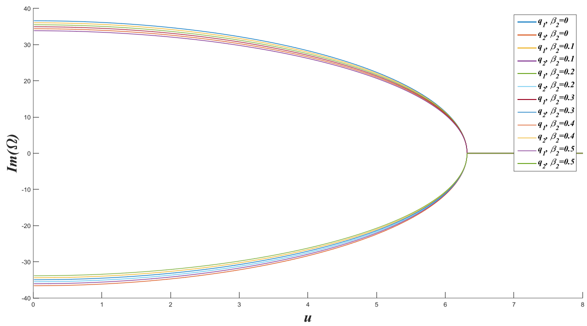
(c) clamp-clamp (C-C) boundary conditions

Fig.2: The effect of β_1 on the imaginary part of the first frequency

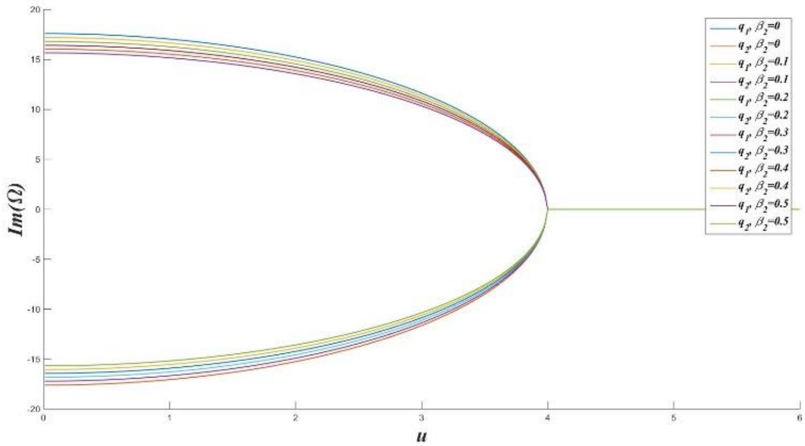
Fig. 2 shows the effect of the variation of β_1 on the critical velocity. As the β_1 value increases, the critical velocity increases. Therefore, the structure will be stable up to higher speeds by increasing β_1.



(a) simply–simply supported (S-S) boundary conditions



(b) simply–simply supported boundary conditions



(c) clamp-clamp (C-C) boundary conditions

Fig.3: The effect of density changes on dimensionless critical flow velocities.

The effect of density changes on the critical velocity can be seen in Fig. 3. At low speeds ($0<u<2$), with increasing beta, the imaginary part of the frequency decreases. This effect fades for higher speeds. As can be seen, density changes do not affect on the critical velocity.

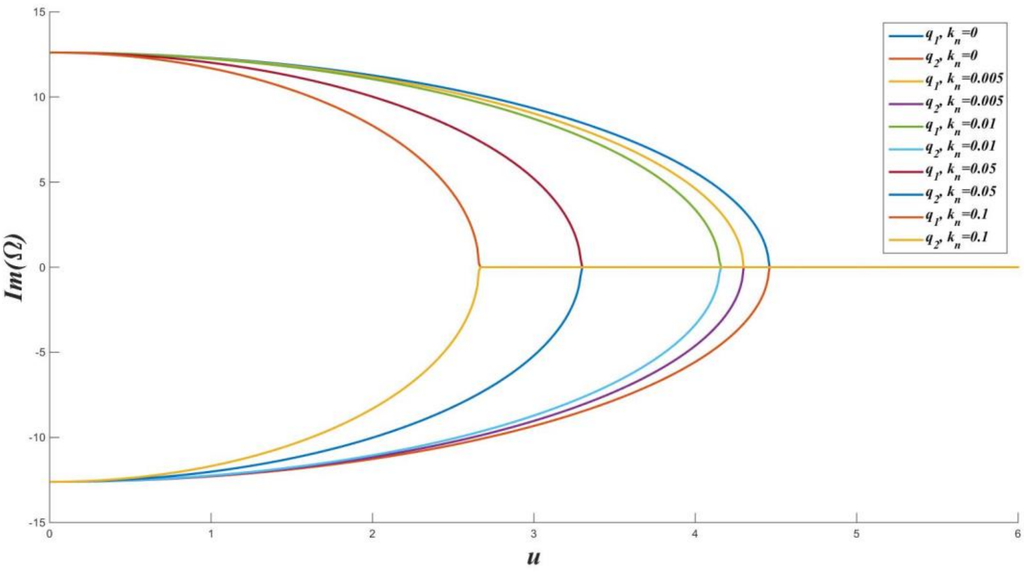


Fig. 4: The effect of the Knudsen number on the dimensionless flow velocities.

Fig. 4 demonstrates the effect of the Knudsen number on the dimensionless flow velocities and dynamic response of a simply–simply supported (S-S) boundary conditions nanotube conveying a liquid. The effect of the Knudsen number is low at low velocities, and at high velocities, the effect of this number is more significant, in contrast to the effect of density. As the Knudsen number increases, the critical velocity decreases.

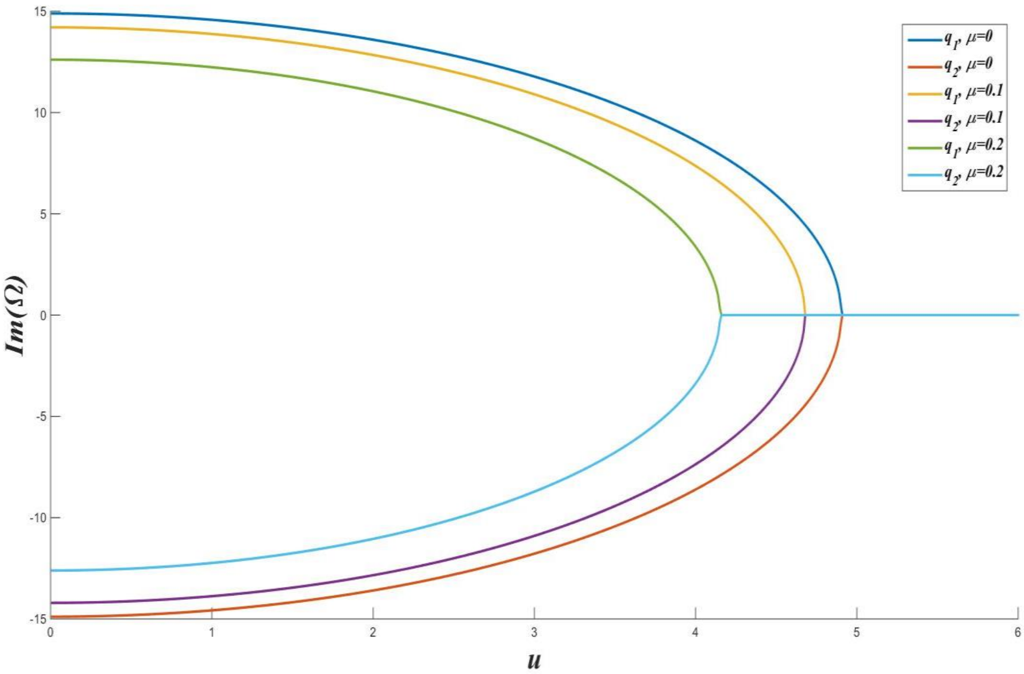


Fig.5: The effect of size parameter μ on the dimensionless flow velocities.

The effect of the size parameter μ on the dimensionless critical flow velocities on the simply–simply supported (S-S) boundary conditions nanotube is shown in Fig. 5. The critical velocity of the fluid and the size parameter are inversely related. The critical velocity will be observed sooner if the size effect parameter is larger. Therefore, increasing the size parameter μ has led to a softer structure.

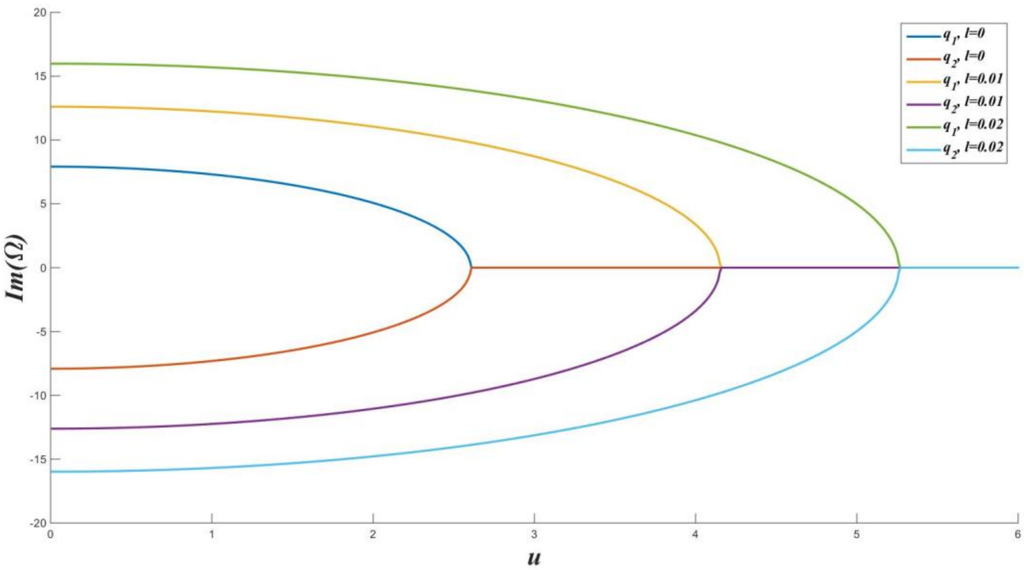


Fig.6: The effect of size parameter l on the dimensionless flow velocities of simply–simply supported (S-S) boundary conditions nanotube.

Contrary to what was seen for the effect of size parameter μ, as the size parameter l increases, the critical velocity will be increased (The size parameter l and critical velocity are directly related). This fact is shown in Fig.

6. The effect of size parameter l on the critical velocity of the fluid is greater than the effect of the size parameter μ. Therefore, increasing size parameter l has led to a harder structure.

According to the results of Figs. 5 and 6, it can be seen that the present theory has created an equilibrium for the effect of the size parameters (μ, l) on the critical velocity. It can be seen that the dynamics of conveying fluid nanotubes has a considerable dependency on the size parameter l.

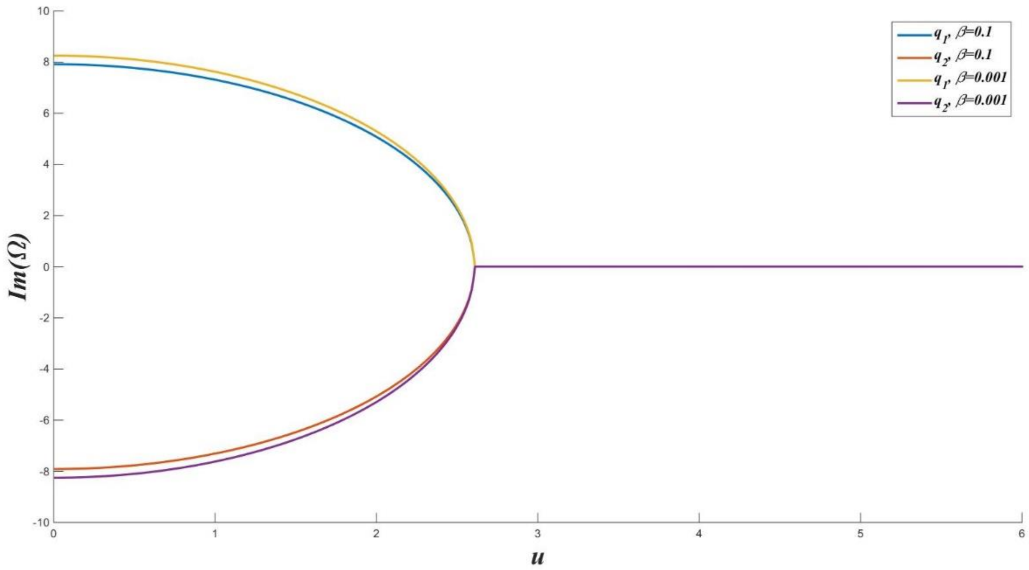


Fig.7: The effect of β on the dimensionless critical flow velocities.

Fig. 7 demonstrates the effect of β on the critical velocity of simply–simply supported (S-S) boundary conditions nanotube. At low speeds ($0<u<2$), with increasing *Beta*, the imaginary part of the frequency decreases. This effect fades for higher speeds. As can be seen, *Beta* changes do not affect on the critical velocity.

5. Conclusion

In the frameworks of nonlocal couple stress theory, and Bernoulli-Euler beam theory a size-dependent functionally graded nanotubes tube conveying fluid is studied to investigate the size effects on flutter and divergence instability. The dimensionless equation of motion and boundary conditions are derived using the variational approach. The frequency equation was derived as a function of small-scale parameters, Knudsen number, and inhomogeneity parameter of FGM. Results were compared with those of other researchers.

According to the results, it can be seen that the present theory has created an equilibrium for the effect of the size parameters (μ, l) on the critical velocity. The smaller value of l may cause sooner divergence and flutter instabilities to happen. The opposite of this truth is true for μ. So, increasing the small-scale parameters l and μ will have stiffness-softening and stiffness-hardening effects, respectively. Also, a detailed investigation is conducted to elucidate the influence of key factors material distribution on the divergence and flutter instability. On the one hand, it is found that the density gradient parameter hasn't a significant effect on the instability. On the other hand, by increasing the β, nanotubes can be used for higher velocities, and instability will occur at higher velocities of fluid. The presented results are expected to provide a way to guide the application of FG nanotubes as nanofluidic devices. By increasing β_2, the divergence fluid velocities reduce; however, the flutter fluid velocity does not vary.

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