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RESEARCH PAPER

A Sustainable Competitive Supply Chain Design Considering Uncertainty of Demand (Case Study in the Battery Manufacturing Industry)

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Abstract

In this paper, the problem of sustainable bi-level closed-loop supply chain network design under uncertainty is modelled and solved. The competition is between two supply chains (existing and new), on the selling price of new products in the forward supply chain and purchasing price of returned products in reverse logistics. Demand is price sensitive and the quantity of returned products is dependent on the price of the competitor. Dealing with uncertainty is possible here using fuzzy theory. The bi-level supply chain model is considered to be a leader-follower game. At the upper level, the leader optimizes decision variables of the network design and At the lower level, the follower deals with a non-linear problem with continuous variables. Then Benders decomposition approach is applied to solve the single-level problem. Finally, a case study in the battery industry and a numerical analysis are presented. Results show that increasing the forward elasticity coefficient entails a decline in supply chain profit and as the elasticity coefficient of the competitor increases, the demand becomes more sensitive to the competitive price, and the demand for the competitor declines, leading to a decrease in the rival's profit and an increase in the current supply chain's profit.

Keywords:
Closed-Loop Supply Chain;
Uncertainty;
Stackelberg Competition;
Sustainability

Introduction

Over the past few years, the enactment of strict environmental laws and regulations, the shortage of raw materials, and the high profitability potential have led to increasing attention to the reverse flow in supply chain management. The reverse flow typically begins with the return of defective or end-of-life products, and continues until the recycled material returns to the forward chain or is disposed [1]. In many cases, the environmental constraints such as the lack of resources required to bury or dispose of the waste and components are more considerable. For example, digital wastes associated with computer systems and digital home equipment at the end of their lifetime have turned out to be a large venture in Taiwan, due to the extreme scarcity of landfills, in addition to the damaging materials found in such waste. As a result, the country's Environmental Protection Agency has adopted a law in 1998, under which manufacturers and importers of computer equipment and home appliances are required to collect expired products. On that basis, they pay $20 per sold product to a fund, responsible for managing the collection of and recycling of these products [2].

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In the reverse chain, several processes including inspection, disassembly, remanufacturing, repair, recycling and safe disposal are performed. In the presence of reverse chains, network design decisions including facility location, capacity determination, number of facilities in the forward and reverse network, and selection of forward and reverse supply chains are interrelated and integrated. Therefore, the existence of closed-loop supply chains and the importance of decisions in this area emphasize the need for further investigation. In many industries, only one or two members in the reverse chain exist; or a combination of operations in the reverse chain is performed in an integrated manner at a designated center. On the one hand, in the real world, supply chains operate in an uncertain environment. This uncertainty becomes even more important because, during the design phase of the supply chain, many strategic decisions will be made that will remain unchanged for a long time. On the other hand, the uncertainty in closed-loop supply chain networks due to the difficulty of evaluating and controlling the quantity and quality of returned products is increasingly important [3].

Supply chains with technological progress, globalization of the economy and unpredictable customer behavior competes in a dynamic environment. These factors, along with the advanced electronic business infrastructure are changing the competition schema from a competition between the firms into a competition between supply chains. Today, many industries and business firms face the problem of the arrival of different new competitors. Competitors are constantly trying to gain greater market share through lower buy and sell prices, higher service levels, better quality, and closer sales facilities. This has resulted in a natural reaction: shortening the lifecycle of products and the creative use of information technology. Furthermore, firms need to create more value in novel ways, get products to market faster, be more flexible to changes in demand and also reduce costs. Access to such quality service has forced many companies to resort to external suppliers to receive new services and capabilities that they lacked, which in turn demands higher levels of collaboration between firms. Therefore, many companies have chosen to operate in a chain schema and have the opportunity to create new value through the integration of the competencies of the chain members. In such a manner, the growing trend of competition between chains is not surprising. Under a plan for integrated supply chain management, the Massachusetts Institute of Technology (MIT) has studied competition in supply chains using the views of more than 30 experts in the fields of consulting, academia and industry. The study revealed that the majority of experts (70%) considered the interchain competition as a clear feature of future markets [4]. In the 21st century, the best production and sale of the best and highest quality products do not seem to be sufficient. Success today lies in the creation of a team of companies that can cooperate beyond traditional forms of negotiations in business relationships and offer the best products to customers at the best possible price [5].

The design of the supply chain network, which leads to the identification of the overall structure of the chain, requires the adoption of decisions that limit the decisions at lower levels, and therefore have a direct impact on the performance of the supply chain, particularly its competitiveness in the market. Moreover, the decision on the design of the network, once taken, is impossible or rather very onerous to modify; therefore, considering the competitiveness factors and the behaviors of current and potential competitors will increase the competitiveness of the supply chain and consequently increase the profitability of the chain in the long run. It is clear that the design must be sufficiently robust to random environmental factors (such as demand, price and exchange rates). Additionally, supply chains can be disrupted, under conditions such as terrorist incidents, terrible storms, etc. Thus, supply chains operate in a dynamic environment and under uncertain conditions, and these conditions will affect the entire planning and activities of the chain. In fact, the complexity of decision-making in the supply chain network design phase is due to the uncertainty of future conditions [3]. A proper design of the supply chain must provide a correct prediction of the future. Therefore, design problems are in conditions of uncertainty, and in addition to the consideration of financial factors such as the return of capital, the robustness of the supply chain network needs to be taken into account as well. Wu et al. [6] proposed an enhanced and robust model for the selection and evaluation of suppliers in a supply chain. The proposed model was able to manage inaccurate information and improve the capability of differentiation between efficient and inefficient suppliers. Nasiri et al. [7] presented a modified genetic algorithm for the capacitated competitive facility location problem with partial demand satisfaction. The presented problem is composed of two competitors (a leader and a follower) who seek to attract customers by establishing new facilities and consequently maximizing their own profit. Ahmadi and Ghezavati [8] developed a new model for a competitive facility location problem where the attractiveness function of each facility is based on the sustainable aspects. Sazvar et al. [9] formulated a scenario-based, multi-purpose linear programming model to design a sustained supply chain according to the uncertainties of demand that examines the reverse flow of expired drugs at three levels (must be disposed of, is possible to be reproduced and recycled). A method is presented with an innovative algorithm and in addition, a real world case study has been investigated to assess the efficacy of the model. Manteghi et al. [10] presented a bi-level optimization approach to address the competition between members of a food supply chain, including two suppliers and a producer in which the producer seeks to determine the price, production and investment in reducing emissions to maximize profits at a high level and the supplier seeks to reduce production costs by maintaining inventory, and of course, by accepting the risk of waste. At a lower level, suppliers compete to determine the price of raw materials and the number of additives to maximize their profits. Two linear approximation methods are proposed for the linearization of the nonlinear model. The results indicate that the increase in dependency of demand to price leads to a 30% reduction in the net profit of the supply chain. The producer faces an average of a 3% decrease in demand by increasing the waste generated in the production process by 4 to 7%. The 18% increase in spending on lowering greenhouse gas emissions will lead to a 5% reduction in the total supply chain profit. Ghomi Avili et al. [11] proposed a robust bi-level model of the problem of multi-purpose network design for a competitive green supply chain, considering pricing decisions and an inventory subject to uncertainties and disruptions. The proposed model simultaneously takes into account the uncertainties of demand and risks arising from the disruption and is able to deal with such uncertainties by implementing resilience strategies such as making inventory and contract decisions with credible suppliers. Esmizadeh and Mellat Parast [12] conducted a systematic literature review to explore several projects in the logistics network and evaluated their performance in terms of cost, quality, delivery and flexibility. Afterwards, they used the literature review results to assess the strengths and weaknesses of each logistics plan for different operations strategies. Yu et al. [13] developed a supply chain network model where companies are actively competing in discrete time periods on the limited planning horizon. They evaluated a set of numerical samples to assess the impact of consumers' preferences for sustainability and environmental policies on firm profitability and environmental footprint. The results showed that firms continue performing sustainable operations as long as they are financially valuable to them, which compels governments and companies to educate consumers about sustainability. Shabbir et al. [14] proposed a model for closed-loop supply chain design and pricing in competitive conditions by considering the variable value of return products using the whale optimization algorithm. Zahedi et al. [15] formulated a closed-loop supply chain network with the sales representatives and customers. The proposed model has four forward levels and five reverse levels. This model not only takes into account several limitations of previous studies, but also investigates new limitations. The findings of the proposed network have shown that the use of features of sales representative centers and customer clusters increases both the overall revenue of the representatives as well as the number of collected returned products. Yousefi et al. [16] presented a modified genetic algorithm for the capacitated competitive facility location problem with partial demand satisfaction. Taghizadeh et al. [17] carried out the design and optimization of a multi-purpose linear programming model as a case study for simultaneous optimization of three functions, including minimizing the total cost of fixed expenses and transportation costs, minimizing total emissions from the transportation network and maximizing the utilization of the full capacity of the suppliers. Rentizelas et al. [18] carried out the formulation and finding the optimal solution of a complex integer linear programming model for the problem of the design of the wind turbine blades supply chain network. Arabi and Gholamian [19] proposed a three-objective multi-period multi-product mixed-integer quadratic programming problem to optimize a sustainable stone supply chain network design. They used an ε-constraint approach to solve the multi-objective model and achieve the non-dominated solutions. Enayati Shiraz et al. [20] designed a system dynamics model in Vensim applying lean and green supply chain Policies in the sustainability of CHOUKA Iran Wood & Paper Industries Inc. Yousefi-Babadi [21] presented a heuristics algorithm to solve a capacitated sustainable resilient closed-loop supply chain network design.

The case study in this paper comes from the battery manufacturing industry, which is environmentally important due to certain raw materials used in its production. The batteries, because of the material they contain, are classified as dangerous goods and thus, are subject to environmental regulations. Some types of batteries have a risk of explosion, and generally contain heavy metals such as lead, mercury, cadmium, etc., which are considered as environmental pollutants, and due to the presence of electrolyte, tend to have a high degree of corrosiveness. Battery waste is, therefore, part of hazardous waste and the collection of worn-out batteries in many countries is faced with strict rules. From an economic point of view, the collection of worn-out batteries because of the existence of mentioned metals is economic and common practice. Accordingly, transboundary transport, import and export of worn acid batteries, as dangerous waste, to the countries of the Basel Convention is subject to the provisions of this convention, which requires the existence of proper environmental management of such waste. On the other hand, competition, gaining a greater share of the market and strategic exchange has forced companies to implement many reverse policies, in order to re-distribute the returned products in similar markets through business firms as is the case with newly manufactured products. The reverse logistics process starts with the collection of products that are returned for any reason. Then, based on the quality inspection performed at the collection centers, they are sent for destruction, recycling or restoration. The goods that can still be functional after repairs are sent back for re-construction. The returned goods that are not repairable but contain reusable parts for other products are disassembled. This is usually experienced with electronic products such as laptops and mobile phones. The remaining products need to either be recycled, such as the lead in batteries and plastics in plastic parts or be safely destroyed. The reverse logistics process might consist of several members and facilities such as collection, processing, disassembly, inspection, recycling and destruction. It should be noted that these members are not necessarily visible or separable in all chains. The battery market is a competitive market and therefore, the importation of batteries into a country or the creation of new battery-producing brands is common among current competitors. The competition takes place between a new chain still in the process of design (arrival of a new battery-producing chain to the market) and a chain currently in the market, while the demand in these chains is faced with uncertainty. Since the supply chain under study makes decisions after the competition's arrival on the creation of one or more of the new distribution centers or the establishment of a new recycling center, we face the problem of network design. On the other hand, the pricing of new products is done in the forward pricing channel and the reverse battery pricing [2]. The success of a supply chain depends on the integration and coordination of all its members to form an efficient network structure. An efficient network will lead to carrying out operations and processes with minimal costs and help quickly respond to customer needs.

Regarding the studied research gap mentioned by Fallah et al. [22], the presented model is developed considering the conditions of sustainability and adding a supplier level. In this study, two closed-loop supply chains are considered to be competing in an uncertain environment. The competition takes place over the retail price in the forward channel and the price of return goods in the reverse channel (the price at which the retail seller buys worn-out goods from the customer). The market demand for each supply chain depends on the price, and the returned quantity is sensitive to the reverse price (price of the returned product). Moreover, the case study of this paper concerns a battery production chain in Iran, which has a high share of the car battery market in the country. This market is competitive because: (1) The demand for batteries in Iran is steadily increasing in light of the number of cars, increasing approximately 15% annually. (2) It is not difficult to enter the market because tax breaks are granted to domestic battery suppliers; the customs laws have become stricter and the production of car batteries does not require advanced technology. (3) There is also fierce competition in the reverse chain of battery production both economically (because of the price of the lead) and environmentally, and around 50% of the lead needed for the production of batteries is usually supplied from the recycling of worn-out batteries. First, the problem of the design of a supply chain network with respect to the uncertainty factor, competition and greenhouse emissions is modeled; afterwards, the problem is studied based on the rank of the chains in the market and the decision-making power of the actors as follows: one of the chains is recognized as the leader according to its position in the market which has more power. In this case, a leader-follower game takes place between the two chains and the follower chain can take the best decision considering the leader's decisions. As a result of this competition, with the identification of the Stackelberg equilibrium point, the optimal leader and follower policies are determined and based on that, it can be decided on the structure of the supply chain network. The decision on the structure is also feasible using a bi-level approach. In this case, the problem of competition and network design is solved in an integrated competitive model that leads to a single-level model.

In the second part, the Mathematical Model is demonstrated, in the third and fourth parts, respectively, conversion of the bi-level model to a single-level problem and solving of the single-level model, in the fifth and sixth sections, Case Study and the Numerical Solution and in the seventh and eighth parts the Managerial Insights and the Conclusion are presented.

Mathematical Modeling

In this paper, two closed-loop supply chains are considered to compete in a non-deterministic space. The competition takes place over the retail price in the forward channel and the price of the returned goods in the reverse channel (price at which the retail seller buys worn-out goods from the customer). The market demand for each supply chain is price-dependent and the return value is sensitive to the reverse price (return price). Uncertainty for this problem is modeled using the theory of fuzzy sets. The competitive structure for closed-loop supply chains is considered to be leader-follower. In this form of competition after the arrival of a new chain in the market, the existing chain will alter its decisions based on the characteristics of the new rival, and then the new chain design is determined based on the observed response from the existing chain about its forward and reverse price. In this case, the existing chain plays a leadership role and the new chain takes on the follower role. The design problem of the closed-loop competitive supply chain network requires the knowledge of the demand volume, the quantity of returned goods and forward and reverse prices. Suppose that there are a number of potential site locations among which a few are expected to be selected as factories. These factories purchase materials from their selected suppliers and send out the final product to the retail sellers. At this phase, these products are sent to retailers who have been selected from potential retailers. The retailers are tasked with the acquisition of the products from the plant and their delivery to customers, and in the next phase, they send back the returned goods to the recycling centers. In the reverse chain, there are a number of potential sites for recycling centers where defective and returned products are taken from the retailers and delivered to the corresponding plants. At the highest level of the supply chain sits the supplier from whom the raw materials required for the plant are acquired. The problem at hand is single-period, meaning that an infinite planning horizon and the mathematical model are provided only for a specific period. Each of the chains presents a product that is similar to or highly replaceable with the product from another chain. In this system, the solution to the problem includes increasing the profit and reducing greenhouse gas emissions. The increase in profit is done by increasing the selling price and reducing the costs of the system.

The costs of the system include the following:

- The transportation cost of the product from the producer to the retailer, from the retailer to the recycler, from the recycler to the producer and the cost of transferring the raw material from suppliers to the factory.
- The fixed cost of construction for the manufacturer, retailer and the recycling center.
- The cost of buying raw materials from the supplier.

The greenhouse emissions consist of the following items:

- The amount of greenhouse gases released due to the transport of the products from the plant to the retailer, from the retailer to recycling centers and from the recycling centers to the plant as well as the emitted greenhouse gas due to transport from suppliers to the plant.
- The fixed amount of greenhouse gases emitted in factories and recycling centers.

In the figure below the schematic of the closed-loop supply chain is depicted.

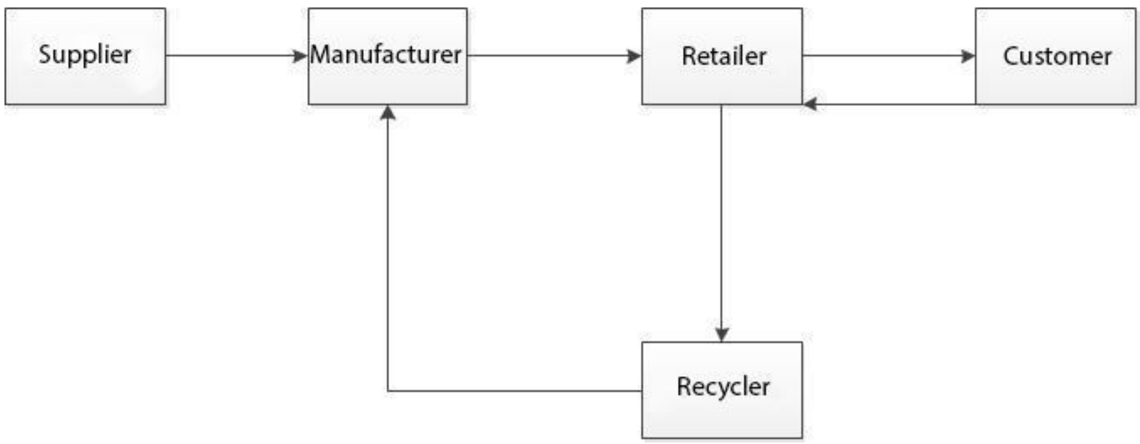

Fig. 1. The schematic of the closed-loop supply chain under study.

Model assumptions:

- The production line costs for all production centers in the supply chain are the same.
- The recycling centers buy the returned products from the retailers at a fixed price.
- All customer demands shall be met and all returned goods are recycled.
- The current problem is single-period and single-product.
- The market demand for each supply chain is price-dependent meaning that an increase in the price leads to a decline in the demand for the chain and the demand for the rival chain increases.
- The quantity of returned products for each chain depends on the price paid to purchase the worn-out products.

In a competitive market, each actor's demand depends on both the offered price and the competitor's price. The higher the price of a product offered by a chain, the demand for that product decreases and shifts to the competing supply chain. In economics and marketing literature, it is common to consider a linear form for demand, in the form of an ascending function of the competitor's price and a descending function of the price of another competitor.

The supply chain demand is defined as follows [22].

$$D_i(P_i, P_t) = \tilde{d}_i - \tilde{\beta}_{1i}P_i + \tilde{\beta}_{2i}P_t \; ; \; i = 1{,}2 \; ; \;\; t = 3 - i \tag{1}$$

- $\tilde{d}_i$ represents a potential or base demand (demand volume with zero price) for the supply chain i.
- P_i is the unit price in supply chain i.
- $\tilde{\beta}_{1i}$ and $\tilde{\beta}_{2i}$ are elasticity coefficients of price signifying the degree of response of demand to price.
- $\tilde{d}_i$, $\tilde{\beta}_{1i}$ and $\tilde{\beta}_{2i}$ are positive and fuzzy parameters.
- $\tilde{\beta}_{1i}$ and $\tilde{\beta}_{2i}$ are considered to be the direct and indirect effects of price on demand.
- The demand for each supply chain is more sensitive to its price than the other. The following relationship holds:

$$E\left(\tilde{\beta}_{1i}\right) \geq E\left(\tilde{\beta}_{2i}\right) \tag{2}$$

The amount of returned products in the reverse chain depends on the amount that retailers pay to the customers. The amount of returned good are assumed to be in the form of a linear function of the reverse price of the chains:

$$R_i(rp_i, rp_t) = \tilde{r}_i + \widetilde{\beta'}_{1i} rp_i - \widetilde{\beta'}_{2i} rp_t \; ; \; i = 1{,}2 \; ; \; t = 3 - i \tag{3}$$

- $\tilde{r}_i$ is the potential return for the chain i, which is a factor of the potential demand of each chain.
- rp_i unit price for the returned product in supply chain i.
- $\widetilde{\beta'}_{2i}$ ، $\widetilde{\beta'}_{1i}$ and $\tilde{r}_i$ are negative and fuzzy parameters:

$$E\left(\widetilde{\beta'}_{1i}\right) \geq E\left(\widetilde{\beta'}_{2i}\right). \tag{4}$$

- **Indexes and model sets**

s: set of suppliers centers
n: set of potential manufacturers centers
j: set of potential retailers centers
k: set of potential recycling centers
i: index of supply chains (i=1, 2)

- **Model parameters**

v_i: cost of manufacturing a unit product in the manufacturing center of the supply chain i
ω: the fixed cost of purchasing one unit product in the recycling center
$cost^{/}_{sn}$: the cost of transport from the supplier s to the manufacturing center n
tp_{nj}: the cost of transport of a unit product from the manufacturer n to the retailer j
tr_{jk}: the cost of transport of a unit product from the retailer j to the recycling center k
tm_{kn}: the cost of a recycled unit product from the recycling center k to the manufacturing center n

$cost_s$: the cost of purchasing one unit of raw material from the supplier s
f_n : the fixed cost of establishing a manufacturing center in potential location n
c_k: the fixed cost of establishing a recycling center in potential location k
b_j: the fixed cost of establishing a retailer in potential location j
φ_k: the cost of recycling one unit product in the recycling center k
cap_s: the maximum production capacity of the supplier s
ct_n: the maximum production capacity of the manufacturer n
cd_j: the maximum capacity of the retailer j
cm_k : the maximum capacity of the recycling center k
cs_{sn}: the amount of greenhouse gas emitted while transporting one unit of raw material from the supplier s to the manufacturer n
ctp_{nj}: the amount of greenhouse gas emitted while transporting one unit product from the manufacturing center n to the retailer j
ctr_{jk}: the amount of greenhouse gas emitted while transporting one unit product from the retailer j to the recycling center k
cts_{kn}: the amount of greenhouse gas emitted while transporting one unit of recycled material from the recycling center k to the manufacturing center n
cp_n: the fixed amount of greenhouse gas emission in the manufacturing center n
cr_k: the fixed amount of greenhouse gas emission in the recycling center k
tax: the tax imposed on one unit of greenhouse emission

- **Decision variables**

X_{nj}: the amount of product which is sent from the factory n to the retailer j.
m_{kn}: the amount of returned product which is sent from the recycling center k to the factory n.
U_{jk}: the amount of products which is sent from the retailer j to the recycling center k.
X'_{sn}: the amount of products which is sent from the supplier ‘s’ to the manufacturer n.
z_n: binary decision variable. If the factory n is constructed equals ‘1’, otherwise ‘0’.
y_k: binary decision variable. If the recycling center ‘k’ is constructed equals ‘1’, otherwise ‘0’.
w_j: binary decision variable. If the retailer ‘j’ is established equals ‘1’, otherwise ‘0’.
p_i: retail unit price of the product in the supply chain ‘i’.
rp_i: unit price of the returned product in the supply chain ‘i’ which retailers pay to the customers returning the product.

The following two objective functions are defined for the problem of the closed-loop supply chain network design for the new or the follower chain in the market (i=2).

$$\begin{aligned} max \sum_n \sum_j (p_2 - v_2) X_{nj} + \sum_j \sum_k (\omega - rp_2) U_{jk} - \sum_n \sum_j tp_{nj} X_{nj} \\ - \sum_j \sum_k tr_{jk} U_{jk} - \sum_k \sum_n tm_{kn} m_{kn} - \sum_n f_n z_n - \sum_k c_k y_k - \sum_j b_j w_j \\ - \sum_j \sum_k \varphi_k U_{jk} - \sum_s \sum_n cost_s X'_{sn} - \sum_s \sum_n cost'_{sn} X'_{sn} \end{aligned} \tag{5}$$

$$\begin{aligned} min \sum_n \sum_j ctp_{nj} X_{nj} + \sum_j \sum_k ctr_{jk} U_{jk} + \sum_k \sum_n cts_{kn} m_{kn} \\ + \sum_n cp_n z_n + \sum_k cr_k y_k + \sum_s \sum_n cs_{sn} X'_{sn} \end{aligned} \tag{6}$$

✓ Notes regarding the first objective function:
Part 1: The profit of the manufacturer.

Part 2: The profit of the retailer.
Part 3: The cost of products transported from the manufacturer to the retailer.
Part 4: The cost of products transported from the retailer to the recycling center.
Part 5: The cost of products transported from the recycling center to the manufacturer.
Part 6 The fixed construction cost of the manufacturer.
Part 7 The fixed construction cost of the recycling center.
Part 8 The fixed construction cost of the retailer.
Part 9: The cost of recycling the products transported from the retailer to the recycling center.
Part 10: The cost of buying raw materials from the supplier.
Part 11: The cost of transporting the raw material from the supplier to the manufacturer.

✓ Notes regarding the second objective function:

Part 1: The amount of greenhouse gas emissions transporting the product from the manufacturer to the retailer.
Part 2: The amount of greenhouse gas emissions transporting the product from the retailer to the recycling center.
Part 3: The amount of greenhouse gas emissions transporting the product from the recycling center to the factory.
Part 4: The amount of greenhouse gas emissions in the factory.
Part 5: The amount of greenhouse gas emissions in the recycling center.
Part 6: The amount of greenhouse gas emissions transporting the raw material from the supplier to the factory.

The second objective function, which concerns the amount of greenhouse gas emissions, is converted to the cost function and added to the first objective function, considering the amount of tax that the supply chain should pay relative to the amount of emission. Then, by substitution of the price-dependent demand functions in the above equations, the objective function of the follower is completed. Thus, there is a bi-level model in which at the higher level the leader determines the optimal values for decision variables of network design, and the amount of production, distribution and return. The problem of the lower level is a continuous non-linear programming problem. The following figure is a schematic of the bi-level modeling approach for the problem of leader-follower competition.

$$\begin{aligned} Max\ Z_2 = {} & \left(\tilde{d}_2 - \tilde{\beta}_{12}P_2 + \tilde{\beta}_{22}P_1\right) P_2 - \sum_n \sum_j v_2 X_{nj} + \sum_j \sum_k (\omega - rp_2) U_{jk} \\ & - \sum_n \sum_j tp_{nj} X_{nj} - \sum_j \sum_k tr_{jk} U_{jk} - \sum_k \sum_n tm_{kn} m_{kn} - \sum_n f_n z_n - \sum_k c_k y_k \\ & - \sum_j b_j w_j - \sum_j \sum_k \varphi_k U_{jk} - \sum_s \sum_n cost_s X'_{sn} - \sum_s \sum_n cost'_{sn} X'_{sn} \\ & - tax[\sum_n \sum_j ctp_{nj} X_{nj} + \sum_j \sum_k ctr_{jk} U_{jk} + \sum_k \sum_n cts_{kn} m_{kn} + \sum_n cp_n z_n \\ & + \textstyle\sum_k cr_k y_k + \sum_s \sum_n cs_{sn} X'_{sn}] \end{aligned} \tag{7}$$

$$\sum_n \sum_j X_{nj} = \tilde{d}_2 - \tilde{\beta}_{12}P_2 + \tilde{\beta}_{22}P_1 \tag{8}$$

$$\sum_j \sum_k U_{jk} = \tilde{r}_2 + \tilde{\beta}'_{12} rp_2 - \tilde{\beta}'_{22} rp_1 \tag{9}$$

$$\sum_k U_{jk} \leq \sum_n X_{nj} \qquad \forall\, j \tag{10}$$

$$\sum_j U_{jk} = \sum_n m_{kn} \quad \forall\, k \tag{11}$$

$$\sum_k m_{kn} \le \sum_j X_{nj} \quad \forall\, n \tag{12}$$

$$\sum_k m_{kn} \le z_n ct_n \quad \forall\, n \tag{13}$$

$$\sum_j U_{jk} \le y_k cm_k \quad \forall\, k \tag{14}$$

$$\sum_n X_{nj} \le w_j cd_j \quad \forall\, j \tag{15}$$

$$\sum_n X'_{sn} \le cap_s \quad \forall\, s \tag{16}$$

$$\sum_n f_n z_n + \sum_k c_k y_k + \sum_j b_j w_j \le budget \tag{17}$$

$$X'_{sn}, m_{kn}, U_{jk}, X_{nj}, P_2, rp_2 \ge 0 \quad \forall\, s,n,j,k \tag{18}$$

$$w_j, y_k, z_n = 0\ or\ 1 \quad \forall\, n, j, k \tag{19}$$

$$Max\ Z_1 = \left(\tilde{d}_1 - \tilde{\beta}_{11}P_1 + \tilde{\beta}_{21}P_2\right)P_1 + \sum_j \sum_k (\omega - rp_1)(\tilde{r}_1 + \tilde{\beta}'_{11}rp_1 - \tilde{\beta}'_{21}rp_2) \tag{20}$$

✓ Notes regarding the constraints:

Part 1: The constraints guarantee the fulfillment of the customers' demands.
Part 2: All of the returned products are required to be transported from the retailers to the recycling centers.
Part 3: The amount of returned product entering the reverse flow by retailers can at most equal the manufactured amount.
Part 4: Make a balance in the flow of recycled and returned products.
Part 5: The amount of recycled material transported from the recycling centers to the manufacturing centers can at most equal the manufactured amount.
Part 6: Guarantees that the maximum capacity of manufacturing centers is respected.
Part 7: Guarantees that the maximum capacity of recycling centers is respected.
Part 8: Guarantees that the maximum capacity of retailers is respected.
Part 9: Guarantees that the limitation of the supplier is respected.
Part 10: The required budget for the construction of the factories, retailers, and recycling centers is taken into account.
Part 11: Determines the positive variables.
Part 12: Determines the binary variables.

Consequently, assuming that non-deterministic parameters are triangular fuzzy numbers, the fuzzy values of the problem can be written in an equivalent form using the Jimenez approach:

$$\tilde{d}_2 = \frac{d_2^p + 2d_2^m + d_2^o}{4}$$
$$\tilde{d}_1 = \frac{d_1^p + 2d_1^m + d_1^o}{4}$$
$$\tilde{r}_2 = \frac{r_2^p + 2r_2^m + r_2^o}{4}$$
$$\tilde{r}_1 = \frac{r_1^p + 2r_1^m + r_1^o}{4}$$
$$\tilde{\beta}_{12} = \frac{\beta_{12}^P + 2\beta_{12}^m + \beta_{12}^o}{4}$$
$$\tilde{\beta}_{22} = \frac{\beta_{22}^P + 2\beta_{22}^m + \beta_{22}^o}{4}$$
$$\widetilde{\beta'}_{12} = \frac{\beta'^P_{12} + 2\beta'^m_{12} + \beta'^o_{12}}{4} \tag{21}$$
$$\widetilde{\beta'}_{22} = \frac{\beta'^P_{22} + 2\beta'^m_{22} + \beta'^o_{22}}{4}$$
$$\tilde{\beta}_{11} = \frac{\beta_{11}^P + 2\beta_{11}^m + \beta_{11}^o}{4}$$
$$\tilde{\beta}_{21} = \frac{\beta_{21}^P + 2\beta_{21}^m + \beta_{21}^o}{4}$$
$$\tilde{\beta}'_{11} = \frac{{\beta'}^P_{11} + 2{\beta'}^m_{11} + {\beta'}^o_{11}}{4}$$
$$\tilde{\beta}'_{21} = \frac{\beta'^P_{21} + 2\beta'^m_{21} + \beta'^o_{21}}{4}$$

Finally, the equivalent values are rewritten in the main model using defuzzy operations.

Conversion of the bi-level model to a single-level problem of MINLP

The problem of the lower level is a continuous non-linear programming problem that is also convex as it has been previously proved. Therefore, using the conditions of the first level, the optimal values of decision variables in lower lever P_1 and $rp_1$are found and consequently, by replacing them in the higher level model, the problem is converted into a mixed-integer nonlinear programming problem.

$$\frac{\delta Z_1}{\delta P_1} = \tilde{d}_1 - 2\tilde{\beta}_{11}P_1 + \tilde{\beta}_{21}P_2 = 0 \tag{22}$$
$$P^*{}_1 = \frac{\tilde{d}_1 + \tilde{\beta}_{21}P_2}{2\tilde{\beta}_{11}} \tag{23}$$
$$\frac{\delta Z_1}{\delta rp_1} = \omega\widetilde{\beta}'_{11} - \tilde{r}_1 - 2\widetilde{\beta'}_{11}rp_1 + \widetilde{\beta'}_{21}rp_2 = 0 \tag{24}$$
$$rp^*{}_1 = \frac{\omega\widetilde{\beta}'_{11} - \tilde{r}_1 + \widetilde{\beta'}_{21}rp_2}{2\widetilde{\beta'}_{11}} \tag{25}$$

Substitution of P_1 and rp_1 in the higher level model yields:

$$\begin{aligned} max\left(\tilde{d}_2 - \tilde{\beta}_{12}P_2 + \tilde{\beta}_{22}\left(\frac{\tilde{d}_1 + \tilde{\beta}_{21}P_2}{2\tilde{\beta}_{11}}\right)\right)P_2 - \sum_n\sum_j v_2X_{nj} + \sum_j\sum_k(\omega - rp_2)U_{jk} \\ - \sum_n\sum_j tp_{nj}X_{nj} - \sum_j\sum_k tr_{jk}U_{jk} - \sum_k\sum_n tm_{kn}m_{kn} - \sum_n f_nz_n - \sum_k c_ky_k \\ - \sum_j b_jw_j - \sum_j\sum_k \varphi_kU_{jk} - \sum_s\sum_n cost_s\,X'_{sn} - \sum_s\sum_n cost'_{sn}X'_{sn} \end{aligned} \tag{26}$$

$$-tax[\sum_n \sum_j ctp_{nj} X_{nj} + \sum_j \sum_k ctr_{jk} U_{jk} + \sum_k \sum_n cts_{kn} m_{kn}$$
$$+\sum_n cp_n z_n + \sum_k cr_k y_k + \sum_s \sum_n cs_{sn} X'_{sn}]$$

$$\sum_n \sum_j x_{nj} = \tilde{d}_2 - \tilde{\beta}_{12} P_2 + \tilde{\beta}_{22} \left(\frac{\tilde{d}_1 + \tilde{\beta}_{21} P_2}{2\tilde{\beta}_{11}} \right) \tag{27}$$

$$\sum_j \sum_k U_{jk} = \tilde{r}_2 + \tilde{\beta}'_{12} rp_2 - \tilde{\beta}'_{22} \left(\frac{\omega \tilde{\beta}'_{11} - \tilde{r}_1 + \tilde{\beta}'_{21} rp_2}{2\tilde{\beta}'_{11}} \right) \tag{28}$$

$$\sum_k U_{jk} \le \sum_n X_{nj} \quad \forall j \tag{29}$$

$$\sum_j U_{jk} = \sum_n m_{kn} \quad \forall k \tag{30}$$

$$\sum_k m_{kn} \le \sum_j X_{nj} \quad \forall n \tag{31}$$

$$\sum_k m_{kn} \le z_n ct_n \quad \forall n \tag{32}$$

$$\sum_j U_{jk} \le y_k cm_k \quad \forall k \tag{33}$$

$$\sum_n X_{nj} \le w_j cd_j \quad \forall j \tag{34}$$

$$\sum_n X'_{sn} \le cap_s \quad \forall s \tag{35}$$

$$\sum_n f_n z_n + \sum_k c_k y_k + \sum_j b_j w_j \le budget \tag{36}$$

$$X'_{sn}, m_{kn}, U_{jk}, X_{nj} \ge 0 \quad \forall s,n,j,k \tag{37}$$

$$w_j, y_k, z_n = 0 \text{ or } 1 \quad \forall n, j, k \tag{38}$$

Solve the Single-Level Model Using the Benders Decomposition Approach

One of the effective tools to solve MINLP problems is the Benders decomposition approach which divides a problem into two subproblems that makes it easier to solve. The *subproblem* of Benders here is a continuous nonlinear programming problem called NLP',and is modeled as follows:

$$Min\ Z_{NLP'} = -\left(\tilde{d}_2 - \tilde{\beta}_{12} P_2 + \tilde{\beta}_{22} \left(\frac{\tilde{d}_1 + \tilde{\beta}_{21} P_2}{2\tilde{\beta}_{11}} \right) \right) P_2 + \sum_n \sum_j v_2 X_{nj} - \sum_j \sum_k (\omega - rp_2) U_{jk} +$$
$$\sum_k c_k \underline{y}_k^{(T)} + \sum_j b_j \underline{w}_j^{(T)} + \sum_n f_n \underline{z}_n^{(T)} + \sum_n \sum_j tp_{nj} X_{nj} + \sum_j \sum_k tr_{jk} U_{jk}$$
$$+ \sum_k \sum_n tm_{kn} m_{kn} + \sum_k \sum_j \varphi_k U_{jk} + + \sum_s \sum_n cost_s X'_{sn} + \sum_s \sum_n cost'_{sn} X'_{sn} \tag{39}$$
$$+tax[\sum_n \sum_j ctp_{nj} X_{nj} + \sum_j \sum_k ctr_{jk} U_{jk} + \sum_k \sum_n cts_{kn} m_{kn} + \sum_n cp_n \underline{z}_n^{(T)} + \sum_k c_k \underline{y}_k^{(T)}$$

$$+\sum_{s}\sum_{n} cs_{sn}X'_{sn}]$$

s.t.

$$\sum_n \sum_j X_{nj} = \tilde{d}_2 - \tilde{\beta}_{12}P_2 + \tilde{\beta}_{22}\left(\frac{\tilde{d}_1+\tilde{\beta}_{21}P_2}{2\tilde{\beta}_{11}}\right) \tag{40}$$

$$\sum_{j}\sum_{k} U_{jk} = \tilde{r}_2 + \tilde{\beta}'_{12}rp_2 - \tilde{\beta}'_{22}\left(\frac{\omega\tilde{\beta}'_{11} - \tilde{r}_1 + \tilde{\beta}'_{21}rp_2}{2\tilde{\beta}'_{11}}\right) \tag{41}$$

$$\sum_k U_{jk} \leq \sum_n X_{nj} \quad \forall j \tag{42}$$

$$\sum_j U_{jk} = \sum_n m_{kn} \quad \forall k \tag{43}$$

$$\sum_k m_{kn} \leq \sum_j X_{nj} \quad \forall n \tag{44}$$

$$\sum_j U_{jk} \leq cm_k \underline{y}_n{}^{(T)} \ \forall k \tag{45}$$

$$\sum_n X_{nj} \leq cd_j \underline{w}_j{}^{(T)} \ \forall j \tag{46}$$

$$\sum_{k} m_{kn} \leq ct_n \underline{z}_n{}^{(T)} \ \ \forall\, n \tag{47}$$

$$\sum_{n} X'_{sn} \leq cap_s \quad \forall\, s \tag{48}$$

$$\sum_{n} f_n \underline{z}_n{}^{(T)} + \sum_{k} c_k \underline{y}_n{}^{(T)} + \sum_{j} b_j \underline{w}_j{}^{(T)} \ \leq budget \tag{49}$$

$$X_{nj}, U_{jk}, m_{kn}, P_2, rp_2 \geq 0 \ \forall n, j, k \tag{50}$$

In the subproblem of NLP', the values for $\underline{y}_k{}^{(T)}$, $\underline{w}_j{}^{(T)}$ and $\underline{z}_n{}^{(T)}$ variables are obtained by solving the main Benders problem. By solving the Benders subproblem, the values for continuous variables and also the dual optimal values for the constraints (45) - (47) and (50) are yielded which are denoted by $\underline{\gamma}^{(T)}$, $\underline{\eta}^{(T)}$, $\underline{\mu}^{(T)}$ and , $\underline{\lambda}^{(T)}$.

The *main* Benders problem is a mixed-integer programming problem denoted by MIP'.

$Min\ Z_{MIP'} = w$

s.t:

$$\begin{aligned}
w \geq & -\left(\tilde{d}_2 - \tilde{\beta}_{12}\underline{P_2}^{(t)} + \tilde{\beta}_{22}P_1\left(\frac{\tilde{d}_1+\tilde{\beta}_{21}\underline{P_2}^{(t)}}{2\tilde{\beta}_{11}}\right)\right)\underline{P_2}^{(t)} + \sum_n \sum_j v_2 \underline{X}_{nj}{}^{(t)} \\
& - \sum_j \sum_k \left(\omega - \underline{rp}_2{}^{(t)}\right)\underline{U}_{jk}{}^{(t)} + \sum_k c_k y_k + \sum_j b_j w_j + \sum_n f_n z_n \\
& + \sum_{n}\sum_{j} tp_{nj}\underline{X}_{nj}{}^{(t)} + \sum_{j}\sum_{k} tr_{jk}\underline{U}_{jk}{}^{(t)} + \sum_{k}\sum_{n} tm_{kn}\underline{m}_{kn}{}^{(t)} \\
& + \sum_{k}\sum_{j} \varphi_k \underline{U}_{jk}{}^{(t)} + \sum_{s}\sum_{n} cost_s\, \underline{X}_{sn}{}^{(t)} + \sum_{s}\sum_{n} cost_{sn}\underline{X}_{sn}{}^{(t)} \\
& + tax[\sum_{n}\sum_{j} ctp_{nj}\underline{X}_{nj}{}^{(t)} + \sum_{j}\sum_{k} ctr_{jk}\underline{U}_{jk}{}^{(t)} + \sum_{k}\sum_{n} cts_{kn}\underline{m}_{kn}{}^{(t)} \\
& + \sum_{n} cp_n z_n + \sum_{k} cr_k y_k + \sum_{s}\sum_{n} cs_{sn}\underline{X'}_{sn}{}^{(t)}] \\
& + \underline{\gamma}^{(t)}\left(-cm_k \underline{y}_k{}^{(t)}\right) + \underline{\eta}^{(t)}\left(-\mathrm{cd}_j\, \underline{w}_j{}^{(t)}\right) + \underline{\mu}^{(t)}\left(-ct_n \underline{z}_n{}^{(t)}\right) + \underline{\lambda}^{(T)}(\sum_n f_n \underline{z}_n{}^{(T)} + \\
& \sum_k c_k \underline{y}_n{}^{(T)} + \sum_j b_j \underline{w}_j{}^{(T)} - budget) \ \ ; \forall t = 1 \ldots, T
\end{aligned} \tag{51}$$

$$\underline{\gamma}'^{(t)}\left(-cm_k \underline{y_k}^{(t)}\right) + \underline{\eta}'^{(t)}\left(-cd_j\ \underline{w_j}^{(t)}\right) + \underline{\mu}'^{(T)}\left(-ct_n \underline{z_n}^{(t)}\right) + \underline{\lambda}^{(T)}\left(\sum_n f_n \underline{z_n}^{(T)} + \sum_k c_k \underline{y_n}^{(T)} + \sum_j b_j \underline{w_j}^{(T)} - budget\right) \leq 0\ ;\ \forall t = 1 \dots, Q \quad (52)$$

The inequalities (51) are known as Optimality Cuts. In case no solution exists for the Benders subproblem, the Feasibility Cuts which are demonstrated in inequalities (52) are used.

Case Study

The mathematical model presented in the previous section is studied in a case study in the battery manufacturing industry. As mentioned before, the problem is concerned with a car battery production chain in Iran, which possesses a high capacity in the car battery market in the country. In this chain, the raw material is sent to the firm by the supplier through the forward route and by the recycling center through the reverse route, the product is sold by the retailer, and the returned product is sent back by the customer to the factory. Without loss of generality, one supplier, three manufacturers, three recycling centers, and thirty-one retailers (equal to the number of provinces in the country) are assumed. The input parameters and outputs of the problem are presented in this section. The case study model is encoded using Benders analysis in GAMS software and has been implemented on a computer with a 2D memory processor. The input parameters and outputs of the problem are presented in this section. To estimate the uncertainty parameters, a triangular probability distribution is used. These values, which are considered fuzzy due to a lack of knowledge and information, are shown after calculating and performing defuzzy operations. The utilized data are achieved from websites, expert officials, field research, and some papers like [22]. Some parameter values are demonstrated in the case study data presented in Table 1.

Table 1. Case study data for some parameters

Parameter	Value	Parameter	Value	Parameter	Value
tp_{11}	4.3	tp_{21}	2.6	tp_{31}	4.5
tp_{12}	3.5	tp_{22}	2.7	tp_{32}	4
tp_{13}	4.5	tp_{23}	2.6	tp_{33}	4.1
tp_{14}	2.5	tp_{24}	3.5	tp_{34}	4
tp_{15}	2.8	tp_{25}	4.5	tp_{35}	4.8
tp_{16}	4.1	tp_{26}	6	tp_{36}	4
tp_{17}	5.8	tp_{27}	3	tp_{37}	5.5
tp_{18}	4.8	tp_{28}	5	tp_{38}	3.3
tp_{19}	4.5	tp_{29}	4.9	tp_{39}	3.1
tp_{110}	5.1	tp_{210}	5.2	tp_{310}	3.2
tp_{111}	2.9	tp_{211}	4	tp_{311}	4.5
tp_{112}	6.1	tp_{212}	5.5	tp_{312}	3.5
tp_{113}	4	tp_{213}	4.5	tp_{313}	2.5
tp_{114}	4	tp_{214}	5.5	tp_{314}	3
tp_{115}	5.1	tp_{215}	3.4	tp_{315}	5.5
tp_{116}	3.5	tp_{216}	4.1	tp_{316}	5.5
tp_{117}	4.7	tp_{217}	4.1	tp_{317}	3.5
tp_{118}	6.8	tp_{218}	5.9	tp_{318}	3.6
tp_{119}	5.6	tp_{219}	3.8	tp_{319}	6.1
tp_{120}	6.1	tp_{220}	4.9	tp_{320}	3.8
tp_{121}	4.8	tp_{221}	2.9	tp_{321}	3.9
tp_{122}	3.1	tp_{222}	3.2	tp_{322}	3.3
tp_{123}	4.6	tp_{223}	3	tp_{323}	5.1
tp_{124}	6.5	tp_{224}	6.5	tp_{324}	5
tp_{125}	5.8	tp_{225}	4.1	tp_{325}	4.9
tp_{126}	5.6	tp_{226}	5.1	tp_{326}	4.3

Parameter	Value	Parameter	Value	Parameter	Value
tp_{127}	6.5	tp_{227}	6.5	tp_{327}	6.5
tp_{128}	6.3	tp_{228}	3.5	tp_{328}	5.6
tp_{129}	3.6	tp_{229}	4.5	tp_{329}	5.6
tp_{130}	6	tp_{230}	7	tp_{330}	7
tp_{131}	5.5	tp_{231}	4	tp_{331}	6.5
tr_{11}	3.5	tr_{12}	3	tr_{13}	4.8
tr_{21}	3.6	tr_{22}	3.1	tr_{23}	5
tr_{31}	4.2	tr_{32}	5	tr_{33}	3.6
tr_{41}	5.1	tr_{42}	4.4	tr_{43}	3.5
tr_{51}	2.9	tr_{52}	4.5	tr_{53}	5.5
tr_{61}	4.5	tr_{62}	3.1	tr_{63}	5.5
tr_{71}	3.3	tr_{72}	3.5	tr_{73}	2.8
tr_{81}	3.9	tr_{82}	3.8	tr_{83}	2.6
tr_{91}	3.2	tr_{92}	4.1	tr_{93}	4.4
tr_{101}	3.4	tr_{102}	5	tr_{103}	3.2
tr_{111}	4.5	tr_{112}	3.3	tr_{113}	4.9
tr_{121}	5.1	tr_{122}	4.5	tr_{123}	3.3
tr_{131}	3	tr_{132}	4.3	tr_{133}	3.4
tr_{141}	2.8	tr_{142}	4.5	tr_{143}	5.1
tr_{151}	4.8	tr_{152}	4.6	tr_{153}	3.2
tr_{161}	3.8	tr_{162}	3.5	tr_{163}	2.8
tr_{171}	2.6	tr_{172}	3.9	tr_{173}	4.8
tr_{181}	4.8	tr_{182}	4.5	tr_{183}	3.5
tr_{191}	3.8	tr_{192}	3	tr_{193}	4.6
tr_{201}	2.9	tr_{202}	3.9	tr_{203}	4.8
tr_{211}	3.1	tr_{212}	4.2	tr_{213}	3.9
tr_{221}	2.8	tr_{222}	4.5	tr_{223}	5.1
tr_{231}	5.4	tr_{232}	3.4	tr_{233}	4.8
tr_{241}	3	tr_{242}	2.9	tr_{243}	4.5
tr_{251}	4.5	tr_{252}	3.5	tr_{253}	5.5
tr_{261}	4.3	tr_{262}	3.7	tr_{263}	4.9
tr_{271}	6.5	tr_{272}	6	tr_{273}	6.5
tr_{281}	2.9	tr_{282}	3.9	tr_{283}	4.1
tr_{291}	3.1	tr_{292}	4.2	tr_{293}	3.9
tr_{301}	4.6	tr_{302}	5.1	tr_{303}	5.2
tr_{311}	4.9	tr_{312}	3.5	tr_{313}	4.8
tm_{11}	3.3	tm_{21}	4.5	tm_{31}	4.9
tm_{12}	5.5	tm_{22}	5.3	tm_{32}	3.5
tm_{13}	6	tm_{23}	4.1	tm_{33}	4.3
$cost_{11}^{/}$	0.3	$cost_{12}^{/}$	0.8	$cost_{13}^{/}$	0.1
f_1	550000000	f_2	550000000	f_3	550000000
c_1	90000000	c_2	90000000	c_3	90000000
b_1	40000000	b_2	40000000	b_3	40000000
b_4	40000000	b_5	40000000	b_6	40000000
b_7	40000000	b_8	40000000	b_9	40000000
b_{10}	40000000	b_{11}	40000000	b_{12}	40000000
b_{13}	40000000	b_{14}	40000000	b_{15}	40000000
b_{16}	40000000	b_{17}	40000000	b_{18}	40000000
b_{19}	40000000	b_{20}	40000000	b_{21}	40000000
b_{22}	40000000	b_{23}	40000000	b_{24}	40000000
b_{25}	40000000	b_{26}	40000000	b_{27}	40000000
b_{28}	40000000	b_{29}	40000000	b_{30}	40000000
b_{31}	40000000	$\tilde{d}_2,\tilde{d}_1$	4000000	v_2	130
φ_1	20	φ_2	20	φ_3	20
ct_1	1500000	ct_2	1500000	ct_3	1500000

Parameter	Value	Parameter	Value	Parameter	Value
cd_1	800000	cd_2	800000	cd_3	800000
cd_4	800000	cd_5	800000	cd_6	800000
cd_7	800000	cd_8	800000	cd_9	800000
cd_{10}	800000	cd_{11}	800000	cd_{12}	800000
cd_{13}	800000	cd_{14}	800000	cd_{15}	800000
cd_{16}	800000	cd_{17}	800000	cd_{18}	800000
cd_{19}	750000	cd_{20}	750000	cd_{21}	750000
cd_{22}	750000	cd_{23}	750000	cd_{24}	750000
cd_{25}	750000	cd_{26}	750000	cd_{27}	750000
cd_{28}	750000	cd_{29}	750000	cd_{30}	750000
cd_{31}	750000	cap_1	1000000	$cost_1$	3000
cm_1	70000	cm_2	70000	cm_3	70000
Tax	0.1			W	80
cs_{11}	2.5	cs_{12}	2.6	cs_{13}	3
ctp_{11}	2.5	ctp_{21}	2.5	ctp_{31}	2.5
ctp_{12}	2.6	ctp_{22}	3	ctp_{32}	3.2
ctp_{13}	2.8	ctp_{23}	3.2	ctp_{33}	3.5
ctp_{14}	3	ctp_{24}	3.5	ctp_{34}	3
ctp_{15}	3.2	ctp_{25}	3.9	ctp_{35}	3.2
ctp_{16}	3.4	ctp_{26}	4.1	ctp_{36}	4.1
ctp_{17}	3.5	ctp_{27}	4	ctp_{37}	3.4
ctp_{18}	3.7	ctp_{28}	4.3	ctp_{38}	4
ctp_{19}	3.5	ctp_{29}	4.6	ctp_{39}	4.3
ctp_{110}	3.8	ctp_{210}	4.8	ctp_{310}	3.8
ctp_{111}	3.9	ctp_{211}	5	ctp_{311}	3
ctp_{112}	4.1	ctp_{212}	5.1	ctp_{312}	4.3
ctp_{113}	4	ctp_{213}	2.5	ctp_{313}	3.2
ctp_{114}	4.3	ctp_{214}	3	ctp_{314}	2.5
ctp_{115}	4.5	ctp_{215}	3.2	ctp_{315}	3.2
ctp_{116}	4.6	ctp_{216}	2.5	ctp_{316}	3.5
ctp_{117}	4.8	ctp_{217}	3.2	ctp_{317}	3
ctp_{118}	4.9	ctp_{218}	3.5	ctp_{318}	3.2
ctp_{119}	5	ctp_{219}	3	ctp_{319}	4.8
ctp_{120}	5.1	ctp_{220}	3.2	ctp_{320}	4.9
ctp_{121}	5.6	ctp_{221}	4.1	ctp_{321}	3.2
ctp_{122}	5.9	ctp_{222}	3.4	ctp_{322}	4.1
ctp_{123}	6.1	ctp_{223}	4	ctp_{323}	4
ctp_{124}	6.2	ctp_{224}	4.3	ctp_{324}	3.8
ctp_{125}	6.3	ctp_{225}	3.8	ctp_{325}	4.3
ctp_{126}	6.8	ctp_{226}	3	ctp_{326}	6.3
ctp_{127}	6.9	ctp_{227}	4.3	ctp_{327}	5.8
ctp_{128}	7	ctp_{228}	6.3	ctp_{328}	6.1
ctp_{129}	7.1	ctp_{229}	6.8	ctp_{329}	6.5
ctp_{130}	7.5	ctp_{230}	7.1	ctp_{330}	6.7
ctp_{131}	7.6	ctp_{231}	7.6	ctp_{331}	7.5
ctr_{11}	2.2	ctr_{12}	2.2	ctr_{13}	3.4
ctr_{21}	2.5	ctr_{22}	2.5	ctr_{23}	3.2
ctr_{31}	2.9	ctr_{32}	3.4	ctr_{33}	3.6
ctr_{41}	3.2	ctr_{42}	3.4	ctr_{43}	4.6
ctr_{51}	3.4	ctr_{52}	4.3	ctr_{53}	3.8
ctr_{61}	3.6	ctr_{62}	4.6	ctr_{63}	4.5
ctr_{71}	3.8	ctr_{72}	4.5	ctr_{73}	3.4
ctr_{81}	3.4	ctr_{82}	4.6	ctr_{83}	4.6
ctr_{91}	4.1	ctr_{92}	4.7	ctr_{93}	4.7
ctr_{101}	4.2	ctr_{102}	4.9	ctr_{103}	4.1
ctr_{111}	4.3	ctr_{112}	5.6	ctr_{113}	4.2

Parameter	Value	Parameter	Value	Parameter	Value
ctr_{121}	4.5	ctr_{122}	6.8	ctr_{123}	6.5
ctr_{131}	4.6	ctr_{132}	6.5	ctr_{133}	4.9
ctr_{141}	4.7	ctr_{142}	2.2	ctr_{143}	4.8
ctr_{151}	4.5	ctr_{152}	2.5	ctr_{153}	5.7
ctr_{161}	4.6	ctr_{162}	3.4	ctr_{163}	5.9
ctr_{171}	4.8	ctr_{172}	3.4	ctr_{173}	6.1
ctr_{181}	4.7	ctr_{182}	4.3	ctr_{183}	6.8
ctr_{191}	4.9	ctr_{192}	4.5	ctr_{193}	6.5
ctr_{201}	4.8	ctr_{202}	6.1	ctr_{203}	3.4
ctr_{211}	5.3	ctr_{212}	4.3	ctr_{213}	3.2
ctr_{221}	5.6	ctr_{222}	6.8	ctr_{223}	3.6
ctr_{231}	5.7	ctr_{232}	5.3	ctr_{233}	4.6
ctr_{241}	5.9	ctr_{242}	5.6	ctr_{243}	3.8
ctr_{251}	6.1	ctr_{252}	5.7	ctr_{253}	6.1
ctr_{261}	6.8	ctr_{262}	4.6	ctr_{263}	6.8
ctr_{271}	6.5	ctr_{272}	4.5	ctr_{273}	6.9
ctr_{281}	6.8	ctr_{282}	6.1	ctr_{283}	7.5
ctr_{291}	6.9	ctr_{292}	4.3	ctr_{293}	7.9
ctr_{301}	7.2	ctr_{302}	6.8	ctr_{303}	6.1
ctr_{311}	7.3	ctr_{312}	6.9	ctr_{313}	6.8
cts_{11}	7.5	cts_{21}	7.2	cts_{31}	6.5
cts_{12}	7.6	cts_{22}	7.5	cts_{32}	6.7
cts_{13}	7.8	cts_{23}	7.8	cts_{33}	6.8
cp_1	7.9	cp_2	7.9	cp_3	7.1
cr_1	8.1	cr_2	6.8	cr_3	7.5

Then the triangular distribution is used to estimate non-deterministic parameters.

Table 2. Values of demand and reverse elasticity coefficients

Demand elasticity coefficient				Reverse elasticity coefficient			
$\tilde{\beta}_{11}$	$\tilde{\beta}_{12}$	$\tilde{\beta}_{21}$	$\tilde{\beta}_{22}$	$\tilde{\beta}'_{11}$	$\tilde{\beta}'_{21}$	$\tilde{\beta}'_{12}$	$\tilde{\beta}'_{22}$
1.1	1	0.9	0.7	2.5	0.3	2.3	0.2

Tables 3 to 5 report the values of decision variables of the current chain including the number of manufactured products (the number of products transported from the manufacturer to the distributor), the number of recycled products, and the number of returned products from the distributor to recycling centers.

Table 3 shows the optimal number of manufactured products in each manufacturing center in the Stackelberg competition condition in the bi-level model. The sum of the manufactured and transported products from the manufacturing center to retailers equals the demand level. On the other hand, it can be seen that transporting from a manufacturing center to all demanding centers is not a must and such a decision is made and optimized based on transportation costs.

Table 3. The number of manufactured and transported products from the factory n to the retailer j, (x_{nj}), bi-level model

j / **n**	1	2	3	4	5	6
1	-	-	-	69212	69264	34379
2	68900	69160	68952	-	-	-
3	-	-	-	-	-	35040
j / **n**	7	8	9	10	11	12
1	-	-	-	-	68120	-
2	68640	-	-	-	-	-
3	-	66300	70200	69550	-	67860
j / **n**	13	14	15	16	17	18
1	-	-	-	68250	-	-
2	-	-	67080	-	-	-
3	66950	69940	-	-	66820	70018
j / **n**	19	20	21	22	23	24
1	-	-	-	23761	-	-
2	67418	-	66768	20751	69056	-
3	-	68588	-	24362	-	69394
j / **n**	25	26	27	28	29	30
1	-	-	-	-	66638	33162
2	68406	-	-	67782	-	-
3	-	68016	-	-	-	-
j / **n**	31					
2	68931	-	-	-	-	-

Table 4. The number of recycled and transported products from the recycling center k to the manufacturer n (m_{kn}) , bi-level model

n / **k**	**1**	**2**	**3**
1	714022	-	-
2	-	-	731040
3	-	491108	81500

Table 4 demonstrates the optimal number of recycled products. As mentioned earlier, the total number of the recycled and taken out from the recycling centers equals the total number brought to the recycling centers. The optimal value for these decisions are determined based on the recycling and transportation costs of products from recycling centers to manufacturing centers under the condition of Stackelberg competition in the bi-level model.

Table 5 reports the number of returned and transported products from the retailers to the recycling centers.

Table 5. The number of returned and transported products from the retailer j to the recycling center k (u_{jk}), bi-level model.

k / **j**	**1**	**2**	**3**
1	6411	62489	-
2	5349	63811	-
3	-	-	68952
4	-	-	69212
5	69264	-	-
6	-	69420	-
7	-	-	68640
8	-	-	66300
9	70200	-	-
10	43254	-	26296
11	-	68120	-
12	-	-	67860
13	66950	-	-
14	69940	-	-
15	-	-	67080
16	-	-	68250
17	66820	-	-
18	-	-	70018
19	-	67418	-
20	68588	-	-
21	66768	-	-
22	68874	-	-
23	-	69056	-
24	6037	63356	-
25	-	68406	-
26	-	68016	-
27	-	-	-
28	5765	62016	-
29	66638	-	-
30	33162	-	-
31	-	68931	-

As noted before, the sum of reverse flow from the retailers to the recycling centers can at most equal the amount of production of the chain as the entire manufactured products never return back to the chain and the chains focus on maximizing the amount of return by setting appropriate prices. The optimal amount of returned products is determined by the model, based on the costs of transporting worn-out products from the retailers to the recycling centers and the offered price to buy these products. The optimal values related to retail and returned prices and the amount of return and demand can be calculated for the leader-follower competition and afterwards, the network design model can be optimized using these optimal values. The values for objective function in the bi-level model where the leader decides on the network design and pricing are presented in Table 6.

Table 6. The output from the bi-level model of the Stackelberg competition - the objective function value

Profit function of the follower	Profit function of the leader	Type of model
9.00986E+12	10.19925E+12	Bi-level

As we can see, the profit of the current chain, also known as the leader, is higher than the new chain which follows the policies of the leader. It is preferable that the current battery manufacturing chain utilizes the current supply chain after the introduction of the new brand and avoids creating a new network; it is also preferable that the new brand utilizes the existing facilities in the current network while examining the pricing problem, investigates the creation of new facilities (e.g. construction of new distribution centers).

Numerical Results and Sensitivity Analysis

In this section, the mathematical model is analyzed using the GAMS software. The data has been randomly generated and without loss of generality, one supplier, three manufacturers, three recycling centers, and six retailers are assumed for numerical samples. In order to estimate the uncertainty parameters, triangular distribution is used. First, a numerical sample is randomly generated and the problem is solved using the GAMS software. The ranges for random numbers for each parameter are listed in Tables 7 and 8. The input parameters and the resulting outputs are presented in this section and, in the end, a comprehensive sensitivity analysis is performed on the important parameters of the model.

Table 7. The random numbers used for some parameters

parameter	value	parameter	value
v_i	$U(80,84)$	ct_n	$U(80000,90000)$
tp_{nj}	$U(5,7)$	cd_j	$U(900000,1000000)$
tr_{jk}	$U(3.5,5.5)$	cm_k	$U(820000,990000)$
tm_{kn}	$U(2.2,4.5)$	cs_{sn}	$U(2,10)$
f_n	$U(520000,560000)$	ctp_{nj}	$U(2,10)$
c_k	$U(81000,95000)$	ctr_{jk}	$U(2,10)$
b_j	$U(39000,57000)$	cts_{kn}	$U(2,10)$
φ_k	$U(17,22)$	cp_n	$U(1,5)$
$cost^{/}_{sn}$	$U(2,4)$	cr_k	$U(2,10)$
$cost_s$	$U(2000,4000)$	$\tilde{d}_i$	$U(3800000,4200000)$
cap_s	$U(950000,1100000)$	Tax	0.1
W	$U(120,145)$		

The demand and reverse elasticity values are demonstrated in Table 8.

Table 8. The demand and reverse elasticity values

Demand Elasticity Coefficient				Reverse Elasticity Coefficient			
$\tilde{\beta}_{11}$	$\tilde{\beta}_{12}$	$\tilde{\beta}_{21}$	$\tilde{\beta}_{22}$	$\tilde{\beta}'_{11}$	$\tilde{\beta}'_{21}$	$\tilde{\beta}'_{12}$	$\tilde{\beta}'_{22}$
1.1	1	0.9	0.7	2.5	0.3	2.3	0.2

In this section, the changes in the objective function for different values of elasticity coefficients are investigated. Without loss of generality, one supplier, three manufacturers, three recycling centers, and six retailers are assumed.

- Analysis of forward elasticity parameter $\tilde{\beta}_{11}$: Different values for this parameter are generated and presented regarding the random sample to analyze the forward elasticity parameter. In doing so, the elasticity parameter of the price $\tilde{\beta}_{11}$ took on different values ranging from 0.5 to 1.5, and the results of the single-level are depicted in Fig. 2.

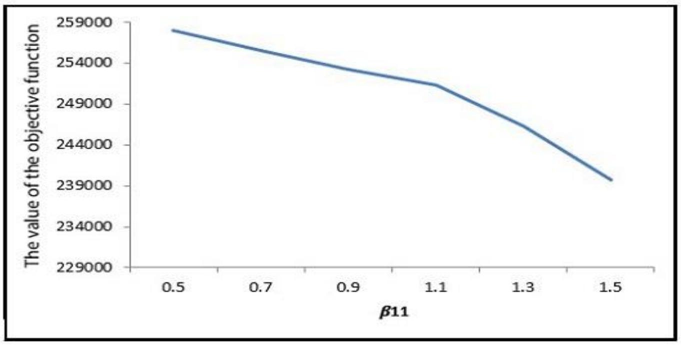

Fig. 2. Analysis of changes in parameter $\tilde{\beta}_{11}$ (direct elasticity coefficient of supply chain 1)

Tabel 9. Analysis of changes in parameter $\tilde{\beta}_{11}$ in Stackelberg competition

	0.5	0.7	0.9	1.1	1.3	1.5
Z2	103250	103198	102958	102856	102835	102632
Z1	156230	156200	156175	155689	155682	155523

As it can be seen, the increase in the elasticity coefficient entails a decline in the profitability of the supply chain. In fact, as the elasticity coefficient increases the market tends to become more competitive which in turn, leads to a reduction in prices and demand. The decline in the supply chain profit occurs because a more competitive market forces the supply chain to offer lower prices. This, accompanied by a lower market share will lead to a decrease in profit. Thus, the more the market becomes competitive (a higher demand elasticity), the lower becomes the profit of the supply chain because to gain a higher market share or at least maintain the current level of market share, the chains are forced to lower prices which takes place simultaneously with the drop in price of the competitor and finally leads to a decline in demand on both sides.

- Analysis of the competitor elasticity $\tilde{\beta}_{21}$: In order to analyze this parameter, different values in the random sample are generated and presented. Then, different values ranging from 0.5 to 1.5 were assigned to the competitor elasticity $\tilde{\beta}_{21}$ the results of which are demonstrated in Fig. 3.

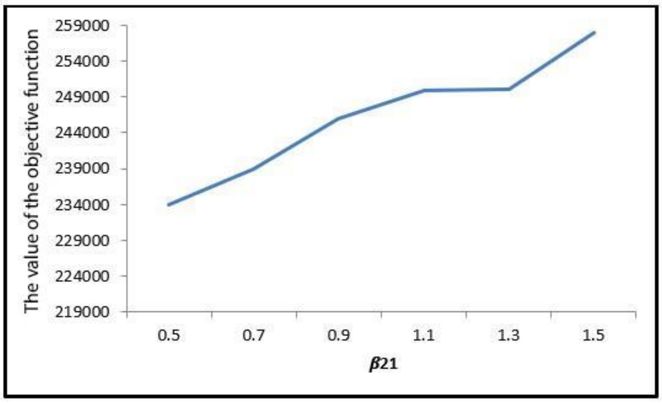

Fig. 3. Analysis of changes in parameter $\tilde{\beta}_{21}$ (indirect elasticity coefficient of supply chain 1)

Table 10. Analysis of changes in parameter $\tilde{\beta}_{21}$ in the Stackelberg competition

	0.5	0.7	0.9	1.1	1.3	1.5
Z2	102610	102725	102938	103125	103103	103290
Z1	155723	155982	155985	156027	156121	156210

As it can be seen, the profit of the current supply chain, considered the leader, is higher than the new chain which follows the policies of the leader. As the competitor elasticity coefficient increases, the demand becomes more sensitive to the competitor's price and therefore the increase in demand decreases for the competitor and rises for the current chain, leading to a growth in market share, and as a result an increase in the profit.

Managerial insights

The following conclusions can be drawn after sensitivity analysis and solving the models:

With the entrance of a new battery brand into the market, it is advantageous for both parties to use the existing supply chain network and decide only on the introduction of new facilities to the existing network and focus solely on setting the selling price of new batteries and buying price of worn-out batteries.

The more competitive a market (i.e. a higher demand elasticity coefficient), the lower the profit of supply chains. This is because, in order to gain more market share, or at least keep the current share of the chains, the chains are forced to lower their prices, which coincides with price decline on the competitor side, leading to a reduction in demand for both sides.

Increasing the price sensitivity parameter of $\widetilde{\beta}'_{11}$ in the reverse chain leads to an increase in the number of returned products. In case the new chain needs to attract more returned goods (e.g. because of the constraint in the raw materials), it should increase this parameter, and it is preferable to choose the Stackelberg policy. This parameter determines the power of competition in the reverse channel which can be increased by using methods such as advertising, environmental protection mottos, incentives for collecting wastes, etc.

Conclusion

With regard to the increasing competition in today's world and the paradigm shift from a competition between companies to chains, and on the other hand, the significant effect of strategic decisions on the competitiveness of a supply chain, in this paper the problem of closed-loop supply chain design with demand uncertainty has been investigated. Supply chain design is carried out with the goals of maximizing profits and minimizing greenhouse emissions, despite the competition between two chains over the price of products in the forward chain and the buying price of returned products in the reverse chain. The competition has been studied in the form of a bi-level leader-follower model and the GAMS software has been used to solve the problem. The problem of the Stackelberg competition has been formulated as a bi-level model and then the bi-level problem is reduced to a single-level problem. To verify the correctness of the results of modeling, several numerical samples are designed using randomly generated data and solved by GAMS software. The single-level problem is solved using GAMS software and the computational data obtained from the solution are presented. In the sensitivity analysis section, it is proved that increasing the forward elasticity coefficient (i.e. a more competitive market) entails a decline in supply chain profit. This reduction in supply chain profitability is because as competition increases, the chain will need to offer lower prices, and on the other hand, the market share is reduced, thus, leading to lower profits. Indeed, as the increase in price sensitivity results in a reduction in the supply chain profit, the chains must strive to reduce this coefficient. As the elasticity coefficient of the competitor increases, the demand becomes more sensitive to the competition price, and the demand for the competitor declines, leading to a decrease in the rival's profit and an increase in the current supply chain's profit; thus the profitability rises as the market share goes up. In cases where the new chain needs to attract more returned products (e. g. because of the constraint in the raw materials), it should increase this parameter, and it is preferable to choose the Stackelberg policy. For the existing chain, the Stackelberg policy would lead to the acquisition of fewer returned products. Following are a number of suggestions for future research. In this paper, the competition only takes place on the price, while it can occur on other features such as distance from the market, quality, service level, etc. Considering that the subject of this research stems from a real case (battery supply chain), the competition takes place between two chains, namely the chain under study and a new chain that enters the market and produces the same product; though in other cases, the competition between the existing chain and two or more new chains in the market can be studied. The studied industry in this paper is based solely on the batteries produced for passenger personal automobiles, which is assumed to be a single-product supply chain because of the similarity of the batteries, but most chains on the competitive market are multi-product, the study of which requires the consideration of dependence or independence of products relative to each other which will result in different models. The inhomogeneous routing and the combination of the supply chain problem with the vehicle routing problem can also result in significant results. The inclusion of time constraints in the production and delivery of goods to the customers can also create a new model. The comparison between the results of this research and the working conditions of supply chains is a useful problem for future research. As a result, the chains can use joint facilities to provide products or services. Simultaneous study of collaboration and competition also leads to a new form of problems that examine the benefits resulting from cooperation in competitive conditions.

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