-RESEARCH PAPER-

International Journal of Mining and Geo-Engineering

IJMGE

IJMGE 56-2 (2022) 181-190

DOI: 10.22059/IJMGE.2021.321818.594907

A new method for determining geochemical anomalies: U-N and U-A fractal models

Seyyed Saeed Ghannadpour [a,*], Ardeshir Hezarkhani [a]

[a] *Department of Mining Engineering, Amirkabir University of Technology (Tehran Polytechnic), Tehran, Iran*

Article History:

Received: 10 April 2021.

Revised: 15 July 2021.

Accepted: 02 September 2021.

ABSTRACT

Undoubtedly, determining the threshold of anomalies and separating geochemical anomalies from background is one of the most important stages of minerals exploration. In the discussion of the separation of geochemical anomalies from background, there are different methods that structural methods have shown much greater efficiency than nonstructural methods. Among structural methods (methods that consider the position and location of samples), U-statistic and fractal methods have a special place. In this study, by using the algorithm of the abovementioned methods and combining them, a new method as U values fractal model (U-N and U-A) is introduced for the first time. Then, the proposed method is employed to determine the boundaries of background and anomalous populations (about the gold (Au) and arsenic (As) elements in Susanvar district). Results show that in U-N and U-A fractal models, the first fracture boundary is much clearer and more accurate than previous fractal models (C-N and C-A) in the same condition. In U-N model, due to the nature of the U method algorithm, there is a discontinuity as the exact threshold between background and anomaly that in U-A model, this does not exist due to the homogenization of U values. In this method, the exact threshold between background and anomaly is determined by the U-statistic method and by its combination with the fractal method, in each population, sub-populations are identified more accurately and simply than the concentration fractal model. Finally, a lithogeochemical map of the study area is provided for Au and As which has been prepared using U-N and U-A fractal methods. In these maps (especially the prepared maps by U-A model), the delineated Au-As mineralization is closely associated with the defined Au ore indications in the study area.

Keywords: *Anomalies delineation, Susanvar, U-Statistics, U-N, U-A fractal*

1. Introduction

Quantitative descriptions of geochemical patterns and providing geochemical anomaly maps are important in applied geochemistry for mineral exploration and environmental assessment. Extraction demands an astronomical amount of money, so in order to have tangible benefits, it is necessary to explore pretty accurately and also avoid dissipating money, time, and energy. To discern geochemical distribution in an area, different statistical approaches have been devised (e.g. [1-21]). These methods ranged from simple approaches to complicated ones, are divided into the nonstructural and structural ones following a general objective which is based on decreasing the error of distinguishing anomaly from background. The latter includes approaches that consider sampling point coordinates and their spatial relation in the estimation of anomalous areas. As examples of structural or objective methods, techniques like discriminant analysis, fractal models, fuzzy clustering, and U-spatial statistics can be noted [6, 9-12, 22-27].

One of the major applications of fractal geometry is in estimating threshold and consequently, separating the geochemical anomalies from background based on the variation of their fractal dimension. If the study area has no anomalies about a geochemical variable and only background exists in it, fewer increases and decreases appear in the distributions diagram of that variable. Therefore, its fractal dimension will be close to 2. However, as soon as it crosses from background range and enters the anomalous populations, due to the appearance of high peaks in the variability of the geochemical variable, the fractal dimension increases relative to the anomalous intensity. Therefore, background and anomalous values could be separated from each other using the difference between the fractal dimensions of the two populations.

There are different algorithms and methods such as variogram analysis, number-size (N-S) model, concentration-area (C-A) model, concentration-volume (C-V) model, concentration-perimeter (C-P) model, Concentration-number (C-N) model, and fractal model of power spectrum– area to calculate the fractal dimension that were investigated in many studies [12, 26-31].

The U-statistic method is one of the structural methods. This method is considered an applicable approach to separate anomalous populations from background [7]. It is a window-based method that applies window dimensions for averaging each particular point by utilizing surrounding points. This method is way effective to separate anomalies from background and has been compared with the other separation methods. One of the advantages of this method is the ability of the U-statistic method in combining with other methods. Because it devotes a new value to each sample as U values.

In the present study, for employing the U-statistic and fractal technique (C-N and C-A models) as two structural methods for separating anomaly from background, U-N and U-A fractal models are introduced. This method is the combination of two powerful methods and the new method will be expected to be very useful for separating background and anomalous populations and their subpopulations.

Susanvar was selected as the case study to evaluate the presented U-N and U-A fractal method because this area and its condition (sample

* Corresponding author. *E-mail address:* s.ghannadpour@aut.ac.ir (S. Ghannadpour).

type, their grid, Au ore indications, etc.) are very suitable for validation of methods performance based on previous studies [28].

2. Geology

Susanvar exploration district is located at a distance of 7 km from north of the Moaleman area, about 120 km from the south of Domghan, 550 km from east of Tehran, and on the Torud-ChahShirin mountain range (TCS) of Semnan Province. Susanvar district r with the coordinates of 54° 31′ 18″ - 54° 34′ 14″ E and 35° 16′ 49″ - 35° 19′ 51″ N, represents an area of approximately 5 km2.

The oldest rock formation in the area is belonging to Silurian and in terms of mineralogy it consists of calcite, dolomite, and serpentine and in terms of rock classification, it includes greenschist, mica-schist, tuff, and shale.

The following rock formations are Devonian - Carboniferous carbonate, dark gray and light marble Permian limestone, brownish-gray dolomite and Triassic marble units as well as dark gray and black Jurassic metamorphic sandstone and shale, respectively.

The stratigraphic sequence has shown that regional metamorphism happened after the lower Jurassic; hence the age of metamorphism should be considered to be between the lower Jurassic and Cretaceous.

Older sediments have been covered by upper Cretaceous rocks containing limestone, sandstone, sandy limestone, green shale, and conglomerate. So a sequence of thin-bedded volcaniclastic rocks, siltstones, and sandstones, with subordinate marlstones and tuffaceous sandstones in the lower part, lapilli tuffs, volcanic breccias, and intermediate lava flows, and rhyolite to rhyodacite domes could be recognized in the area (Fig. 1) [28].

Felsic pyroclastic rocks, intermediate lava flows and volcanic breccias, and silicic epizonal intrusions are existent in the outcrops around the study area.

In the current study, we have considered arsenic (As) as an important factor to detect gold (Au) because the As-Au correlation has drawn great attention. Because they have similar complexes which means that by breaking down of As complex, Au is released and As itself remains as well, native gold would be increased by increasing As, and in fact, they are proportional; so arsenic could be an extremely significant sign of Au. Yet, one should be noted is about the fact that Au is a stable element; therefore it would not be leached out by another hydrothermal circulation but As could easily react with the other hydrothermal solution and leave the system out and then, disturbs the balance. So, in order to remain balanced, the system must stay away from entering anything and in this situation, the high cooperation between Au and As would be seen.

Therefore, because arsenic is a suitable indicator of deposits of not only Au but also other elements such as Cu, Ag, Zn, it is particularly practical in geochemical surveys [29]. In order to understand gold, recognition of the occurrence and the geochemical behavior and also mineral forms of arsenic could be effective [30].

Increasing the solubility of gold with the concentration of As in alkaline fluid has been proved by experimental work [31]. Therefore, in order to survey Au concentrations in Susanvar district, As has been selected according to the high proportional relationship between Au and As geochemical behavior. So, it has been shown that because of the positive correlation between Au- and As-bearing minerals (pyrite and arsenopyrite), the concentration of Au is highly affected by As considered as a significant factor [32].

3. Material

To confine the area of auriferous, 29 geochemical samples and 29 heavy mineral samples have been collected from areas of gold anomaly. The next stage belonged to optimally and accurately design a lithogeochemical network by using the results of samples' analysis and also using the analysis of 17 samples collected from different parts of heavy mineral samples.

In order to optimize network design, the first 29 samples of stream sediments have been picked up and afterward sent to company ALS Chemex in Canada to analyze 44 elements by applying the ICP-MS method, dissolving in 4 acids, and analyzing Au via the Fire Assay method.

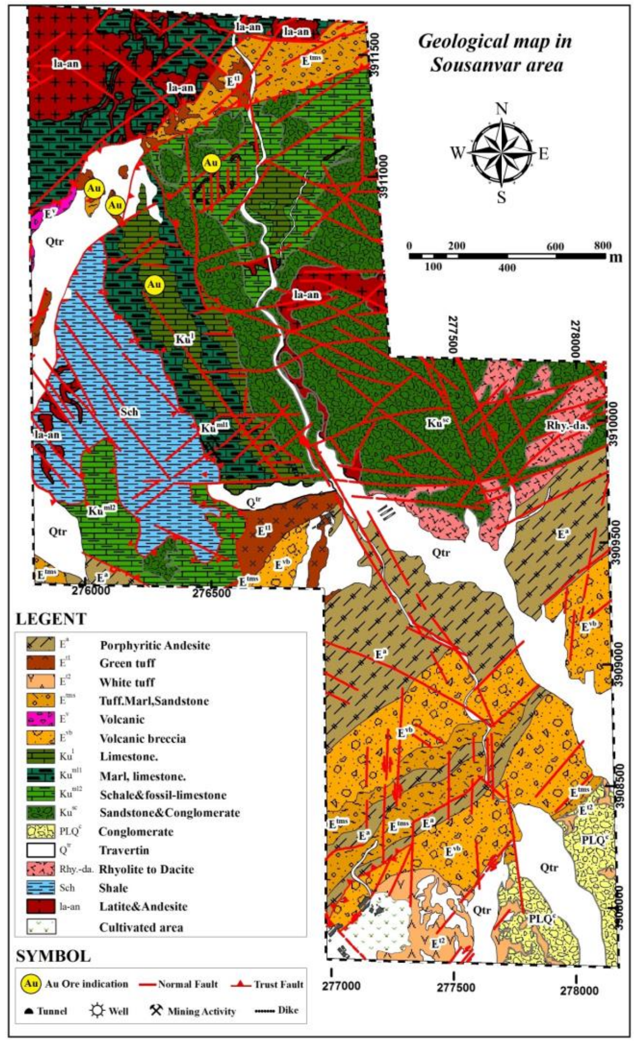

Fig. 1. Geologic map of the Susanvar exploration district.

So by comparing the results, an acceptable correlation has been observed, and also non-magnetic part of the heavy mineral was analyzed and had self-identity with the other parts. By considering the entire results, up lines of samples that were relatively enriched by Au have been considered as the gravity center of anomaly and observed more. According to this fact, designing of the network belonging to lithogeochemical sampling was done; so in this centers of anomaly gravity, networks of 50 by 50 meters in dimensions, and in the areas where tracers of detector elements were observed, 50 by 100 meters and 100 by 100 meters and finally in background area, networks of 100 by 200 meters in dimensions were designed.

Regarding this matter, 603 lithogeochemical samples were collected by the Geological Survey of Iran (GSI) from 603 numbers of designed cells in the incipient stage of the field operations illustrated in Fig. 2 (Fig. 3 displays their spatial distribution in the MATLAB environment).

The method of collecting lithogeochemical samples was row-random that was picking up 40 pieces of stone with 100 to 150 gr weight from each cell of network and all of the existent outcrops with chip sampling method and at last, a mixture of them would be the samples of the cells.

The whole of collected rock samples was ground by jaw crusher to the size of 1 mm and afterward 200 gr of that result were comminuted to -200 mesh. It is necessary to be wary about any pollution of samples while they are prepared. All of the preparations had been done in the Zar Azma laboratory (in Tehran, Iran) before they were sent to Amdel Mineral Laboratories, Adelaide, South Australia to be analyzed for major and trace element concentration by ICP-OES and ICP-MS (detection limits of ICP device are reported for different elements in Table 1).

Table 1. Detection limits of ICP device for different elements.

Element	Ag	Al	As	Au	B	Ba	Be	Bl	Ca	Cd	Ce
UNITS	ppm	ppm	ppm	ppb	ppm	ppm	ppm	ppm	ppm	ppm	ppm
DETECTION	0.01	10	0.5	1	0.5	0.2	0.2	0.1	10	0.1	0.5
Element	Co	Cr	Cs	Cu	Fe	Hg	K	La	Li	Mg	Mn
UNITS	ppm	ppm	ppm	ppm	ppm	ppm	ppm	ppm	ppm	ppm	ppm
DETECTION	0.2	2	0.1	0.2	100	0.05	10	10	0.5	10	2
Element	Mo	Na	Nb	Ni	P	Pb	Rb	S	Sb	Sc	Sn
UNITS	ppm	ppm	ppm	ppm	ppm	ppm	ppm	ppm	ppm	ppm	ppm
DETECTION	0.1	10	0.5	2	5	0. 2	0.1	50	0.1	1	0.2
Element	Sr	Te	Th	Ti	Tl	U	V	W	Y	Zn	Zr
UNITS	ppm	ppm	ppm	ppm	ppm	ppm	ppm	ppm	ppm	ppm	ppm
DETECTION	0.1	0.2	0.02	10	0.1	0.02	2	0.1	0.05	0.2	5

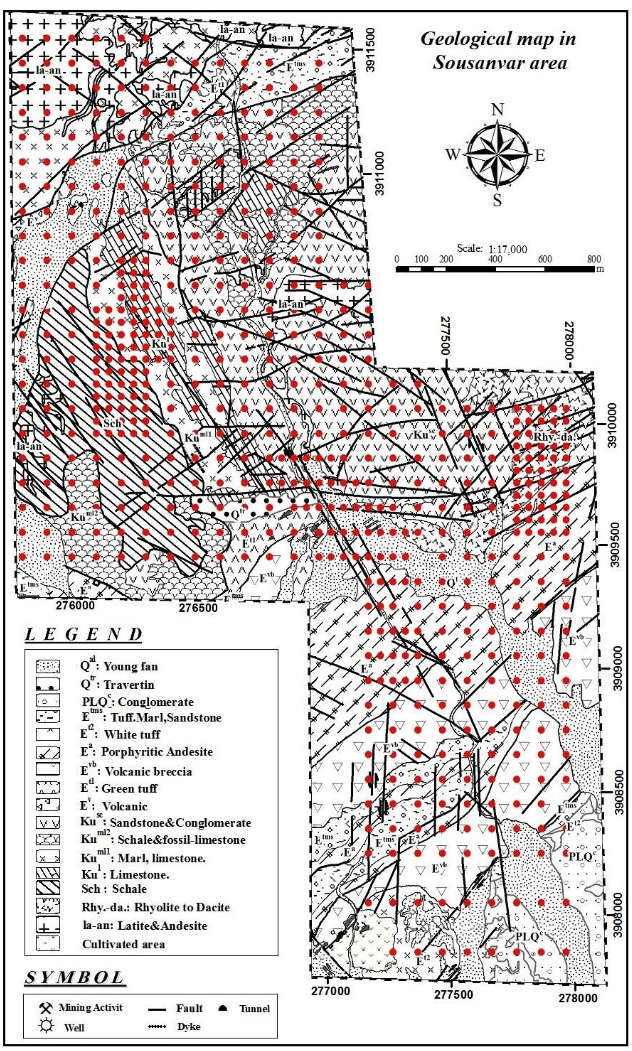

Fig. 2. Displaying the sampling locations.

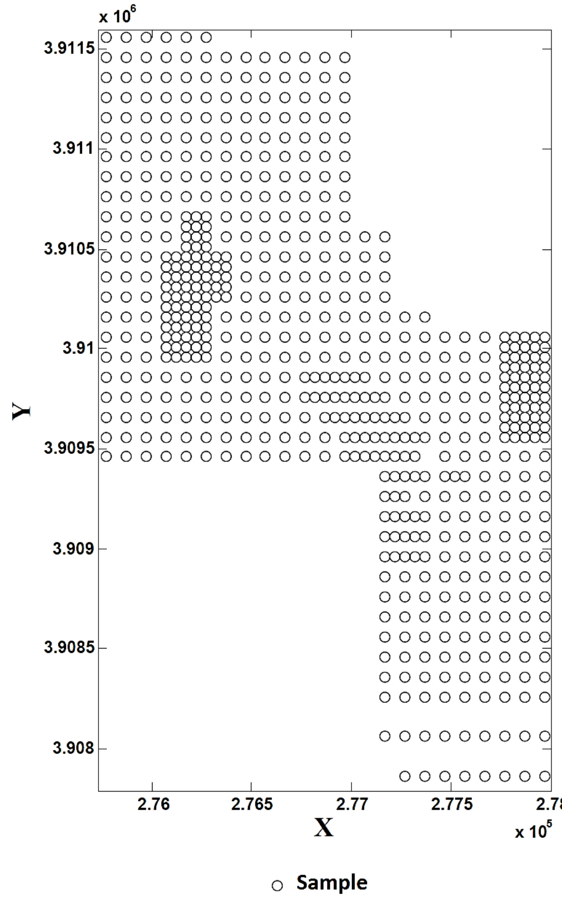

Fig. 3. Spatial distribution of the samples in the Susanvar district.

4. Methodology

4.1. U-statistics

This method introduced for the first time by Cheng et al. in 1996 [7] is considered a structural approach. Which has been presented as a powerful method to separate anomalous area from background is a kind of moving averaging method that changes dimensions of window which is under average. Therefore, in each special point, some U-statistics values would be prepared by using surrounding points [7].

This method could separate populations according to distribution models without any consideration of geochemists by analyzing data statistically.

The number of samples is considered as the controlling factor for separation errors so by increasing that better results would be achieved [7].

Arbitrary variable U is defined by:

$$U = \frac{1}{n}\sum x_i \tag{1}$$

But if the weighted average is considered instead definition is as follows:

$$U = \sum_{i=1}^{n} W_i x_i \tag{2}$$

Which is applied under two restrictions:

$$0 \le W_i \le 1 \qquad \sum W_i = 1 \tag{3}$$

So the distance criterion should be determined based on the below quantity which introduces the distance between two points:

$$\mu_{(\alpha_i,r)}(\alpha_j) = \frac{r - d(\alpha_i, \alpha_j)}{r} \tag{4}$$

r is defined as the radius which is around i point with α_i coordinate then the weight of j point is as follows:

$$W_j(r) = \frac{\mu_{(\alpha_i,r)}(\alpha_j)}{\sum_{j=1}^{n} \mu_{(\alpha_i,r)}(\alpha_j)} \tag{5}$$

n is the number of samples included in the search area. So the weights are dependent on the search radius. Afterward, weighted average values of the points in the search area for each fixed point could be calculated by:

$$\bar{x}_i(r) = \sum_{j=1}^{n} w_j(r) x_j \tag{6}$$

So it is obvious that the sample dispersions in the search area should be considered as:

$$S_i(r) = \sqrt{\sum_{j=1}^{n} w_j^2(r)} \tag{7}$$

At last, U value is acquired as follows [7-8];

$$U_i(r) = \frac{\bar{x}_i(r) - \mu}{\sigma} \tag{8}$$

In which μ and σ are the mean and standard deviation of all data respectively. According to the above equation U value depends on the r-value so by varying r different values of U will be appeared. Hence for each unknown point, different U values should be calculated and finally, the greatest absolute value of them should be selected as U value for the mentioned point [7-8].

At the final step, for separating anomalous samples from background ones, the zero among the U values of samples is used. It means that U values greater than 0 (positive U values) are considered anomalous samples and negative U values are devoted to background population.

4.2. Fractal method

4.2.1. C-N model

The concentration-Number fractal method (C-N) is one of the types of N-S fractal models [33]. The basis of this method is based on the inverse relationship between grade and the cumulative frequency of each grade along with larger grades. This method is introduced based on the following equation [33-37].

$$N(\geq \rho) \propto \rho^{-\beta} \tag{9}$$

Where ρ indicates the concentration of the studied elements and N (⩾ ρ) is equal to the cumulative frequency of samples that are equal to or higher than ρ . β is also equal to the fractal dimension of the element distribution.

4.2.2. C-A model

The Concentration-Area fractal method (C-A) is one of the most common separation methods for preparing and drawing contour maps, which was introduced by Cheng et al. in 1994. If the value of each contour is considered ρ value, a power equation could be presented as follows [38-39].

$$A(\rho \leq \upsilon) \propto \rho^{-\alpha_1};\ A(\rho \geq \upsilon) \propto \rho^{-\alpha_2} \tag{10}$$

In the above equation, ρ represents the concentration of the element (contour value ρ), A (ρ) denotes the area with concentration values greater than contour value ρ, υ is the threshold; values and α_1 and α_2 are the fractal dimensions. By plotting the concentration area on a logarithmic scale, the dimension of each population could be calculated through the line fitted to it [10].

5. Results and Discussion

5.1. Data preparation

In this section, results from the chemical analysis are prepared to be processed first and then anomalous samples are separated and finally, anomalous values are forecasted in the next sections. First of all, it is necessary to survey the distribution of raw data; so their statistical parameters have been studied (Table 2) [40-42]. According to the results (W Index), there is not any normal distribution.

Table 2. Statistical parameters of elements in Susanvar (original

Element	Average Grade (ppm)	Variance (ppm²)	Maximum (ppm)	W Index
Gold (ppb)	8.88	824.31	548	234.05
Arsenic (ppm)	29.43	2280.39	416	24.84

But after taking the logarithm of data and conducting research on the population distribution type, the resulting data were available to be processed. By applying some applications [43-45], the distribution of gold and arsenic data is lognormal and W indexes approve it (Table 3). So statistical parameters of data prepared for processing are presented in Table 2, at a confidence level of 95 percent.

Table 3. Statistical parameters of elements in Susanvar (logarithmic values).

Element	Distribution Type	Additive Constant	Average	Real Average	Variance	Real Variance	W Index
Gold	Bivariate	0	1.02	6.62	1.75	207.52	2.01
Arsenic	Bivariate	0	2.69	27.68	1.26	1944.52	1.13

Therefore, the Au and As data could be considered as a bivariate lognormal population.

In the next section, by considering the prepared data, we combine the U-statistics method with the C-N and C-A fractal methods for separating anomaly from background. For this purpose, after calculating the U values of Au and As grades, C-N and C-A fractal methods will be applied to U values of Au and As.

5.2. Assessment of threshold using C-N and C-A fractal

At the first, the C-N fractal method was applied to the gold and arsenic the results are shown in Fig. 4.

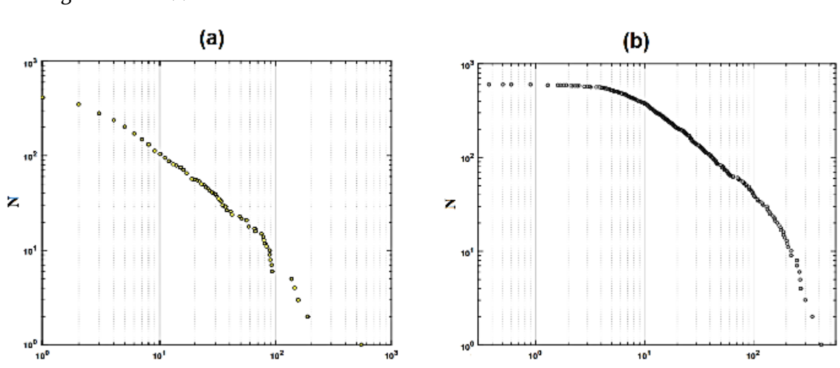

Fig. 4. C-N log-log plot for the concentration of Au (a) and As (b).

In the following, the different line segments related to geochemical populations were fitted to the above models in order to define geochemical populations (based on least-square regression). The straight fitted lines can be seen in Fig. 5.

Accordingly, there are four populations for the copper and molybdenum data that were reported in Table 4.

Then, at the second step, concentration values in places for which there is no data, are generated using applying regular networking by analogy and interpolation, which are possible by different methods and software.

For this purpose, Inverse Distance Weighting (IDW) interpolation and Surfer software were selected, and finally, by considering 50×50 m^2 pixels, a univariate geochemical surface map of the study area was prepared based on the IDW method and finally, concentrations related to each pixel were calculated.

In the last step, by applying the C-A fractal method to the data related to gold and arsenic elements (reported from Surfer software based on IDW), the C-A fractal model, their threshold, and finally the concentration ranges related to populations were resulted. The C-A fractal model and concentration populations are respectively observed in Fig. 6. and Table 5.

Table 4. Susanvar mineralization zones based on three thresholds of Au and As contents defined from the C-N fractal model.

Population	Au (ppb)	fractal dimension (Au)	As (ppm)	fractal dimension (As)
1	<4	0.342	<3.9	0.023
2	4-11	0.914	3.9-11.1	0.461
3	11-48	0.991	11.1-72.3	0.95
4	>48	1.48	>72.3	2.00

Table 5. Susanvar mineralization zones based on three thresholds of Au and As contents defined from the C-A fractal model.

Population	Au (ppb)	fractal dimension (Au)	As (ppm)	fractal dimension (As)
1	<3.05	0.481	<13.45	0.143
2	3.05-19.68	1.223	13.45-38.136	1.565
3	19.68-47.403	1.931	38.136-94.68	2.1
4	>47.403	1.752	>94.68	2.381

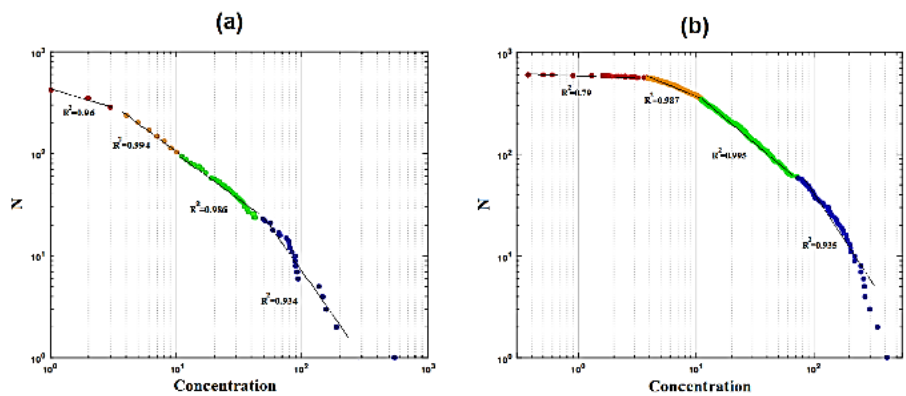

Fig. 5. The straight fitted lines to geochemical populations of C-N log-log plot for the concentration of Au (a) and As (b).

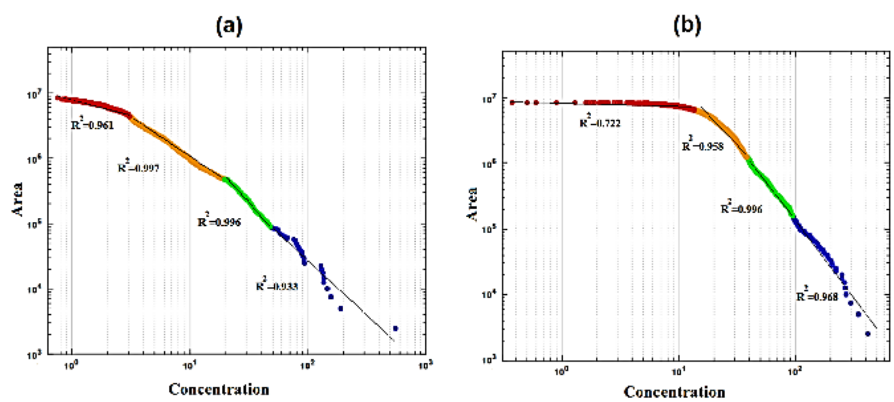

Fig. 6. C-A log-log plot for the concentration of Au (a) and As (b).

5.3. Separating Anomaly using U-statistics

In order to combine two separation methods (to achieve U-N and U-A models), first, the U-statistics method should be applied to the Au and As grades. So the initial radius is 30 meters and its rate of increase is 30 meters, too. The histograms of calculated U values of each point are illustrated in Fig. 7.

In the next section, C-N and C-A fractal methods will be used in order to apply the U value of gold and arsenic for introducing the proposed method.

5.4. Assessment of threshold using U-N and U-A Fractal

In fact, in the proposed method, with a simple shift, U values replace the amount of raw data (elements concentration), and finally, the effective and efficient U-N and U-A fractal methods are created.

In order to achieve this purpose, the algorithm of C-N and C-A fractal methods will be applied to the U values calculated in Section 5.3 by the U-statistical method.

It should be noted that due to the existence of negative values between the values of U and finally the inability to draw these values in logarithmic diagrams, before running the algorithm, a constant value (C) is added to the matrix of U values. This causes all the matrix values to be positive. (In this study, positive values of C=16.1 and C=12 are considered for gold and arsenic elements, respectively). These results are shown in Fig. 8.

As expected, a clear breakpoint can be seen in the above diagrams, which in addition to this breakpoint, there is also a discontinuity that represents the boundary between background and anomaly populations. It shows the performance of this method compared to the C-N fractal method.

Other advantages of this method include other breakpoints in these diagrams, which are much clearer than the C-N model (Fig. 4). Actually, in original fractal models of raw data, the breakdown boundaries and, consequently, the thresholds are not very clear and are determined visually and by taste. While in this method, by applying two structural separation methods simultaneously on the data, the breakpoints become clearer and their determination will be more accurate and much simpler.

In the following, the different line segments related to geochemical populations are fitted to the above models in order to define geochemical populations (based on least-square regression), and the straight fitted lines and their exact boundaries can be seen in Fig. 9 and Table 6, respectively.

In order to see the performance of the U-N fractal method, first, the latitude and longitude of samples along with the calculated U values + C, are provided to the Surfer software, and then, prospective areas are mapped based on reported information in Table 7 (Fig. 10).

Table 6. Susanvar mineralization zones based on four and three thresholds of Au and As contents defined from the U-N fractal model.

Population	Au (U+C)	fractal dimension (Au)	As (U+C)	fractal dimension (As)
1	<6.02	0.005	<7.01	0.021
2	6.02-12.02	0.61	7.01-12	3.133
3	12.02-12.68	22.62	12-20.11	3.392
4	19.5-20.01	10.54	>20.11	9.87
5	>20.01	5.51	-	-

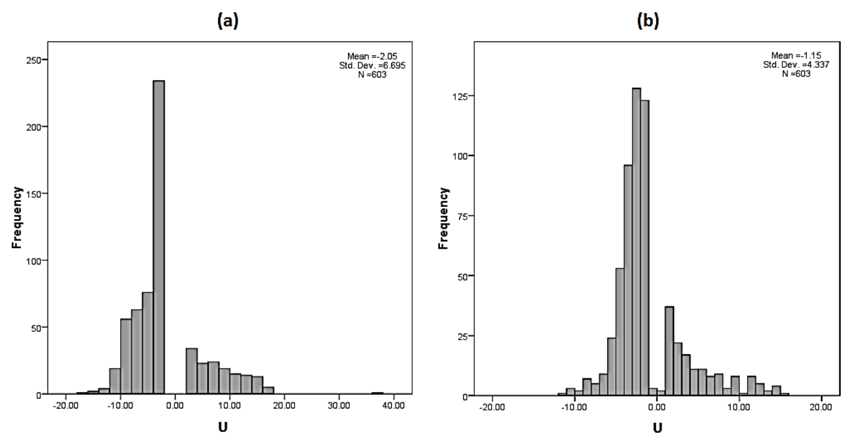

Fig. 7. Frequency diagram of U values for Au (a) and As (b).

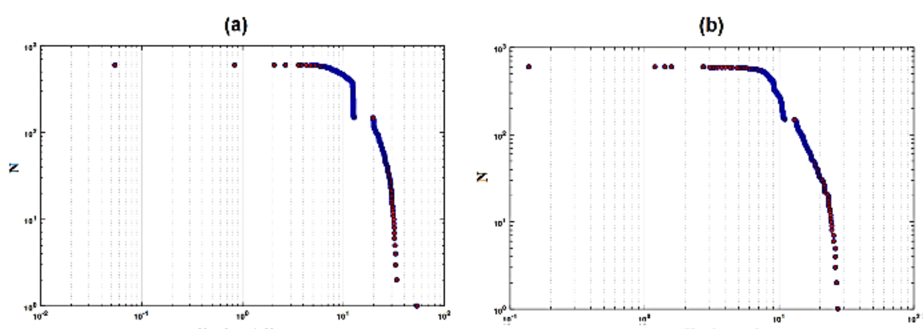

Fig. 8. U-N log-log plot for the concentration of Au (a) and As (b).

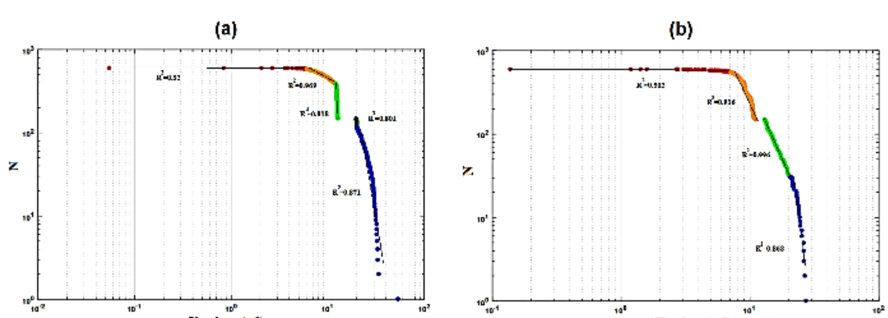

Fig. 9. The straight fitted lines to geochemical populations of U-N log-log plot for the concentration of Au (a) and As (b).

Table 7. Gridding method and grid line geometry information.

Gridding method	Gridline geometry			
	X direction		Y direction	
Kriging	Minimum	275770	Minimum	3907860
	Maximum	277970	Maximum	3911560
	Spacing	25	Spacing	25
	Notes No.	89	Notes No.	149

Then, at the second step, by using IDW interpolation, U values related to each pixel (50×50 m^2) were calculated, and then, by applying the C-A fractal method to U values related to Au and As elements, the U-A fractal model, their threshold and finally the concentration ranges related to populations resulted. However, it should be noted that in this section, due to the homogeneity of the U values by the Surfer software, especially at the zero point where the separation was caused by the separation nature of the U statistic, the discontinuity (which was created in the U-N model) may not be observed in its fractal model (U-A). However, it is still more appropriate to consider the threshold between background and anomalous populations at this point (i.e., equal to the constant amount (C) of additive added to the U values).

The U-A fractal model and concentration populations are respectively observed in Fig. 11 and Table 8.

Table 8. Susanvar mineralization zones based on four and three thresholds of Au and As contents defined from the U-A fractal model.

Population	Au (U+C)	fractal dimension (Au)	As (U+C)	fractal dimension (As)
1	<11.5	0.116	<8.5	0.046
2	11.5-16	7.548	8.5-12.5	5.095
3	16-19.5	1.818	12.5-18.5	4.683
4	19.5-26	3.838	>18.5	7.359
5	>26	13.08	-	-

As said above about the absence of a discontinuity in the U-A fractal model and as can be seen in the above shown fractal models, the U-A fractal models for gold and arsenic did not have a discontinuity equal to the additive constant (C). But according to what has been said in the past, in the fractal method of U-A, even if we do not see a discontinuity at point zero (among the raw values of U) or a constant amount of additive C (among the values of U + C), we continue to consider the threshold between background and anomalous populations according to the defined thresholds by the U-statistic method. This threshold is approximately equal to the breakpoint that is created near the constant value of the additive (even if the breakpoint is mild and imperceptible). Because this boundary as a threshold is much closer to reality than the considered boundary by fractal models (at the first breakpoint).

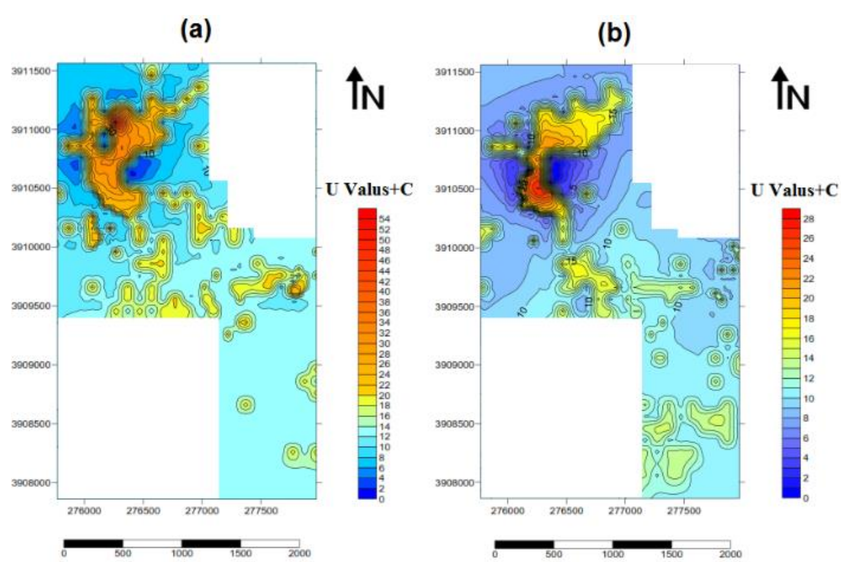

Fig. 10. Prospective map of the study area using U-N fractal method for Au (a) and As (b).

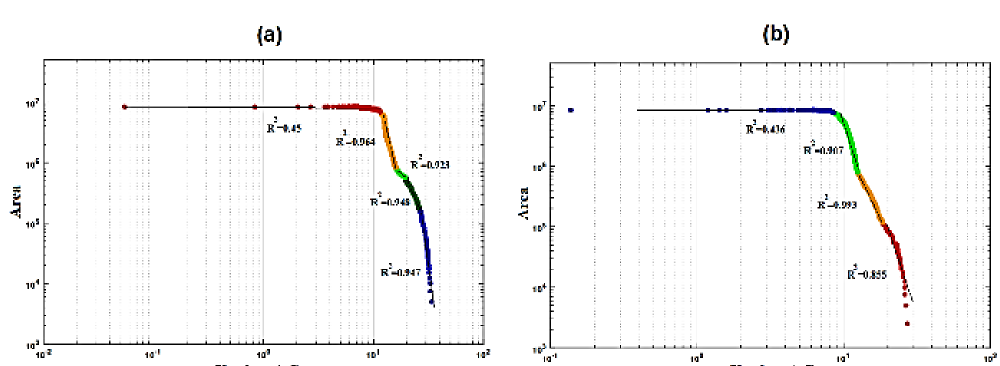

Fig. 11. U-A log-log plot for the concentration of Au (a) and As (b).

Finally, in order to see the performance of the U-A fractal method (as seen for the UN model), prospective areas are mapped based on reported information in Table 7 by using Surfer software (Fig. 12).

As can be seen, the boundary between background and anomaly in the above maps corresponds very closely to the Au ore indications in the Susanvar district (Geologic map in Fig. 1). Therefore, the proposed fractal methods could be introduced as effective tools in this field, so that determining the threshold between background and anomaly and also other fracture boundaries by using them, is much simpler and more accurate than concentration fractal models (C-N and C-A) in a similar situation. Therefore, it could be observed that the combination of U-statistics and fractal technique, is a far more powerful combination than the others and this combination could be introduced as U-N and U-A fractal methods in the field of separation anomaly from background. Also, it should be noted that the other advantage of using this combination is detecting the anomalous values according to considering the location of samples (based on the spatial structure of data).

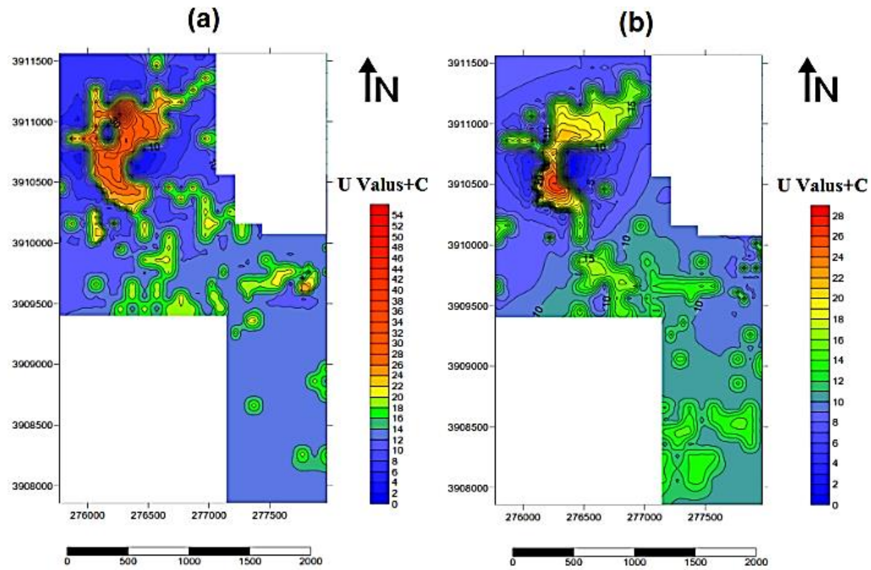

Fig. 12. Prospective map of the study area using U-A fractal method for Au (a) and As (b).

6. Conclusion

In this paper, U-statistics theory and also fractal technique as two structural methods were combined to determine anomalous areas of Au and As mineralization in the Susanvar district as U-N and U-A fractal model at the first time. By applying U-statistics and fractal method as two powerful methods, samples with anomalous Au and As concentrations were specified. Results coming up from combining these two methods are more accurate than using just one of them.

Because the U-statistic method is very powerful in determining threshold between background and anomaly populations and the fractal technique in identifying their subpopulations. For this reason, in results, it is seen that fracture boundaries are much clearer and more accurate than previous fractal models (C-N and C-A) in the same condition. Moreover, in U-N model, due to the nature of the U method algorithm, there is a discontinuity as exalt threshold between background and anomaly that however, in U-A model, this item was not seen due to the homogenization of U values.

On the other hand, it was seen in lithogeochemical maps that the delineated Au mineralization is closely associated with the defined Au ore indications in the study area (see Fig. 1 and Fig. 12). Moreover, they are associated with the delineated As mineralization in lithogeochemical maps.

Finally, it could also be said that because the U-statistic method devotes a new value to each sample, it could be combined with other methods such as the C-V fractal model (as U-V fractal model) to distinguish supergene enrichment and hypogene zones, from oxidation zones and barren host rocks, based on the distribution of elements grades such as Cu grades in three-dimensional condition.

Funding sources

This Research did not receive any specific grant from funding agencies in the public, commercial or not-for-profit sectors.

REFERENCES

[1] Hawkes H E and Webb J S 1962 Geochemistry in mineral exploration, Harper and Row, New York, 415.

[2] Miesch A T 1981 Estimation of the geochemical threshold and its statistical significance; Journal of Geochemical Exploration. 16, 49-76.

[3] Sinclair A J 1989 Application of Probability Graphs in Mineral Exploration, the association of exploration geochemists, the association of exploration geochemists, 4th Edition.

[4] Sinclair A J 1991 A fundamental approach to threshold estimation in exploration geochemistry: Probability plots revisited; Journal of Geochemical Exploration. 41(1), 1–22.

[5] Rousseeuw P J, Croux C 1993 Alternatives to the Median Absolute Deviation; Journal of the American Statistical Association. 88(424), 1273-1283.

[6] Cheng Q, Agterberg F P and Ballantyne S B 1994 The separation of geochemical anomalies from background by fractal methods; Journal of Geochemical Exploration. 51, 109–130.

[7] Cheng Q, Agterberg F P and Bonham-Carter G F 1996 A spatial analysis method for geochemical anomaly separation; Journal of Geochemical Exploration. 56, 183–I95.

[8] Cheng Q 1999 Spatial and scaling modelling for geochemical anomaly separation; Journal of Geochemical Exploration Journal of Geochemical Exploration. 65(3), 175–194.

[9] Goncalves M A, Mateus A and Oliveria V 2001 Geochemical anomaly separation by multifractal modeling; Journal of Geochemical Exploration. 72, 91–114.

[10] Li C, Ma T and Shi J 2003 Application of a fractal method relating

concentrations and distances for separation of geochemical anomalies from background; Journal of Geochemical Exploration. 77, 167–175.

[11] Zuo R 2011 Identifying geochemical anomalies associated with Cu and Pb–Zn skarn mineralization using principal component analysis and spectrum-area fractal modeling in the Gangdese Belt, Tibet (China); Journal of Geochemical Exploration. 111, 13–22.

[12] Afzal P, Khakzad A, Moarefvand P, Rashidnejad Omran N, Esfandiari B and Alghalandis Y F 2010 Geochemical anomaly separation by multifractal modeling in Kahang (Gor Gor) porphyry system, central Iran; Journal of Geochemical Exploration. 104, 34–46.

[13] Jodeiri Shokri B, Ramazi H, Doulati Ardejani F and Moradzadeh A 2014 A statistical model to relate pyrite oxidation and oxygen transport within a coal waste pile: case study, Alborz Sharghi, northeast of Iran; Environ. Earth. Sci. 71, 4693–4702.

[14] Ghannadpour S S, Hezarkhani A, Sharifzadeh M 2017 A method for extracting anomaly map of Au and As using combination of U-statistic and Euclidean distance methods in Susanvar district, Iran; Journal of Central South University. 24(11), 2693–2704.

[15] Afzal P, Fadakar Alghalandis Y, Khakzad A, Moarefvand P, Rashidnejad Omran N 2011 Delineation of mineralization zones in porphyry Cu deposits by fractal concentration–volume modelling; Journal of Geochemical Exploration. 108, 220–232.

[16] Shahbazi S, Ghaderi M, Afzal P 2021 Prognosis of gold mineralization phases by multifractal modeling in the Zehabad epithermal deposit, NW Iran; Iranian Journal of Earth Sciences. 13, 31-40.

[17] Kouhestani H, Ghaderi M, Afzal P, Zaw K 2020 Classification of pyrite types using fractal and stepwise factor analyses in the Chah Zard gold-silver epithermal deposit, central Iran; Geochemistry: Exploration, Environment, Analysis 20, 496-508.

[18] Shamseddin Meigooni M, Lotfi M, Afzal P, Nezafati N, Kargar Razi M 2021 Detection of rare earth element anomalies in Esfordi phosphate deposit of Central Iran, using geostatistical-fractal simulation; Geopersia. 11(1), 115-130.

[19] Nazarpour A 2018 Application of C-A fractal model and exploratory data analysis (EDA) to delineate geochemical anomalies in the: Takab 1:25,000 geochemical sheet, NW Iran; Iranian Journal of Earth Sciences. 10, 173-180.

[20] Yasrebi A B Hezarkhani A 2019 Resources classification using fractal modelling in Eastern Kahang Cu-Mo porphyry deposit, Central Iran. Iranian Journal of Earth Sciences 11, 56-67.

[21] Abdoli Sereshgi H, Ganji A, Ashja Ardalan A, Torshizian H, Taheri J 2019. Detection of metallic prospects using staged factor and fractal analysis in Zouzan region, NE Iran. Iranian Journal of Earth Sciences. 11(4), 256-266.

[22] Zadmehr F, Shahrokhi S V, 2019 Separation of geochemical anomalies by concentration-area and concentration-number methods in the Saqez 1: 100,000 sheet, Kurdistan. Iranian Journal of Earth Sciences. 11(3), 196-204.

[23] Kramar U 1995 Application of limited fuzzy clusters to anomaly recognition in complex geological environments; Journal of Geochemical Exploration. 55, 81–92.

[24] Cheng Q, Yaguang X and Eric G 2000 Integrated spatial and spectrum method for geochemical anomaly separation; Natural Resources Research. 9(1), 43–52.

[25] Ghannadpour S S, Hezarkhani A, Sabetmobarhan A 2017 The Parkam exploration district (Kerman, Iran): Geology, alterations, and delineation of Cu- and Mo-mineralized zones using U-spatial statistic with associated software development. Journal of Earth Sciences 28(2), 283–294.

[26] Ghannadpour S S, Hezarkhani A, Maghsoudi A, Farahbakhsh E 2015 Assessment of prospective areas for providing the geochemical anomaly maps of lead and zinc in Parkam district, Kerman, Iran; Geosciences Journal. 19(3), 431-440.

[27] Ghannadpour S S, Hezarkhani A 2016 Introducing 3D U-statistic method for separating anomaly from background in exploration geochemical data with associated software development; Journal of Earth System Science. 125(2), 387–401.

[28] Ghannadpour S S, Hezarkhani A 2020 Mineral potential mapping for Au and As using Gap statistic method in multivariate mode; Carbonates and Evaporites. 35(1), 1-2.

[29] Boyle R W, Jonasson I R 1973 The geochemistry of arsenic and its use as an indicator element in geochemical prospecting; Journal of Geochemical Exploration. 2, 251-296.

[30] Thornton I, Farago M 1997 The geochemistry of arsenic; In Arsenic Exposure and health effects: C.O. Abernathy; R.L. CALDERON, W.R. Chappell. (eds.), Chapman & Hall, London, UK, pp. 1-16.

[31] God R, Zemann J 2000 Native arsenic-realgar mineralization in marbles from Saualpe, Carinthia, Austria. Mineralogy and Petrology. 70, 37-53.

[32] Zhu Y, An F, Tan J 2011 Geochemistry of hydrothermal gold deposits: A review; Geoscience Frontiers. 2, 367-374.

[33] Hassanpour S, Afzal P 2013 Application of concentration–number (C-N) multifractal modeling for geochemical anomaly separation in Haftcheshmeh porphyry system, NW Iran; Arabian Journal of Geosciences, 6, 957–970.

[34] Nazarpour A, Omran N R, Paydar G R 2015 Application of multifractal models to identify geochemical anomalies in Zarshuran Au deposit, NW Iran; Arabian Journal of Geosciences, 8, 877-889.

[35] Momeni S, Shahrokhi S V, Afzal P, Sadeghi B, Farhadinejad T, Nikzad M R 2016 Delineation of the Cr mineralization based on the stream sediment data utilizing fractal modeling and factor analysis in the Khoy 1:100,000 sheet, NW Iran; Bulletin of the Mineral Research and Exploration. 152, 1–17.

[36] Ahmadfaraj M, Mirmohammadi M, Afzal P 2016 Application of fractal modeling and PCA method for hydrothermal alteration mapping in the Saveh area (Central Iran) based on ASTER multispectral data; International Journal of Mining and Geo-Engineering. 50(1), 37-48.

[37] Mandelbrot B B1983 The fractal geometry of nature. W.H. Freeman and company, San Francisco, New York, 468 p.

[38] Daya A A 2015 Comparative study of C-A, C-P, and N-S fractal methods for separating geochemical anomalies from background: a case study of Kamoshgaran region, northwest of Iran; Journal of Geochemical Exploration. 150, 52-63.

[39] Afzal P, Ahmadi K, Rahbar K 2017 Application of fractal-wavelet analysis for separation of geochemical anomalies; African Journal of Earth Science. 128, 27-36.

[40] Ghannadpour S S, Hezarkhani A 2015 Lead and zinc geochemical behavior based on geological characteristics in Parkam Porphyry Copper System, Kerman, Iran; Journal of Centeral of South University. 22, 4274–4290.

[41] Ghannadpour S S, Hezarkhani A, Sabetmobarhan A 2015 Some statistical analyses of Cu and Mo variates and geological interpretations for Parkam porphyry copper system, Kerman, Iran; Arabian Journal of Geosciences. 8(1), 345–355.

[42] Ghannadpour S S, Hezarkhani A 2015 Investigation of Cu, Mo, Pb, and Zn geochemical behavior and geological interpretations for Parkam porphyry copper system, Kerman, Iran; Arabian Journal of Geosciences. 8(9), 7273–7284.

[43] Ghannadpour S S, Hezarkhani A 2012 Determine the initial statistical specifications of Copper and molybdenum elements in Porphyry Copper ore deposit in Kerman. International Mining Congress and Exploration, Iran. (in Persian)

[44] Hezarkhani A, Ghannadpour S S 2015 Exploration Information Analysis, first ed. Amirkabir University of Technology (Tehran Polytechnic) press, Tehran. (In Persian)

[45] Hezarkhani A, Ghannadpour S S 2015 Geochemical behavior investigation based on K-Means Clustering (Basics, Concepts and Case Study), first ed. LAP LAMBERT Academic Publishing, Germany.