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RESEARCH PAPER

Coupled Non-Stationary Thermoelastic Fields In A Rigidly Fixed Round Plate

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Abstract,

The mathematical formulation of thermoelasticity problems includes coupled non-self-adjoint differential equations of motion and heat conduction. The problem of integrating them and constructing a general solution leads, as a rule, to the study of only the heat conduction equation or to the analysis of thermoelasticity problems in an unconnected formulation. However, for a better assessment of thermomechanical fields, it becomes necessary to construct coupled analytical solutions in a three-dimensional formulation. Therefore, the development of effective analytical methods and algorithms for calculating elastic systems is currently one of the urgent problems of modern science. In this problem, a mathematical calculation model is developed and a closed solution of the coupled axisymmetric non-stationary problem of the theory of thermoelasticity for a rigidly fixed isotropic plate is constructed. Design ratios are obtained by the method of finite biorthogonal transformations and are valid for an external temperature effect arbitrary in time (boundary conditions for thermal conductivity of the 1st kind). Software that allows to analyze the effect of coupled thermoelastic fields on the temperature field and the stress-strain state of the structure has been developed. Numerical analysis of the results shows that for a given external temperature effect, the rigidity of an elastic system (physical and mechanical characteristics and geometric dimensions) has a significant effect on its thermoelastic field. The developed calculation algorithm finds its application in the design of enclosing structures in the form of single-layer and multi-layer plates.

Keywords: round plate, theory of thermoelasticity, non-stationary temperature action, finite integral transformations.

1. Introduction

Uneven non-stationary heating of structures for various purposes leads to thermal deformations and stress occurrence, which must be taken into account in the case of a comprehensive analysis of the strength characteristics of elastic systems of finite dimensions [1-5]. At present, various theories of thermoelasticity have been developed (CTE, GHI – GHIII, LS) [8, 20, 49, 55], which solve this problem with varying accuracy degrees. Various theories and assumptions are also widely used in modern works [6-15].

The mathematical formulation of the considered initial-boundary value problems in a linear three-dimensional formulation includes coupled non-self-adjoint differential equations of motion and heat conduction. As a rule, this

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system of differential equations is considered in an unrelated setting [16-22] this case, when an external non-stationary heat load acts on the elastic system, the effect of the rate of change in the volume of the body on the temperature field is not taken into account.

In a related formulation, closed solutions of dynamic problems of thermoelasticity are presented in a few works. In particular, studies [23-26] were carried out for a finite isotropic cylinder with membrane fixing of its end surfaces. In [27] using the generalized method of finite integral transformations [28] and [24, 26] the biorthogonal integral transform [23, 29].

Research [30, 31] was carried out using hyperbolic (GHII, GHIII) theories of thermoelasticity and helps to analyze the frequency equations, as well as the forms of harmonic waves in an infinite cylindrical waveguide.

Modern works devoted to the analysis and stability of structural elements under the influence of various thermal, thermomechanical, and electrical loads include the works [3, 4, 6-8, 10-14, 16, 32-52].

In this work, a rigidly fixed round isotropic plate is investigated. The case of the action on the upper and lower surfaces of an unsteady axisymmetric temperature load (boundary conditions of the 1st kind) is considered. The numerical results of calculating this problem in an unconnected formulation [53] allow us to conclude that the inertial forces of an elastic system affect its stress-strain state only in very thin structures ($h^*/b \le 0.01$, $h^*, b -$ thickness and radius of the plate) under the action of a high-frequency load. Taking into account these results, the inertia forces are not taken into account when solving the system of non-self-adjoint differential equations of the classical (CTE) theory of thermoelasticity, i.e., the constraint is used for the considered constructions $h^*/b \le 0.01$.

The constructed solution of the coupled problem in a three-dimensional formulation makes it possible to take into account the effect of the rate of change in its volume (rate of dilatation) on the nature of the distribution of the temperature field and the stress strain.

2. Materials and Methods

As noted earlier, the purpose of this work is to construct a closed solution to the unsteady axisymmetric problem of thermoelasticity for an isotropic rigidly fixed disk in the case of uneven temperature heating of its surfaces. In this regard, the main tasks of the study are to solve the heat conductivity problem and determine the temperature load function by the method of finite integral transformations, then solve the thermoelasticity problem taking into account the known temperature function, construct the final expressions for determining the functions of displacement and temperature, and then confirm the analytical calculation by experiment.

As an example, consider an isotropic plate, on the end surfaces of which a temperature load acts. Let a round rigidly fixed plate occupy the region Ω: $\{0 \le r_* \le b, 0 \le \theta \le 2\pi, 0 \le z_* \le h^*\}$ in the cylindrical coordinate system (r_*, θ, z_*). On the upper and lower surfaces, the temperature is set, the value of which depends on the radial coordinate r_* and time t_*: at $z_* = 0$ $\omega_1^*(r_*, t_*)$, at $z_* = h^*$ $\omega_2^*(r_*, t_*)$ (Fig. 1).

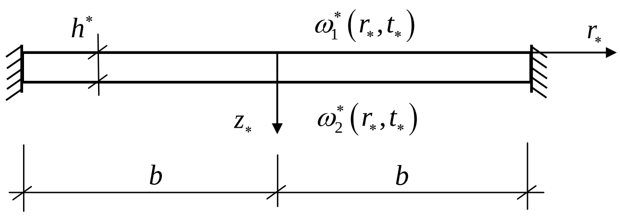


Fig. 1: Calculation scheme. "Compiled by the authors"

The mathematical formulation of the initial boundary value problem under consideration in a dimensionless form includes

- a system of linear axisymmetric non-self-adjoint differential equations for the components of the displacement vector $U(r,z,t), W(r,z,t)$ and temperature $\Theta(r,z,t)$:

$$\frac{\partial}{\partial r}\nabla U + a_1 \frac{\partial^2 U}{\partial z^2} + a_2 \frac{\partial^2 W}{\partial r \partial z} - \frac{\partial \Theta}{\partial r} = 0 \tag{1}$$

$$a_1\nabla\frac{\partial W}{\partial r}+\frac{\partial^2 W}{\partial z^2}+a_2\frac{\partial}{\partial z}\nabla U-\frac{\partial\Theta}{\partial z}=0$$

$$\nabla\frac{\partial\Theta}{\partial r}+\frac{\partial^2\Theta}{\partial z^2}-\frac{\partial\Theta}{\partial t}-a_3\frac{\partial}{\partial t}\left(\nabla U+\frac{\partial W}{\partial z}\right)=0$$

- boundary conditions:

$$r=0,1 \qquad \{U(0,z,t),W(0,z,t),\Theta(0,z,t)\}<\infty,\ \frac{\partial\Theta}{\partial r}_{|r=1}=0,\ \{U(1,z,t),W(1,z,t)\}=0; \tag{2}$$

$$z=0,h \qquad \frac{\nu}{1-\nu}\nabla U+\frac{\partial W}{\partial z}=\{\omega_1,\omega_2\},\ \frac{\partial W}{\partial r}+\frac{\partial U}{\partial z}=0,\ \Theta(r,z,t)_{|z=0,h}=\{\omega_1,\omega_2\};$$

- initial conditions:

$$t=0 \qquad \Theta(r,z,0)=0. \tag{3}$$

where $\{U,W,r,z,h\}=\{U^*,W^*,r_*,z_*,h^*\}/b$, $\{\Theta,\omega_1,\omega_2\}=a_4\{\Theta^*,\omega_1^*-T_0,\omega_2^*-T_0\}$, $a_1=a_2(1-2\nu)$, $a_2=0.5(1-\nu)^{-1}$, $a_3=\frac{\gamma^2(1+\nu)(1-2\nu)}{E(1-\nu)c_\varepsilon}T_0$, $a_4=\frac{1+\nu}{1-\nu}\alpha_t$, $t=t_*\frac{\Lambda}{b^2c_\varepsilon}$, $U^*(r,z,t),W^*(r,z,t),\Theta^*(r,z,t)-$ displacement vector components and temperature increment in dimensional form; $\Theta^*=T-T_0$, Θ^*,T,T_0- current temperature and temperature of the initial state of the body, in which there are no mechanical stresses; $E,\nu-$ elastic modulus and Poisson's ratio of the material; $\alpha_t,c_\varepsilon,\Lambda-$ coefficients of linear thermal expansion, volumetric heat capacity and thermal conductivity of the material.

3. Results

The initial boundary value problem (1) – (3) is solved by the method of integral transformations, using successively the Hankel transform [18] with finite limits in the variable and the degenerate biorthogonal finite transformation [29] in the coordinate z. At each stage of the solution, the procedure of standardization of the corresponding boundary conditions is carried out [54].

Transformants $R(n,z,t)$, $G(\lambda_{in},n,t)$ and the inversion formulas of the corresponding transformations have the following form:

$$R(n,z,t)=\int_0^1 N(r,z,t)P(n,r)rdr,\quad N(r,z,t)=\sum_{n=0}^{\infty}\Omega_n^{-1}RP; \tag{4}$$

$$G(\lambda_{in},n,t)=\int_0^t R(n,z,t)Y(\lambda_{in},z)dz,\quad R=\sum_{i=1}^{\infty}GH\|K_{in}\|^{-1}; \tag{5}$$

where $N(r,z,t)=\left[U(r,z,t),W(r,z,t),\Theta(r,z,t)\right]^T$, $P=\left[s_{mp}\right]-$ 3-order diagonal matrix, $\left(s_{11}=J_1(j_nr),s_{22}=s_{33}=J_0(j_nr)\right)$, $Y(\lambda_{in},z)=\left[0,0,K_3(\lambda_{in},z)\right]^T$, $H(\lambda_{in},z)=\left[N_1(\lambda_{in},z),N_2(\lambda_{in},z),N_3(\lambda_{in},z)\right]^T$, K_3,N_1,N_2,N_3- components of the vector-function of biorthogonal transformations; $\Omega_n,\|K_{in}\|-$ square of the norm of transformation kernels; $j_n,\lambda_{in}-$ eigenvalues ($n=0,1,2...$, $i=1,2,3...$).

As a result, we obtain an expression for the functions $U(r,z,t),W(r,z,t)$, $\Theta(r,z,t)$ in the form of spectral expansions:

$$N(r,z,t)=\sum_{n=0}^{\infty}\Omega_n^{-1}P(n,r)\left[F_k+\sum_{i=1}^{\infty}G(\lambda_{in},n,t)H(\lambda_{in},z)\|K_{in}\|^{-1}\right], \tag{6}$$

where F_k- matrix is a column of standardizing functions.

The algorithm for solving the initial – boundary value problem of thermoelasticity (1) - (3) is described in detail in [53].

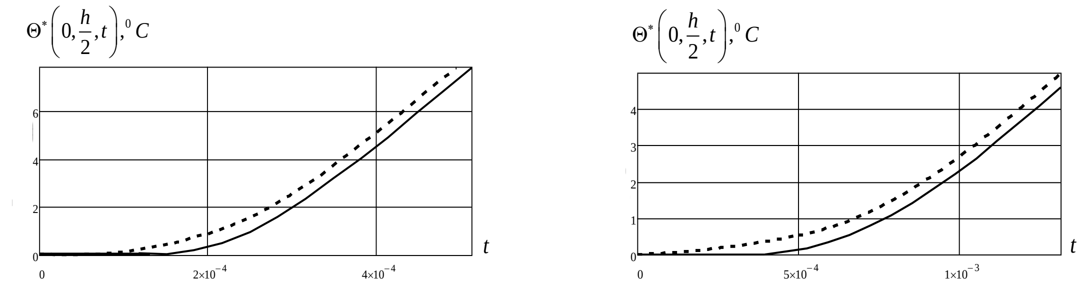
As an example, we consider a rigidly fixed round plate ($b=1$m) made of steel, which has the following physical and mechanical characteristics of the material: $E=2\times10^{11}$ Pa, $\Lambda=50$, W/(m^0K), $\nu=0.28$, $c_\varepsilon=3.8\times10^6$ J/(m^3 ^{0}K), $\alpha_t=1.2\times10^{-5}$ 1/^{0}K.

The case of the action of ($z_*=0$) a temperature load on the upper front surface in the form of:

$$\omega_1^*(r_*,t_*)=(1-r_*)T_{\max}\left[\sin\left(\frac{\pi}{2t_{\max}}t_*\right)H\left(t^*_{\max}-t_*\right)+H\left(t_*-t^*_{\max}\right)\right],\quad \omega_2^*(r_*,t_*)=0, \tag{7}$$

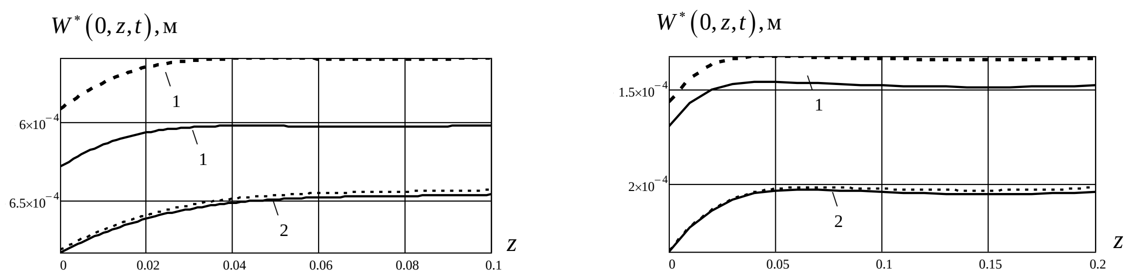
where $H(\tilde{t})-$ the single function of Heaviside ($H(\tilde{t})=1$ at $\tilde{t}\geq0$, $H(\tilde{t})=0$ at $\tilde{t}<0$), $T_{\max}=T^*_{\max}-T_0$, $T^*_{\max},t^*_{\max}-$ the maximum value of the external temperature effect and the corresponding time in the dimensional form ($T^*_{\max}=373\ {}^0K$ ($100\ {}^0C$), $T_0=293\ {}^0K$, ($20\ {}^0C$)).

Figures 2–4 show graphs of temperature changes $\Theta^*\left(0,h/2,t\right)$, components of the vector of displacements $U^*(0.5,z,t),W^*(0,z,t)$ in time and axial coordinate with allowance for (solid line) and also without (dashed line) for the coupling of thermoelastic fields ($t^*_{\max}=10$ c, $t_{\max}=\frac{\Lambda}{b^2c_\varepsilon}t^*_{\max}=1.3\times10^{-4}$). The temperature field and stress-strain state are analyzed for plates with a thickness $h=0.1,0.2$.



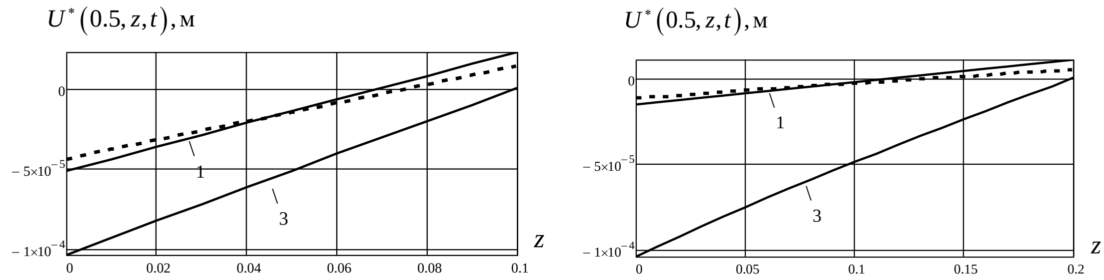
a) $h=0.1$ b) $h=0.2$

Fig. 2: Graphs of temperature $\Theta^*\left(0,\frac{h}{2},t\right)$ changes over time t (Compiled by the authors)



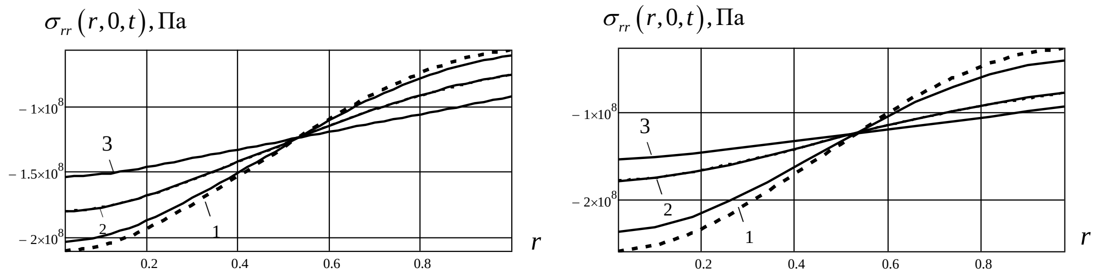
a) $h=0.1$ b) $h=0.2$

Fig. 3: Graphs of the $W^*(0,z,t)$ change in the height of the plate (1– $t=t_{\max}$,2– $t=5t_{\max}$) (Compiled by the authors)



a) $h = 0.1$ b) $h = 0.2$

Fig. 4: Graphs of change $U^*(0.5, z, t)$ along the height of the plate (1–$t = t_{\max}$,3–$t = 100t_{\max}$) (Compiled by the authors)



a) $t^*_{\max} = 10\ c$ $(t_{\max} = 1.3\times10^{-4})$, 1–$t = t_{\max}$,
2–$t = 5t_{\max}$,3–$t = 100t_{\max}$

b) $t^*_{\max} = 1\ c$ $(t_{\max} = 1.3\times10^{-5})$, 1–$t = t_{\max}$,
2–$t = 50t_{\max}$,3–$t = 500t_{\max}$

Fig. 5: Graphs of changes $\sigma_{rr}(r, 0, t)$ in the radial coordinate ($h = 0.1$) (Compiled by the authors)

4. Discussion

Analysis of the calculation results allows us to draw the following conclusions:

- the connectivity of thermoelastic fields at a given temperature load (7) leads to a slower heating of the plate over time (Fig. 2). In this case, the rate of change in the volume of the body, which is taken into account in the heat conduction equation (1), has a significant effect at the first stage of the study of the temperature regime, when $t_{\max} < t < 10t_{\max}$. In the future, this effect is not observed;
- a decrease in the rate of temperature change inside the plate due to its dilation gives an increase in the gradients $\partial\Theta/\partial r, \partial\Theta/\partial z$ that are used in the initial differential equations of thermoelasticity (1). As a result, there is an increase in the numerical values of the axial component of the displacement vector (Fig. 3, graphs 1, 2);
- at a given temperature load, the coupling of thermoelastic fields decreases over time (Fig. 3). In addition, as a result of warming up the structure, an increase in displacements is observed (Fig. 3,4), and with a steady temperature regime on the lower surface there are no radial displacements (Fig. 4, graph 3);
- the linear nature of the change in the radial component of the displacement vector along the height of the plate, allows us to conclude that when solving thermoelasticity problems for homogeneous elastic systems with the help of applied theories, it is possible to use the kinematic hypothesis of plane sections;
- the numerical values of radial displacements $U^*(0.5, z, t)$ at a steady temperature regime do not depend on the thickness of the plate (Fig. 4, graphs 3).

Figure 5 shows graphs of changes in normal mechanical stresses along $\sigma_{rr}(r, 0, t)$ the radial coordinate at different points in time taking into account (solid line), as well as disregarding (dashed line) the connectivity of thermoelastic fields.

5. Conclusion

As a conclusion, we can note the following:

- the greatest influence of the field coupling on the stress tensor component $\sigma_{rr}(r,0,t)$ is observed at $t = t_{\max}$. Subsequently, this effect sharply decreases (fig. 5);
- in the process of warming up $t > t_{\max}$ the structure, when there is a decrease in normal stresses $\sigma_{rr}(r,0,t)$. At $t = t_{\max}$ this component of the stress tensor is higher at a faster temperature loading of the plate;
- it should be noted that when the condition is met $\omega_2^*(r_*, t_*) = 0$ the lower face plane $(z = h)$ is a neutral surface $\sigma_{rr}(r,h,t) = 0$, since $\partial W^* / \partial z_{|z=h} = 0$ (fig. 3) and (fig. 4).

In conclusion, we can conclude that when calculating structures of finite dimensions in the case of a high-speed thermal load, the coupling of temperature and elastic fields has a significant effect on its stress-strain state. Moreover, this feature is more pronounced in thin plates. As a result of the work performed, it will be possible to conduct a thermoelastic calculation of the design in question, which allows you to choose the geometric dimensions of the plate, as well as the physical characteristics of the material that ensure its most efficient operation.

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