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Extreme Value Chart & Analysis of MeansBased on Linear Failure Rate Distribution

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Abstract

The Probability model of a quality characteristic is assumed to follow the linear failure rate distribution. Control charts based on the extreme values of each subgroup are constructed. The control chart constants depend on the probability model of the extreme order statistics of each subgroup and the size of the subgroup. Accordingly, the proposed chart is known as extreme value chart. The technique of analysis of the means for a skewed population is applied with respect to linear failure rate distribution. The results are illustrated by example on live data.

Keywords: Analysis of means (ANOM); Statistical quality control (SQC); Average run length (ARL); Linear failure rate distribution (LFRD).

Introduction

The probability density function (pdf) and cumulative distribution function (cdf) of Linear failure rate distribution are given by

$$f(x)=(a+bx)e^{-\left(ax+\frac{b}{2}x^2\right)};x>0,a,b>0 \quad (1)$$

$$F(x)=1-e^{-\left(ax+\frac{b}{2}x^2\right)};x>0,a,b>0 \quad (2)$$

Linear Failure Rate Distribution (LFRD) is a skewed and non-zero unimodal distribution with $a/\sqrt{b}<1$.

The Mean and Variance of LFRD are respectively

$$\mu=\sqrt{\frac{2\pi}{b}}e^{\left(\frac{a^2}{2b}\right)\left(1-\phi\left(\frac{a}{\sqrt{b}}\right)\right)} \quad (3)$$

$$\sigma^2=\frac{2}{b}(1-a\mu)-\mu^2 \quad (4)$$

In order to construct extreme value control chart based on the extreme observations of a subgroup. This sub group must be drawn from a production process with the quality vitiate follows LFRD. In this context we consider the percentiles of extreme order statistics of the subgroups. For these extreme value control charts we consider the test statistics as the original sample vector $X=(x_1,x_2,...,x_n)$. In this chart instated of finding statistics simply all the individual sample observations are plotted in chart in view of statistics. A corrective action is taken if one or either of extreme values x_1 and x_n of the sample respectively falls within or outside the two specified limits. Based on all these reasons the chart is named asextreme value control chart. The Shewart charts are the basis for many statisticians working with

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statistical quality control tools. When these charts indicate that there is an assignable cause, adjust the process when remedial action is known. Otherwise, attributable causes are assumed to be indicative of the heterogeneity of the subgroup statistic for which the control chart is being generated. For example, if the statistic is the sample mean, this can lead to heterogeneity in the process mean, indicating a deviation from the target mean. This analysis is usually done by using means to divide a set of means of a certain number of subgroups into categories so that the means are homogeneous within the categories and heterogeneous among the categories, a process called Analysis of Means (ANOM)[1] For using the ANOM technique the concept of the control chart for means is viewed in a deferent way grouping of plotted means to fall within the control limits or some outside the control limits. For the homogeneity of all the means, it is necessary that all the means should fall within the control limits. The probability of all the subgroup means to fall within the control limit $(1-\alpha)$. If the subgroups are independent the probability of all subgroups means to fall within the control limit of $(1-\alpha)^{1/n}$ i.e., in the sampling distribution of $\bar{x}$ the confidence interval for $\bar{x}$ to lie between two specified limits should be equal to $(1-\alpha)^{1/n}$. The same principle is adopted in the rest of this paper through LFRD. Because this paper aims at exploring ANOM using control limits of extreme value statistics we have considered only the control chart aspects but not any recently developed ANOM tables or techniques. However, a detailed literature about ANOM is available in [2] and some related works in this direction are [3-16, 8, 10-16] and references there in.

Materials and Methods

1. Extreme Value Charts

The given sample observations are assumed to follow LFRD. The control lines are determined by the theory of extreme order statistics of LFRD. The control lines are determined in such a way that an arbitrarily chosen x_i of $X=(x_1,x_{2,}...,x_n)$ lies with probability inequality in the following way. $P(x_1 \le L)=\alpha/2$ and $P(x_n \ge U)=\alpha/2$. The theory of order statistics say that the cdf of theleast and highest order statistics in the sample of size n from any continuous population are $\left[F(x)\right]^n$ and $1-\left[1-F(x)\right]^n$ respectively where $F(x)$ is the cdf of the population. If $1-\alpha$ is desired at

Table 1. Control Limits of Extreme value chart

n	$Z_{(1)0.00135}$	$Z_{(n)0.99865}$
2	0.00067	2.2478
3	0.00045	2.3206
4	0.00033	2.3711
5	0.00027	2.4097
6	0.00022	2.4408
7	0.00019	2.4669
8	0.00016	2.4893
9	0.00015	2.5090
10	0.00013	2.5264

Table 2. Constants of Extreme value chart

n	***L***	***U***
2	0.00040	1.3459
3	0.00027	1.3895
4	0.00020	1.4198
5	0.00016	1.4429
6	0.00013	1.4616
7	0.00011	1.4772
8	0.00010	1.4906
9	0.00009	1.5024
10	0.00008	1.5128

0.9973 then α would be 0.0027. Taking $F(x)$ as the cdf of a LFRD model we can get solutions of the two equations $1-\left[1-F(x)\right]^n=0.00135$ and $\left[F(x)\right]^n=0.99865$ which in turn can be used to develop the control limits of extreme value chart. The solutions of the above equations for $n=2,3,...,10$ are given in Table 1 denoted as $Z_{(1)0.00135}$ and $Z_{(n)0.99865}$.

The values of Table 1 indicate the following probability statement

$$P\left(Z_{(1)0.00135}<Z_i<Z_{(n)0.99865}\right)=0.9973\ \forall\ i=1,2,...,n \quad (5)$$

$$P\left(\sigma\bar{x}\right)=0.9973\ \forall\ i=1,2,...,n\sqrt{b^2-4ac} \quad (6)$$

Taking $\bar{x}\times1.67$ as an unbiased estimate of σ, the above equation becomes

$$P\left(L\bar{x}<x_i<U\bar{x}\right)=0.9973\ \forall\ i=1,2,...,n \quad (7)$$

where $L=\frac{Z_{(1)0.00135}}{1.67}$ and $U=\frac{Z_{(n)0.99865}}{1.67}$. Thus L & U would constitute the control chart constants for the extreme value charts. These are given in Table 2 for *n*=2(1)10.

2. Average Run Length (ARL) of the Extreme Value Chart

For a given parametric combination *a=1, b=2* of LFRD (with the condition $a/\sqrt{b}<1$) and a given sample size '*n'* and if L & U respectively stand for the lower control limit (LCL) and upper control limit(UCL) of the extreme value control chart, then the ARL of that control chart is given by, $ARL = \frac{1-p}{p}$ where $p = 2 - \left[F(U)\right]^n - \left[1-F(L)\right]^n$. Here $F(x)$ denotes the distribution function of LFRD with parameters *a=1, b=2*. We have to substitute L & U to get the value of p and hence to get ARL. Generally, ARL decreases as *n* increases.

Table 3. Average Run Length

n	*ARL*
2	10.8981
3	8.4929
4	7.0960
5	6.1614
6	5.4830
7	4.9631
8	4.5494
9	4.2103
10	3.9263

3. Analysis of Means (ANOM) - LFRD

If the data variables LFRD follow, we assume $\overline{x}_1, \overline{x}_2, ..., \overline{x}_k$ are the arithmetic mean of *k* subgroups of size *n*, each from an LFRD model. When these subgroup means are used to develop control charts to assess whether the population from which these subgroups are derived operates within acceptable quality deviations. Depending on the basic population model, we can use the control chart constants we developed, or the popular Shewart constants given in any SQC textbook. In general, a process is said to be in control if all subgroup meansare within the control range. Otherwise we say the process is uncontrollable. If α is the significance level of the above decision, we can make the following probability statements.

$$P\left(LCL < \overline{x}_i < UCL\right) = 1-\alpha \;\; \forall i = 1,2,...,k$$

(8)Using the notation of independent subgroups (8) becomes

$$P\left(LCL < \overline{x}_i < UCL\right) = (1-\alpha)^{1/k} \qquad (9)$$

With equi-tailed probability for each subgroup mean, we can find two constants say L^* and U^* such that

$$P\left(\overline{x}_i < L^*\right) = P\left(\overline{x}_i > U^*\right) = \frac{1-(1-\alpha)^{1/k}}{2} \qquad (10)$$

In the case of normal population L^* and U^* satisfy $U^* = -L^*$. For the skewed populations like LFRD we have to calculate L^*, U^* separately from the sampling distribution of $\overline{x}_i$. Accordingly these depend on the subgroup size '*n*' and the number of subgroups '*k*'.

Results and Discussion

The percentiles of sampling distribution of $\overline{x}$ in samples from LFRD are worked out through Monte-Carlo simulation and are given in the Table 3.1. We make use of its percentiles in equation(3.3) for specified '*n*' and '*k*' to get L^* and U^* for $\alpha = 0.01$, 0.05 and 0.10. These are given in Tables 3.1, 3.2 and 3.3. A control chart for averages giving 'In Control' conclusion indicates that all the subgroup means though vary among themselves are

Table 3.1. LFRD constants for Analysis of Means (1−α)=0.99

n	k=1	2	3	4	5	6	7	8	9	10
2	0.1084	0.1053	0.1032	0.1019	0.1016	0.1010	0.1009	0.1009	0.1003	0.1003
	2.1012	2.1852	2.2178	2.2454	2.2475	2.2945	2.2952	2.3046	2.3083	2.3083
3	0.1246	0.1181	0.1162	0.1146	0.1137	0.1127	0.1126	0.1124	0.1120	0.1120
	1.9128	2.0182	2.0388	2.0771	2.0921	2.1105	2.1204	2.1263	2.1703	2.1703
4	0.1347	0.1259	0.1230	0.1216	0.1212	0.1210	0.1208	0.1187	0.1186	0.1186
	1.8243	1.8609	1.8879	1.9031	1.9229	1.9353	1.9434	1.9644	1.9753	1.9753
5	0.1432	0.1365	0.1343	0.1309	0.1292	0.1288	0.1287	0.1285	0.1282	0.1282
	1.7638	1.8066	1.8422	1.8594	1.8710	1.8754	1.8821	1.8828	1.8934	1.8934
6	0.1502	0.1446	0.1403	0.1372	0.1348	0.1327	0.1297	0.1296	0.1285	0.1285
	1.7235	1.7869	1.8223	1.8465	1.8494	1.8538	1.8555	1.8656	1.8820	1.8820
7	0.1575	0.1513	0.1483	0.1436	0.1389	0.1369	0.1358	0.1357	0.1331	0.1331
	1.6855	1.7144	1.7446	1.7499	1.7607	1.7788	1.7874	1.7972	1.7990	1.7990
8	0.1615	0.1535	0.1504	0.1487	0.1471	0.1462	0.1461	0.1453	0.1448	0.1448
	1.6575	1.6983	1.7366	1.7481	1.7516	1.7727	1.7753	1.7840	1.7867	1.7867
9	0.1685	0.1627	0.1594	0.1576	0.1559	0.1535	0.1534	0.1509	0.1504	0.1504
	1.6386	1.6745	1.6962	1.7062	1.7246	1.7317	1.7386	1.7580	1.7590	1.7590
10	0.1715	0.1658	0.1621	0.1599	0.1579	0.1576	0.1566	0.1563	0.1557	0.1557
	1.6079	1.6407	1.6665	1.6734	1.6813	1.6915	1.6931	1.6932	1.7067	1.7067

Table 3.1. Continued

n	15	20	25	30	35	40	45	50
2	0.0989	0.0989	0.0989	0.0933	0.0933	0.0933	0.0933	0.0933
	2.4740	2.6251	2.6251	2.6512	2.6512	2.6512	2.6512	2.6512
3	0.1100	0.1082	0.1082	0.1053	0.1053	0.1053	0.1053	0.1053
	2.2049	2.2174	2.2174	2.2634	2.2634	2.2634	2.2634	2.2634
4	0.1161	0.1136	0.1136	0.1061	0.1061	0.1061	0.1061	0.1061
	1.9820	2.0178	2.0178	2.0689	2.0689	2.0689	2.0689	2.0689
5	0.1273	0.1260	0.1260	0.1257	0.1257	0.1257	0.1257	0.1257
	1.9072	1.9383	1.9383	1.9432	1.9432	1.9432	1.9432	1.9432
6	0.1239	0.1234	0.1234	0.1233	0.1233	0.1233	0.1233	0.1233
	1.8923	1.9101	1.9101	1.9318	1.9318	1.9318	1.9318	1.9318
7	0.1297	0.1292	0.1292	0.1275	0.1275	0.1275	0.1275	0.1275
	1.8483	1.8496	1.8496	1.8820	1.8820	1.8820	1.8820	1.8820
8	0.1393	0.1388	0.1388	0.1381	0.1381	0.1381	0.1381	0.1381
	1.8173	1.8288	1.8288	1.8310	1.8310	1.8310	1.8310	1.8310
9	0.1487	0.1458	0.1458	0.1454	0.1454	0.1454	0.1454	0.1454
	1.7912	1.8051	1.8051	1.8057	1.8057	1.8057	1.8057	1.8057
10	0.1539	0.1529	0.1529	0.1460	0.1460	0.1460	0.1460	0.1460
	1.7310	1.7333	1.7333	1.7449	1.7449	1.7449	1.7449	1.7449

Table 3.2. LFRD constants for Analysis of Means (1−α)=0.95

n	k=1	2	3	4	5	6	7	8	9	10
2	0.1263	0.1166	0.1128	0.1108	0.1087	0.1073	0.1068	0.1063	0.1060	0.1053
	1.8476	1.9446	2.0120	2.0407	2.0952	2.1245	2.1411	2.1587	2.1812	2.1852
3	0.1428	0.1329	0.1298	0.1267	0.1248	0.1228	0.1212	0.1200	0.1191	0.1181
	1.7370	1.8143	1.8693	1.8919	1.9092	1.9471	1.9693	1.9820	1.9962	2.0182
4	0.1563	0.1458	0.1414	0.1383	0.1349	0.1334	0.1317	0.1299	0.1285	0.1259
	1.6657	1.7371	1.7768	1.8017	1.8242	1.8314	1.8407	1.8472	1.8549	1.8609
5	0.1630	0.1537	0.1494	0.1462	0.1444	0.1425	0.1412	0.1391	0.1372	0.1365
	1.6267	1.6883	1.7157	1.7486	1.7633	1.7741	1.7824	1.7952	1.7967	1.8066
6	0.1696	0.1597	0.1565	0.1528	0.1504	0.1489	0.1472	0.1461	0.1451	0.1446
	1.6058	1.6573	1.6872	1.7080	1.7219	1.7376	1.7453	1.7573	1.7694	1.7869
7	0.1758	0.1674	0.1640	0.1603	0.1584	0.1557	0.1536	0.1525	0.1520	0.1513
	1.5777	1.6273	1.6532	1.6706	1.6853	1.6965	1.7028	1.7081	1.7120	1.7144
8	0.1801	0.1712	0.1682	0.1637	0.1620	0.1596	0.1579	0.1564	0.1543	0.1535
	1.5613	1.6041	1.6317	1.6453	1.6573	1.6672	1.6772	1.6877	1.6940	1.6983
9	0.1866	0.1765	0.1726	0.1704	0.1686	0.1676	0.1662	0.1652	0.1632	0.1627
	1.5472	1.5915	1.6114	1.6280	1.6379	1.6481	1.6560	1.6667	1.6690	1.6745
10	0.1885	0.1795	0.1757	0.1738	0.1719	0.1708	0.1698	0.1687	0.1677	0.1658
	1.5270	1.5628	1.5821	1.5940	1.6078	1.6129	1.6174	1.6343	1.6377	1.6407

Table 3.2. Continued

n	15	20	25	30	35	40	45	50
2	0.1034	0.1019	0.1016	0.1010	0.1009	0.1009	0.1003	0.1003
	2.2172	2.2454	2.2475	2.2945	2.2952	2.3046	2.3083	2.3083
3	0.1162	0.1146	0.1137	0.1127	0.1126	0.1124	0.1120	0.1120
	2.0335	2.0771	2.0921	2.1105	2.1204	2.1263	2.1703	2.1703
4	0.1234	0.1216	0.1212	0.1210	0.1208	0.1187	0.1186	0.1186
	1.8868	1.9031	1.9229	1.9353	1.9434	1.9644	1.9753	1.9753
5	0.1345	0.1309	0.1292	0.1288	0.1287	0.1285	0.1282	0.1282
	1.8417	1.8594	1.8710	1.8754	1.8821	1.8828	1.8934	1.8934
6	0.1406	0.1372	0.1348	0.1327	0.1297	0.1296	0.1285	0.1285
	1.8210	1.8465	1.8494	1.8538	1.8555	1.8656	1.8820	1.8820
7	0.1487	0.1436	0.1389	0.1369	0.1358	0.1357	0.1331	0.1331
	1.7354	1.7499	1.7607	1.7788	1.7874	1.7972	1.7990	1.7990
8	0.1509	0.1487	0.1471	0.1462	0.1461	0.1453	0.1448	0.1448
	1.7334	1.7481	1.7516	1.7727	1.7753	1.7840	1.7867	1.7867
9	0.1604	0.1576	0.1559	0.1535	0.1534	0.1509	0.1504	0.1504
	1.6949	1.7062	1.7246	1.7317	1.7386	1.7580	1.7590	1.7590
10	0.1622	0.1599	0.1579	0.1576	0.1566	0.1563	0.1557	0.1557
	1.6609	1.6734	1.6813	1.6915	1.6931	1.6932	1.7067	1.7067

homogenous in some sense. This is exactly the null hypothesis in an analysis of variance technique. Hence the constants of Tables 3.1, 3.2 and 3.3 can be used as an alternative to analysis of variance technique. For a

Table 3.3. LFRD constants for Analysis of Means (1−α)= 0.90

n	*k*=1	2	3	4	5	6	7	8	9	10
2	0.1396	0.1265	0.1206	0.1169	0.1152	0.1132	0.1120	0.1110	0.1098	0.1088
	1.7314	1.8449	1.9004	1.9407	1.9829	2.0078	2.0267	2.0402	2.0505	2.0932
3	0.1553	0.1432	0.1374	0.1334	0.1310	0.1301	0.1282	0.1269	0.1257	0.1250
	1.6556	1.7338	1.7801	1.8123	1.8374	1.8636	1.8810	1.8891	1.9000	1.9080
4	0.1676	0.1567	0.1509	0.1465	0.1435	0.1416	0.1398	0.1386	0.1373	0.1352
	1.5884	1.6639	1.7105	1.7355	1.7609	1.7729	1.7896	1.8013	1.8156	1.8238
5	0.1747	0.1636	0.1582	0.1539	0.1518	0.1497	0.1473	0.1463	0.1459	0.1446
	1.5666	1.6236	1.6571	1.6836	1.7066	1.7150	1.7273	1.7447	1.7545	1.7620
6	0.1812	0.1701	0.1643	0.1598	0.1578	0.1569	0.1541	0.1533	0.1524	0.1510
	1.5486	1.6035	1.6321	1.6565	1.6718	1.6849	1.6970	1.7050	1.7145	1.7212
7	0.1862	0.1762	0.1708	0.1674	0.1659	0.1646	0.1621	0.1605	0.1598	0.1585
	1.5225	1.5767	1.6039	1.6267	1.6406	1.6507	1.6598	1.6656	1.6767	1.6827
8	0.1911	0.1804	0.1755	0.1713	0.1700	0.1686	0.1666	0.1641	0.1632	0.1621
	1.5122	1.5588	1.5891	1.6022	1.6222	1.6307	1.6402	1.6446	1.6508	1.6567
9	0.1977	0.1869	0.1800	0.1768	0.1744	0.1731	0.1718	0.1710	0.1698	0.1688
	1.5008	1.5452	1.5722	1.5911	1.6002	1.6099	1.6204	1.6269	1.6342	1.6369
10	0.2007	0.1893	0.1839	0.1799	0.1775	0.1761	0.1751	0.1739	0.1733	0.1723
	1.4863	1.5253	1.5463	1.5618	1.5723	1.5819	1.5880	1.5921	1.5974	1.6076

Table 3.3. Continued

n	15	20	25	30	35	40	45	50
2	0.1067	0.1053	0.1045	0.1034	0.1028	0.1024	0.1017	0.1016
	2.1416	2.1848	2.1995	2.2172	1.2218	1.2443	1.2457	1.2475
3	0.1207	0.1186	0.1172	0.1162	0.1151	0.1148	0.1144	0.1137
	1.9706	1.9966	2.0246	2.0335	2.0471	2.0733	2.0882	2.0921
4	0.1313	0.1261	0.1249	0.1234	0.1222	0.1217	0.1214	0.1212
	1.8425	1.8604	1.8735	1.8868	1.8891	1.8925	1.9047	1.9229
5	0.1396	0.1368	0.1353	0.1345	0.1335	0.1324	0.1307	0.1292
	1.7863	1.8063	1.8266	1.8417	1.8514	1.8592	1.8619	1.8710
6	0.1467	0.1449	0.1426	0.1406	0.1393	0.1378	0.1354	0.1348
	1.7513	1.7719	1.8095	1.8210	1.8329	1.8402	1.8475	1.8494
7	0.1531	0.1516	0.1500	0.1487	0.1481	0.1441	0.1401	0.1389
	1.7042	1.7143	1.7232	1.7354	1.7454	1.7493	1.7548	1.7607
8	0.1574	0.1543	0.1524	0.1509	0.1498	0.1487	0.1481	0.1471
	1.6793	1.6952	1.7192	1.7334	1.7390	1.7465	1.7499	1.7516
9	0.1654	0.1630	0.1617	0.1604	0.1593	0.1585	0.1570	0.1559
	1.6619	1.6735	1.6933	1.6949	1.6985	1.7042	1.7068	1.7246
10	0.1693	0.1666	0.1636	0.1622	0.1612	0.1607	0.1596	0.1579
	1.6317	1.6399	1.6587	1.6609	1.6719	1.6725	1.6751	1.6813

normal population one can use the tables of [1]. For a LFRD our tables can be used. We therefore present below some examples for which the goodness of fit of LFRD model is assessed with Q-Q plot technique (strength of linearity between observed and theoretical quintiles of a model) and tested the homogeneity of means involved in each case. These are given in Tables 3.1, 3.2 and 3.3. A chart of means, which provides a "controlled" conclusion, shows that all subgroup means, although different from each other, are homogeneous in some sense. This is exactly the null hypothesis in the ANOVA technique. Therefore, the constants of Tables 3.1, 3.2 and 3.3 can be used as an alternative to the ANOVA technique. For normal populations, [7] table can be used. Our table is available for LFRD. We therefore show below some examples given in Tables 3.6, 3.7 and 3.8 that evaluate the goodness of fit of LFRD models using the QQ plot technique (linear strength between the observed and theoretical fifths of the model), and in each case. The following tests both test the homogeneity of the means involved.

***Example 1**:* Consider the following data of 25 observations on manufacture of metal merchandise that suspect variations in iron content of raw material furnished by five suppliers. Five ingots have been randomly selected from every of the 5 suppliers. The Table 3.6 consists of the information for the iron determinations on each ingot in percent by weight.

***Example 2**:* Three battery brands are checked. The three brands have different life spans (in weeks). Five batteries of each brand were tested and the values are given in Table 3.7.

***Example 3**:* Four catalysts that may affect the

Table 3.4. Normal Distribution

			No. of Counts			
Example		[LDL,UDL]	In	P=in/k	Out	Out/k
1.	**n=5, k=5, α=0.01**	[3.517,3.879]	3	0.6	2	0.4
2.	**n=5, k=3, α=0.05**	[87.82,95.52]	2	0.7	1	0.3
3.	**n=4, k=4, α=0.10**	[26.14,82.84]	2	0.5	2	0.5

Table 3.5. LFRD

			No. of Counts			
Example		[LDL,UDL]	In	P=in/k	Out	Out/k
1.	**n=5, k=5, α=0.01**	[0.4454,6.4504]	5	1	0	0
2.	**n=5, k=3, α=0.05**	[13.6950,157.2725]	3	1	0	0
3.	**n=4, k=4, α=0.10**	[7.9805,94.5413]	4	1	0	0

Table 3.6. Iron content present in the raw material

Supplier				
1	**2**	**3**	**4**	**5**
3.46	3.59	3.51	3.38	3.29
3.48	3.46	3.64	3.40	3.46
3.56	3.42	3.46	3.37	3.37
3.39	3.49	3.52	3.46	3.32
3.40	3.50	3.49	3.39	3.38

Table 3.7. Life period of Electric batteries

Weeks of Life		
B1	**B2**	**B3**
100	76	108
96	80	100
92	75	96
96	84	98
92	82	100

Table 3.8. Concentration of Catalysts

Catalyst			
1	**2**	**3**	**4**
58.2	56.3	50.1	52.9
57.2	54.5	54.2	49.9
58.4	57.0	55.4	50.0
55.8	55.3	54.9	51.7

Table 3.9. Comparison of Correlations coefficients

Example	LFRD	Normal
1	0.9787	0.2067
2	0.9309	0.4149
3	0.9382	0.4447

concentration of one component in a three component liquid mixture are being investigated. The concentrations obtained are given in Table 3.8.

For these three examples the goodness of fit as revealed by Q-Q plot are summarized in the following table. This shows that LFRD is a better model, exhibiting linear relation between sample and population quantiles. Graphs indicating QQ plot for the data in the illustrated examples are given in the Figures 1.1, 1.2 and 1.3.

Treating these observations in the data as samples, we calculated decision limits for the normal population and the LFRD population and present them in Tables 3.4 and 3.5, respectively.

Conclusion

The decision lines using Normal distribution yield that the number of homogenous means for each data set are 3,2 and 2 respectively. And those away from homogeneity are 2,1 and 2 respectively. On the other hand when the ANOM tables of our model(LFRD)are

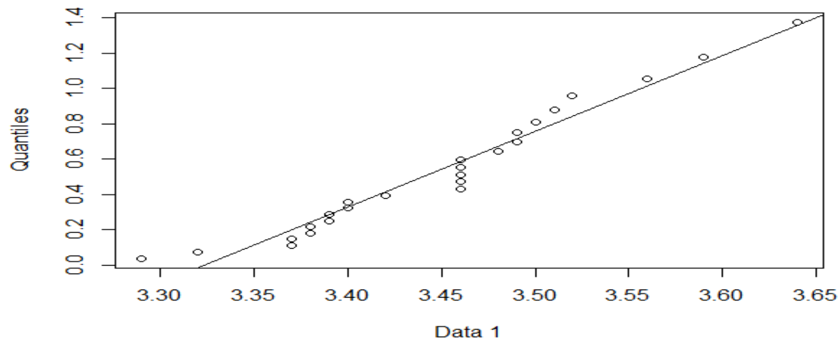

Figure 1.1. Graph of QQ-plot for the data in example 1

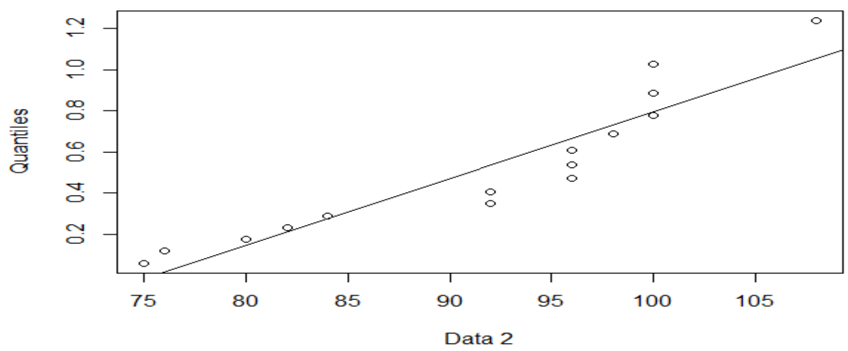

Figure 1.2. Graph of QQ-plot for the data in example 2

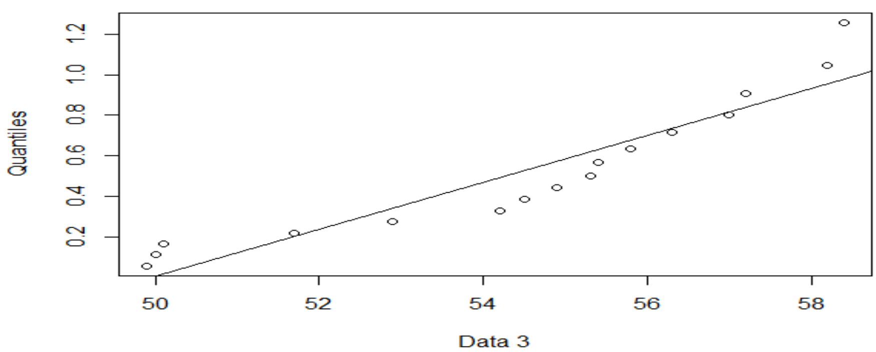

Figure 1.3. Graph of QQ-plot for the data in example 3

used for the same data sets we get the number of homogenous means to be 5,3,4 respectively without exhibiting deviation of any mean from homogeneity. Thus usage of normal model resulted in homogeneity for some means and deviation for some other means, indicating a possible rejection of those means. This decision is valid if Normal distribution is a good fit to the data. For comparison, we have determined from the QQ plot that LFRD is a better model than Normal, which is supported by the correlation coefficients of the Q-Q plots with Normal and LFRD respectively for each dataset and are given in Tables 3.9. Therefore, we assume that there may be more errors in the normally distributed decision process. Therefore, any means of achieving homogeneity using LFRD (Table 3.5) is better than means deviating from homogeneity using the normal ANOM method.

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