-RESEARCH PAPER-

International Journal of Mining and Geo-Engineering IJMGE

IJMGE 56-3 (2022) 249-255 DOI: 10.22059/IJMGE.2022.296377.594841

Failure modeling of alluvial foundations due to boiling: numerical modeling versus experiments

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Article History:
Received: 01 February 2020.
Revised: 22 February 2022.
Accepted: 04 March 2022.

ABSTRACT

The sheet piles are used below hydraulic structures to reduce the seepage flow rate and hydraulic gradient at the outlet of such structures rested on permeable foundations. Up to now, for analysis of seepage under hydraulic structures much research work has been carried out in the form of numerical models. However, less field and laboratory works have been performed to study and compare boiling phenomena for evaluation of the numerical models. By simulating an experimental model of a sheet pile inserted in a sand foundation by computer code FLAC based on the finite difference method, the soil behavior mechanism flow has been investigated under the seepage effect. The results indicate that the FLAC computer code underestimated uplift pressures compared to the experimental data. In order to study the boiling, the soil treatment was analyzed at the most critical condition as well. The numerical model presented in FLAC is properly able to simulate the soil and foundation behavior. Comparing the results obtained from the numerical model with experimental data also confirms its suitability, because this model predicts the boiling observations with reasonable accuracy and it was possible to predict the heaving mechanism based on the stress analysis before performing the plan.

Keywords: *Alluvial foundation, Boiling, Sheet pile, Laboratory model, hydraulic structures, Failure, Computer code FLAC.*

1. Introduction

Investigation of the behavior of saturated sandy soils has always been of interest to researchers working in different areas of civil engineering. The first studies about optimizing the seepage flow from soil dams were done by Tarzaghi (1946). He presented his results by analyzing the seepage from body and soil dam foundations according to the permeability coefficient of the materials. The result was that the lower the score permeability than foundation and shell, the more seepage declined. Considering the depths of the sheet pile located in the foundation dam, he analyzed the seepage from shell and foundation as well. His analysis was done based on the flow network method illustrated for each condition. During the past decades, many researchers have reviewed seepage issues analytically and numerically. In most researches, the correctness of numerical model results with experimental data was evaluated by Sedghi-Asl et al (2010, 2012).

Boiling happens when a small prism of soil at the drilled surface does not have enough resistance to neutralize the uplift pressure. Terzaghi (1943) defined the critical hydraulic gradient (i_c) to control the boiling. McNami (1949), according to Equation 1, defined the safety factor coefficient against boiling as the ratio of a critical hydraulic gradient to exit hydraulic gradient.

$$F = \frac{i_c}{i_e} \tag{1}$$

Lane (1912) presumed that the flow creep length from the foundation must be equal to the formula total length and one-third of the horizontal length of the seepage path. According to this, he suggested some coefficients for different foundations to properly control the boiling and internal erosion. He defined the weighted creeping factor as ($c = L/\Delta h$), where L is the creep length and Δh is the difference between upstream and downstream water levels and c is Lane's weighted index. If the calculated weighted index for a dike or weir is higher than the value suggested by Lane, then the dike would be safe against boiling and internal erosion.

Neumann & Witherspoon (1970) simulated the seepage flow through an earthen dam by finite element method which is more complicated mathematically. Javan & Farjood (1993) suggested a computer model that was able to estimate the uplift pressure in hydraulic structures with several approaches. He compared his results with piezometric data of Doroodzan Dam and found that there is an error of 4 percent between model results and field data.

Earthquake-induced soil liquefaction has been distinguished as a risk for over forty years (e.g. Seed and Lee, 1966; Ishihara, 1985) and static liquefaction for longer (e.g. Koppejan et al., 1948). Loose saturated sands attempting to settle during shaking are unable to dissipate fluid from the contracting void spaces quickly enough, and individual soil grains lose contact with each other and go into suspension (as observed by, e.g., Sasaki et al., 2001). Deep liquefied sand is acknowledged to be able to burst through overlying impermeable layers. However, the exact conditions required to produce sand boils are unclear, as is their relative importance to engineering structures (Brennan 2008).

Sedghi-Asl et al (2005) reviewed the effect of optimal position of sheet pile on seepage discharge and flow velocity decrease under hydraulic structures by using a numerical model and concluded that the best location for controlling the seepage and boiling phenomenon is at the heel and toe of the dam.

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Urban development makes it necessary to build constructions with high depth which may be used in constructing parking and urban transportation. Deep digging design is often followed by water flow around the sheet pile (Benmebarek et al 2005). Benmebarek et al (2005) were the first researchers to study numerically the failure caused by seepage at the lower side of sheet pile installed in sand foundation. They used the FLAC model which is based on explicit finite difference to analyze the boiling caused by seepage. The base of their works is determining the condition for failure made by phenomena like boiling. They indicated that the rectangle prism of boiling in their simulation is smaller than Terzaghi's. However, the results obtained in their research have not been compared with experimental data.

Sedghi-Asl et al (2012) studied the effective factors on seepage from coastal foundation dikes. They built an experimental model that was 9 m long, width and height of 1 meter, with steel skeleton and glass and plexiglass walls and coastal clean sand was selected as foundation materials. Their results showed that the ratio of blanket optimal length and sheet pile depth to upstream water depth and foundation thickness in order to minimize the seepage and control the internal erosion, are 8 and 0.8, respectively.

The critical hydraulic head differences for possible seepage failure were analyzed using the prismatic failure concept by Tanaka & Verruijt (1999). After that, Tanaka & Yokoyama (2006) investigated the effect of supplementary injection of jet grouting under sheet piles for single and double-sheet-pile-wall and circular cutoff walls. They concluded that the critical hydraulic head differences between up-and downstream sides go up with an increase in injected length of jet grouting. Recently Tanaka et al (2012) investigated seepage failure of bottom soil within a double-sheet-pile wall for a case study. They studied seepage and boiling by means of the finite element method (FEM) and stability analysis and then calculated safety factors against seepage failure. The case study for the failure was a ditch border.

Many researchers have already done studies on the seepage, whether from inside the body or from the dam foundation. The previous studies have been mainly based on the mathematical and numerical analysis of seepage issues. Since the comparison between results of numerical models and field or experimental models has not been performed yet, therefore, the numerical results would be uncertain and it is not reasonable to solely rely on the numerical data. In this research, FLAC computer code based on a finite difference approach has simulated the boiling phenomenon for an experimental model of sheet pile.

In the present study, by using FLAC software, the numerical modeling has been performed where seepage path and boiling were studied and the results were compared with the experimental data of Sedghi-Asl et al (2012).

2. Experimental set-up

Laboratory experiments were conducted in a flume 2.2m long, 0.40m wide, and 0.80m deep with sidewalls made of a combination of Plexiglas and steel sheets. As mentioned before, non-cohesive fine clean sand exemplifying a material extremely sensitive to piping and boiling was used to establish the most potentially critical situation subject to erosion. The bottom of the flume was filled with this material to a depth of 40cm. Figure 1 shows the general view of the laboratory model and the schematic view is presented by Figure (2). The flume consists of the following components: a water supply system, joints and connectors, sheet pile element, a tank for regulating upstream and downstream water surface levels, piezometer tubes, data registering panel, and discharge measuring device. In total, 24 piezometer tubes in 3 rows and at depths of 10, 20, and 30cm from the bed of the flume were installed to measure piezometric pressure at downstream and upstream sides of the sheet pile. Also, the volumetric method was used to measure the flow discharge. For easy readings of the pressures, the piezometers were installed on a scaled piezometric board as shown in Figure 1. In order to prevent the material from washing away from the outlet, a #150 wire mesh was provided at the end of the flume. The sheet pile depths were adjusted at various depths of 10, 15, 16, 17.5, 18, 20, and 30 cm from the bottom of the flume. Before conducting any experiments, it was necessary to fully saturate the foundation material. A vacuum pump was used to drain any air bubbles inside the piezometer tubes. The measurements were initiated after steady-state seepage was established and in total, 24 hours was spent for each experiment (reading piezometers and measuring the seepage discharge).

The upstream water level was set at the 7 levels of 5, 10, 15, 20, 25, 30, and 35 cm from the surface of the foundation while the downstream water level was set at zero level. Figure 3 shows the grading curve of fine clean sand used in this research.

Figure 1. General view of the laboratory model.

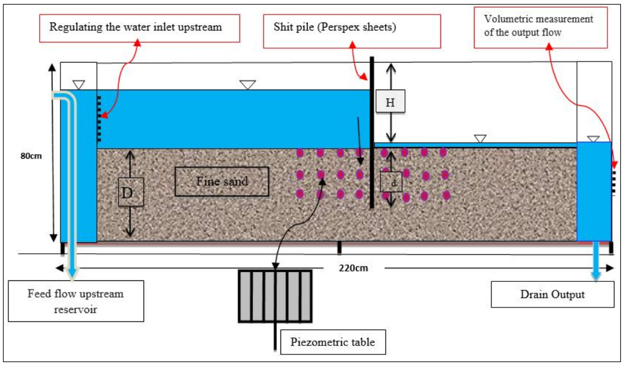

Figure 2. Schematic design of the laboratory model.

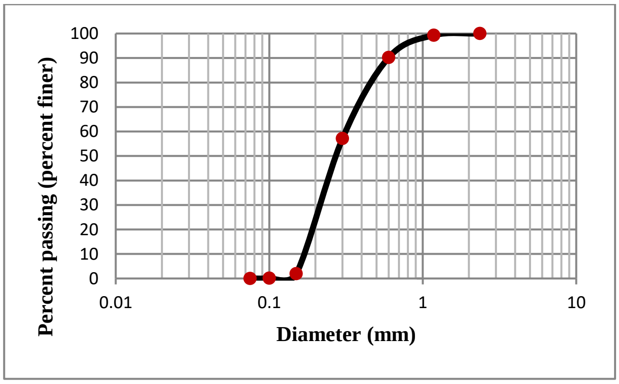

Figure 3. Grading curves of fine clean sand used in this research.

3. Modeling

In order to analyze experimental data obtained during the tests, some normalized parameters were defined using Buckingham theorem as follows:

$$F(h, H, d, D, h_m, p) = 0$$
$$F(h/H, h/h_m, d/D) = 0 \quad (2)$$

In which h is the upstream water level, sheet pile d water level, upstream the maximum h_m depth which ranges from 10 to 30cm, D is the thickness of alluvial foundation which is 40cm.

Modeling in FLAC has been done according to the experimental model, with Figure 1 showing the schematic picture of the experimental model.

The purpose is to obtain the numerical approach based on the finite difference method in order to predict the heaving mechanism under hydraulic structures that are possible to boil. The analysis was performed by the FLAC^{2D} program which is an explicit finite difference method based on Lagrangian analysis. In this program, an explicit time-dependent approach was used to solve the algebraic equations. Generally speaking, the finite differences approach has been used to regular zones and boundaries. In FLAC, based on Wilkins' studies, it is possible to calculate differential equations based on the finite difference method from areas with irregular zones and boundaries (Wilkins 1946). FLAC usage mechanism includes 1- creating a finite-difference grid; 2- applying the treatment model and material properties, and 3- applying boundary conditions.

Fluid flow passing through porous media is modeled by the FLAC computer code. According to Tarzaghi and Taylor's equation, three forces are applied on the soil volume unit in stiffness matrix that includes: soil weight, buoyancy force, and seepage force. This classification has been mentioned by Bear (1972). Soil behavior is modeled by elastic-perfectly plastic nonassociative Mohr-Coulomb model encoded in FLAC code. FLAC also operates based on this classification. The forces have been considered automatically in FLAC and the governing equation in the numerical package is as the following (Bear 1972):

$$\frac{\partial \sigma_{ij}}{\partial X_j} + \rho_s . g_j = 0 \tag{3}$$

Where ρ_s is non-drained density, g_j is a gravitational vector, and σ_{ij} is effective stress.

After replacing and calculating, Eq. (4) is obtained.

$$\frac{\partial \sigma'_{ij}}{\partial X_j} + \rho_d . g_j - (1-n)\frac{\partial p}{\partial X_j} - n\gamma_w \frac{\partial \phi}{\partial X_i} = 0 \tag{4}$$

In which ϕ is piezometric pressure and n is porosity, $\rho_d.g_j$ is soil weight, $(1-n)\frac{\partial P}{\partial X_j}$ is floating or buoyancy force, and $n\gamma_w \frac{\partial \phi}{\partial X_i}$ shows the seepage force (FLAC-2D, 2000). In the first step of sample modeling, girding was performed to create the mesh according to Figure 4.

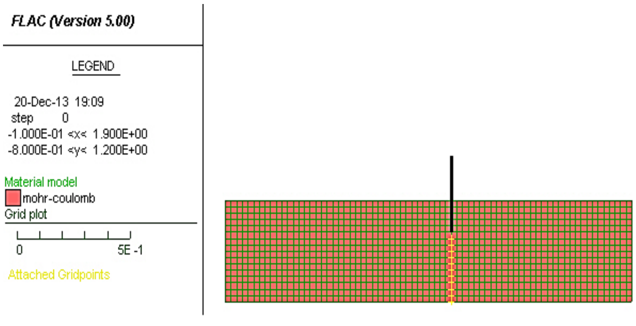

Figure 4. Numerical model grid in numerical setting.

In the next step, considering that sheet pile and foundation materials are different elements, modeling the location between wall and soil is an important part of the analysis. In other words, the friction force between sand and sheet pile is important in the calculating process. In this research, the material properties and stiffness of each section were separately specified. In Figure 5, by using Coulomb's rule, the joint between wall and soil has been simplified. The connection at two sides of the element has been shown by soil and the two different elements are reasonably shown. The spring is tangent on the element border that shows the Coulomb cutting resistance measure.

To apply the characteristic related to the joint surface of two elements, the vertical stiffness K_n and shear stiffness K_s of side zones should be calculated. According to Eq. (5), Es parameters are primarily calculated and then, based on Eqs. (6) and (7), the vertical stiffness and shear stiffness are determined in connected joint surface position. The equated stiffness introduces the stiffness of the most rigid zone in the studied border (FLAC-2D, 2000). Furthermore, for modeling the place between sheet pile and sand, the friction angle between Plexiglas screen and sand should be determined. By using direct shear tests, the friction angle was determined from 22 to 26 degrees. The most critical condition in this study was chosen 22 degrees.

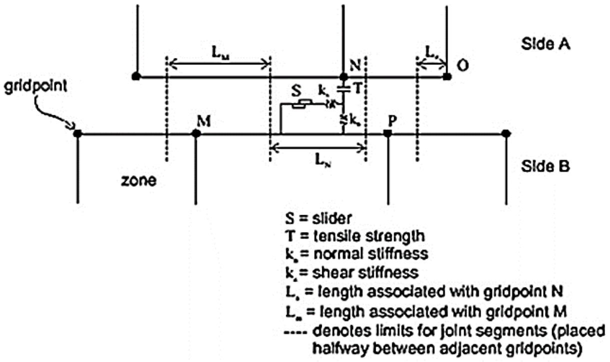

Figure 5.. Modeling the two different element connections in FLAC (FLAC-2D, 2000)

$$Es = \frac{\left(K + \frac{4}{3}G\right)}{\Delta Z_{\min}} \tag{5}$$

$$K_n = 10Es \tag{6}$$

$$K_s = 10Es \tag{7}$$

$\Delta Z_{\min}$ is the minimum size of element dimension.

Mohr Coulomb's model has been used as an elastic-plastic model in this computer code. The special density of the dry sand used in the foundation is γ=1350 (kg/m^3) for fine sand, elasticity module E=10 – 25 Mpa, and Poisson ratio ν= 0.25. Considering that the purpose is studying the most critical condition of the foundation and that very fine sand is considered as E= 10 Mpa (Barchard 2002, Das 1941, Subramanian 2008).

In the next step, the fluid flow and mechanical changes of the porous medium are simultaneously considered. According to experimental set-up, boundary conditions have been applied in a way that upstream head was at d/D=0.375, 0.4, 0.425, 0.438 with 0.43, 0.57, 0.71, 0.86, 1.

4. Results and discussion

Modeling ad then the analysis was performed in FLAC. At the first step, the fine sand of the foundation was saturated during saturation according to Figure 6, it can be observed that the flow direction moves exactly around the sheet pile and at the downstream side of the sheet pile moves upward. For example, Figure 6 schematically shows water direction through sheet pile foundation for d=10cm of sheet pile. Moving upward at the downstream side causes effective stress of $P=iz\gamma_w$, where i is hydraulic gradient, z is the distance from the surface, and γ_w is water density. Now, if decreasing of the effective stress is enough to come down to zero, in such conditions the soil is unstable and the boiling is possible to occur. In this condition, the hydraulic gradient is called critical hydraulic gradient i_{cr} and is equal to 1 in most soils.

A better comparison can be observed, at the same location of laboratory and numerical mesh, the piezometric pressures were recorded in FLAC. According to Figs.7 to 11, the results were compared with experimental data for sheet piles with the depth of 15 cm and upstream water levels varying 15, 20, 25, 30, and 35 cm.

These figures show a good agreement between experimental and numerical results while in the experimental work done by Sedghi-Asl et al., (2012) the empirical method significantly underestimated pore pressures. In the above-mentioned figures, the depth of the sheet pile is constant and is the threshold depth to onset boiling. Therefore, we selected this depth as the most critical depth to compare the piezometric pressures obtained from numerical modeling underestimated piezometric pressures while at the downstream part it was overestimated. In fact, the downstream side of the sheet pile is more critical than the upstream one, since the boiling and heaving mechanism started from the outlet prism. Overestimation of the piezometric pressures for technical design is suitable while from the economic design it is not optimistic.

Based on the present study, increasing the friction and dilation angles may reduce the boiling possibility. It is worth mentioning that the dilation angle range for sand is 0 to 15 degrees (FLAC^{2D}, 2000). By choosing ϕ=8°, φ=40°, the numerical model is exactly simulated for sheet pile depth of 15 cm, upstream reservoir water of 30, 35 cm. Theoretically, the model is balanced when the applied force estimations in each model are zero. However, generally, the balanced force in the model would never be zero in numerical analysis and when the unbalanced force with a determined ratio is against small model loading, the balance condition is acceptable. When the boiling occurs, the system will be practically unbalanced and the maximum unbalanced force is not close to zero (Figure 12).

Figure 13 indicates that effective stress at downstream is near zero. Theoretically, according to soil mechanic principles, sand boiling occurs when the pore water pressure or piezometric pressure increases until it is equal to the particle weights; under these circumstances, the effective stress is zero, and sand particles, because of loss of effective stress would float and particle replacements occur.

When boiling happens, according to Figure 14, the soil shear stress downstream decreases significantly, and soil failure happens.

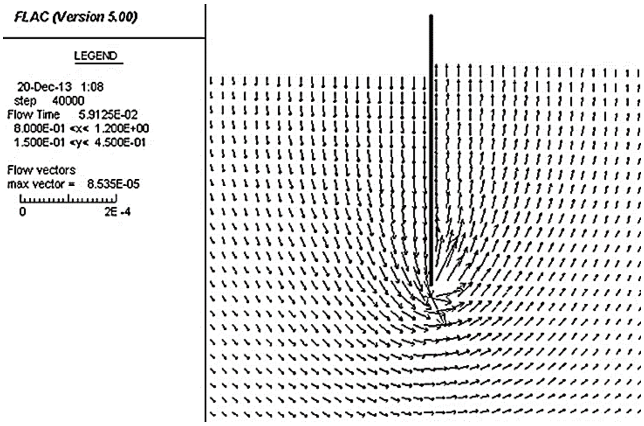

Figure 6. Simulated flow around sheet pile installed at soil foundation.

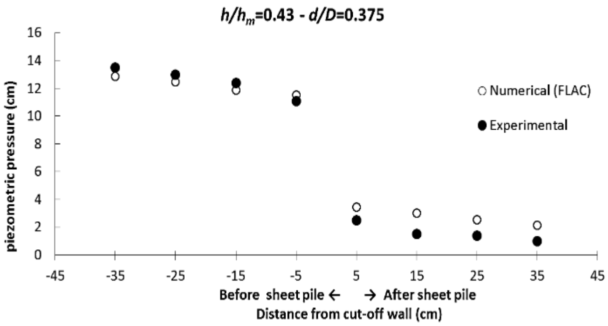

Figure 7. Comparing piezometric pressure between numerical and experimental results for d= 15 cm, h= 15 cm.

Figure 15 shows grid transformations at the downstream part of the sheet pile. This transformation of the grids may be interpreted as the occurrence of boiling. When boiling occurs, the soil shear stress at downstream is reduced considerably and failure happens. Figure 15 shows the grid transformations at downstream, causing upheaving as depicted in Figure 16. Figure 16 indicates heaving caused by this phenomenon and Figure 17 indicates the displacement vector when boiling happens.

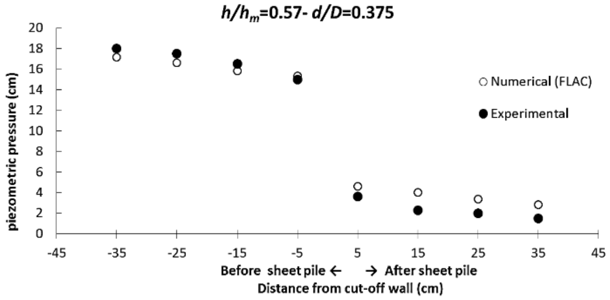

Figure 8. Comparing piezometric pressure between numerical and experimental results for d= 15 cm, h= 20 cm.

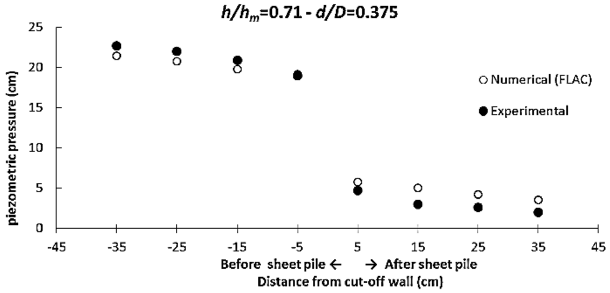

Figure 9. Comparing piezometric pressure between numerical and experimental results for d= 15 cm, h= 25 cm

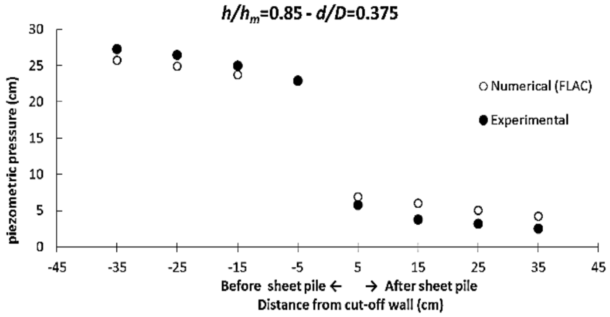

Figure 10. Comparing piezometric pressures between numerical and experimental results for d= 15 cm, h= 30 cm.

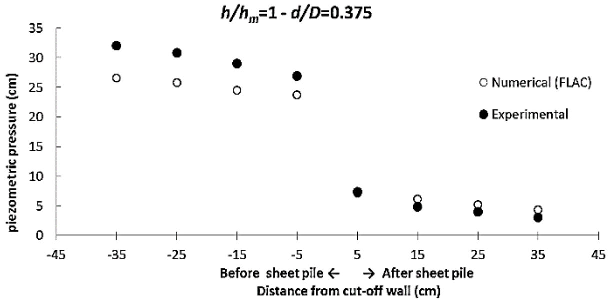

Figure 11. Comparing piezometric pressures between numerical and experimental results for d= 15 cm, h= 35 cm.

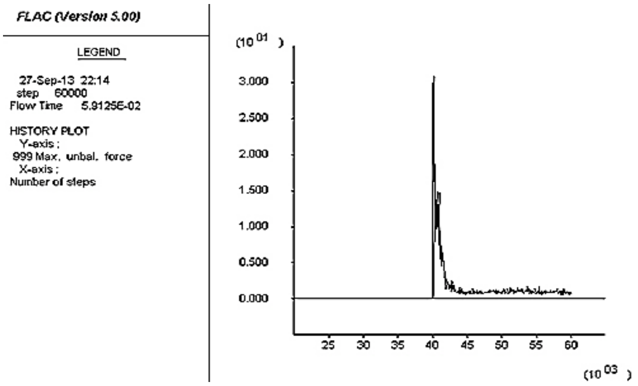

Figure 12. Maximum unbalanced force diagram in boiling (for Ø= 41° and dilation angle=8°).

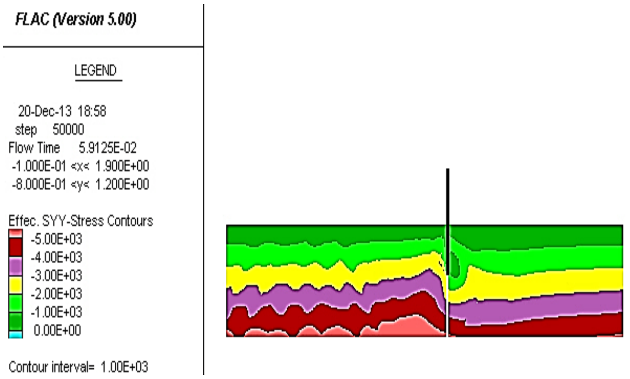

Figure 13. Effective stress in downstream of sheet pile when boiling happens (for Ø= 41° and dilation angle=8°).

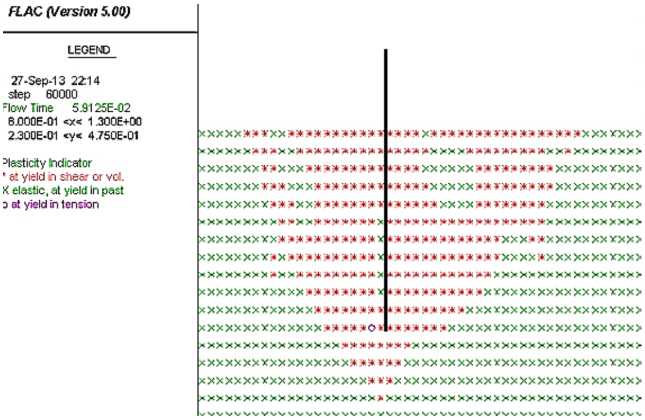

Figure 14. Yield in shear around sheet pile when boiling happens (for Ø= 41° and dilation angle=8°).

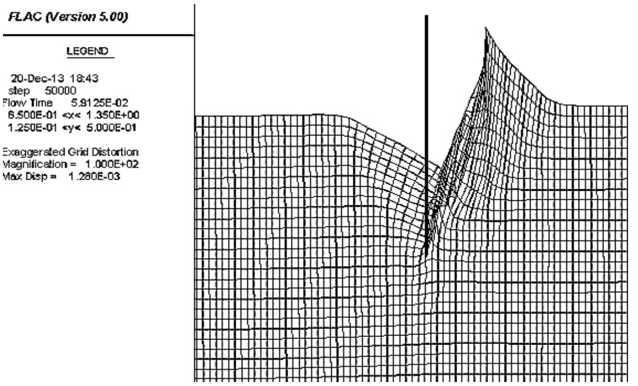

Figure 15. Grid transformations at downstream when boiling happens at failure place (for Ø= 41° and dilation angle=8°).

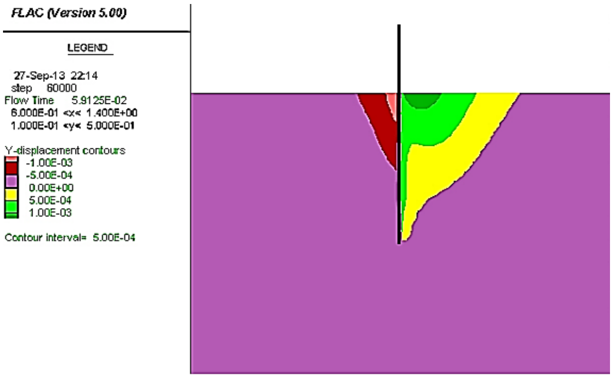

Figure 16. Heaving at downstream when boiling happens (for Ø= 41° and dilation angle=8°).

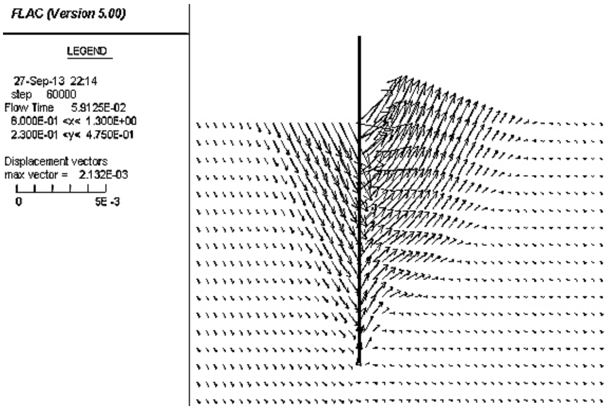

Figure 17. Displacement vector when boiling happens (for Ø= 41° and dilation angle=8°).

Comparing the results obtained from FLAC with experimental observations indicates that there is a good agreement between them. In other words, FLAC has modeled boiling and has predicted its behavior properly. In FLAC in such conditions, the numerical model acts properly and according to Figure 18, the maximum unbalanced force is close to zero. Figure 19 shows grid transformations at downstream when boiling does not happen. There is no boiling occurrence, a point which is shown by experimental observation.

Figure 20 indicates that low heaving caused by this phenomenon does not happen and figure 21 indicates that the displacement vector when boiling does not happen and there is no fluctuation and irregular displacement.

The results presented here are related to the most critical fine clean sand soil, which is the most compatible case with the experimental results. Table 1 indicates a comparison of numerical model results with experimental data for predicting boiling phenomena. It is observed that computer code FLAC has suitably simulated boiling phenomenon. Therefore, this table is presented as a verification tool for further application of this computer code in scientific and engineering practice works.

Results of numerical and experiment modeling show that according to the engineering and economic point of view the ratio $\frac{d}{D} \cong 0.44$ can be selected as optimum depth in hydraulic structures.

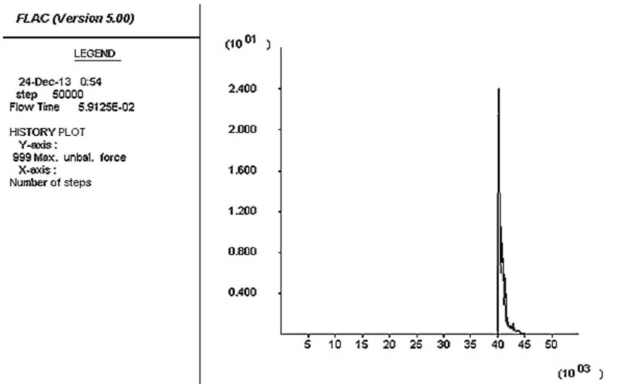

Figure 18. Maximum unbalanced force diagram when boiling doesn't happen (for Ø= 41° and dilation angle=8°).

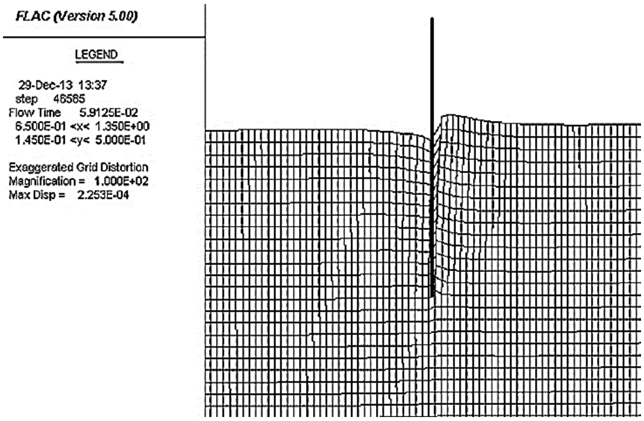

Figure 19. Grid transformations at downstream when boiling does not happen (for Ø= 41° and dilation angle=8°).

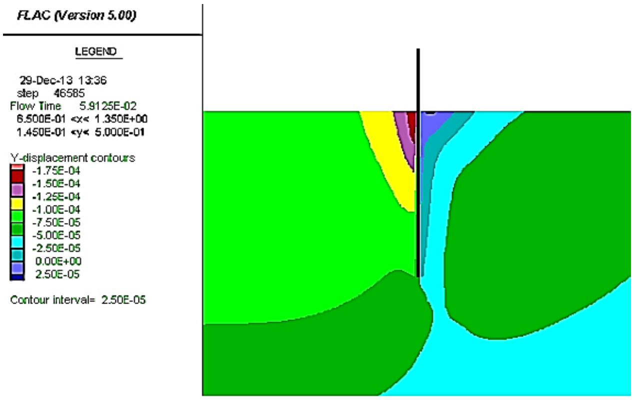

Figure 20. Heaving at downstream when boiling does not happen (for Ø= 41° and dilation angle=8°).

5. Concluding remarks

Boiling is a highly destructive phenomenon for hydraulic structures rested on alluvial foundations. This current research indicated that proper modeling of such phenomenon in FLAC makes it possible to predict heaving mechanism based on the stress analysis. Based on the current numerical and experimental observations, the following remarks can be concluded:

1. The analysis indicates that the sheet pile dept ratio $\frac{d}{D} \cong 0.44$ is the optimum value to effectively reduce the boiling phenomenon under hydraulic structures.

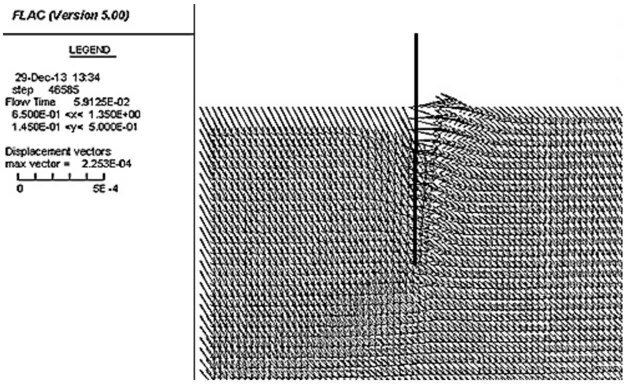

Figure 21. Displacement vector when boiling does not happen (for Ø= 41° and dilation angle=8°).

Table 1. Comparison of numerical results with experimental observations for predicting boiling phenomena.

d/D		0.375	0.4	0.425	0.438
	h/h_m	Occurrence of Boiling or Not			
Experiment	0.43	†	†	†	†
Numerical (FLAC)		†	†	†	†
Experiment	0.57	†	†	†	†
Numerical (FLAC)		†	†	†	†
Experiment	0.71	¤	†	†	†
Numerical (FLAC)		†	†	†	†
Experiment	0.86	¤	†	†	†
Numerical (FLAC)		¤	†	†	†
Experiment	1	¤	†	†	†
Numerical (FLAC)		¤	†	†	†

Boiling (¤), No Boiling (†)

2. The selection of a proper depth for sheet pile depends on the technical and economic factors and it may not always be feasible to employ the deepest one.

3. Comparison between the results of laboratory experiments and numerical methods performed by FLAC indicate that numerical modeling of the boiling phenomenon was carried out properly and this modeling package works well.

4. The results presented in this study for coastal structures, diversion dams, and embankments rested on alluvial foundations are applicable to safely remove the boiling phenomenon.

5. The results show a good agreement between experimental and numerical results of pore pressures while in the experimental work done by Sedghi-Asl et al., (2012) the empirical method significantly underestimated pore pressures.

6. Hypothetically, according to soil mechanic principles, quicksand boiling happens when the pore pressure enlarges until it is equal to the aggregates weight; in such a case, the effective stress is zero, sand particles would float and particle replacements happens, because of loss in effective stress.

Acknowledgments

The authors would like to thank the support of Yasouj University.

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