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RESEARCH PAPER

Dynamic Allocation of Hospital Beds During the COVID-19 Pandemic Outbreak: A Possibilistic Programming Approach

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Abstract

The health service network has problems such as a shortage of medical equipment and human resources. Due to the need for high expertise in supplying these facilities, this problem is much harder to be solved than other industrial ones. In the COVID-19 pandemic, maintaining tranquility in society is the most important factor. The tranquility is obtained by providing medical facilities in the health care network. Also, the COVID-19 pandemic imposes new restrictions on the network because of preventive guidelines. In this situation, the problem of resource allocation will become more sophisticated and will reduce system efficiency. In this paper, the problem of transferring hospital beds to patients infected by COVID-19 considering a predetermined capacity level is considered. To cope with these problems, a mixed-integer mathematical programming model is suggested. In addition, to consider the uncertainty in the demand of patients that occurs in the pandemic, the fuzzy programming approach is used. The suggested model is solved with the Benders decomposition algorithm (BDA) and applied for assigning beds in two samples. The results show that proper management of resources in crisis situations such as the COVID-19 outbreak is very effective. As a result, this issue causes to overcome pressure on medical staff and lack of hospital facilities, during pandemic conditions.

Keywords:
Health Care;
Capacity Expansion;
Dynamic Allocation;
Pandemic, Demand Uncertainty;
Benders Decomposition

Introduction

In December 2019, several patients were first diagnosed with an infectious disease in Wuhan, China, and reported to the World Health Organization [11].The disease, called COVID-19, spread rapidly throughout China and from China to other parts of the world. Given the advancement of medical science and the increasing access of all human societies to health in the 21st century, as well as the eradication of epidemic diseases, at least in the last hundred years, the occurrence and prevalence of pandemics such as COVID-19 seem unpredictable. Even after it happened, many countries did not believe in its occurrence and severity until the COVID-19 pandemic took an incredible toll.

Although it was the site of the emergence of COVID-19 in Wuhan, China, in December 2019, it quickly became a global pandemic, and on March 31, 2020, the United States of America surpassed China with more than 3,900 deaths. Even Italy and Spain surpassed China in terms of the death toll, as in the United States. In one of the COVID-19 pandemic centers in

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northern Italy, the conditions of the healthcare system deteriorate. Equipment shortages went so far that doctors had to make impossible decisions about which patients to survive. Doctors who were involved in the treatment of patients were forced to consider factors such as age, underlying diseases, and weight regarding the possibility of people surviving to decide which patients to receive mechanical ventilation [21]. So, due to the large number of patients affected by the contagion of COVID-19 and the deterioration of many patients when referred, communities around the world faced a new challenge of scarce hospital equipment. The challenge of shortages of hospital equipment such as ventilators was partially offset by production with urgency. However, this method itself has problems such as lack of time, manufacturing machines and labor due to unpredictable conditions. Therefore, it is necessary to use the equipment in the best possible way to maximize the lifesaving rate of patients.

Our contribution

In response to the mentioned challenge, this research presents a mathematical model for creating a system to consider the allocation of patients as fair as possible with considering the distance of the hospital. Achieving this goal prevents patients from concentrating in certain hospitals which reduces the rate of treatment and increases the rate of burnout of the medical staff and also provides a system for referring patients fairly to all existing hospitals, taking the patient's distance from the hospital as a factor. Also, the capacity of each hospital is considered as a decision-making variable that by considering the capacity occupied by the patients, the possibility of providing services to other non-emergency patients by each hospital is also determined.

Organization

This paper is organized into 5 sections. In Section 2, a literature review related to the subject is presented. In Section 3, the allocation model and handling of the uncertainty of the patient population are presented. Also, in Section 4, our solution approach and computational results in COVID-19 outbreak conditions are presented. In Section 5, the article will be ended with some concluding remarks.

Literature Review

The importance of the location of each facility which has an effect on the quality of presenting services is considered [14]. Scheduling is considered for using equipment due to the lack of capacity in intensive care units of hospitals that is an important issue and should be considered in presenting medical services [12].

The allocation of equipment that is the main resource in the crisis was addressed and the deterioration of victims' health conditions has been modeled as a Markov chain. In this study, the rate of expected health improvement increased and reduced waiting time was considered [24]. The allocation of scarce medical resources after a crisis occurred is investigated with using event simulation [5].

Resource allocation was investigated in emergency situations within the framework of multi-objective optimization and simulation [6]. Simulation and queuing models for hospital bed allocation was examined [23,8]. Optimal allocation of sanitary equipment in developing countries is employed [7]. The system modeling approach has been expressed to distribute influenza vaccines in pandemic conditions [2].

Kaplan addressed the effects of quarantine and compliance with other health protocols announced to prevent further COVID-19 outbreaks [13]. The allocation of ventilators in emergency situations has been proposed [26]. Before the emergence and prevalence of COVID-19, in a situation where the problem of the influenza pandemic had caused the need for ventilators, its demand in the United States was considered [15]. By strictly considering the scenarios of an influenza outbreak, it was estimated a need for 35,000-60,500 additional ventilators to avert 178,000-308,000 deaths. Robust models were studied [4]. They assess the lack of hospital staff with the aim of minimizing the impact of emergency situations . The effect of this approach was shown through experimentation and comparison with realistic data [4]. Huang et al optimized the storage of ventilators needed to treat patients with influenza alignment, in which they considered all possible pandemic conditions in the state of Texas [10].

However the distribution of ventilators is not considered while in the case of COVID-19, one of the essential issues is considering the distribution of the ventilator, which should be considered due to the peak or drop in the need at different times [10].

A hybrid approach was presented for routing and scheduling in home health care services [20]. The home health care industry which is necessary due to the lack of beds in hospitals and the danger of getting involved in hospitals is assessed [16]. This method is also useful for diseases like COVID-19 although it was presented for all the patients who can be at home and don't need scarce equipment. A multi-objective optimization approach to resource management in crisis scenarios in uncertain situations was presented in 2021 [9].

A dynamic operating room scheduling model was offered [27]. In this paper, a model for scheduling and sequencing the assignment of the operating room to surgeons is considered. To solve the model, a hybrid metaheuristic of Grey Wolf Optimizer (GWO) with Variable Neighborhood Search (VNS) was applied. There is a research gap to model dynamic emerging situations of COVID-19 due to a robust approach aimed at providing fast and efficient responses to pandemic situations reviewed [25]. The problem of activating and assigning rooms to the COVID-19 patients and emergency patients, and also scheduling the operations of these patients is studied [1].

Table 1. comparison of studies conducted in different researches

reference	X-objective	Model	Uncertainty handling	Solution method	Case study	Scheduling	Routing	Location	Allocation	COVID-19
Mahmoodzade et al. (2015).	Two	MIP	-	Lingo	Tehran			*		
Feng et al. (2017)	Multi	MIP	-	Genetic Algorithm	Taiwan				*	
Jafari eskandari et al. (2018)	Single	MIP	-	GAMS	-	*				
Zhu et al. (2020)	Single	MIP	-	Hybrid metaheuristic	China	*				
Hallaji and Ramezanian (2021)	Single	MIP	-	GAMS	Tehran	*	*		*	
Mirabnejad et al. (2021)	Single	MIP	-	Genetic Algorithm	-	*	*			
Hernández-Pérez (2021)	Multi	MIP	*	GAMS	USA				*	*
Arab Momeni et al. (2022).	Single	MIP	*	GAMS	Tehran	*				*
This study	Single	MIP	*	Benders decomposition	-				*	*

Based on the literature review, we present research gaps that justify the motivation for the present study. Modeling in COVID-19 pandemic conditions with the aim of creating effectiveness as a basic need should be considered. At the same time, developing a fuzzy programming approach, which can take into account the degree of deviation from facilities in hospital resources and provide the possibility of managing hospital beds in COVID-19 pandemic conditions, is considered a research gap that provides valuable insight into the management of hospital resources dynamically.

In addition, it has been tried to consider dependence on medical resources in the proposed model. Finally, in this paper, decisions will be made in time periods (days), under uncertain demand conditions. Our model is formulated as a fuzzy programming model, and the developed model will be solved in its uncertain form of demand.

The main contributions of this research that makes it different from the other papers could be summarized as follows:

- The model considers expanding all the hospitals in the network with attention due to budget constraints. This feature creates a real-world adaptation to the model due to the importance of expanding capacity as soon as possible in the pandemic situation which should be able to respond to the demand that is not expectable.
- The objective function is oriented toward preventing capacity completion from the specified limit, as much as possible to improve the quality of health care services.
- Paying attention to the patient's distance from the hospital is considered a parameter in the model which is not the main goal and so it is considered a facilitator factor.
- Applying the uncertainty programming method (i.e. the fuzzy programming approach) to cope with the source of uncertainty which is the demand.
- Developing Benders decomposition method for solving mixed-integer linear programming (MILP) in large-scale problems.

Problem Definition and Mathematical Modeling

It should be noted that in times of crisis epidemic outbreak, the possibility of community involvement is exponentially high and as a result, the number of patients is increasing compared to normal conditions. Therefore, it is necessary to use the equipment in the best possible way to maximize the lifesaving rate of patients which is considered in our study that is what we will assess below.

In general, in crisis situations, patients referred to the hospital have three physical conditions. First, patients with “normal” symptoms can be hospitalized at home and only due to epidemic conditions of the disease need to be quarantined to avoid the spread of COVID-19 to others. Second, patients with “serious” symptoms who have special physical conditions and it is necessary to be admitted to the normal ward of the hospital before their conditions become acute. Third, patients who come to the hospital with “acute” physical conditions and therefore it is necessary to use special hospital equipment due to their high risk and thus be admitted to the intensive care unit. According to the mentioned classification, patients with critical conditions need a hospital intensive care unit and special equipment. Due to the lack of facilities in the intensive care unit of hospitals in pandemic conditions and the importance of providing facilities to patients with acute conditions to better care for them, this issue has been investigated in this article. Fig. 1 illustrates the structure of the problem.

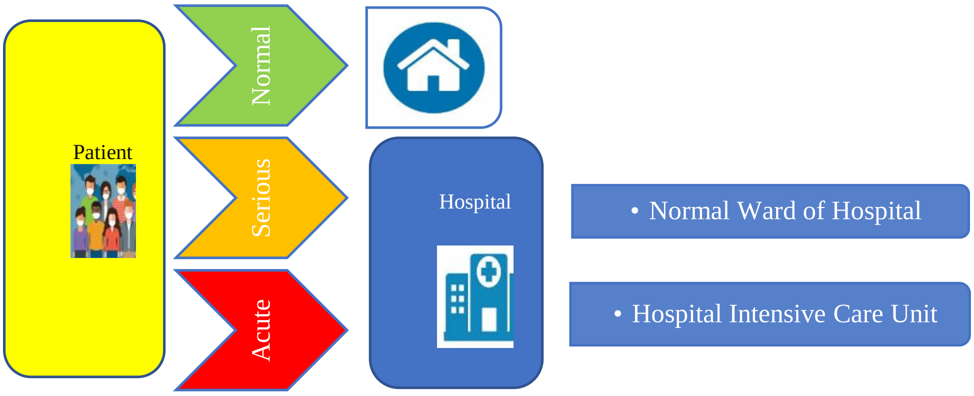

Fig. 1. The structure of problem

In such critical situations, due to the completion of hospitalization capacity and lack of scarce equipment in many hospitals, patients were forced to receive health care services with reduced quality and also even forced to go to multiple centers to find hospitals with empty capacity. This challenge endangers the safety of society both in terms of physical and mental health. So that the allocation of patients in a fixed percentage of capacity occurs in all hospitals and if there is still patient, allocation of patients to the capacity of the hospital is done. Considering this goal indirectly determines the possibility of hospitals serving other patients such as non-urgent patients in each period.

In critical situations, it is sometimes necessary for hospitals to allocate the capacity of other wards that are not in the priority of treatment to patients involved in the pandemic, which is considered by defining capacity as a variable and not an input parameter with the maximum possible.

On the other side, preventing the movement of people with symptoms of the disease in the city, despite the capacities being completed, can cause mental relaxation at the community level and also prevent the spread of the virus in the community. Hence considering the radius of coverage of patients simultaneously with other factors is very important to manage the allocation of patients to hospitals and in our research, paying attention to the patient's distance from the hospital is considered a parameter in the model. As a result of modeling this condition, the health care network that refers the patients to the hospitals directly, can be obtained.

Assumptions

The assumptions used for formulating the mathematical model are, as follows:

- The population classification of patients in each period is based on the location where they are located so that the patient can be assigned to available medical centers in terms of location as a goal and also be able to calculate the coverage radius. It is assumed that the patient population of each area is located in the center of that area.
- All patients can be referred to a single hospital.
- The number of direct visits to each hospital is assumed to be one for each patient from location h in period *t*.
- The gradual coverage is defined by taking circles to a radius of $\tau_i^{\min}$ and $\tau_i^{\max}$ (minimum and maximum coverage radius of hospital *(i)*) to the centrality of each hospital. In this definition, if the patient is placed inside the circle with a smaller radius, the amount of radius coverage equals one and in the case of being outside the larger radius, the amount of coverage equals zero and if placed in the outside area of the smaller circle and inside the larger circle is calculated linearly based on formula (1a) (Berman et al., 2003). if

parameter ψ_{hi} stands for the distance between the patient zone (*h*) and the hospital (*i*), then the coverage score of hospital (*i*) for the patient zone (*h*) which is shown by (a'_{hi}), could be calculated as follows:

$$a'_{hi} = \begin{cases} 1 & \tau_i^{\min} \geq \psi_{hi} \\ \dfrac{\tau_i^{\max} - \psi_{hi}}{\tau_i^{\max} - \tau_i^{\min}} & \tau_i^{\min} \leq \psi_{hi} \leq \tau_i^{\max} \\ 0 & \psi_{hi} \geq \tau_i^{\max} \end{cases} \tag{1a}$$

Now, according to the defining a'_{hi}, definition of the binary parameter of the coverage score is as follows:

$$a_{hi} = \begin{cases} 1 & a'_{hi} \leq 1 \\ 0 & a'_{hi} = 0 \end{cases} \tag{1b}$$

Mathematical model

The notations used in the mathematical model for designing a health care network are as follows:

Indices and sets:

$h \in H, i \in I; I \subseteq H$	The set of patient zone and hospital locations respectively
$t, t' \in T$	The set of periods

Parameters:

e_{it}	Capacity expansion cost of hospital *i* at period *t* (per unit)
o_{it}	Operating cost of hospital *i* at period *t* (per unit)
p_{ht}	population of the patient zone *h* at period *t*
v_i	Maximum allowable percentage of admission to hospital *i*
b_t	Available budget at period *t*
l_i, u_i	Minimum and maximum number of beds for allocating at hospital *i*
a_{hi}	Binary parameter indicating point *h* is covered by a hospital at point *i* or not
M	Relatively large number

Variables:

y_{it}	Binary variables indicating capacity expansion of hospital *i* at period *t*
c_{it}	Operating capacity of hospital *i* at period *t*
c_{it}^{e}	The amount of capacity expansion of hospital *i* at period *t*
s_{hit}	Binary variables implying assignment of patients at patient zone *h* to hospital *i* at period *t*
q_{hit}	The flow of patients from patient zone *h* to hospital *i* at period *t* which is lower than v_i percent of the hospital capacity

q'_{hit}	The flow of patients from patient zone *h* to hospital *i* at period *t* which is upper than v_i percent of the hospital capacity until the completion of capacity

The model is designed in such a way that in *t*=0, the corresponding binary variable value of existing operational hospitals is equal to one and the corresponding continuous variable is equal to the positive value. Also, if the goal is to create a network from the beginning, the initial value of all decision variables in *t*=0 is equal to 0.

Considering the notations, the mixed-integer non-linear programming (MINLP) model of the concerned problem is as follows:

$$\text{Max} \sum_h \sum_i \sum_t q_{hit} - \sum_h \sum_i \sum_t q'_{hit} \tag{2}$$

s.t.

$$\sum_h q_{hit} \le v_i c_{it} \qquad \forall i,t \tag{3a}$$

$$\sum_h q'_{hit} \le (1-v_i) c_{it} \qquad \forall i,t \tag{3b}$$

$$c_{it} = c_{i(t-1)} + c_{it}{}^{e} \qquad \forall t > 0 \tag{4}$$

$$c_{it} \le u_i \qquad \forall i, t > 0 \tag{5}$$

$$c_{it}{}^{e} \le u_i y_{it} \qquad \forall i, t > 0 \tag{6}$$

$$\sum_i s_{hit} \ge 1 \qquad \forall h,t \tag{7}$$

$$q_{hit} + q'_{hit} \le u_i (a_{hi} s_{hit}) \qquad \forall h,i,t \tag{8}$$

$$\sum_i (q_{hit} + q'_{hit}) \le p_{ht} \qquad \forall t,h \tag{9}$$

$$\sum_{t=1}^{t'} \sum_i (o_{it} c_{it} + e_{it} c_{it}{}^{e} y_{it}) \le \sum_{t=1}^{t'} b_t \qquad \forall t' > 0 \tag{10}$$

$$q_{hit} + q'_{hit} \ge (1/M) s_{hit} \qquad \forall h,i,t \tag{11}$$

$$c_{it}{}^{e} \ge (1/M) y_{it} \qquad \forall i,t \tag{12}$$

Objective function (2) is developed to maximize the total flow of patients from patient zone h to hospital *i* at period *t* which is lower than v_i percent of the filled hospital capacity so that as long as there is no need, the patient will not be allocated more than the specified limit.

Constraint (3a) creates the condition that the allocation exceeds the specified limit (v_i) if this is possible in terms of the number of patients. Constraint (3b) controls patient allocation to hospitals until capacity completion if needed. The reason why Constraints (3a) and (3b) are discrete is the priority of handling possibility of the capacity completion which is the main aim of the objective function. Constraint (4) calculates the capacity of each hospital, in each period**.** Constraint (5) put a limitation on the hospital capacity. Constraint (6) indicates that hospital capacity can be expanded in this period. Constraint (7) ensures that the patient zone should be assigned to the hospitals. Constraint (8) indicates that a flow of patients between two points in the network can happen if the point of the patient zone is located in the hospital coverage radius. Constraint (9) indicates that in each period, the total flow of patients from each patient zone to all of the hospitals is lower than the all of the population of patients which we call it p_{ht}. Constraint (10) ensure that the total cost of expanding the capacity of hospitals, cannot exceed the cumulative budget until the end of each period. Constraint (11) creates the relation between q_{hit}, q'_{hit} and the corresponding binary variable of them (s_{hit}). Constraint (12) also creates the relation between c^e_{it} and the corresponding binary variable of it which is called (y_{it}).

Linearization

The obtained model due to the product of a continuous variable in a binary variable in constraint (10) is a mixed-integer non-linear programming and according to the nonlinear nature, it is possible to be linearized that is explained [3]. It is enough to put:

$$c_{it}^{\ e} y_{it} = \tau_{it} \geq 0 \qquad \forall i,t > 0 \tag{13}$$

Constraint (10) will be rewritten as follows:

$$\sum_{t=1}^{t'} \sum_{i} (o_{it} c_{it} + e_{it} \tau_{it}) \leq \sum_{t=1}^{t'} b_t \qquad \forall t' > 0 \tag{14}$$

$$\tau_{it} \leq c_{it}^{\ e} \qquad \forall i,t > 0 \tag{15}$$

$$\tau_{it} \leq M * y_{it} \qquad \forall i,t > 0 \tag{16}$$

$$\tau_{it} \geq c_{it}^{\ e} - M(1 - y_{it}) \qquad \forall i,t > 0 \tag{17}$$

According to the above constraints, we've been able to equate the mixed-integer nonlinear programming with the mixed-integer linear programming which has an objective function as follows:

$$\text{Max} \quad \sum_h \sum_i \sum_t q_{hit} - \sum_h \sum_i \sum_t q'_{hit} \tag{18}$$

s.t.

Constraints (3a), (3b), (4)-(9), (11), (12), (14)-(17)

A possibilistic programming approach

For handling both epistemic uncertainty in input data and elasticity in constraints and/or flexibility in goals, the fuzzy programming approach is applied. Fuzzy mathematical programming is divided into two main categories: (1) possibilistic programming and (2) flexible programming [17,18]. The possibilistic programming approach controls epistemic uncertainty in input parameters due to lack of knowledge and/or even due to using the historical data, that have the dynamic nature. On the other hand, the flexible programming approach copes with soft constraints and flexibility on the target values of goals. As fuzzy programming approach is appropriate approach for handling the uncertainty that the exact value of parameter is not available and only the approximating as a triangular number for the population of the patients is accessible. Due to the nature of COVID-19 that is an unknown phenomenon, the possibilistic programming approach is consistent with the concerned problem.

In detail, let us assume the following fuzzy model including imprecise parameters:

$$\begin{aligned}&\text{Max } z = cx\\&\text{s.t.}\\&\quad Ax \geq \tilde{b} \qquad\qquad x \geq 0\end{aligned} \tag{19}$$

where x denotes decision variables and c; A; $\tilde{b}$ are model parameters. It is assumed that parameter b is tainted with uncertainty.

The possibilistic chance-constrained programming (PCCP) approach requires that the decision makers (DMs) be able to declare the minimum level of proper satisfaction for each possibilistic chance-constraint [19]. In this paper, the necessity measure instead of possibility is used because it's stricter than other measures to handle of healthcare network epistemic uncertainty which is related to the society health status and actually in emergency cases that is dealing the human lives. Now, by having the following model:

$$\begin{aligned}&\text{Max } z = cx\\&\text{s.t.}\\&\quad Nec\{Ax \leq \tilde{b}\} \geq \alpha \qquad\qquad x \geq 0\end{aligned} \tag{20}$$

The crisp counterpart of model (18) is as follows:

$$\begin{aligned}&\text{Max } z = cx\\&\text{s.t.}\\&\quad Ax \leq \underline{b} - \alpha L \qquad\qquad x \geq 0\end{aligned} \tag{21}$$

In the crisp counterpart of model (21), as an assumption, the uncertain parameter $\tilde{b}$ has triangular fuzzy parameter and could be represented LR fuzzy number: $b = \prec \underline{b}, \overline{b}, L, R \succ$. In this paper the population of the patient zone h at period t (p_{ht}) is regarded with epistemic uncertainty and could be represented $p_{ht} = \prec \underline{p}_{ht}, \overline{p}_{ht}, pl_{ht}, pr_{ht} \succ$. Parameters $0.5 \prec \alpha \leq 1$ correspond to minimum confidence level of uncertain parameters of constraint. DMs determine value of minimum confidence level of uncertainty parameters base on their risk aversion. Increasing confidence level leads to maximum risk aversion of output decisions of model. In detail, according to the above mentioned, the model will be rewritten eventually as follows:

$$\text{Max } \sum_h \sum_i \sum_t q_{hit} - \sum_h \sum_i \sum_t q'_{hit} \tag{22}$$

s.t.

$$\sum_i (q_{hit} + q'_{hit}) \leq (\underline{p}_{ht} - \alpha * pl_{ht}) \qquad \forall i, t \tag{23}$$

Constraints (3a), (3b), (4)-(8), (11), (12), (14)-(17).

Solution Approach

In this section, a Benders decomposition algorithm is developed for solving large scale Mixed-integer programming (MIP) problems. BDA was first introduced by Benders (1962). In BDA, the master problem is decomposed into a problem which is called a relaxed master problem (RMP), and a linear problem which is called a sub-problem (SP). The feasible space of the primal SP is dependent on the given values of binary variables which is not suitable for handling the SP. For this reason, the sub-problem is dualized and its solution generates coefficients for inequalities known as Benders optimality cuts or feasibility cuts in the RMP. An optimal dual sub problem (DSP) generates coefficients for an optimality cut, while if it is infeasible, then a feasibility cut is generated. RMP and DSP find the lower and upper bounds for the optimal value of the problem, which improves in different iterations until they converge to the optimal value or an early stopping criterion is reached. Clearly, the Benders decomposition is successful if it completes the solution much earlier than the total number of master problem solution points. To implement a BDA, it is necessary to formulate the DSP and RMP first and the compact form of the master problem is considered. The Benders primal sub-problem (PSP) is formulated as follows:

$$\text{Max} \quad \sum_h \sum_i \sum_t q_{hit} - \sum_h \sum_i \sum_t q'_{hit} \tag{24}$$

s.t.

Constraints (3a), (3b), (8), (9),(11)

Let consider the variables to the fixed values $(s_{hit} = \bar{s}_{hit}, y_{it} = \bar{y}_{it}, c_{it} = \bar{c}_{it}, c_{it}^{e} = \bar{c}_{it}^{e})$ so that they can be applied to the constraints of the RMP. If λ represent the dual variables of the constraints of Benders PSP, then the DSP, is formulated as follows:

$$\begin{aligned} \text{Min} \quad & \sum_i \sum_t v_i \bar{c}_{it} \lambda_{it}^{3a} + \sum_i \sum_t (1-v_i)\bar{c}_{it} \lambda_{it}^{3b} + \sum_h \sum_i \sum_t u_i a_{hi} \bar{s}_{hit} \lambda_{hit}^{8} \\ & + \sum_h \sum_t (\underline{p}_{ht} - \alpha * pl_{ht}) \lambda_{ht}^{9} + \sum_h \sum_i \sum_t (1/M) a_{hi} \bar{s}_{hit} \lambda_{hit}^{11} \end{aligned} \tag{25}$$

s.t.

$$\lambda_{it}^{3a} + \lambda_{hit}^{8} + \lambda_{ht}^{9} + \lambda_{hit}^{11} \geq 1 \qquad \forall i,t,h \tag{26}$$

$$\lambda_{it}^{3b} + \lambda_{hit}^{8} + \lambda_{ht}^{9} + \lambda_{hit}^{11} \geq -1 \qquad \forall i,t,h \tag{27}$$

Now, according to the obtained solution of DSP, the RMP is represented as follows:

$$\text{Max} \quad \Xi \tag{28}$$

s.t.

feasibility cut:

$$\begin{aligned} & \sum_i \sum_t v_i \bar{c}_{it} \lambda_{it}^{3a} + \sum_i \sum_t (1-v_i)\bar{c}_{it} \lambda_{it}^{3b} + \sum_h \sum_i \sum_t u_i a_{hi} \bar{s}_{hit} \lambda_{hit}^{8} \\ & + \sum_h \sum_t (\underline{p}_{ht} - \alpha * pl_{ht}) \lambda_{ht}^{9} + \sum_h \sum_i \sum_t (1/M) a_{hi} \bar{s}_{hit} \lambda_{hit}^{11} \geq 0 \end{aligned} \tag{29}$$

Optimality cut:

$$\begin{aligned} \Xi \leq & \sum_i \sum_t v_i \bar{c}_{it} \lambda_{it}^{3a} + \sum_i \sum_t (1-v_i)\bar{c}_{it} \lambda_{it}^{3b} + \sum_h \sum_i \sum_t u_i a_{hi} \bar{s}_{hit} \lambda_{hit}^{8} \\ & + \sum_h \sum_t (\underline{p}_{ht} - \alpha * pl_{ht}) \lambda_{ht}^{9} + \sum_h \sum_i \sum_t (1/M) a_{hi} \bar{s}_{hit} \lambda_{hit}^{11} \end{aligned} \tag{30}$$

Constraints: (4), (5), (6), (7), (12), (14)-(17)

The RMP and DSP give an upper and lower bound for the objective function of the master problem at each iteration respectively. The solution which is resulted from DSP and also modifies DSP, are indicating extreme points and extreme rays that are needed to find optimality cuts and feasibility cuts. In the RMP, Constraints (29) and (30) represent feasibility and optimality cuts. Although considering the upper and lower bounds at the beginning of the BDA can lead to a rapid reach of the optimal value, but due to the difficulty of defining a tighter interval, they are usually considered $+\infty$ and $-\infty$, respectively.

In the initial form of BDA, a large number of algorithm iterations are usually needed to achieve convergence, especially when MIP is complex, so several strategies have been proposed by researchers to accelerate the implementation of the algorithm in the literature. Here one of them which is the valid inequalities, will be mentioned.

Valid inequalities

For avoiding slow convergence of the BDA which is because of the low quality of the RMP solution in the initial iterations of the algorithm, some inequalities may be added to the RMP. These valid inequalities restrict the feasible region. Also, these valid inequalities have the feature of creating the equivalent model relative to the main problem [22]. These inequalities obtain from useful information that is from the concept of the problem. Therefore, according to the defined problem, it is necessary that patients first be within the hospital coverage radius, Constraint (31) can be defined. According to this constraint, if the patient is not within the radius of hospital coverage, allocation to that hospital is not done. Because of considering the coverage radius of each hospital in Constraint (8) in the base model, Constraint (31) can act as an accelerator in solving the model:

$$a_{hi} \geq s_{hit} \qquad \forall h,i,t \tag{31}$$

Implementation and validation

The obtained mixed integer linear programming model was implemented using the GAMS software version 24.2.1 in a personal computer with a processor of 2 GHz and 4 GB of RAM using the syllable solution method. In order to validate the model and to get a logical answer from the constructed model, two numerical examples are designed.

In the first designed example, 20 locations for COVID-19 patients and 20 hospitals in different locations with a maximum operating hospital capacity equal to 220 are considered. In this example, 60 periods (days) are considered to show the changes in patients' conditions and capacity. The amount of available budget in each period in this example is 20 thousand million Toman. In Table 2, the other parameters of the problem are also introduced.

Table 2.The value of model parameters

Symbol	Quantity
o_{it}	Uniform (1,8)
e_{it}	Uniform (2,6)
$\underline{p}_{ht}$	0.8*Uniform(10,560)
$\overline{p}_{ht}$	0.88* Uniform(10,560)
pl_{ht}	0.08* Uniform(10,560)
pr_{ht}	0.16* Uniform(10,560)
v_i	0.8
α	0.5
M	100,000

v_i, maximum allowable percentage of admission to hospital i, is considered 0.8, which is proposed as a confident value for each period to prevent a lack of the capacity which needs for emergency situations and to increase the rate of recovery which directly depends on the rate of bed occupation in hospitals. By placing the coverage radius as a binary random number, indicating the possibility or impossibility of allocating patients to active hospitals, the input parameters of the model were completed. The existing model is solved with and without BDA in The GAMS software. As it is obvious, the objective function value was 272.76. This value was obtained without using BDA in 6:04 minutes, but using BDA in 18 seconds and in two iterations.

In the second numerical example, for better representation of convergence resulting from the implementation of BDA and also the alteration in capacity expansion, the problem is performed for 90 locations for COVID-19 patients and 12 hospitals in different locations with maximum operating hospital capacity that is equal to 220 in 60 periods (days).it should be noted that some hospitals are not activated because of defining the coverage radius. In each period the budget is 5000, and the following results showed convergence to the objective function of 38,654.81 that is obtained after seven iterations which the details are mentioned in Table 3.

Table 3. The lower bound and upper bound of the optimal solution.

Iteration	Lower bound	Upper bound
iter1		41,043.06
iter2	12,144.00	39,933.37
iter3	12,144.00	39,398.90
iter4	22,000.00	38,890.25
iter5	25,872.00	38,726.82
iter6	27,104.00	38,654.81
iter7	38,654.51	38,654.81

The overall results of each iteration can be seen in Fig. 2.

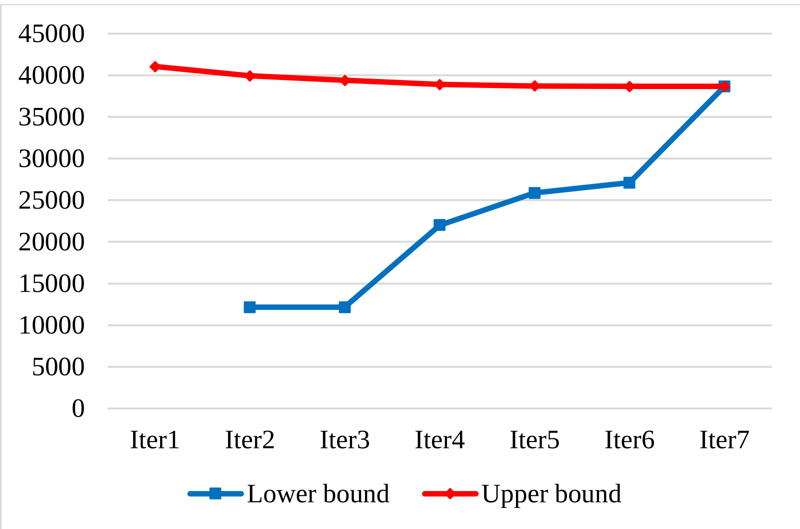

Fig. 2. Convergence of BDA implementation

As a result, in all periods, it has been tried to complete 80% of the hospital capacity and if necessary, the rest of the capacity is completed. According to the amount of budget in each period and the number of patients, there are changes in the capacity of hospitals, for example, hospital 1 in the 46th period needs to increase the capacity from 102 to 220, which is the maximum possible amount. Two samples of these capacity changes in different periods are shown in Fig. 3.

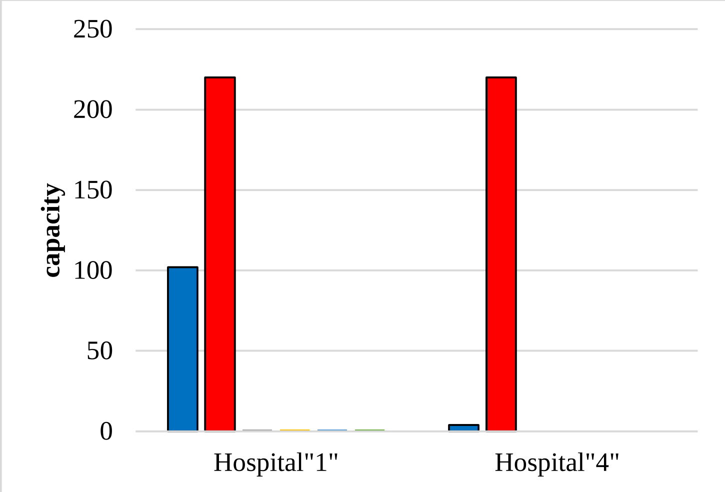

Fig. 3. Alteration of hospital capacity

As can be seen in Fig. 4, hospitals in different periods need to expand their capacity to better cover COVID-19 patients. As is evident from Fig. 4, as an example of cases, it can be seen that hospital “7” needed to develop its capacity in the 53rd period. It should be noted that as a management result before this expansion, the hospital capacity in intensive care units, can be used for other non-COVID-19 patients.

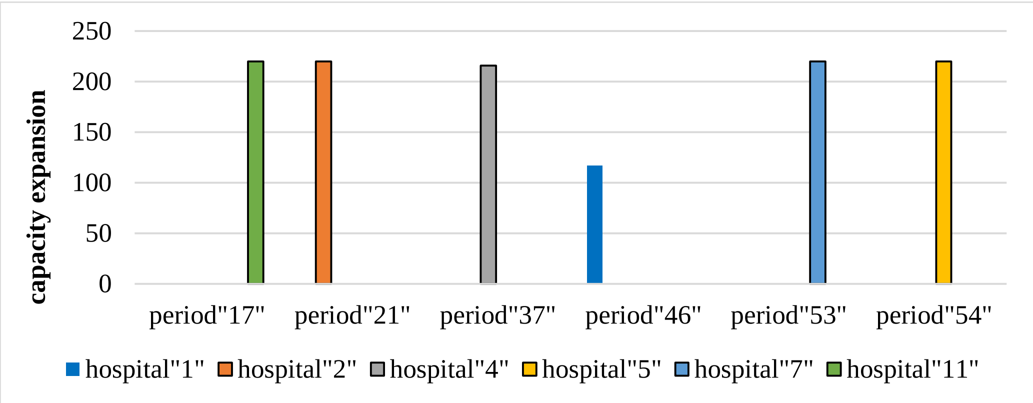

Fig. 4. Capacity expansion quantity

Conclusion Remarks and Suggestions

In this paper, a mixed-integer non-linear programming model is developed for solving the concerned problem which then became its equivalent linear form to be solved efficiently. The model incorporates prominent decisions, i.e. the initial capacity and the expansion over the planning horizon, proportional to the budget which was shown in the results. The objective function prevents concentrating patients in some special hospitals by allocating capacity up to a pre-specified percentage. This situation increases the quality of health care services and as a result, it increases the satisfaction level of the health network in the community and, also leads to the satisfaction of the medical staff. For dealing with the epistemic uncertainty of demand in pandemic outbreak, a possibilistic programming approach is applied. Since the model includes one objective function, we have proposed an improved BDA which enhances the performance of the solving on large scales significantly. Also, validation is provided to illustrate the performance of the solution methodology. As a suggestion, a scenario-based formulation could be provided to enhance the resilience against probable disruption risks which occur in pandemic situations. Also, other objectives could be regarded, i.e. considering the fully/partially radius coverage as an objective function. At the end of the pandemic due to factors such as general vaccination, equipment such as ventilators are useless. So it is a basic requirement to manage in a way that equipment can cover peak conditions and also provide reasonable preparations such that considering the fine in the objective function which is considering the conditions of the post-pandemic period. Also, the compulsion of the model to send a certain number of patients to medical centers can be considered in studies, as has been the case with regard to the establishment of health networks in previous research.

Managerial insight

As assigning scarce hospital equipment is a vital requirement that has an effect directly on the recovery rate, the lack of equipment for the future should be considered. The first step is estimating the deficiency in each period which is obtained from solving the model. Therefore, if it is possible, the budget, the producers and the apparatus should increase to supply the scarce equipment to prevent occurring a disaster. It should be noted that supplying equipment is a time-consuming process and estimating the deficiency is essential for overcoming it. On the other hand, in all the periods there are non-COVID-19 unwell patients who cannot wait and should be treated or determined. So estimating the equipment needed for COVID-19 patients can also determine the possibility of providing services in each hospital for these emergency patients. Finally, as a result of this study, the intending for the future and creating a service of referring patients to hospitals with the aim of preventing the concentration of patients in a few special hospitals is obtained and could be applied.

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