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Turbulent Supply Chain Control with Entropy Minimization Approach

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ABSTRACT

Entropy is a measure of disorder in a system and is widely used in other scientific and engineering disciplines such as statistical mechanics and information theory. In a chaotic supply chain, the goal is to reduce chaotic behaviors and predict the future of the supply chain. In this case, relationships in a three-tier supply chain are considered in a continuous-time environment. In this paper, the chaotic supply chain is investigated and controlled using the entropy minimization method. Due to the dynamic nature of the supply chain and its chaotic behavior, Poincaré mapping of the system has been prepared by the stroboscopic method. Then, by defining Shannon entropy on the map, the entropy of the system is significantly reduced by the gradient descent algorithm.

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1 Introduction

Why is the supply chain (SC) important and why do companies, even with long-term experience, consider a strong SC a valuable merit and hire logistics and SC specialists? The answer lies in the fact that supply and demand never balance in the real world. On the one hand, keeping high inventory levels in warehouses imposes considerable costs on storage, manpower, decays, obsolescence, and so on, and on the other hand, goods deficiency leads to customer and credit loss.

SC entities are echelon-based, do not have a specific and fixed number and can be totally different from one chain to another. In its most primitive form, a supply chain is single-echelon and includes a producer and a customer who are directly related, and depending on the issue being studied, it can be multi-echelon including distributor, wholesaler, retailer, dealer, and so on; obviously, the conceptual and structural complexities of multi-echelon SCs are far greater than those with fewer echelons. This research will examine a three-echelon SC with a supplier, a distributor and a retailer.

Chaotic systems lie between cause-and-effect and random systems; their behavior and output are similar to those of random systems, but they are highly sensitive to the initial conditions. In them, the starting points of the processes are very important because they can cause many deviations in the path as well as in the final destination. Importance of disorder/chaos control in a system becomes clearer when we understand that by implementing the related controlling measures, we will have more resources leveled in the system. In other words, controlling chaos and disorder in a dynamic system will help the realization of this issue by reducing data dispersion. In chaotic SCs, the objective is to reduce chaotic behavior and predict the future of the SC; this is more important especially regarding the multi-echelon SC discussed in this research. Previous methods assumed all system equations were available and all related variables were measurable to control the supply chain, but since these conditions are rarely realized, effort was made in this study to address the SC controlling by only having information about the amount of production and without having the system chaotic behavior equations. To this end, “entropy” has been defined as a criterion for system disorders and unpredictability, and a desirable objective has been considered to control the chaotic system and reduce entropy.

In [6], a controller has been designed and implemented to reduce the entropy of the supply chain. This problem considers the relationships of a three-echelon financing chain in a time continuous environment, prepares the Poincare map of the system by the stroboscopic method considering the SC's dynamic nature and chaotic behavior and reduces the system entropy significantly by defining the Shannon entropy on the mentioned mapping and using the gradient reduction algorithmic.

Next, Section (2) reviews the papers on chaotic SCs and shows the related research gaps, Section (3) defines the problem and states its assumptions, Section (4) addresses the problem solution and analysis and, finally, Section (5) presents the related conclusions.

2 Literature review

As competition is really serious in global markets and business companies try to meet the customers' expectations, many of them have recently focused more on the performance optimization through more efficient supply chain network management [11]. In an irregular supply chain network, the system can be protected against increased supply risks and longer order supply times only by increasing the confidence level at each echelon or faster service provisions, but since this is not in line with more business profits, planning proper multi-echelon supply chains and having strategic goals in management are necessary to increase profits, reduce costs and improve customer satisfaction. SC systems have many similarities to complex dynamic systems (e.g., stock markets), which involve chaotic, irregular performance and cannot be managed and controlled even with the best available computational software and basic model. SC networks show complex dynamic behavior changing over time with various uncertainties due to demand/inventory/shortage level fluctuations, forecast deviations and unexpected natural/man-made disasters. The system theory enables using a set of advanced control methods to ensure acceptable performance in dealing with chaotic dynamic system against uncertainties. This is why many researchers have investigated the dynamic analysis of the supply chain model in logistics studies through the system dynamics technique and then applied the appropriate control structure to optimize the complex system. Studies on SC systems, since the 1950s, show that the chains' internal systems can create fluctuations in the demand and inventory in the supply process. Uncertainties can be due to late deliveries and order cancellation or increased reliance on inventory to control the effects. Since traditional methods have not succeeded in controlling/reducing uncertainty-related fluctuations, the chaos theory has been more popular nowadays. SCs can determine some key characteristics of chaotic systems and show that chaos is sensitive to initial conditions [33].

Since 1990, various algorithms have been proposed to control the chaos by fixing periodic circuits or fixed points of the chaotic system. Pyragas [26] used the delayed feedback signal to present a chaos control method, and other nonlinear control methods such as the feedback linearization [2], sliding mode [12] and adaptive Liapanov-based control [27] have been used to suppress chaos in many physical systems.

Salarieh and Alasti [2] presented a minimum-entropy-chaos-control algorithm for a discrete-time chaotic-behavior dynamic system using the (Minimum Entropy) ME control method through delayed feedback, where the feedback is obtained adaptively so that the system entropy converges to zero and, hence, a fixed point is stabilized in the system.

Wikner et al. [32] proposed a system simplification method of analyzing complex production-distribution models, which was first introduced by [5] as the visualization of the Bullwhip effect that describes the phenomenon of enhancing demand diversity and distortion across a supply chain network. Spiegler et al. [28] proposed a simpler Forrester-based linear model to reduce the complexity of dynamic behavior. Considering the Lorenz equation, a three-echelon nonlinear framework has been modeled to show a chaotic supply chain involving information fluctuations ([73]; [3]; [6]; [10]). This study uses a set of independent differential equations to show the bullwhip effects on a supply chain, where the system's overall dynamics show some unpredictable features that lead to branching and turbulence due to nonlinearity and uncertainty. Assuming that the product demand does not increase uniformly with an increase in the inventory, Mondal [23] proposed a novel SC model that assumes demands have a saturation level. He used the Roth-Horwitz criteria to determine the model's fixed points and their stability and showed its dynamic variations for different parameters numerically by drawing time charts. To quantify the supplier's disruption wave effect, Hosseini et al. [8] proposed a model by integrating the discrete-time Markov chain (DTMC) and a dynamic Bayesian network.

Recent studies, too, have addressed the optimization of chaotic SC systems; Lane et al. [15] studied an integrated SC model and the performance criteria of the bullwhip effect and showed that the latter could be reduced by prioritizing the service-capacity adjustment time. Maiyar and Takar [22] presented a strong mixed integer nonlinear optimization model for the food grain transportation problem that used the particle swarm optimization (PSO) to address the system chaos-related problems.

Dolgi et al. [4] studied the optimal control applications for scheduling in industry, SC and production systems. Considering the SCs dynamic behavior, since traditional methods cannot reduce undesirable bullwhip effects on non-organizational SCs, the chaos theory may offer new solutions to problems in the SC management. Wilding [33] examined five identified key features of a chaos system that had serious consequences for the SC management. Modern control theories identified as potentially great initiatives in suppressing and harmonizing chaotic SCs include adaptive [3], sliding mode [29], intelligent [10] and robust ([9]; [24]; [35]). Hajipour and Baliano [7] designed an adaptive sliding mode controller to stabilize the chaotic deficit-order financial system and showed that it synchronized the system well. Considering a multi-layer SC system with parametric disturbances and turbulences to show the dynamic nonlinear behavior, Xu et al. [35] used the STW algorithm for the chaotic SC system management where the control algorithm presented a satisfactory performance in suppressing chaos and coordinating the chaotic SC based on the system theory.

Lu et al. [17] studied a supply chain system with one producer and two retailers (traditional and online), examined the bullwhip effect in a complex chain with competitive sales and customer return, used the delay feedback control method to control the system entropy and showed that high return rates involved damage for both retailers, and the sales expectation rate consistency had great impact on the whip effects.

As the demand, inventory and production unpredictability make the system behave chaotically, this study presents a three-echelon SC mathematical model to stabilize the chaotic system by adding a linear control parameter, where the system instability and collapse are prevented by increasing the production rate. In studying the system behavior variations by adjusting the levels of linear control parameters, Poincare's analyses show that when production increases, the chain chaotic state is controlled. Control parameters reduce the demand uncertainty-related effects by increasing the production flexibility and improve the chain performance [1].

Using a mathematical model Göksu et al. [6] coordinated the chaotic SC by analyzing the

effects of active controllers on two identical chaotic SC systems, and controlled the chaotic fluctuations by adding linear feedback controllers to the chaotic nonlinear system. To this end, the Lyapunov theory and then computer simulation were used to control/confirm the outputs.
Ma et al. [18] studied closed-loop SC models with two-channel recycling and one producer, analyzed the system output using the "game theory" and system chaos and simulated the problem for entropy assuming that customers were not familiar with product reproduction. Results showed that the high speed of adjusting the recycling cost by the producer/middleman caused system chaos and increased entropy.
Ma and Li [19] addressed the numerical solution of fractional differential equations and the chaos method to provide a mathematical foundation for further studies on chaotic systems, and considered a three-echelon SC to analyze the dynamic behavior effects on the output by simulating the system's complex behavior. Finally, using the controlling parameter method, they balanced the chaotic system, presented a new financial model of fractional time delay, analyzed the system dynamics on its basis using the Lyapunov's stability theory, designed a controller to control the system entropy and presented it in the financial market as a guide for sustainable development.
Chaotic SC controlling has always been highly challenging due to the limitations on the input controllers, and SC controlling through input restrictions has been almost ignored in the related literature. Wang et al. [30] presented a novel controlling plan, where the input data and their uncertainties are controlled better and the uncertainty-induced chaos is examined by simulation tools.
An analysis has been conducted by Zhang et al. [38] to analyze the complex dynamic behavior of the supply chain inventory system. Their study investigated different perspectives on decision making using a simulation approach. In another study, Peng et al. [25] represented a new nonlinear supply chain model which is sensitive to uncertainties. Using a synchronization method, they introduced simulation experiments that were conducted for the purpose of demonstrating the effectiveness of their proposed analytical result. By presenting a new control scheme to overcome control input limitations, Wang et al. [31] presented a new fixed-time super-twisting sliding mode technique while taking into consideration the control input limitations. A novel four-echelon chaotic supply chain model was proposed by Xu et al. [36], where uncertainties and disturbances have a detrimental effect on the closed loop supply chain performance as a result of uncertainties and disturbances. An innovative control technique for fractional-order supply chain systems has been presented by Liu et al. [16]. The researchers propose that the sliding mode of the system they used is equipped with a mechanism that allows the system to be adaptive.
With regard to the applications, various investigations can be found in different categories like marine transportation (Lee et al. [14], Lee et al. [13]), knowledge systems (Ma et al. [21]), bargaining systems (Ma et al. [20]), etc. It can be concluded that the subject of this research is practical and is attracting much attention from a wide range of researchers. A summary of the literature review in addition to a comparison is presented in Table 1.

A review of the literature reveals that although studies on the dynamics of SC systems

Table 1: Summary of literature review

StudyFeature	**Equations Unavailability**	**Independence of all parameter measurement**	**Robustness**	**Uncomplicated Computations**	**Real World Applicability**
Ma et al. [18]				*	
Kocamaz et al. [10]			*	*	
Wang et al. [31]			*	*	
Xu et al. [34]			*		
Current Study	*	*	*	*	*

are many, research on chaotic SCs still has many gaps because the field is vast and complexities are great. Therefore, this research has studied and analyzed the subject from a perspective closer to the real world, where access to basic control equations is none.
The rest of the text has been so organized as to explain the classic, chaotic, SC-related equations in Section 2, describe the entropy and entropy-control-related relations in Section 3, define, explain and implement the entropy minimization controller on a chaotic SC system, using the gradient descent algorithm, and present the simulation results of the above system in Section 4 and present the final conclusions in Section 5.

3 Problem definition

3.1 Equations of the chaotic supply chain

As mentioned earlier, SC management is a complex dynamic system consisting of several entities that involve business activities from the very beginning (raw-material supply) to the final step (customer delivery). SC networks have different forms, where interactions are generally complicated because various chain components usually form a nonlinear dynamic system that exhibits chaotic behavior against external disturbances and parameter variations. Traditional linear programming-based management strategies may not be able to solve, for a variety of reasons, problems caused by product complexity, demand/inventory fluctuations, and customer expectations because all the risks and variables of business activities in today's world follow nonlinear patterns with disorders and parameter uncertainty. According to Fig. 1, a complete SC network involves an information flow that describes the order flow among the producer, distributor and retailer, and a product flow that describes the entire product delivery process from the producer to the retailer through a distributor. In such a chain, the state variable vector $(x(t)\epsilon R^3)$ defines the temporal variations of three state variables, based on which a set of three first-order independent differential equations can be used to describe the chaotic behavior of the SC model, in a state space representation as follows ([37]; [3]).
In fact, the SC model is described by a system of three paired nonlinear differential equations and state variables, each of which changes over time (t), are represented as follows:

$$
\begin{aligned}
x_1 &= mx_2 - (n+1)x_1 \\
x_2 &= rx_1 - x_2 - x_1x_3 \\
x_3 &= x_1x_2 + (k-1)x_3
\end{aligned}
\tag{1}
$$

$x_1(t)$: Customer demand
$x_2(t)$: Distributor inventory
$x_3(t)$: Producer's produced product
System parameters m, n, r, and k show the delivery efficiency, customer demand satisfaction, distortion coefficient and safety storage coefficient, respectively; in real SC activities, m, n and r should be interpreted as positive percentages (m=10, n=9, r=28, k=-5/3) (initial condition = [0;-0.11; 9]) [34].
Considering the above relationships, the SC activities in a time interval are: m% products are delivered from the distributor to the retailers, n% of the retailer orders are satisfied, distributor receives the order-related information from the retailer at a distortion rate of r% and producer produces k orders more than the distributor demand to avoid the effects of uncertain factors on inventory. State variables can acquire negative values too; negative values of variables x_1, x_2, and x_3 indicate less supply than demand, serious information entropy and excess inventory, respectively.

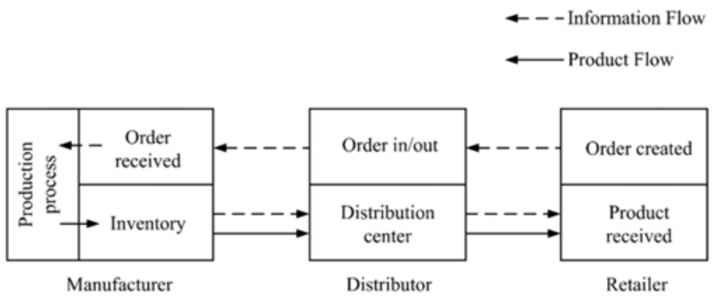

Figure 1: General structure of the multi-level supply chain [34]

This dynamic model considers the comprehensive effects of the retailer and distributor on the producer as well as those of retailer and producer on the distributor. SC networks involve different types of system uncertainties in the demand, delivery and production, and since small variations in the system parameters - delivery efficiency, customer demand satisfaction rate, distortion, safety inventory, etc. - in the SC result in great, qualitative variations in the system dynamic behavior, the SC system, where parameter variations create uncertainties, can exhibit strange, undesirable states such as chaos. Analyzing the specific value of the equilibrium points can help better understand local dynamic behavior of the nonlinear SC system. Three equilibrium points can be calculated for such a system by Eq. (2):

$$E_0 = (0,0,0)$$
$$E_{1,2} = (\pm\frac{\sqrt{(k-1)(n+1)(n-mr+1)}}{n+1}, \pm\frac{\sqrt{(k-1)(n+1)(n-mr+1)}}{m}, -\frac{(n-mr+1)}{m}) \quad (2)$$

The Jacobean matrix at equilibrium point Ei is calculated by Eq. (3):

$$J_i = J|E_i = \begin{bmatrix} -(n+1) & m & 0 \\ r - E_{i,3} & -1 & -E_{i,1} \\ E_{i,2} & E_{i,1} & k-1 \end{bmatrix} \quad (3)$$

where $E_{i,j}$ is the coordinates of the j^{th} helping point at equilibrium point E_i. Using 10, 9, 28, -5/3 for parameters m, n, r, k, equilibrium points are found to be $E_0 = (0,0,0)$ and $E_{1,2} = (\pm 8.4853, \pm 8.4853, 27)$. J_0 specific values for equilibrium point E_0 are calculated by Eq. (4):

$$\lambda_{0,1} = -2.6667, \lambda_{0,2} = 11.8277, \lambda_{0,3} = -22.8277, \quad (4)$$

Since $\lambda_{0,2} = 11.8277 > 0$, the Routh-Hurwitz criterion shows that E_0 is unstable, where fluctuations grow slowly. Similarly, J_1 and J_2 specific values are calculated for, respectively, equilibrium points E_1 and E_2 by Eq. (5):

$$\begin{aligned} \lambda_{1,1} &= -13.8545, \lambda_{1,2}, \lambda_{1,3} = 0.0939 \pm 10.1942i, \\ \lambda_{2,1} &= -13.8545, \lambda_{2,2}, \lambda_{2,3} = 0.0939 \pm 10.1942i, \end{aligned} \quad (5)$$

Obviously, equilibrium points E_1 and E_2 too are unstable and this SC system shows, depending on specific parameters and initial conditions, chaotic behavior, which are quite definite although seem irregular.

3.2 Minimum entropy control

Entropy is an important statistical quantity that describes the concept of order and disorder. If a set of states, possible to occur in a system, is evenly distributed among the stages, disorder is at its highest level, and the system does not prefer to move towards a particular arrangement; therefore, if the system assumes a state that leads to finality, entropy will decrease. In a chaotic SC too, a controller is aimed to achieve certainty and move towards a regular arrangement to plan the chain better.

This section presents the ME control method [2] in brief. Consider a nonlinear mapping (Eq. 6) that represents a chaotic dynamic system:

$$q(t+1) = f(q(t), u(t)) \quad (6)$$

where $q(t)$ is a state vector, $u(t)$ is the control operation and $f(.,.)$ is a nonlinear map (function) assumed to be unknown, but system states are accessible. Here, the main

objective is to design the control law $u(t)$ to remove the system chaos by fixing one of its unstable points, for which the control operation $u(t)$ is so determined that the system entropy converges to 0.
Shannon defines the entropy as follows (Eq. 7):

$$E(u) = -\int_{q\epsilon\Omega} p(q,u) ln\ p(q,u)\, dq \tag{7}$$

where, E Is the entropy, $p(.,.)$ is the system probability density function and Ω is a phase space occupied by the state variables and γ is the waiting factor of the gradient descent algorithm (γ=0.00001). In Eq. (7), it is obvious that when the system states converge to a fixed point, its entropy converges to zero, and vice versa.
To reduce the system entropy by designing a control signal, the latter is obtained, according to [12], as follows (Eq. 8):

$$\mu(n+1) = \mu(n) + \frac{\gamma}{n}\frac{1}{\mu(n)-\mu(n-1)}[(1+ln(\frac{N_k(n)}{n})) - \sum_{i=1}^{N}(1+ln(\frac{N_i(n)}{n}))\frac{N_i(n)}{n}] \tag{8}$$

4 Problem solution and discussion

Controlling chaos in an economic system requires a method relatively independent of the system equations. Among the mentioned control methods, the error feedback control, first proposed by Pyragas [26], is the one widely used for this purpose. If the system dynamic equation is known, finding a suitable feedback for the Pyragas method is easily possible by examining the Jacobian or the characteristic system matrix; otherwise, the search for a confirmatory feedback needs to be done through trials and errors. To reduce chaos in dynamic systems, a new control method has been recently proposed, which is independent of the system equations, minimizes its entropy function and eliminates its chaotic behavior [12]. Since the control operation is through an entropy minimization algorithm, the system becomes regular when its entropy converges to a small value.
To implement the entropy minimization method, it is necessary to first find the Poincare's mapping of the SC system. This operation is needed to discretize the continuous time system to form the entropy as follows (Eq. 9):

$$E(u) = -\sum_{i=1}^{N} p_i\ ln\ pi \tag{9}$$

There are two approaches to form a Poincaré mapping for discretization purposes: 1) cutting the phase space of the chaotic system with a single plane, which, despite having the advantage of reducing the system dimension by one unit, it also has a negative feature; it may contact some movement circuits (because processing is based on a selected procedure) and make the Poincare mapping erroneous in interpreting the system dynamics, and 2) sampling at fixed intervals, which is simpler and does not face the mentioned problem.

This paper has applied the second approach considering a 0.5-time-unit sampling period and, assuming that the system equations do not exist and only 1 (out of 3) state variables is available, has obtained the entropy for the third variable (production) because the latter is more controllable and allows for a more realistic control of the SC system than the other two. The first variable (customer demand) depends on many factors and considering the controller for it is not suggested because its implementation in real environments is difficult, and the second variable (inventory in the distribution center) too may be affected by market variations and other related factors. It is worth noting that the fewer controllers are used to control the system chaos and guide towards stability, the more it will be popular with managers in terms of cost and time as well as the operational feasibility.

In entropy calculations, the more information is available from the system's previous periods, the more accurate will be its chaos estimation to study and apply the controller because it starts the controlling process once it has reached a good estimate of the system entropy. To form entropy, the third variable's variations interval is divided into equal parts so that each event's occurrence probability in small intervals is greater than zero and calculable. To apply a control signal in a continuous time system, the one obtained discretely is applied to the system, in fixed fragments, between two consecutive points of the mapping; the optimal point considered for system stabilization is the unstable equilibrium point $E_1 = (+8.4853, +8.4853, 27)$ (Zhao et al., 2020).

While previous studies have considered the controller mode for all three variables, this study has considered it, as stated before, for only the third variable (production) and added it to the system equations because the system chaos is controlled with only one state variable, which can be considered as an advantage. Consider Eqs. (10):

$$
\begin{aligned}
x_1 &= mx_2 - (n+1)x_1 \\
x_2 &= rx_1 - x_2 - x_1x_3 \\
x_3 &= x_1x_2 + (k-1)x_3 + u \\
u &= \mu(x_3 - x_f)
\end{aligned}
\tag{10}
$$

These are the same equations of the classical SC system applied to the third state variable, controller μ, the value of which can be found from Eq. (8) as the error feedback factor and then other variables are calculated based on their mutual inter-relations. This process has been simulated through successive iterations under the gradient descent method to finally converge to a stable, controllable system point.

MATLAB Software, Ver. 2020A, was used to simulate the chaotic SC model, the length of each interval, where the third variable is divided into smaller intervals, was 0.21 and $\gamma = 0.00001$ considering the relations of the gradient descent method [4]s; the simulation results are shown below.

Figure 2 shows the fluctuations of variable $X1$ versus time, which was converged to 8.314 (fix) after 720 simulation iterations

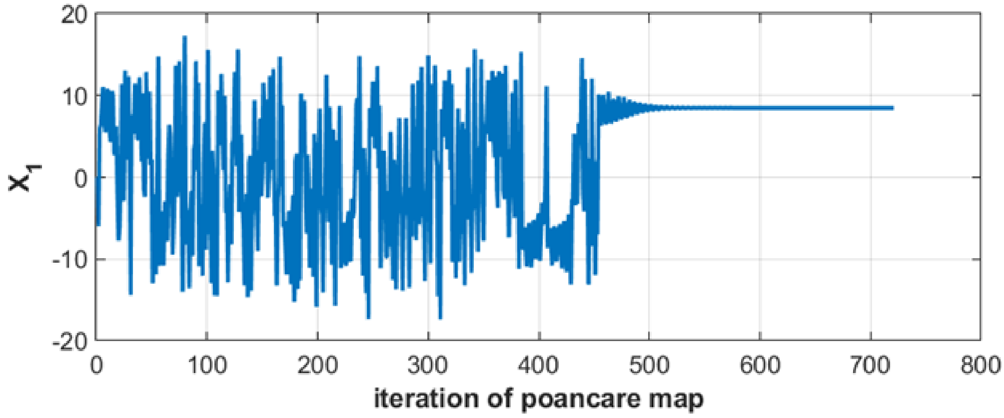

Figure 2: Fluctuations of variable X_1 versus iterations of Poincare mapping of a continuous time system

Figure 3 shows the fluctuations of variable X_2 versus time, which was converged to 9.20 (fix) after 720 simulation iterations.

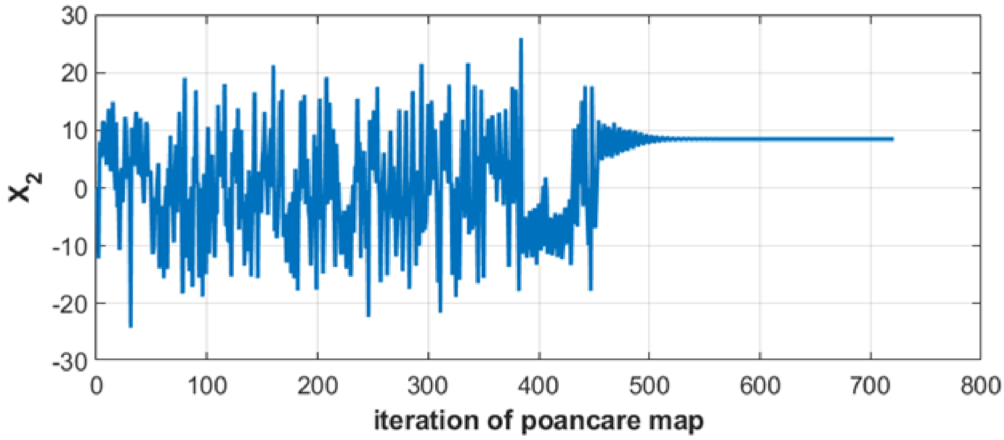

Figure 3: Fluctuations of variable X_2 versus iterations of Poincare mapping of a continuous time system

Figure 4 shows the fluctuations of variable X_3 versus time, which was converged to 27 (fix) after 720 simulation iterations.

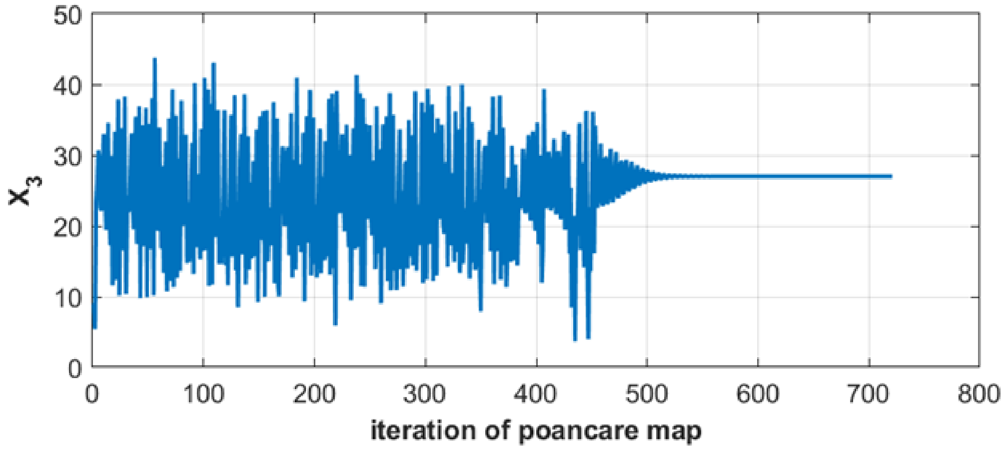

Figure 4: Fluctuations of variable X_3 versus iterations of Poincare mapping of a continuous time system

Poincare's mapping diagrams are presented in Figures 5, 6 and 7.

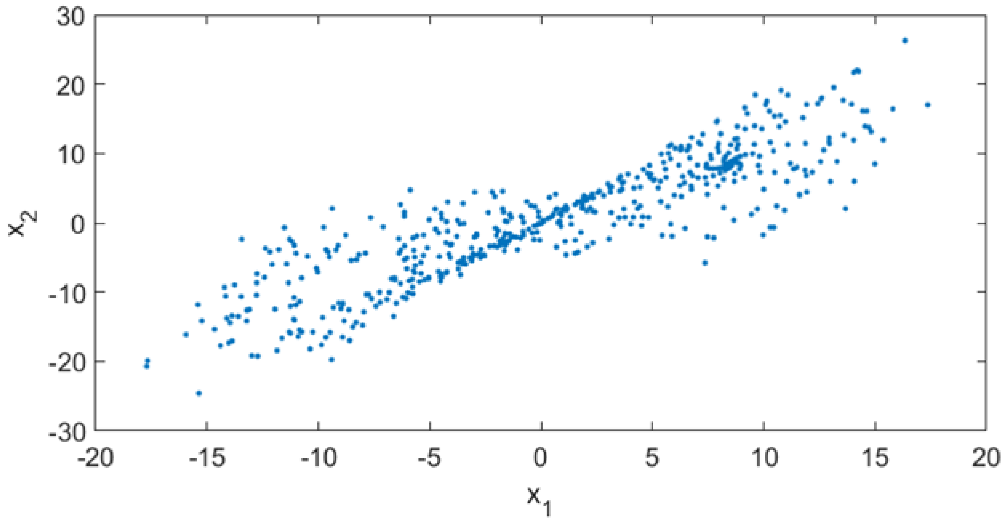

Figure 5: Poincare mapping of variables X_1 and X_2

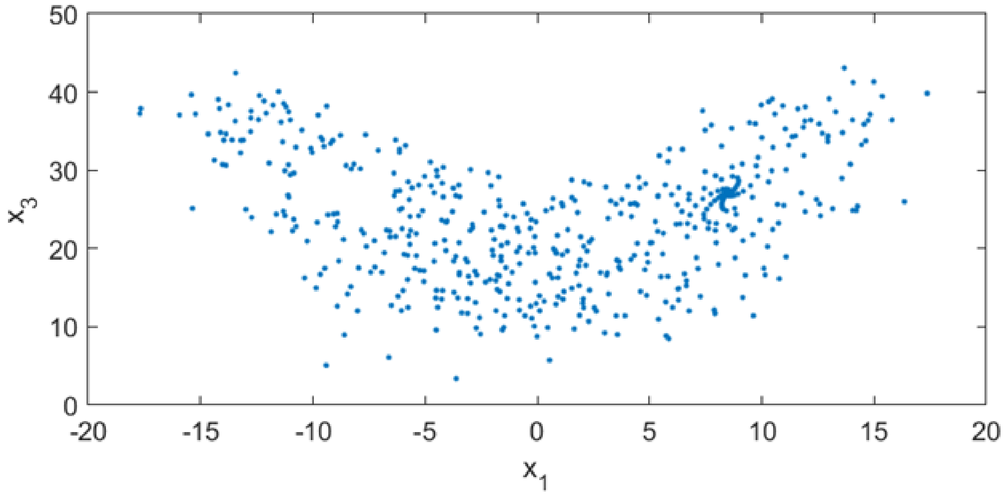

Figure 6: Poincare mapping of variables X_1 and X_3

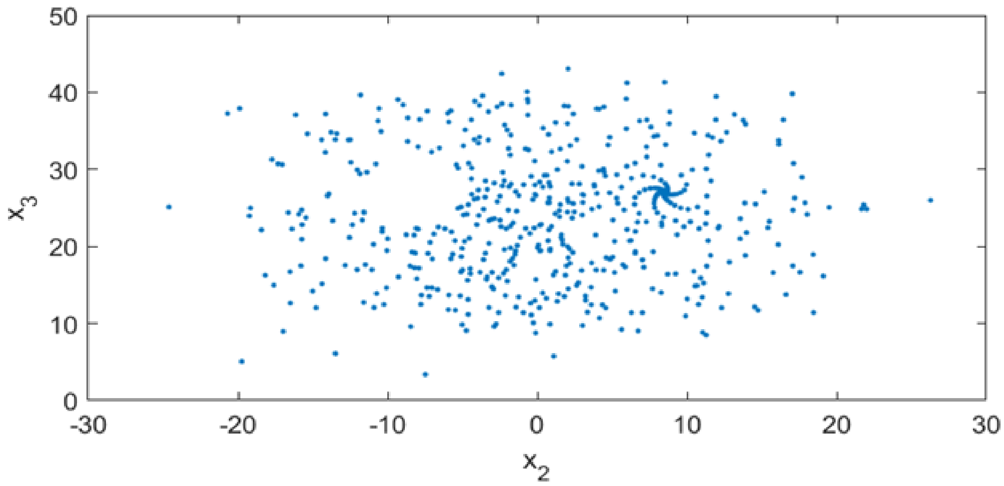

Figure 7: Poincare mapping of variables X_2 and X_3

As shown in all three figures, a decrease in entropy causes the system behavior to converge towards stable points – the star-shaped part in the center of the figure; over time, the system behavior adopts a regular shape and tends towards the coordinates of the center of the shape. These coordinates are 8.314 for X_1, 9.20 for X_2 and 27 for X_3 (Figures 2, 3, and 4).

In Figure 8, entropy has reduced from 4.6 (initial value) to 4.1, during various simulation steps and controller application, indicating the stability of the chaotic SC system.

In Figure 9 that shows the U controller behavior during the simulation iterations, it has moved from the vicinity of an unstable equilibrium point towards a stable one.

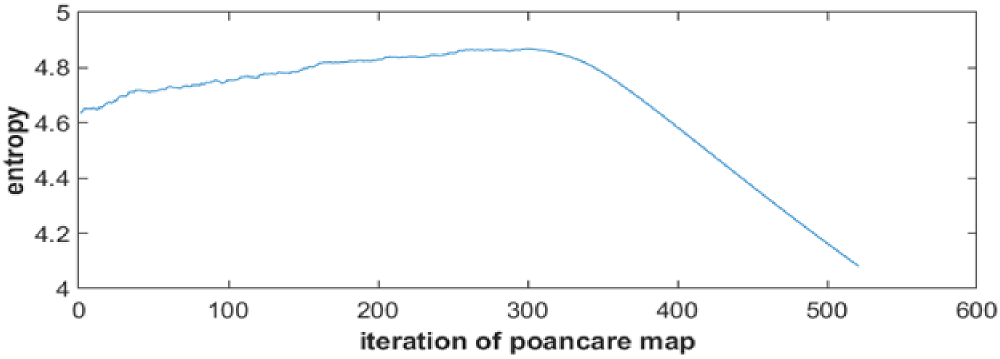

Figure 8: Entropy variations versus simulation iterations

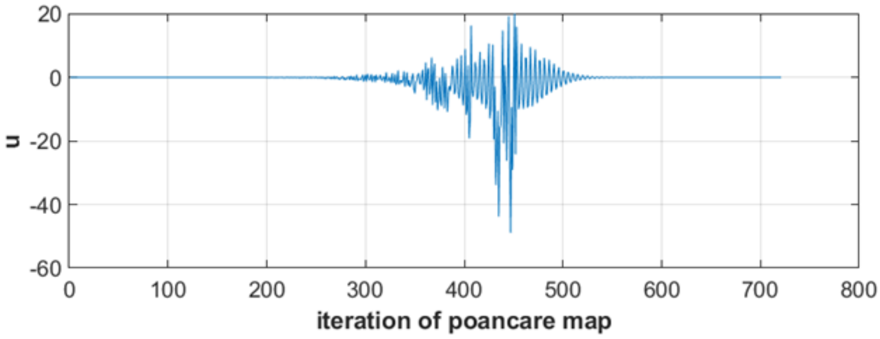

Figure 9: U controller behavior in iterations of Poincare mappings of a continuous time system

As shown, the system has turned stable at unstable equilibrium point $E_1 = (+8.4853, +8.4853, 27)$.

5 Conclusions

This paper tried the control a chaotic supply chain by the entropy minimization method that assumes the system equations do not exist and only the third variable (production) of the main system is measurable. These assumptions make the problem closer to reality because other control methods implemented on chaotic supply chains assume system equations do exist and all variables are measurable. Although the entropy minimization method only needs data to first find the entropy and then reduce it, it is slower and requires more time than other control methods for system stabilization. As mentioned in the main text, the designed controller can control the supply chain and achieve the controlling objectives. In conclusion, there are a number of major outputs of this study that can be summed up as follows:

- It is assumed that the supply chain equations are not available.
- It is assumed that not all variables are measured in the study.

- It is a much simpler computation in comparison to previous related studies that have been conducted in the past.
- Compared to previous studies, this study has a greater level of applicability to the real world.

Future studies may apply the method, introduced in this paper, to other supply chain areas. As mentioned in the text, since a wide range of supply chain-related issues are involved in chaos problems, the methods and techniques investigated in this research can perform well in those fields as well. There are also various other probability functions that can be used based on the nature of the system under investigation. As mentioned, since chaotic systems are very similar to random systems, future studies may try to answer this question: "Do systems assumed random really have a random nature, or are they chaotic?" A thorough understanding of this issue can help solve many problems.

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