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An analytical approach for estimating the bearing capacity of slopes under loading on the upper surface

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ABSTRACT

Bearing capacity plays a significant role in evaluating the safety of the foundations rest on the slope. Many solutions have been proposed to assess the ultimate bearing capacity of the foundation adjacent to the slope, however, the available analytical and empirical methods are associated with some shortcomings in view of slope material properties and geometry. Also, numerical methods suffer from rigorous computational effort, and the accuracy of the outcome depends on the mesh and boundary effect. Therefore, a new analysis is employed in this research work that is able to consider all the effective parameters on the evaluation of ultimate bearing capacity. The results are compared with the existing numerical one in the literature and show good agreements. Also, in order to facilitate the use of the proposed method a Mathematica package code has been proposed to help the researcher to evaluate the bearing capacity of a shallow foundation that rests on the slope.

Keywords: *Bearing capacity, Footing, Shallow foundation, Slope stability, Elasticity*

1. Introduction

Bearing capacity plays a significant role in evaluating the safety of the foundations rest on the slope. Terzaghi's investigation on the foundation resulted in the first approach to evaluate the ultimate bearing capacity [1]. After this study, an equation has been proposed by Meyerhof (1957) to evaluate the bearing capacity by introducing the depth, shape, and inclination factor [2, 3]. The Terzaghi (1943) and Meyerhof (1957) theories focused on the bearing capacity of the footing placed on the horizontal ground surface, and there is no exact solution exists to evaluate the bearing capacity of the foundations rested adjacent to the sloping grounds [4]. In many cases, especially in the mountainous environment, the foundation should be built on or near to the slope crest, and therefore, to increase the safety of such structures it is necessary to study the ultimate bearing capacity. Accordingly, empirical factors for sloping grounds have been proposed by Hansen [5] and Vesic [6] for the estimation of ultimate bearing capacity. However, the proposed methods are limited to considering the footing exactly on the crest and the spacing between the crest and foundation was ignored. Also, the slope height is not involved in their equations. Another limitation is that the equation proposed by Hansen (1961) is independent of the slope's material properties. The shortcoming of the spacing between the foundation and the slope crest has been covered by Bowles [7] empirical equation. However, the limitations of considering the slope height and material properties have remained. The bearing capacity of foundation placed on the purely frictional or purely cohesive soil slopes has been investigated by Meyerhof [2] and some design charts were introduced which are the basis of many design manuals [4]. Also, by employing the upper bound method, Kusakabe et al. [8] presented some design charts to study the bearing capacity of a footing rested on an infinite soil slope.

Numerical methods have been gradually used to evaluate the ultimate bearing capacity. Accordingly, Georgiadis [4], using the finite element method, investigated the undrained bearing capacity of the shallow foundations adjacent to the slopes by considering the footing placement, slope height, and the size of the footing. By means of the upper-bound limit state plasticity failure discretization scheme, and taking the slope angle, footing size, and slope material properties into account, Leshchinsky [9] investigated the bearing capacity of a footing placed near the slope. This method was also used by Zhou et al. [10] to study the ultimate bearing capacity of a vertically loaded footing placed on a slope.

From the mentioned literature it is clear that the existing analytical and empirical approaches have some shortcomings in considering the slope material properties, as well as footing and slope geometry. Although numerical methods do have not the aforementioned limitations, they need more computational effort and they should be verified with experimental or analytical methods [11]. To tackle the above-mentioned limitations, the authors presented a semi-analytical solution and studied the ultimate bearing capacity of a shallow foundation rest on a cohesionless or frictionless soil slope [12]. In the current research work, based on the author's previous research [12-14], an attempt will be made to evaluate the ultimate bearing capacity of a foundation rest on a slope by considering all the effective parameters.

2. Details of the study

The stress distribution within the slope due to foundation load is examined in order to investigate the bearing capacity of a foundation rest near the slope. For this purpose, a newly proposed method by authors [13] has been employed. In this method, by considering the Airy stress function, the stress state within the slope will be investigated. Then, by employing the limit equilibrium method the ultimate bearing

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capacity of the foundation will be studied. The proposed method proved its accuracy in evaluating the ultimate bearing capacity of shallow foundation rest on the cohesionless or frictionless slope by considering Mohr-Coulomb criterion for slope material [12]. In this research, an attempt will be made to extend the previous research work in order to investigate the ultimate bearing capacity resting on a slope. Also, the Hoek-Brown and Drucker-Prager failure criterion will be incorporated into the proposed method. Finally, in order to facilitate the use of the proposed method, a Mathematica package code will be introduced. This package code can be used to find the bearing capacity of the footing (the package is available free for interested researchers on http://www.inscribe.ir/bearing-capacity). It should be noted that in this study, the slope's material was considered as a homogeneous linear elastic. Also, the footing was considered as a shallow foundation with no embedment, and the interface between the foundation and the slope was assumed to be rough.

3. Proposed Analytical Method

One of the best approaches to solving the two-dimensional problem in geotechnical engineering is to use the Airy stress function. The corresponding governing equation in Airy stress function depends on a single unknown variable and mathematical methods should be used in order to solve the equation [15]. The well-known bi-harmonic equation, which satisfies both compatibility and equilibrium equation, would be expressed as equation (1). In Equation (1), ϕ denotes the Airy stress function.

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)^2 \phi = 0 \qquad (1)$$

Based on the given Airy stress function, the components of stress can be derived by equation (2).

$$\begin{aligned}
\sigma_\theta &= \frac{\partial^2 \phi}{\partial r^2} \\
\sigma_r &= \frac{1}{r}\frac{\partial \phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} \\
\tau_{r\theta} &= -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \phi}{\partial \theta}\right)
\end{aligned} \qquad (2)$$

Considering a footing rested adjacent to the crest of a slope as shown in Fig. 1. The authors proposed an analytical solution [13] based on the Mellin transformation scheme and Airy stress function in order to compute the stress state in the slope due to footing load [12]. Definition of Mellin transformation rule and its inversion equation are presented in Equations (3) and (4) respectively.

$$\phi(z,\theta)=\int_0^\infty \varphi(r,\theta)\, r^{z-1} dr \qquad (3)$$

$$\varphi(r,\theta)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} r^{-z}\phi(z,\theta)\, dz \qquad (4)$$

In equations (3) and (4), $\Phi(z,\theta)$ is the transformed form of $\phi(r,\theta)$.

By considering the equations (2) and (4), the stress components can be represented by Equation (5) as follows,

$$\begin{aligned}
\sigma_\theta &= \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} z\,(z+1)\phi\, r^{-z-2} dz \\
\sigma_r &= \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \left[\frac{d^2\phi}{d\theta^2}-z\,\phi\right] r^{-z-2} dz \\
\tau_{r\theta} &= \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} (z+1)\frac{d\phi}{d\theta}\, r^{-z-2} dz
\end{aligned} \qquad (5)$$

The transformed Airy stress function in Mellin space could also be found as Equation (6) [13].

$$\begin{aligned}
&\Phi(z,\theta) \\
&= \frac{-F(z)}{2z(z+1)}\left[\frac{z\cos(z\alpha)\sin(z+2)\theta-(z+2)\cos(z+2)\alpha\sin(z\theta)}{(z+1)\sin(2\alpha)-\sin2(z+1)\alpha}\right. \\
&\left.+\frac{(z+2)\sin(z+2)\alpha\cos(z\theta)-z\sin(z\alpha)\cos(z+2)\theta}{(z+1)\sin(2\alpha)+\sin2(z+1)\alpha}\right]
\end{aligned} \qquad (6)$$

The foundation load may be considered as a step function, s, and represented in Equation (7),

$$F(z)=\int_0^1\left[\sum_{i=0}^{1}(-1)^i s\,(r-r_i)\right] r^{z+1} dr = \frac{1}{z+2}\sum_{i=0}^{1}(-1)^i\, r_i^{\,z+2} \qquad (7)$$

Where r_0 and r_1 are the coordinate of the foundation edge, z represents the complex number and F(z) is transformed surcharge loading function in Mellin space. The variables θ, a, α, and r are shown in Fig. 1.

By considering the equations (5) to (7) and doing some algebra, the stress components are achieved as Equation (8).

$$\begin{aligned}
\sigma_\theta &= \frac{-1}{4\pi i}\int_{c-i\infty}^{c+i\infty}\left(\frac{(\lambda x)^{z+2}-a^{z+2}}{r^{z+2}}\right)\left[\frac{\frac{z}{z+2}\cos(z\alpha)\sin((z+2)\theta)-\cos((z+2)\alpha)\sin(z\theta)}{(z+1)\sin(2\alpha)-\sin(2(z+1)\alpha)}+\frac{\sin((z+2)\alpha)\cos(z\theta)-\frac{z}{z+2}\sin(z\alpha)\cos((z+2)\theta)}{(z+1)\sin(2\alpha)+\sin(2(z+1)\alpha)}\right]dz \\
\sigma_r &= \frac{-1}{4\pi i}\int_{c-i\infty}^{c+i\infty}\left(\frac{(\lambda x)^{z+2}-a^{z+2}}{r^{z+2}}\right)\left[\frac{\cos((z+2)\alpha)\sin(z\theta)-\frac{z+4}{z+2}\cos(z\alpha)\sin((z+2)\theta)}{(z+1)\sin(2\alpha)-\sin(2(z+1)\alpha)}+\frac{\frac{z+4}{z+2}\sin(z\alpha)\cos((z+2)\theta)-\sin((z+2)\alpha)\cos(z\theta)}{(z+1)\sin(2\alpha)+\sin(2(z+1)\alpha)}\right]dz \\
\tau_{r\theta} &= \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\left(\frac{(\lambda x)^{z+2}-a^{z+2}}{r^{z+2}}\right)\left[\frac{\cos(z\alpha)\cos((z+2)\theta)-\cos((z+2)\alpha)\cos(z\theta)}{(z+1)\sin(2\alpha)-\sin(2(z+1)\alpha)}+\frac{\sin(z\alpha)\sin((z+2)\theta)-\sin((z+2)\alpha)\sin(z\theta)}{(z+1)\sin(2\alpha)+\sin(2(z+1)\alpha)}\right]dz
\end{aligned} \qquad (8)$$

By taking the line integration along z=-1 imaginary path and considering the residue of the function, stress components can be obtained as Equation (9),

$$\begin{aligned}
(\sigma_\theta-\sigma_r) &= -\frac{\boldsymbol{a}}{r\pi}\left[\int_0^\infty (f_1+f_2)\left[Sin\left(yLog\left(\frac{a}{r}\right)\right)\right]dy-Residu\right]+\frac{\boldsymbol{\lambda x}}{r\pi}\left[\int_0^\infty (f_1+f_2)\left[Sin\left(yLog\left(\frac{\boldsymbol{\lambda x}}{r}\right)\right)\right]dy-Residu\right] \\
(\sigma_\theta+\sigma_r) &= -\frac{\boldsymbol{a}}{r\pi}\left[\int_0^\infty \frac{(f_3y-f_5)+(f_4y-f_6)}{1+y^2}Sin\left(yLog\left(\frac{a}{r}\right)\right)dy+\int_0^\infty \frac{(f_5y+f_3)+(f_6y+f_4)}{1+y^2}Cos\left(yLog\left(\frac{a}{r}\right)\right)dy+Residu\right] \\
&+\frac{\boldsymbol{\lambda x}}{r\pi}\left[\int_0^\infty \frac{(f_3y-f_5)+(f_4y-f_6)}{1+y^2}Sin\left(yLog\left(\frac{\boldsymbol{\lambda x}}{r}\right)\right)dy+\int_0^\infty \frac{(f_5y+f_3)+(f_6y+f_4)}{1+y^2}Cos\left(yLog\left(\frac{\boldsymbol{\lambda x}}{r}\right)\right)dy+Residu\right] \\
\tau_{r_\theta} &= \left[\frac{-\boldsymbol{a}}{2r\pi}\int_0^\infty (f_7-f_8)Cos\left(yLog\left(\frac{a}{r}\right)\right)dy\right]+\left[\frac{\lambda x}{2r\pi}\int_0^\infty (f_7-f_8)Cos\left(yLog\left(\frac{\lambda x}{r}\right)\right)dy\right]
\end{aligned} \qquad (9)$$

The functions f_1 to f_8, as well as residue, are introduced in Appendix A. For more elaborate details on deriving the transformed Airy stress function and the way of evaluating the stress components within the slope due to footing load, the interested readers are referred to a related article [12, 13].

Since there is no exact solution exists to compute the high oscillating integrals of Equations (4), here Fillon's numerical integration method is employed [16]. Moreover, it is amply clear that gravitational loading plays a significant role in slope stability, and to taking the unit weight of the slope's material into account, the gravitational loading suggested by Goodman and Brown included in the current method based on the superposition scheme. The Goodman and Brown gravitation stress components are presented in Equation (10) [17].

$$
\begin{aligned}
\sigma_{xx} &= \frac{\rho g}{2\alpha-\tan(2\alpha)}\Big[Y\left(2\alpha-\sin(2\alpha)\cos(2\alpha)\right)-2\sin(2\alpha)\cos^2(2\alpha)\left(-Y\cos(2\alpha)+X\sin(2\alpha)\right)\log\left(\frac{\sin(2\alpha-\beta)}{\sin(2\alpha)}\right)-\\
&\quad \beta\sin(2\alpha)\left[Y\sin(2\alpha)\left(1+2\cos^2(2\alpha)\right)+X\cos(2\alpha)\left(1-2\sin^2(2\alpha)\right)\right]\Big]\\
\sigma_{yy} &= \rho g y-\frac{\rho g}{2\alpha-\tan(2\alpha)}\Big[-Y\cos(2\alpha)\sin(2\alpha)+2\sin^3(2\alpha)\left(-Y\cos(2\alpha)+X\sin(2\alpha)\right)\log\left(\frac{\sin(2\alpha-\beta)}{\sin(2\alpha)}\right)\\
&\quad +\beta\sin(2\alpha)\left[Y\sin(2\alpha)\left(1-2\cos^2(2\alpha)\right)+X\left(1+2\sin^2(2\alpha)\right)\cos(2\alpha)\right]\Big]\\
\sigma_{xy} &= \frac{\rho g}{\tan(2\alpha)-2\alpha}\Big[Y\sin^2(2\alpha)+2\sin^2(2\alpha)\cos(2\alpha)\left(-Y\cos(2\alpha)+X\sin(2\alpha)\right)\log\left(\frac{\sin(2\alpha-\beta)}{\sin(2\alpha)}\right)\\
&\quad +\beta\sin(2\alpha)\left(1-2\sin^2(2\alpha)\right)\left(-Y\cos(2\alpha)+X\sin(2\alpha)\right)\Big]
\end{aligned}
\qquad (10)
$$

Where,

$\beta = Tan^{-1}\frac{Y}{X}$

Fig. 2 shows the accuracy of the proposed method in evaluating the stress distribution in the slope. As can be seen, the outcome of the proposed solution is in good agreement with the FEM results.

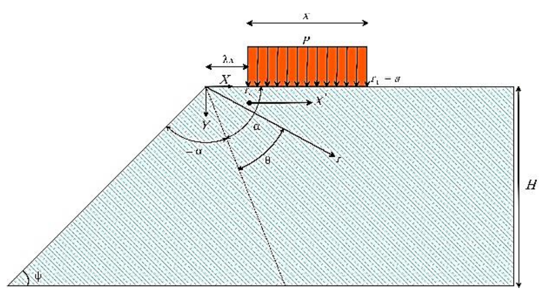

Fig. 1: Schematic representation of slopes under loading on the upper surface.

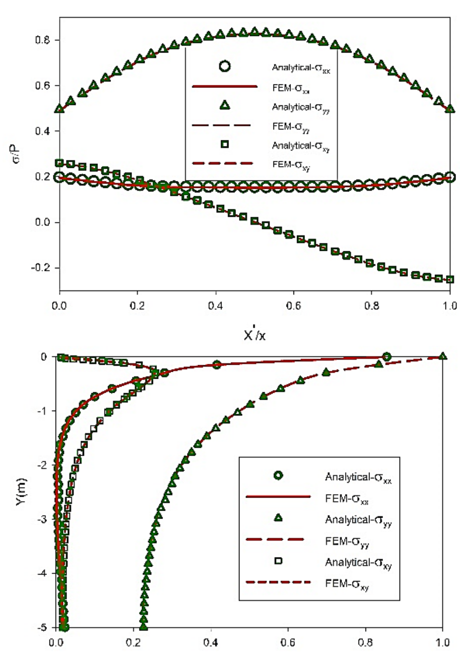

Fig. 2: Stress distribution within a 45o slope and λ=0.5 at (a) Y=-0.5 (b)

In order to study the ultimate bearing capacity of a shallow foundation rest near the slope, the failure criterion should be assigned to the slope's material. Equations (11) to (13) denote the Mohr-Coulomb, Hoek-Brown, and Drucker-Prager constitutive models based on invariants of the stress tensor, respectively [18, 19].

$$\frac{I_1}{3}Sin\varphi + c\,Cos\varphi + \sqrt{J_2}\left(Cos\theta + \frac{Sin\theta\,Sin\varphi}{\sqrt{3}}\right) = 0 \qquad (11)$$

$$-\frac{I_1}{3}m\sigma_c + 4J_2cos^2\theta + \sqrt{J_2}m\sigma_c\left(cos\theta + \frac{sin\theta}{\sqrt{3}}\right) - s\sigma_c{}^2 = 0 \qquad (12)$$

$$k + q\frac{I_1}{3} - \sqrt{J_2} = 0 \qquad (13)$$

Where,

$$I_1 = \sigma_{ii} \qquad (14\text{-a})$$

$$I_2 = \frac{1}{2}\left(\sigma_{ii}\sigma_{jj} - \sigma_{ij}\sigma_{ij}\right) \qquad (14\text{-b})$$

$$I_3 = \det\left(\sigma_{ij}\right) \qquad (14\text{-c})$$

$$J_2 = \frac{1}{3}\left(I_1{}^2 - 3I_2\right) \qquad (14\text{-d})$$

$$J_3 = \frac{1}{27}\left(2I_1{}^3 - 9I_1I_2 + 27I_3\right) \qquad (14\text{-e})$$

$$\theta = \frac{1}{3}Sin^{-1}\left(\frac{-3\sqrt{3}J_3}{2J_2{}^{3/2}}\right) \qquad (14\text{-f})$$

In these equations, the first, second, and third invariant of stress tensor represents by I_1, I_2, and I_3 respectively. Also, J_2 and J_3 are the second and third invariant of the deviatoric stress tensor respectively. Moreover, θ represents the Lode angle, C and φ are the cohesion and friction angle of slope's material; m and s are the Hoek-Brown failure criterion parameters, σ_c is the uniaxial compressive strength of slop's material; k and q are the Drucker-Prager failure criterion parameters. Based on the plain strain assumption, the normal stress can be considered as $\nu(\sigma_1 + \sigma_3)$. By rearranging the Equations (11) to (13), the equations (15) to (17) are obtained.

$$\sqrt{J_2} = \frac{\frac{I_1}{3}Sin\varphi + c\,Cos\varphi}{Cos\theta + \frac{Sin\theta\,Sin\varphi}{\sqrt{3}}} \qquad (15)$$

$$\sqrt{J_2} = \frac{\sqrt{\left(\frac{m\sigma_c}{8}\right)^2\left(1+\frac{tan\theta}{\sqrt{3}}\right)^2 + \frac{s\sigma_c{}^2}{4} + \frac{I_1m\sigma_c}{12}} - \left(1+\frac{tan\theta}{\sqrt{3}}\right)\frac{m\sigma_c}{8}}{cos\theta} \qquad (16)$$

$$\sqrt{J_2} = k + q\frac{I_1}{3} \qquad (17)$$

Stress state can be determined based on Equation (5), and therefore, at any point Equations (15) to (17) as well as Equation (14-d) can be computed based on the slope's material properties and stress state within the slope. In Equation (18), the numerator would be assigned by any of Equations (15) to (17), based on the desired failure criterion, and Equation (14-d) can be considered as the denominator. The process of finding the ultimate bearing capacity is that at each point within a search box near the slope, circles with different radius were drawn, and Equation (18) was computed throughout the length of these circles which overlap with the part of the slope. The minimum foundation load that causes the onset of shear failure in the slope can be considered as ultimate bearing capacity. Therefore, the footing load resulting in the safety factor of 1 was considered as the ultimate bearing capacity of the foundation

$$FS = \frac{\Sigma(\sqrt{J_2})_{Strength}}{\Sigma(\sqrt{J_2})_{Stress}} \tag{18}$$

It should be noted that due to the need of updating the stress state, an iterative process is required to obtain the ultimate bearing capacity. In this regard, a Mathematica package code was developed and all of the abovementioned failure criteria are considered in this package. The process is that, at the first step, Equation (4) will be computed by considering Filon's numerical integration method, and at any point, within the slope, the summation of stress due to foundation load and gravity will be calculated. Then, Equations (14-a) to (14-f) and Equation (18) would be calculated along the predefined curves. Finally, the ultimate bearing capacity of the footing would be computed. The process of computing the ultimate bearing capacity of the footing is provided as a flow chart in Fig. 3. Fig. 4 shows $\sqrt{J_2}$ the distribution and the ultimate bearing capacity load obtained by Mathematica package code. In this case, the Mohr-Coulomb failure criterion was chosen and c=40 kPa, $\varphi = 20$, H/x=5, $\lambda = 1$, $\psi = 30$, $\gamma = 20$ kN/m^3 was considered as the slope's material properties and geometry. The ultimate load was obtained 0.492 MPa, and the sliding surface was also reported.

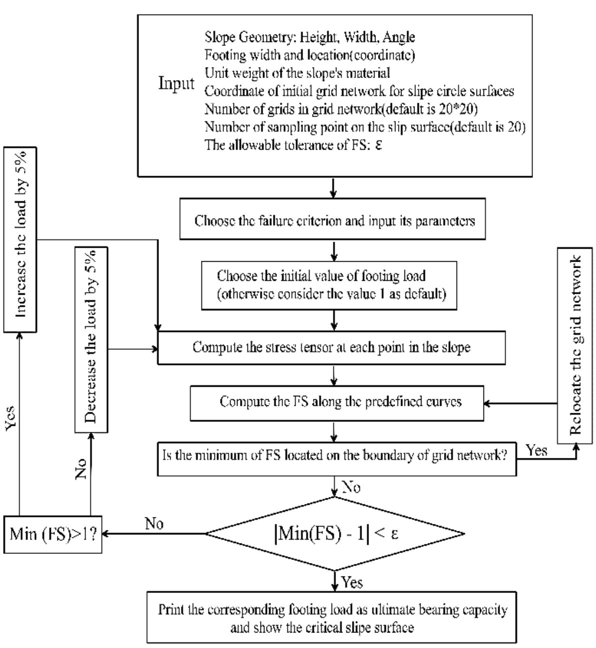

Fig. 3: Flow chart of computing the ultimate bearing capacity

In this study, elastic stress was assumed, and stress redistribution due to developing the plastic zone within the slope was ignored. Although considering the elastic-plastic behavior of materials is more realistic, Krahn [20] and Stianson et al. [21] proved that when the overall safety factor of the slope along the critical failure surface is a matter of concern, the outcome of the elastic and plastic analysis is the same. Therefore, in engineering practice, it is reasonable to consider just elastic analysis to evaluate the safety factor of the slope [21].

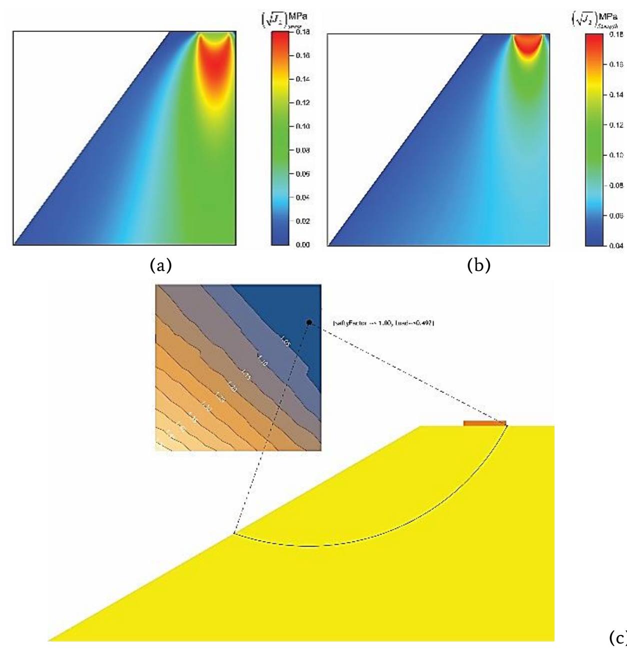

Fig. 4: The outcome of developed Mathematica package (a) Contour of $\sqrt{J_2}$ computed by Equation(14-d) (b) Contour of $\sqrt{J_2}$ computed by Equation (15) (c) Ultimate bearing capacity and the slip surface at 30-degree slope, c=40Kpa, φ=20°, λ=1, H/x=5 and γ=20 KN/m^3

4. Comparison of the proposed solution with available results

To examine the accuracy of the suggested method in evaluating the ultimate bearing capacity of a foundation, the outcome of the proposed approach was compared with the available published results. Zhou et al. (2018) provided a complete set of design charts to evaluate the ultimate bearing capacity of a shallow foundation rest on a c-φ slope by assigning the Mohr-Coulomb behavior model to the materials [10]. The outcome of the proposed method was compared with the numerical results of Zhou et al. (2018) and the comparison was illustrated in Figures 5 and 6.

As can be seen, good agreement is evident between the outcome of the proposed solution and those published by Zhou et al. (2018). Figures 5-a and 6-a, which denote the lower material properties, indicate that by increasing the distance of the footing from the crest, all the curves tend to converge. Indeed, by increasing the distance of the footing from the crest, the effect of the slope geometry on ultimate bearing capacity will decrease and the Prandtl-type failure, occurred on half-plane, and therefore, independent of λ, will be dominant. Therefore, it can be concluded that, in a slope consisting of a weak material, the well-known Prandtl-type failure will occur at lower λ.

Since different slopes material and geometry, as well as foundation width and placement, were considered in these graphs, it can be concluded that the proposed method is able to consider all the effective parameters in evaluating the ultimate bearing capacity. It should be noted that the authors' attempts to find published results that consider Hoek-Brown failure criterion for materials was unsuccessful. However, the proposed method is able to evaluate the bearing capacity of a footing rested on a homogenous rock slope as well.

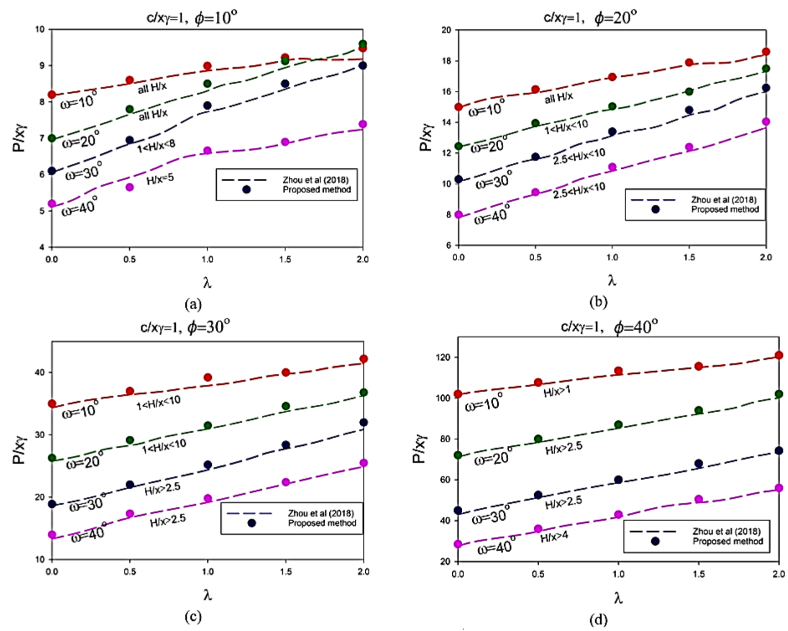

Fig. 5: Variation of P/xγ with λ for different slope's angle and normalized slope's height and c/xγ=1 (a) φ=10° (b) φ=20° (c) φ=30° (d) φ=40°

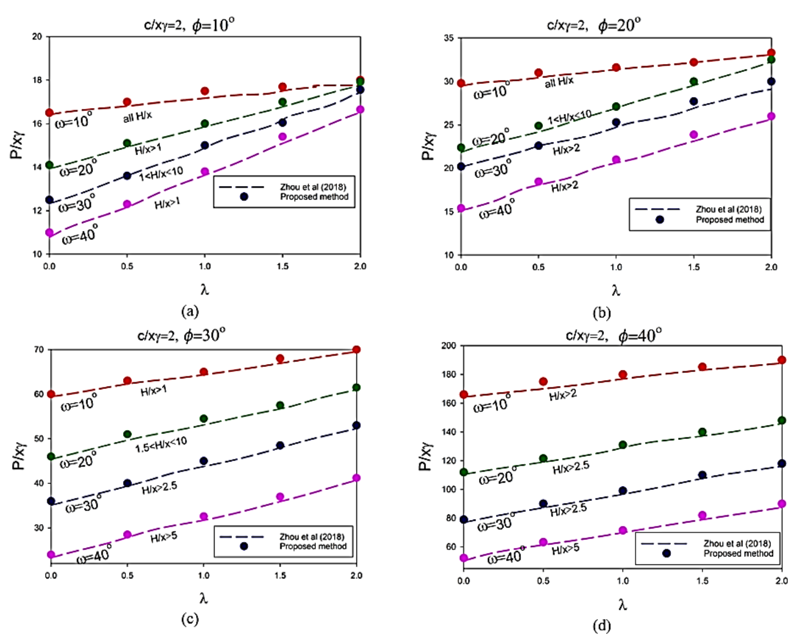

Fig. 6: Variation of P/xγ with λ for different slope's angle and normalized slope's height and c/xγ=2 (a) φ=10° (b) φ=20° (c) φ=30° (d) φ=40°

5. Conclusion

In many cases, especially in a mountainous environment, the foundation should be built on or near to the slope crest, and therefore, to increase the safety of such structures it is necessary to study the ultimate bearing capacity. Since the available analytical and empirical methods are associated with some shortcomings in view of slope material properties and geometry, and numerical methods suffer from the rigorous computational effort, a new analysis is proposed in this research work. The suggested approach is able to consider all the effective parameters that play role in the evaluation of ultimate bearing capacity. Different failure criteria were included in the proposed method, and researchers are able to use this method in order to evaluate the ultimate bearing capacity of a shallow foundation rest on a rock mass or soil slope based on the chosen failure criterion (i.e. Mohr-Coulomb and Drucker-Prager for soil, or Hoek-Brown for the rock case). The results are compared with the existing numerical one in the literature and show good agreements. Also, in order to facilitate the use of the proposed method a Mathematica package code has been proposed to help the researcher evaluate the bearing capacity of shallow foundations rest on the slope. The results indicated that by increasing the distance between the footing and the slope crest or decreasing the slope angle the ultimate bearing capacity will increase.

Appendix A

$$f_1 = \frac{\sin(\alpha+\theta)\cosh(\alpha-\theta)y - \sin(\alpha-\theta)\cosh(\alpha+\theta)y}{(y\sin 2\alpha - \sinh(2\alpha y))}$$

$$f_2 = \frac{\sin(\alpha-\theta)\cosh(\alpha+\theta)\,y + \sin(\alpha+\theta)\cosh(\alpha-\theta)\,y}{(y\sin 2\,\alpha + \sinh(2\alpha y))} \tag{A-1}$$

$$f_3 = \frac{cos(\alpha+\theta)\, sinh(\alpha-\theta)\, y - cos(\alpha-\theta)\, sinh(\alpha+\theta)\, y}{(y\, sin\, 2\, \alpha - sinh(2\alpha y))}$$

$$f_4 = \frac{cos(\alpha-\theta)\, sinh(\alpha+\theta)\, y + cos(\alpha+\theta)\, sinh(\alpha-\theta)\, y}{(y\, sin\, 2\, \alpha + sinh(2\alpha y))}$$

$$f_5 = \frac{sin(\alpha+\theta)\, cosh(\alpha-\theta)\, y - sin(\alpha-\theta)\, cosh(\alpha+\theta)\, y}{(y\, sin\, 2\, \alpha - sinh(2\alpha y))}$$

$$f_6 = \frac{sin(\alpha-\theta)\, cosh(\alpha+\theta)\, y + sin(\alpha+\theta)\, cosh(\alpha-\theta)\, y}{(y\, sin\, 2\, \alpha + sinh(2\alpha y))}$$

$$f_7 = \frac{sin(\alpha-\theta)\, sinh(\alpha+\theta)\, y + sin(\alpha+\theta)\, sinh(\alpha-\theta)\, y}{y\, sin(2\alpha) - sinh(2\alpha y)}$$

$$f_8 = \frac{sin(\alpha-\theta)\, sinh(\alpha+\theta)\, y - sin(\alpha+\theta)\, sinh(\alpha-\theta)\, y}{y\, sin(2\alpha) + sinh(2\alpha y)}$$

$$Residue = [\frac{\pi\, sin\alpha\, cos\theta}{sin2\alpha - 2\alpha} + \frac{\pi\, sin\alpha\, cos\theta}{sin2\alpha + 2\alpha}]$$

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