Journal of Chemical and Petroleum Engineering (JChPE)
Print ISSN: 2423-673X Online ISSN: 2423-6721

New Relative Permeability Correlations for Carbonate Reservoirs through Data-Driven Modeling

Ehsan Bahonar *, **Yaser Ghalenoei**, **Sadegh Salmani**

1. Faculty of Petroleum and Natural Gas Engineering, Sahand University of Technology, Tabriz, Iran. E-mail: e_bahonar95@sut.ac.ir
2. Faculty of Petroleum and Natural Gas Engineering, Sahand University of Technology, Tabriz, Iran. E-mail: ya_ghalenoiee@sut.ac.ir
3. Ahwaz Faculty of Petroleum Engineering, Petroleum University of Technology, Ahvaz, Iran. E-mail: s.salmani@ut.ac.ir

ARTICLE INFO

Article History:
Received: 25 October 2021
Revised: 25 December 2021
Accepted: 26 December 2021

Article type: Research

Keywords:
Correlation,
Data Mining,
End Point,
Relative Permeability,
Wettability

ABSTRACT

Relative permeability is a crucial input to reservoir simulators for modeling reservoir performance. Conventional methods of measuring relative permeability rely on either laboratory core-flooding experiments or fine-scale computer simulations. The former method is expensive and time-consuming, and the latter often does not represent the complex characteristics of existing systems. Data mining algorithms can be implemented to estimate relative permeability with reasonable accuracy for real applications without running laboratory or computer simulation experiments. This paper aims at presenting predictive correlations for relative permeability for carbonate rocks using data-driven approaches. To achieve this aim, a scatter plot matrix was applied to analyze 225 experimental datasets, including almost 3800 relative permeability data points (observations), for predicting relative permeability. Since relative permeability measures are often unavailable exactly at residual oil saturation and connate water saturation (known as endpoints); consequently, cubic equations were fitted and solved to precisely determine these points. Next, a symbolic regression algorithm was developed to predict relative permeability in different situations: when endpoints are available or unavailable and when the rock wettability is clear or not. For this purpose, all 225 datasets were divided into training and testing groups. The correlations were tested to predict testing data, which the symbolic regression algorithm has never seen before. Finally, the most accurate correlations were presented, and a detailed analysis was carried out. The results showed a good agreement between the real and the predicted data. The developed correlations proved to be very efficient in predicting the relative permeability accurately.

Introduction

Relative permeability is defined as the ratio of effective permeability to a base permeability. The base permeability is usually the absolute permeability or oil permeability at irreducible water saturation [1-3]. Any mathematical calculation related to the movement of different fluids in porous media needs relative permeability data [4]. The most reliable method to obtain relative

* Corresponding Author: E. Bahonar (E-mail address: e_bahonar95@sut.ac.ir)

Journal of Chemical and Petroleum Engineering, 2022, 56 (1): 15- 35.
Publisher: University of Tehran, College of Engineering *DOI: 10.22059/JCHPE.2021.332924.1369*

permeability is special core analysis (SCAL). However, since this method is expensive and time-consuming, empirical correlations have been developed for the estimation of relative permeability curves which accept basic rock and fluid properties as input parameters [5]. This category of relative permeability correlations is the subject of this study.

The first step for developing an appropriate correlation for predicting the relative permeability is to determine key effective factors on this petrophysical parameter. Several works made efforts to recognize crucial parameters on relative permeability. These studies investigated the effect of parameters such as temperature [6-8], interfacial tension [9], fluid viscosity and displacement rate [10], pore size distribution [11], and capillary number [12]; on relative permeability. They showed that some parameters such as fluid rate, viscosity, interfacial tension, and temperature have a small effect on predicting relative permeability. Conversely, other parameters like saturation (especially at end-points) and wettability significantly affect relative permeability values. In the following, a few investigations in this domain are brought up in detail. Gates and Lietz [13] developed an expression based on Purcell's model for wetting phase relative permeability. Corey et al. [14] observed a linear relationship between the relative permeability of oil and effective water saturation in a wide range of saturation in the sandstone reservoir. Hereby, considering this observation and more simplifications, Corey presented an equation to calculate relative permeability based on effective saturations for water-oil and gas-oil systems. Torcaso and Wyllie [15] extended a simple expression based on Corey's results to predict the oil phase's relative permeability in a gas-oil system. Pirson [16] derived correlations for wetting and non-wetting phase relative permeability for both drainage and imbibition processes. Naar and Henderson [17] developed a mathematical image (model) of consolidated porous rock for two-phase imbibition relative permeability and considered saturation history at predicting relative permeability. It should be noted that saturation history focused on end-point data availability. Brooks and Corey combined the modified equation with Burdines's equation for developing the new expression that estimates drainage relative permeability for any pore size distribution [18].

Land calculated imbibition relative permeability for two and three-phase flow based on the dependency of relative permeability on pore size distribution [19]. Owen et al. [20] concluded that rock wetting has a significant and predictable effect on oil-water relative permeability measurements. F.M. Carlson [21] presented a method allowing the calculation of imbibition relative permeability at any saturation. Honarpour [22] applied linear regression analysis techniques to develop an empirical equation. Timmerman [23] suggested the equations based on the water-oil drainage capillary pressure to calculate low values of water-oil relative permeability. A few years later, Ertekin et al. [24] investigated the influence of capillary number on the two-phase oil-water relative permeability curve. Alpak et al. [25] have exceptional attention to surface areas of fluids and rock interfaces, and tortuosity. They claimed that acquired results from modified Carman-Kozeny (MCK) expression had shown similar results to modified Brooks and Corey (MBC). M. Ibrahim [26] applied a stepwise multiple linear regression model to develop correlations for water-oil, gas-oil, gas-water, and gas-condensate relative permeability. He presented an improved equation based on formation type and wettability. Behrenbruch et al. [27] identified that the pore structure's shortcoming affects relative permeability prediction by the MBC model based on the CK equation. Ghanbarian [28] experimentally confirmed that providing access to electrical conductivity obtained a more accurate estimation of relative permeability.

Recently, M. Andrew [29] claimed that applying the tunning factor used in Kozeny-Carman's based models is subject to enormous challenges for actual conditions. To decline these challenges, he presents a new approach for predicting Stokes flow permeability from pore-scale images using multivariant statistical regression coupled with pore-scale computational simulation. Besides correlations based upon BC and CK, computational methods such as pore network models (PNM) and direct numerical simulation (NMS) have been presented recently [30, 31]. Although these methods are used as a fast relative permeability data generation, they are subject to uncertainties originating from idealization or limited to a rock or special fluid [32]. In this respect, Raoof et al. [33] introduced a pore-network model that used complex formulations to accurately model transport problems. Nevertheless, gained results of the pore-network model only applied for theoretical studies and be able to study the qualitative trend of relative permeability. Zhao et al. [34] developed a machine learning-based model and applied pore network simulation to produce relative permeability data. They stated that the Euler number (a parameter for measuring fluid connectivity/distribution) is a first-order predictor for relative permeability. They used pore-network simulation to generate requisite data to evaluate the Euler number. Another way to obtain requisite data for the Euler number is the displacement experiment under micro CT. Also, P.Purswani et al. [4] developed a general equation of state approach for relative permeability based on normalized Euler number. They used PNM simulations similar to Zhao to acquire vital data for the Euler number. As mentioned, most traditional correlations, such as Corey, Alpak, etc., considered some of the influential parameters but ignored others.

On the other hand, as the era of easy-to-find oil and gas comes to an end, oil companies are developing new technologies to reduce costs. There are many applications of artificial intelligence being used today to enhance oil recovery in the oil industry. In recent years, as we pointed out in brief, several studies have been done on the use of machine learning in the oil industry, including the production optimization [35], geological studies [36], prediction of PVT properties like viscosity [37, 38], permeability and porosity estimation [39], etc. some studies have been conducted on relative permeability prediction using computational methods [4] and CNM [34], etc. These kinds of correlations either are applicable in theoretical conditions or need extra experimental facilities like micro-CT to determine crucial parameters (Euler number) to predict relative permeability that caused high experimental costs. Also, several studies have been conducted to predict relative permeability only using linear regression while the relationship between crucial parameters such as saturation, wettability, and endpoints are non-linear. However, machine learning algorithms have the merit of developing correlation by considering multiple useful parameters simultaneously [34]. Hereby, along with this growth in applying machine learning, a considerable amount of literature has been published on the prediction of multi-variant parameter functions like relative permeability. For example, Al-Fattah [40] utilized ANN (artificial neural network) in characterizing relative permeability in two-phase flow. In another attempt to predict the association between porosity, permeability, and tortuosity, K.M. Graczyk and M.Matyka [41] have recently implemented CNN (convolutional neural networks) to obtain these values. Although they have shown that CNN gained good results for predicting permeability, porosity, and tortuosity, they have only focused on static, not dynamic parameters. More recently, S. Kalam et al. [42] claimed that presented new correlations to predict relative permeability by using ANN and ANFIS (adaptive neuro-fuzzy inference system) algorithms either in sandstone or in carbonate porous media. Even though they urged that their correlation have exceptional attention to wettability, porosity, water saturation, and end-point data, in their correlation, however, far too little attention has been paid to tortuosity and pore structure's effect on relative permeability. Investigations of M. Adibifard et al. [43] applying GA (genetic algorithm) and EnKf to build the relative permeability curves; revealed that cumulative gas and oil production data shows the complex two-phase flow in porous media.

In this study, we considered all points mentioned above and implemented symbolic regression to develop proper correlations regarding noted issues. This research consists of several steps. Firstly, we analyzed the datasets from a wide range of Iranian carbonate fields and applied a matrix plot to find correlated parameters. Then, we developed a symbolic regression algorithm to predict relative permeability. Due to the strong correlation between relative permeability and wettability, we took into account this matter and developed new correlations both when wettability is known and unknown, and when relative permeability at endpoints is available or not. The rest of this paper is organized as follows: in section 2, we briefly review the datasets and methodology of this work. Section 3 describes the wettability of samples. In section 4, we suggest correlations to predict relative permeability in all possible conditions. Finally, the last section presents our conclusions.

Methodology

The main goal of this study is to suggest new correlation equations that can be used by other researchers who may not have access to our dataset and algorithms. So, we used a symbolic regression algorithm that provides accurate non-linear correlations. 225 relative permeability datasets, including almost 3800 relative permeability data points from Iranian carbonate reservoirs, have been used. Therefore, we first describe the available datasets and summarize the steps to gain precise correlations in a flowchart. Then, we introduce a symbolic regression algorithm.

Dataset Description

In SCAL analysis, relative oil permeability (k_{ro}) at connate water saturation (S_{wc}) have a maximum value known as k_{roend}. In addition to that, the maximum value of $\mathrm{k_{rw}}$ place at residual oil saturation (S_{orw}) is called $\mathrm{k_{rwend}}$ (Fig. 1).

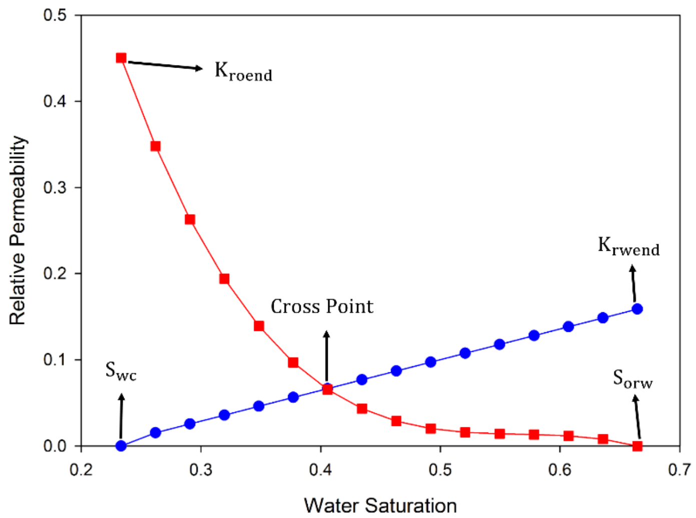


Fig. 1. Key points of a typical water-oil relative permeability curve

For illustration purposes, some relative permeability curves of some used samples are shown in Figs. 2 and 3.

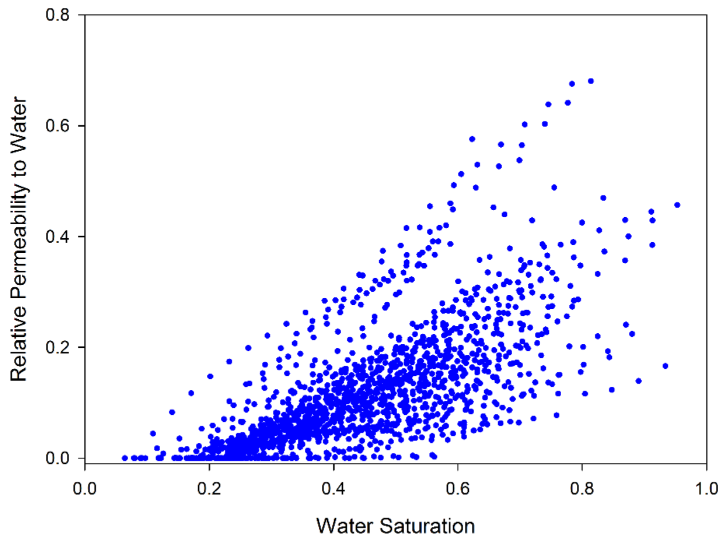


Fig. 2. k_{rw} vs. S_w for some samples

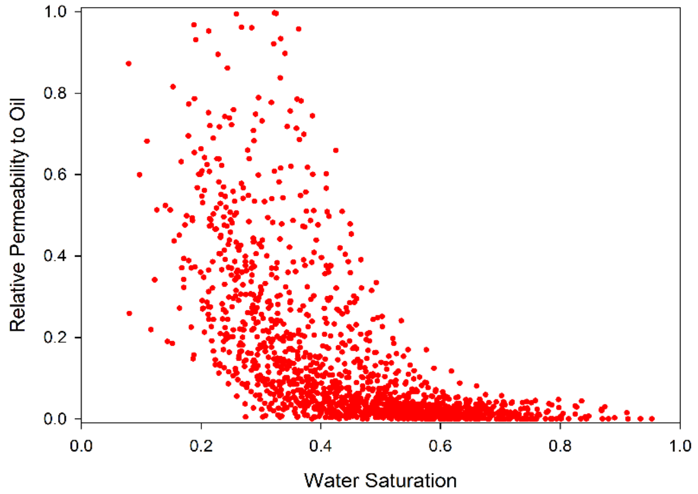


Fig. 3. k_{ro} vs. S_w for some samples

As mentioned earlier, 225 datasets consisting of 3800 relative permeability data are available. Variables and statistical properties of them are as follows:

Table 1. Statistical properties of the dataset.

Statistical Characteristics	Air Permeability (md)	Base Permeability (md)	Porosity	Oil Viscosity (cp)	Water Viscosity (cp)	S_{wc}
Min	0.107	0.0005	0.04284	1.3	1.21	0.0429
1st Qu	1.32	0.238	0.1138	1.3	1.4	0.1889
Median	3.012	0.5247	0.15385	1.3	1.5	0.2256
Mean	7.659	2.2247	0.14976	3.385	1.484	0.227
3rd Qu	7.977	2.334	0.1847	2.5	1.55	0.259
Max	122.673	39.5556	0.2569	20	1.55	0.4491

Statistical Characteristics	S_{orw}	K_{rwend}	K_{roend}	K_{rw}	K_{ro}
Min	0.0470	0.005126	0.005674	0	0

1st Qu	0.2408	0.152051	0.298729	0.045231	0.014154
Median	0.3014	0.221655	0.490026	0.099053	0.044799
Mean	0.3080	0.255084	0.503842	0.126844	0.129565
3rd Qu	0.3935	0.348244	0.694762	0.174266	0.171662
Max	0.6639	0.815393	0.996777	0.815393	0.996777

K_{air} and K_{water} are absolute permeability to air and water. Moreover, S_{wc} is connate water saturation and S_{orw} is residual oil saturation in the water-oil system. It is remarkable to note that the base permeability (K_{base}) can be absolute liquid or air permeability or effective oil permeability at S_{orw} to normalize the relative permeability. In this dataset, the base relative permeability is the absolute permeability of water. Also, the exact measure of the endpoint data was unavailable in some datasets. Therefore, cubic equations have been fitted and solved for each curve of oil and water relative permeability as follows:

$$K_{rw} = aS_w^3 + bS_w^2 + cS_w + d \quad (1)$$

$$K_{ro} = eS_w^3 + fS_w^2 + gS_w + h \quad (2)$$

An example of obtained coefficients for a relative permeability curve has been shown in Fig. 4.

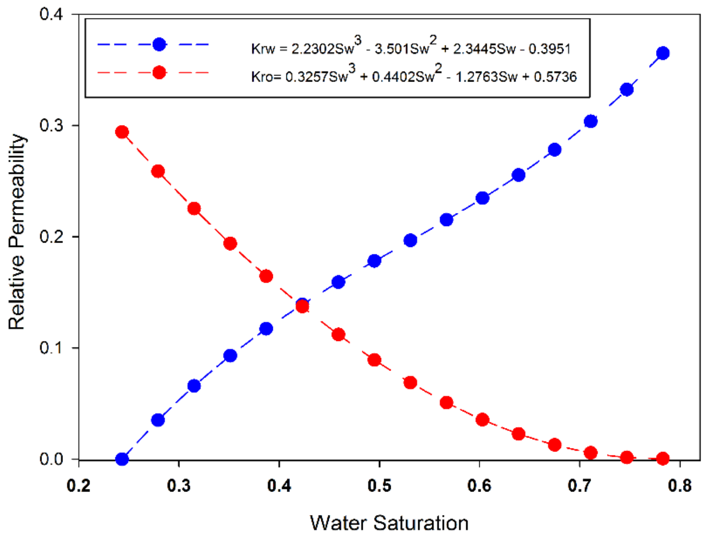


Fig. 4. Fitting and determining the coefficients of a cubic equation

In most cases, the quadratic equation has been fitted adequately with experimental data; nevertheless, the cubic equation is used to reach maximum accuracy. All acquired equations have been solved, and then the exact value of K_{roend} , K_{rwend} , S_{orw} , S_{wc} were calculated.

A scatter plot matrix was prepared to investigate the relations between parameters (Fig. 5). This plot draws scatter plots between all parameters and calculates the Pearson correlation coefficients. The value of the Pearson correlation coefficient represents the linear relations between the two parameters. The values range between -1.0 and 1.0. A value of -1.0 shows a perfect negative correlation, a value of 1.0 indicates a perfect positive correlation, and 0 shows no linear relation between the variables. It is worth mentioning that the Pearson correlation coefficient only calculates linear relations, so to investigate non-linear relations, other correlation coefficients can be used. Using this plot, some helpful information can be obtained: how each parameter is linearly correlated with others? Is there any outlier data? How the data is distributed, and has it normal distribution or not? And so on. As shown in Fig. 5, oil and water relative permeabilities are strongly correlated with water saturation and endpoint relative permeability. On the other hand, it is also clear that oil and water relative permeabilities have poor relations with other parameters. Therefore, in many cases where the K_{rend} data is unavailable, predicting relative permeability will be challenging and complicated. It is worth mentioning that endpoint tests provide the values of K_{rend}, S_{orw} and S_{wc}. These tests are less expensive than the SCAL tests; however, they can provide fruitful information.

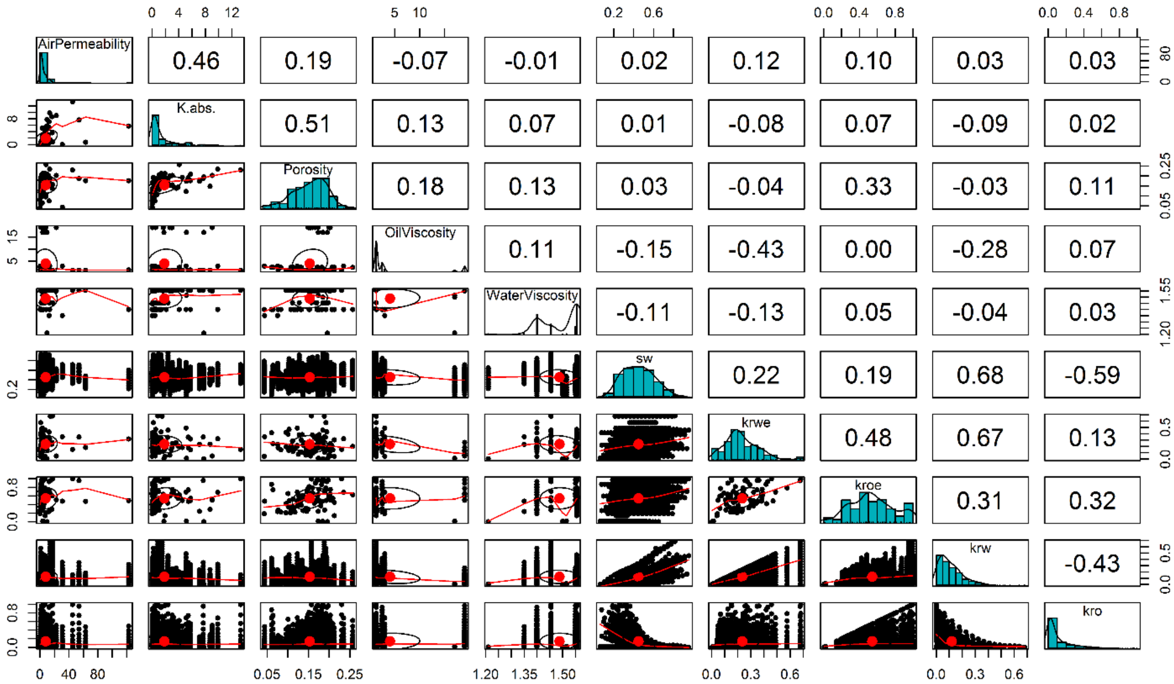


Fig. 5. Scatter plot matrix for the variables

Data Mining

The first phase of data mining projects is preprocessing, which prepares input data for machine learning algorithms. The missing values in each variable were filled by mean value, and outlier data were identified and deleted. These misleading data are comprised of wrong experimental measurements or computational errors.

As mentioned in this work, symbolic regression is utilized to consider the non-linear relations between the dependent and independent parameters. In the first step, 75% of the SCAL datasets (i.e., 168 datasets) were used for training, and 25 % (i.e., 57 datasets) for testing to evaluate the algorithm's performance. In the following, the accuracy and errors of unseen data are presented. It is worth bearing in mind that if the observations are divided randomly into training and testing groups since the algorithm has seen a part of a relative permeability curve in the training step (i.e., it might have seen 10 points of 15 points of a whole curve), it also caused that the accuracy of the predicted value in the testing step to increase dramatically. Therefore, the correlations were tested on complete unseen curves. These datasets have been held back from training the model, and they have not been seen by the model whatsoever. The main steps of this study were drawn in a flowchart, see Fig. 6:

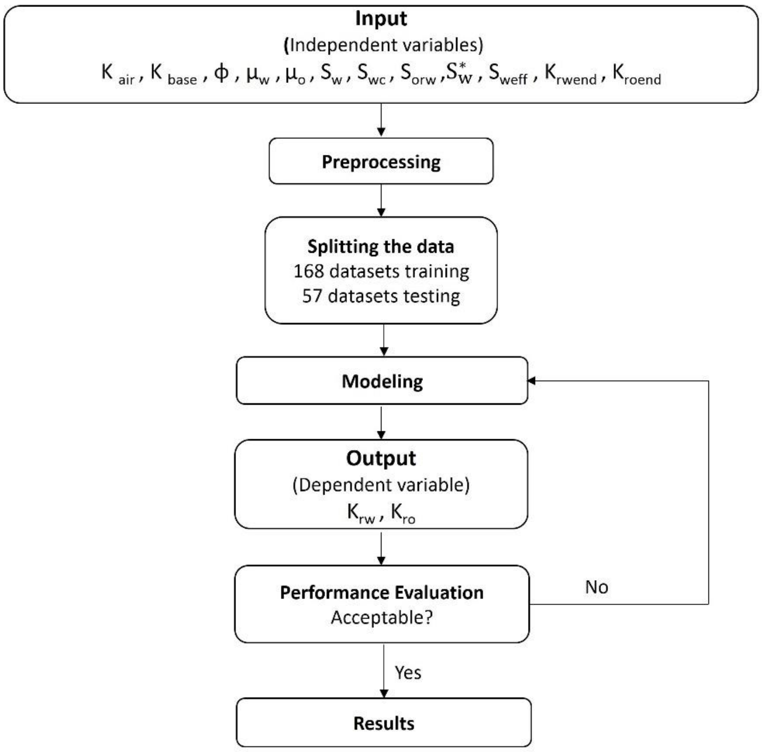


Fig. 6. Relative permeability prediction steps by using data mining algorithms

Nowadays, many powerful algorithms are being used to build predictive models. Symbolic regression can suggest non-linear correlation equations, which can use by other researchers in their projects.

Symbolic Regression

In a general sense, genetic programming (GP) is a method in machine learning for optimizing programs to find a plan that performs the given task well. This work uses symbolic regression as a method among GP's tools to find correlations to predict relative permeability. Symbolic regression uses a population of operators such as arithmetic, trigonometric, exponential, and building the inverted tree (a graph-based representation). The operator set is in the middle and top of the tree. In the end, the phenotype is the function created from this tree, see Fig. 7:

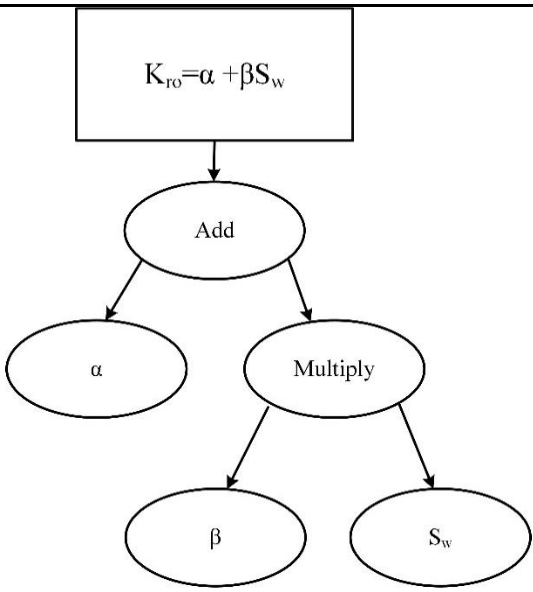


Fig. 7. An example of a symbolic regression tree

It should be noted that symbolic regression keeps iterating to achieve the most accurate model; therefore, we need to choose criteria to accomplish the procedure so that it neither stops early without adequate accuracy nor spend a long time searching to make any improvement.

Wettability of Samples

Lak wettability index is a new index to determine wettability just by applying relative permeability curves. This index was obtained by combining Graig's first rule and modified Griag's second rule by A.Mirzaei-Paiaman [44]. This method has been defined as follows:

$$I_{kr} = I_{kr1} + I_{kr2} = \alpha\left(\frac{0.3 - K_{rw@ROS}}{0.3}\right) + \beta\left(\frac{0.5 - K_{rw@ROS)}}{0.5}\right) + \left(\frac{CS - RCS}{1 - S_{or} - S_{wc}}\right) \quad (3)$$

where $K_{rw@ROS}$ is the value of water relative permeability at residual oil Saturation (K_{rwend}). CS is the water saturation value at the crossover point of the relative permeability curve. RCS is reference crossover saturation which equals $\frac{1}{2} + \frac{S_{wc}-S_{or}}{2}$ for water-oil systems.

In general, I_{kr} have a value between -1.0 to +1.0. The values of 1.0 implied a strongly water-wet system, whereas values approaching -1.0 represent an oil-wet system. Positive measures of the Lak index represent water-wet rocks, and negative measures are for oil-wet rocks [44].

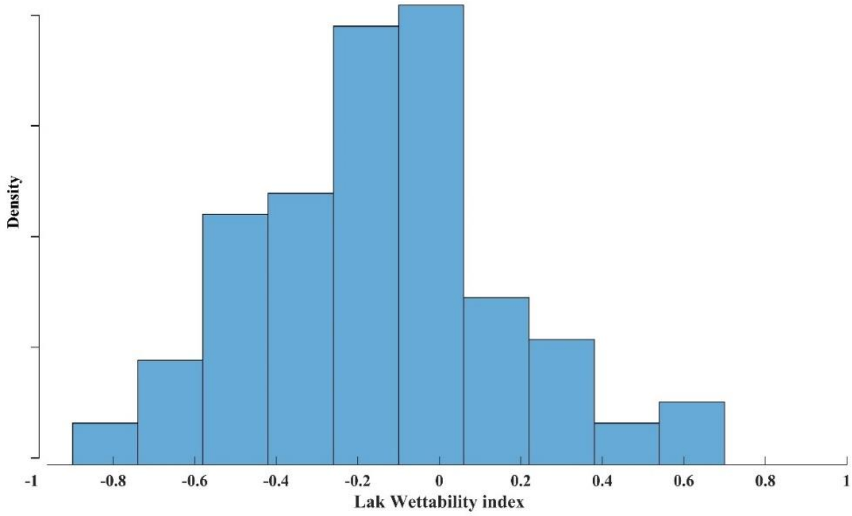


Fig. 8. Lak wettability index distribution

As Fig. 8 illustrates, in most samples, the Lak wettability index is frequently between 0 to -1; as mentioned earlier, these values implied oil-wet wettability in oil-water systems. It is necessary to mention that 172 of the samples are oil-wet, and 53 are water-wet.

Results and Discussion

In this paper, some correlations were developed to predict relative permeability in all possible situations, whether the wettability of the sample is known or not, or whether the endpoint permeability is available or unavailable. Relative permeability strongly correlates with endpoint data, so predicting relative permeability will be associated with remarkable errors without these values. On the other hand, these data are unavailable under many conditions; therefore, having an acceptable estimate of full relative permeability curves without having these parameters appears necessary and useful. So, all the above states will be considered and pointed out in the following separately.

Estimation of Relative Permeability for Oil-Wet Systems

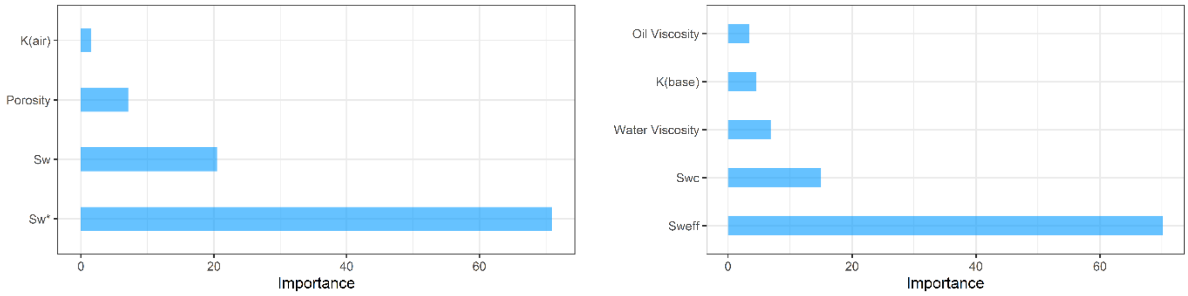
Wettability can be one of the influential parameters in predicting relative permeability, and there is a mutual relation between them. In a case where wettability is clear, two classes of correlations are provided: predicting relative permeability either in the existence of endpoint data or in its absence. Moreover, several previously developed correlations suffer from the inability to predict permeability in the lack of endpoints data, which caused authors to present new correlations for this condition. The newly developed correlations for oil and water relative permeability when endpoint data is unavailable and also wettability is oil-wet are presented as follows:

$$(k_{ro})_{o-w} = \frac{7.345\varphi S_w \mathrm{IF}(\varphi \leq (6.7928\varphi)^{K_{air}}, 1,0) + \mathrm{maximum}(S_w, 0.2363)}{e^{(5.8311 S_w^*)}} \quad (4)$$

$$\begin{aligned}(k_{rw})_{o-w} = {} & \mathrm{minimum}(S_{weff}, 0.0143 + 0.0661 S_{weff} \\ & + \mathrm{maximum}(0.2961 S_{weff}, 0.6521 S_{wc} S_{weff} \mu_w^2) \\ & - 0.0036 K_{base} \mu_{o@25^\circ} S_{weff}\end{aligned} \quad (5)$$

where $IF(\varphi \leq (6.7928\varphi)^{K_{air}}, 1,0)$ returns 1 if $\varphi \leq (6.7928\varphi)^{K_{air}}$, otherwise 0. Furthermore maximum $(S_w, 0.2363)$ returns S_w if $S_w > 0.2363$, otherwise 0.2363. Moreover, $S_{weff} = \frac{(S_w - S_{wc})}{(1-S_{wc})}$) and $S_w^* = \frac{S_w - S_{wc}}{1 - S_{wc} - S_{orw}}$. It should be noted that the porosity and saturations are fractions (between 0 and 1.0).

Fig. 9 displays the relative impact within this model that a variable has on the target variable. As illustrated, water and normalized water saturation greatly impact the relative permeability of oil in oil-wet systems, whereas effective water saturation and connate water saturation significantly contribute to predicting the relative permeability of water in this system.



A B

Fig. 9. The relative importance of each predictor A)$(k_{ro})_{o-w}$, B) $(k_{rw})_{o-w}$

Fig. 10 draws a comparison between actual data and predicted relative permeability obtained by Eq. 4 and Eq. 5. It displays that even if we do not have endpoints, acceptable results can be obtained by presented correlation.

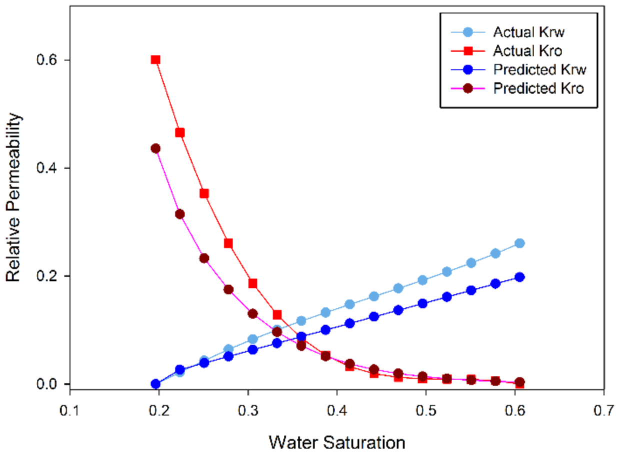


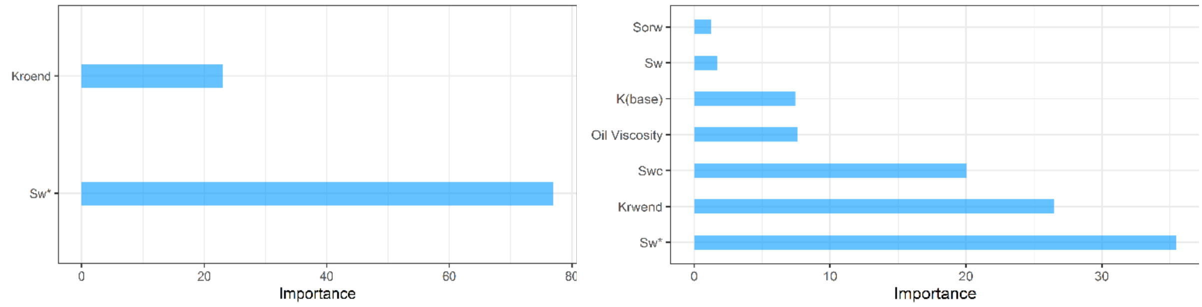
Fig. 10. Comparison between new correlations and real data, oil-wet system, K_{rend} is unavailable

The authors, with the respect that endpoint data have a noteworthy effect on the prediction of relative permeability, developed new correlations for conditions in which these data are available to increase predicted relative permeability accuracy in the following. The developed correlations for oil and water relative permeability have been presented for oil-wet systems when endpoint data is available in the following:

$$((K_{ro})_{o-w})_e = K_{roend} + 3.9100\, S_w^* K_{roend} \mathrm{minimum}(\,0.3842\,, S_w^*) - 1.6558 S_w^* K_{roend} - 1.9540\, K_{roend} \mathrm{minimum}(0.4331\,, S_w^*) \quad (6)$$

$$((K_{rw})_{o-w})_e = K_{rwend} S_W^{*\,\mathrm{maximum}\left(0.8187\,,0.5596+4.6857 S_{wc} S_W^* - \frac{1.9124 S_W}{\mu_{o@25^\circ}{}^2} + \mathrm{minimum}(K_{base}, S_{orw})\right)} \quad (7)$$

It is observed in Fig. 11 that the most important parameters that play an essential role in determining the relative permeability of oil are normalized water saturation and endpoint data while to determine the relative permeability of water other parameters such as connate water, oil viscosity, and base permeability show a significant contribution as well.



A B

Fig. 11. The relative importance of each predictor (A)$((K_{ro})_{o-w})_e$ (B) $((K_{rw})_{o-w})_e$

Fig. 12 has provided a comparison between acquired relative permeability against actual data by applying Eq. 6 and Eq. 7. As expected, the accuracy of the predicted value remarkably increased using endpoint data, and obtained results show a close prediction to actual data.

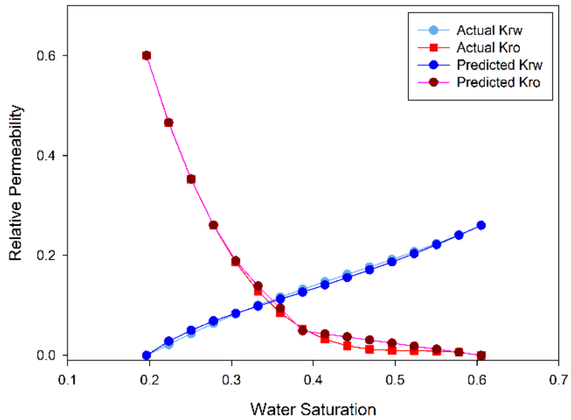


Fig. 12. Comparison between new correlations and real data, oil-wet system, K_{rend} is available

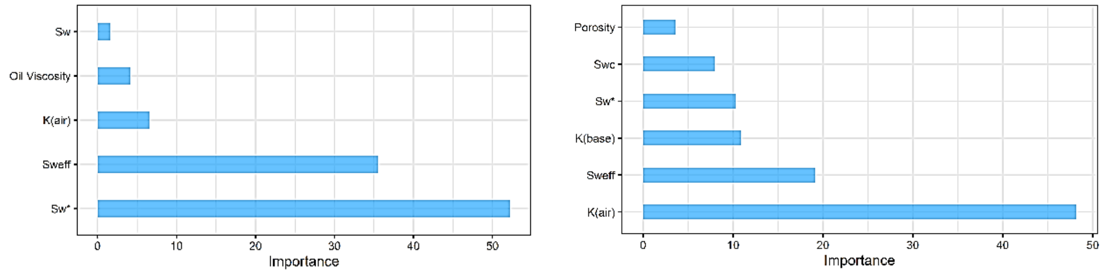
Estimation of Relative Permeability for Water-Wet Systems

The previous section is repeated for water-wet wettability samples to predict relative permeability. In this step, we presented new correlations in conditions where endpoint data is unavailable .Besides, the accuracy of newly developed correlations was strengthened by considering endpoint data. New correlations for water-wet systems, for oil and water, are presented as follows:

$$(K_{ro})_{w-w} = 8.3634 \times 10^{-16} \times 0.0004^{S_W^*} S_{weff}^2 \, \mathrm{Factorial}(\mu_{o@25°}) + \mathrm{minimum}\left(0.7050\,, \mathrm{minimum}\left(K_{air}, 4.4751 S_w (0.0011 S_{weff})^{S_W^*}\right)\right) \quad (8)$$

$$(K_{rw})_{w-w} = 0.0029 + 0.9132 \, \mathrm{minimum}\,(S_{wc} S_W^*\,, \mathrm{minimum}(0.4982 S_{weff}\,, \varphi\, S_{weff} + 0.0597 S_{wc} K_{air}^2)) - 0.0141\, K_{base} S_W^* \quad (9)$$

As explained in the previous section, the author determined the relative impact of parameters for predicting relative permeability in water-wet systems, see Fig. 13.



A B

Fig. 13. The relative importance of each predictor A)$(K_{ro})_{w-w}$, B)$(K_{rw})_{w-w}$

As shown in Fig. 14, the presented correlations almost show close prediction to actual data. These correlations were presented without endpoint data and demonstrated reasonable accuracy for the water-wet system.

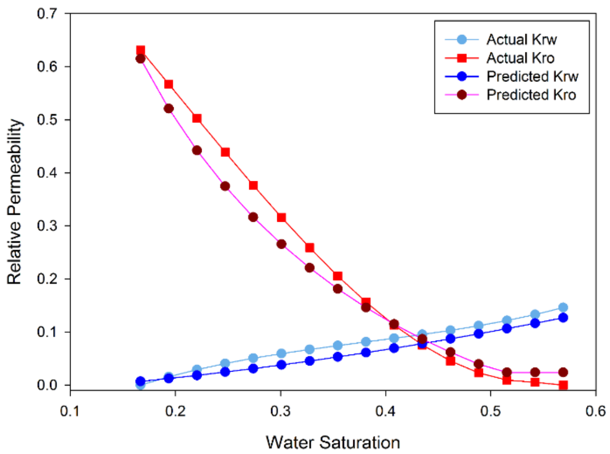


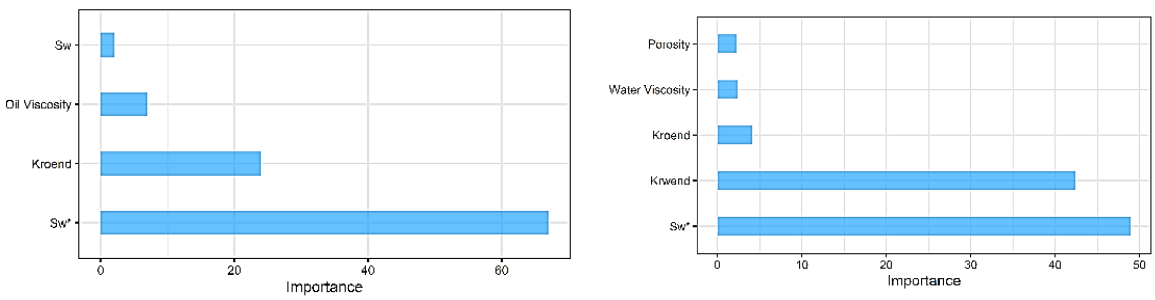
Fig. 14. Comparison between new correlations and real data, water-wet system, K_{rend} is unavailable

However, new correlations when endpoint data is available are:

$$((K_{ro})_{w-w})_e = 0.0057 + 1.0646K_{roend}(0.0052\mu_{o@25^\circ})^{S_W^*} - 0.0055\mu_{o@25^\circ}K_{roend} \quad (10)$$

$$((K_{rw})_{w-w})_e = \text{minimum}(0.0691\varphi K_{roend} + 0.596\mu_w S_W^* K_{rwend}\ , S_W^*) \quad (11)$$

Also, the importance of diverse parameters for predicting relative permeability when endpoint data is available can be seen in Fig. 15.



A B

Fig. 15. The relative importance of each predictor A)$((\mathbf{K_{ro}})_{w-w})_{e}$, B)$((\mathbf{K_{rw}})_{w-w})_{e}$

By comparing Fig. 16 and Fig. 14, we realize when endpoint data is available, the accuracy of the predicted value increased, most especially for the predicted relative permeability of water that shows a remarkable adaption with actual data. It is also expected from Fig. 15, which showed the endpoint data have a great contribution to relative permeability of water, therefore; considering this parameter gives rise to an increase in the accuracy of relative permeability of water.

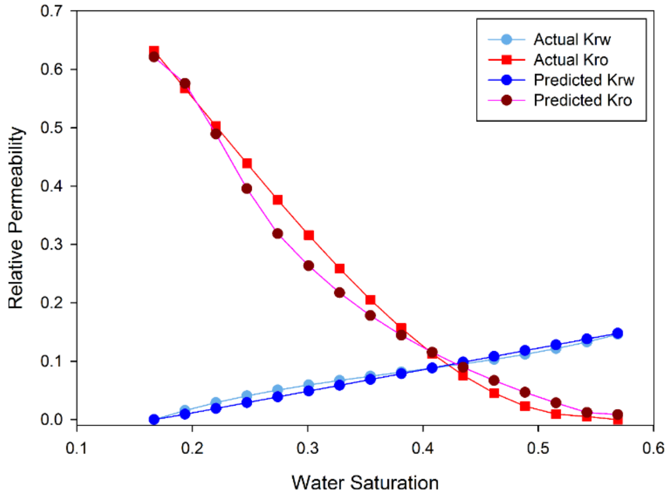


Fig. 16. Comparison between new correlations and real data, water-wet system, K_{rend} is available

Although access to wettability type led to increasing predicted accuracy in most cases, sometimes this parameter is unknown. Alternatively, previously developed correlations suffered from this shortcoming inspired the authors to present new correlations in the absence of wettability.

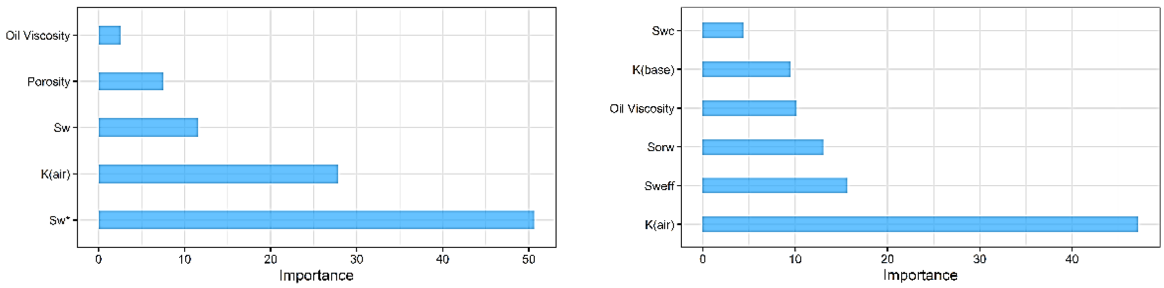
Estimation of Relative Permeability when Wettability Type is Unknown

Finally, general correlations by paying attention to unknowing wettability conditions have been developed. New correlations to predict relative permeability in the absence of endpoint data are as follows:

$$K_{ro} = 0.0047 + \text{minimum}\left(K_{air}, 1.2035 \frac{1}{1+e^{-(18.9880\,\varphi S_w - IF(596\,\varphi^2 > \mu_{o@25°},1,0) - 7.3786 S_w^*)}}\right) \tag{12}$$

$$K_{rw} = 0.7428 S_{wc} S_{weff}\,\text{minimum}\,(1.8057, K_{air}) + \text{minimum}\left(0.3534 S_{weff}, \text{minimum}\left(\frac{0.0159 K_{air}}{K_{base}}, \text{minimum}\left(\frac{S_{wc}}{K_{base}}, S_{orw}^{\mu_{o@25°}}\right)\right)\right)$$

Quite similar to the early section, Fig. 17 displays the relative feature importance of each predictor in these correlations.



A B

Fig. 17. The relative importance of each predictor (A) K_{ro}, (B) K_{rw}

Observation in Fig. 17 goes to show that among the main parameter for predicting relative water permeability, the K_{air} has a massive contribution than to other parameters like Effective water saturation. Whereas normalized water saturation, in addition to K_{air}, plays a major role in predicting the relative permeability of oil. Fig. 18 plotted the predicted relative permeability using Eq. 12 and Eq. 13 versus actual data when both endpoint data and wettability type is unclear. As shown, although we access limited data in this condition, it is certified predicted relative permeability by new correlations has acceptable results.

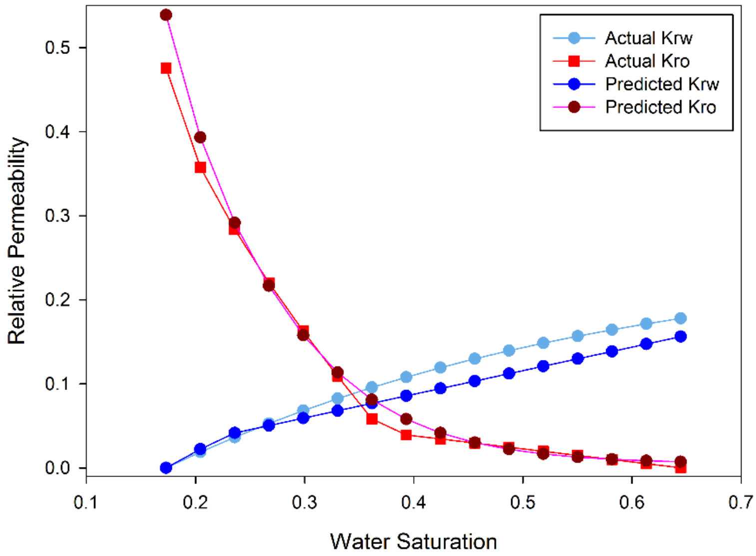


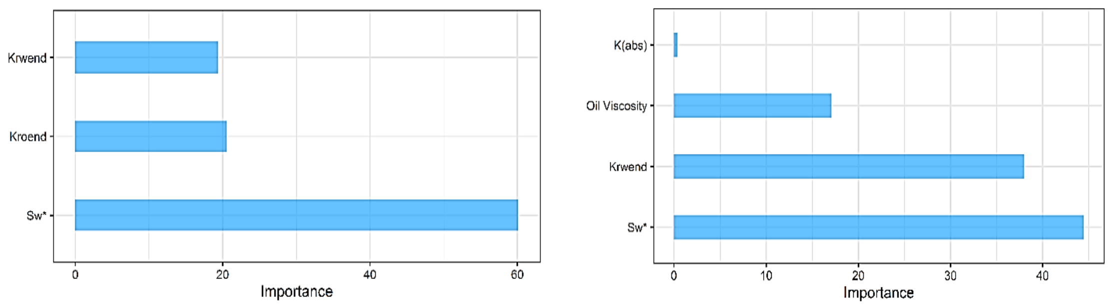
Fig. 18. Comparison between new correlations and real data, unknown wettability, K_{rend} is unavailable

After all, provided correlations when endpoint data is available while wettability is unknown are presented for oil and water relative permeability as follows:

$$(K_{ro})_e = K_{roend} 0.05290^{S_W^*} - K_{roend} \text{minimum}(2.4952 K_{roend}^2, 0.0975^{S_W^*}) \times \text{minimum}(S_W^* (\text{IF}(0.1491 k_{roend} < K_{rwend,}, 1{,}0), 0.4582) \tag{12}$$

$$(K_{rw})_e = K_{rwend} {S_W^*}^{0.6277} - K_{rwend} \text{minimum}(S_W^* - {S_W^*}^{2.2473}, \text{minimum}(0.0317 {\mu_{oil}}^2 \times e^{\text{IF}(K_{abs} > K_{rwend}, 1, o)}, K_{rwend}), K_{rwend}) \tag{13}$$

We assessed to recognize the most influential parameter on the measure of relative permeability when endpoint data is available similar to previous sections, see Fig. 19.



A B

Fig. 19. The relative importance of each predictor (A)$(K_{ro})_e$, (B)$(K_{rw})_e$

We checked the correlation's accuracy versus actual data on two full relative permeability curves. As seen in Fig. 20, the predicted value compared with actual data showed a noteworthy accuracy despite the absence of wettability.

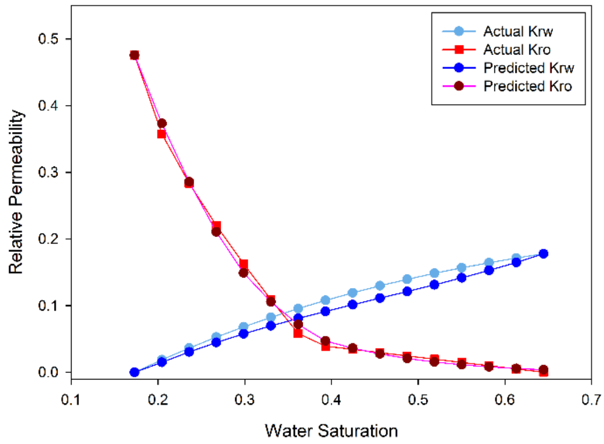


Fig. 20. Comparison between new correlations and real data, unknown wettability, K_{rend} is available

To compare all presented correlations' accuracy, the R^2, MAE, MSE, and MAPE were calculated for testing (unseen) data, and the results are shown in Table 2.

Table 2. The accuracy and errors of presented correlations

Relative Permeability	Wettability	K_{rend}	R^2	MAE	MSE	MAPE
K_{rw}	Oil Wet	Unavailable	0.6615	0.0442	0.0054	0.8909
K_{rw}	Oil Wet	Available	0.9607	0.0155	0.0005	0.1925
K_{ro}	Oil Wet	Unavailable	0.82147	0.0385	0.0061	0.9326
K_{ro}	Oil Wet	Available	0.9906	0.0123	0.0003	0.5138
K_{rw}	Water Wet	Unavailable	0.8651	0.0119	0.0003	1.6505
K_{rw}	Water Wet	Available	0.9311	0.0078	0.0003	0.2289
K_{ro}	Water Wet	Unavailable	0.9232	0.0326	0.0036	0.7210
K_{ro}	Water Wet	Available	0.9697	0.02487	0.0014	0.4521
K_{rw}	Unknown	Unavailable	0.6524	0.0402	0.0041	1.7889
K_{rw}	Unknown	Available	0.9556	0.0151	0.0005	0.2734
K_{ro}	Unknown	Unavailable	0.8212	0.0441	0.0066	1.0413
K_{ro}	Unknown	Available	0.9799	0.0164	0.0007	0.4921

In summary, the correlations presented in this study for all possible situations are as follows:

Oil-wet wettability, K_{rend} is unavailable:

$$(k_{ro})_{o-w} = \frac{7.345\varphi S_w \mathrm{IF}(\varphi \leq (6.7928\varphi)^{K_{air}}, 1, 0) + \mathrm{maximum}(S_w, 0.2363)}{e^{(5.8311 S_w^*)}}$$

$$(k_{rw})_{o-w} = \mathrm{minimum}(S_{weff}, 0.0143 + 0.0661 S_{weff} + \mathrm{maximum}(0.2961 S_{weff}, 0.6521 S_{wc} S_{weff} \mu_w^2) - 0.0036 K_{base} \mu_{o@25^\circ} S_{weff}$$

Oil-wet wettability, K_{rend} is available:

$$((K_{ro})_{o-w})_e = K_{roend} + 3.9100\, S_w^* K_{roend} \text{minimum}(\, 0.3842\, , S_w^*) - 1.6558 S_w^* K_{roend} - 1.9540\, K_{roend} \text{minimum}(0.4331\, , S_w^*)$$

$$((K_{rw})_{o-w})_e = K_{rwend} S_W^{*\ \text{maximum}\left(0.8187\, , 0.5596 + 4.6857 S_{wc} S_W^* - \frac{1.9124 S_w}{\mu_{o@25^\circ}{}^2} + \text{minimum}(K_{base}, S_{orw})\right)}$$

Water-wet wettability, K_{rend} is unavailable:

$$(K_{ro})_{w-w} = 8.3634 \times 10^{-16} \times 0.0004^{S_W^*} S_{weff}^2\, \text{Factorial}(\mu_{o@25^\circ}) + \text{minimum}(0.7050\, , \text{minimum}\big(K_{air}, 4.4751 S_w (0.0011 S_{weff})^{S_W^*}\big))$$

$$(K_{rw})_{w-w} = 0.0029 + 0.9132\, \text{minimum}(S_{wc} S_W^*\, , \text{minimum}(0.4982 S_{weff}\, , \varphi\, S_{weff} + 0.0597 S_{wc} K_{air}^2)) - 0.0141\, K_{base} S_W^*$$

Water-wet wettability, K_{rend} is available:

$$((K_{ro})_{w-w})_e = 0.0057 + 1.0646 K_{roend} (0.0052 \mu_{o@25^\circ})^{S_W^*} - 0.0055 \mu_{o@25^\circ} K_{roend}$$

$$((K_{rw})_{w-w})_e = \text{minimum}(0.0691 \varphi K_{roend} + 0.596 \mu_w S_W^* K_{rwend}\ \ , S_W^*)$$

Unknown wettability, K_{rend} is unavailable:

$$K_{ro} = 0.0047 + \text{minimum}\ (\ K_{air}\, , 1.2035 \frac{1}{1 + e^{-(18.9880\ \varphi S_w - \text{IF}(596\ \varphi^2 > \mu_{o@25^\circ}, 1,0) - 7.3786 S_w^*)}})$$

$$K_{rw} = 0.7428 S_{wc} S_{weff}\, \text{minimum}\ (1.8057\, , K_{air}) + \text{minimum}\ (0.3534 S_{weff}\, , \text{minimum}\left(\frac{0.0159 K_{air}}{K_{base}}, \text{minimum}\ \left(\frac{S_{wc}}{K_{base}}, S_{orw}^{\mu_{o@25^\circ}}\right)\right))$$

Unknown wettability, K_{rend} is available:

$$(K_{ro})_e = K_{roend} 0.05290^{S_W^*} - K_{roend} \text{minimum}(2.4952 K_{roend}^2, 0.0975^{S_W^*}) \times \text{minimum}(S_W^* \big(\text{IF}(0.1491 k_{roend} < K_{rwend,}, 1,0\big), 0.4582)$$

$$(K_{rw})_e = K_{rwend} S_W^*{}^{0.6277} - K_{rwend} \text{minimum}(S_W^* - S_W^*{}^{2.2473}, \text{minimum}(0.0317 \mu_{oil}{}^2 \times e^{\text{IF}(K_{abs} > K_{rwend}, 1, 0)}, K_{rwend})\, , K_{rwend})$$

Conclusion

Twelve utterly new prediction equations for two-phase (water-oil) relative permeability have been developed successfully through data mining modeling using a symbolic regression algorithm for twelve possible situations that commonly exist in petroleum engineering. Prediction correlations for oil-wet, water-wet, and unknown wettability in addition to the availability of endpoint data or not were provided by using 225 SCAL datasets and almost 3800 observations from Iranian carbonated reservoirs. The scatter matrix plot is applied to analyze all variables. Moreover, the relative impact within these correlations that a variable has on the relative permeability has been investigated. It was concluded that the availability of endpoint values has much impact on the accuracy of correlations. Although it is advantageous to have endpoints, without them, an acceptable prediction can be made. Overall, this approach showed precise results to predict relative permeability curves, so that it can be a good candidate for usage in any petroleum engineering software.

Nomenclature

K_{rw}	Relative permeability to water
K_{ro}	Relative permeability to oil
K_{abs}	Absolute permeability
K_{rwend}	End point relative permeability to water
K_{roend}	End point relative permeability to oil
K_{rend}	End point relative permeability
K_{air}	Absolute permeability to air
K_{water}	Absolute permeability to water
K_{base}	Base permeability, in this study K_{water} used as K_{base}
$(k_{ro})_{o-w}$	Relative permeability to oil, oil-wet system, and end point data is unavailable
$(k_{rw})_{o-w}$	Relative permeability to water, oil-wet system, and end point data is unavailable
$((K_{ro})_{o-w})_e$	Relative permeability to oil, oil-wet system, and end point data is available
$((K_{rw})_{o-w})_e$	Relative permeability to water, oil-wet system, and end point data is available
$(K_{ro})_{w-w}$	Relative permeability to oil, water-wet system, and end point data is unavailable
$(K_{rw})_{w-w}$	Relative permeability to water, water-wet system, and end point data is unavailable
$((K_{ro})_{w-w})_e$	Relative permeability to oil, water-wet system, and end point data is available
$((K_{rw})_{w-w})_e$	Relative permeability to water, water-wet system, and end point data is available
$(K_{ro})_e$	Relative permeability to oil, wettability is unknown, and end point data is available
$(K_{rw})_e$	Relative permeability to water, wettability is unknown, and end point data is available
S_{wc}	Connate (irreducible) water saturation
S_{orw}	Residual oil saturation in the water-oil system
$S_w^*=$	Normalized water saturation
S_{weff}	Effective water saturation
μ_w	Water viscosity (cp)
μ_o	Oil viscosity (cp)
SVM	Support vector machine
ANN	Artificial neural network
MLP:	Multilayer perceptron
R^2	R squared (coefficient of determination in statistics)
MAE	Mean absolute error
MAPE	Mean absolute percent error
MSE	Mean squared error

References

[1] Honarpour MM, Koederitz L, Harvey AH. Relative permeability of petroleum reservoirs. Relative Permeability of Petroleum Reservoirs. CRC press; 2018. 1–143.

[2] Heaviside J, Black CJJJ, Berry JF. Fundamentals of relative permeability: experimental and theoretical considerations. In: SPE Annual Technical Conference and Exhibition. Society of Petroleum Engineers; 1983.

[3] Benjamin WJ, Ahmed T. Reservoir engineering handbook. Vol. 27, Cardiovascular Imaging. Gulf professional publishing; 2018. 11–13, 11f, 537, 992–1001 p. Available from: http://www.lavoisier.fr/notice/fr283071.
[4] Purswani P, Johns RT, Karpyn ZT, Blunt MJ. Predictive Modeling of Relative Permeability Using a Generalized Equation of State. SPE J. 2021;26(01):191–205.
[5] Khaz'ali AR, Emamjomeh A, Andayesh M. An accuracy comparison between artificial neural network and some conventional empirical relationships in estimation of relative permeability. Pet Sci Technol. 2011;29(15):1603–14.
[6] Miller MA, Ramey HJ, Ramey Jr HJ. Effect of temperature on oil/water relative permeabilities of unconsolidated and consolidated sands. Soc Pet Eng J. 1985;25(06):945–53.
[7] Sedaee Sola B, Rashidi F, Babadagli T. Temperature effects on the heavy oil/water relative permeabilities of carbonate rocks. J Pet Sci Eng. 2007;59(1–2):27–42.
[8] Esmaeili S, Modaresghazani J, Sarma H, Harding T, Maini B. Effect of temperature on relative permeability – Role of viscosity ratio. Fuel. 2020;278(June):118318. Available from: https://doi.org/10.1016/j.fuel.2020.
[9] Jahanbakhsh A, Sohrabi M, Fatemi SM, Shahverdi H. A comparative study of the effect of gas/oil IFT variation on two- and three-phase relative permeability and the performance of WAG injection at laboratory scale. Proc - SPE Symp Improv Oil Recover. 2016;2016-Janua.
[10] Mai A, Kantzas A. Heavy oil waterflooding: effects of flow rate and oil viscosity. J Can Pet Technol. 2009;48(03):42–51.
[11] Morgan JT, Gordon DT. Influence of pore geometry on water- oil relative permeability. JPT, J Pet Technol. 1970;22(10):1199–208.
[12] Zhao B, Ratnakar R, Dindoruk B, Mohanty K. A Hybrid Approach for the Prediction of Relative Permeability Using Machine Learning of Experimental and Numerical SCAL Data. In: SPE Annual Technical Conference and Exhibition. Society of Petroleum Engineers; 2019.
[13] Gates I, Gates JI, Lietz WT. Relative permeabilities of California cores by the capillary-pressure method. In: Drilling and production practice. American Petroleum Institute; 1950.
[14] Chwang AT, Corey AT, Rathjens CH. Effect of stratification on relative permeability. J Pet Technol. 1956;8(12):69–71.
[15] Torcaso MA, Wyllie MRJRJ. A comparison of Calculated krg/kro Ratios with a Correlation of Field Data. J Pet Technol. 1958;10(12):57–8.
[16] Pirson SJ. Petrophysics. Oil Reserv Eng ed) New York, USA McGraw-Hill B Co, Inc. 1958.
[17] Naar J, Henderson JHH. An imbibition model-its application to flow behavior and the prediction of oil recovery. Soc Pet Eng J. 1961;1(02):61–70.
[18] Brooks RH, Corey AT, Brentrup G, Moawad HMM, Jain H. Hydraulic properties of porous media. Phys Chem Glas Eur J Glas Sci Technol Part B. 1964;49(6):293–6.
[19] Land CS. Comparison of calculated with experimental imbibition relative permeability. Soc Pet Eng J. 1971;11(04):419–25.
[20] Owens WW, Archer Dl. The effect of rock wettability on oil-water relative permeability relationships. J Pet Technol. 1971;23(07):873–8.
[21] Carlson FM. Simulation of relative permeability hysteresis to the nonwetting phase. In: SPE annual technical conference and exhibition. Society of Petroleum Engineers; 1981.
[22] Honarpour M, Koederitz LF, Harvey AH. Empirical equations for estimating two-phase relative permeability in consolidated rock. J Pet Technol. 1982;34(12):2–905.
[23] Timmerman EH. Practical Reservoir Engineering, Part II, Methods for Analyzing Output from Equations and Computers. Penn Well Books, Penn Well Publishing Company, Tulsa, OK, USA; 1982.
[24] Fulcher RA, Ertekin T, Stahl CD, Fulcher Jr RA, Ertekin T, Stahl CD. Effect of capillary number and its constituents on two-phase relative permeability curves. J Pet Technol. 1985;37(02):249–60.
[25] Alpak FO, Lake LW, Embid SM. Validation of a modified Carman-Kozeny equation to model two-phase relative permeabilities. In: SPE Annual Technical Conference and Exhibition. Society of Petroleum Engineers; 1999. p. 1–11.
[26] Mohamad Ibrahim MN, Koederitz LF, Ibrahim MN, Koederitz LF. Two-phase relative permeability prediction using a linear regression model. In: SPE eastern regional meeting. Society of Petroleum Engineers; 2000. p. 111–21.

[27] Behrenbruch P, Goda HM. Two-Phase Relative Permeability Prediction: A comparison of the modified Brooks-Corey methodology with a new Carman-Kozeny based flow formulation. In: SPE Asia Pacific Oil & Gas Conference and Exhibition. Society of Petroleum Engineers; 2006. p. 810–27.

[28] Ghanbarian B, Ioannidis MA, Hunt AG, Perry LK. Theoretical insight into the empirical tortuosity-connectivity factor in the Burdine-Brooks-Corey water relative permeability model. Water Resour Res. 2017;53(12):10395–410.

[29] Andrew M. Permeability prediction using multivariant structural regression. In: E3S Web of Conferences. EDP Sciences; 2020. p. 4001.

[30] Jung S, Sabharwal M, Jarauta A, Wei F, Gingras M, Gostick J, et al. Estimation of Relative Transport Properties in Porous Transport Layers Using Pore-Scale and Pore-Network Simulations. J Electrochem Soc. 2021;168(6):64501.

[31] Mehmani Y, Tchelepi H. Pore-network modeling vs. direct numerical simulation: A comparative study. In: Agu fall meeting abstracts. 2016. p. H51D-1519.

[32] Sorbie KS, Skauge A. Can network modeling predict two-phase flow functions? Petrophysics. 2012;53(06):401–9.

[33] Raoof A, Nick HM, Hassanizadeh SM, Spiers CJ. PoreFlow: A complex pore-network model for simulation of reactive transport in variably saturated porous media. Comput Geosci. 2013;61:160–74.

[34] Zhao B, Ratnakar R, Dindoruk B, Mohanty K. A Hybrid Approach for the Prediction of Relative Permeability Using Machine Learning of Experimental and Numerical SCAL Data. SPE Annu Tech Conf Exhib. 2019;25(5):2749–64.

[35] Luo G, Tian Y, Bychina M, Ehlig-Economides C. Production optimization using machine learning in Bakken shale. In: Unconventional Resources Technology Conference, Houston, Texas, 23-25 July 2018. Society of Exploration Geophysicists, American Association of Petroleum …; 2018. p. 2174–97.

[36] Bestagini P, Lipari V, Tubaro S. A machine learning approach to facies classification using well logs. In: Seg technical program expanded abstracts 2017. Society of Exploration Geophysicists; 2017. p. 2137–42.

[37] Bahonar E, Ghalenoei Y, Chahardowli M, Simjoo M. New correlations to predict oil viscosity using data mining techniques. J Pet Sci Eng. 2021;109736.

[38] Ramirez AM, Valle GA, Romero F, Jaimes M. Prediction of PVT properties in crude oil using machine learning techniques MLT. In: SPE Latin America and Caribbean Petroleum Engineering Conference. Society of Petroleum Engineers; 2017.

[39] Ahmadi MA, Chen Z. Analysis of gas production data via an intelligent model: Application to natural gas production. First Break. 2018;36(12):91–8.

[40] Al-Fattah SM, Al-Naim HA. Artificial-intelligence technology predicts relative permeability of giant carbonate reservoirs. SPE Reserv Eval Eng. 2009;12(1):96–103.

[41] Graczyk KM, Matyka M. Predicting porosity, permeability, and tortuosity of porous media from images by deep learning. Sci Rep. 2020;10(1):1–11.

[42] Kalam S, Khan M, Khan RA, Alam MMM, Sadeed A, Mahmoud M, et al. Advancing Relative Permeability Estimation Through Data-Driven Modeling. In: Abu Dhabi International Petroleum Exhibition & Conference. OnePetro; 2020.

[43] Adibifard M, Talebkeikhah M, Sharifi M, Hemmati-Sarapardeh A. Iterative Ensemble Kalman Filter and genetic algorithm for automatic reconstruction of relative permeability curves in the subsurface multi-phase flow. J Pet Sci Eng. 2020;192(March 2019):107264.

[44] Mirzaei-Paiaman A. New methods for qualitative and quantitative determination of wettability from relative permeability curves: Revisiting Craig's rules of thumb and introducing Lak wettability index. Fuel. 2021;288(November 2020):119623.

How to cite: Bahonar E, Ghalenoei Y, Salmani S. New Relative Permeability Correlations for Carbonate Reservoirs through Data-Driven Modeling. Journal of Chemical and Petroleum Engineering. 2022; 56(1): 15-35.

Appendices

Definition of Statistical Parameters:

Pearson Correlation Coefficient

$$r = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2 \sum(y_i - \bar{y})^2}}$$

r = correlation coefficient
x_i = variable values in x axis
$\bar{x}$ = mean values of variable in x axis
y_i = variable values in y axis
$\bar{y}$ = mean values of varible in y axis

Coefficient of Determination R^2 (Graphically)

$$R^2 = 1 - \frac{SSE}{SS_{yy}}$$

SSE: Deviation of experimental data from predicted viscosity

$$SSE = \sum_{i-1}^{n} \left(\mu_{experimental} - \mu_{est}\right)^2$$

SS_{yy} : Deviation of experimental data from mean value of viscosity that defines as follows:

$$SS_{yy} = \sum_{i-1}^{n} \left(\mu_{experimental} - \mu_{mean}\right)^2$$

Mean Absolute Error (MAE)

$$MAE = \frac{\sum_{i=1}^{n} |y_i - x_i|}{n} = \frac{\sum_{i=1}^{n} |e_i|}{n}$$

Mean Squared Error (MSE)

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (y_i - y)^2$$

y_i = actual value
y = predicted value
N = number of data points

Absolute Percentage Error (APE)

$$E_i = \left| \frac{\mu_{exp} - \mu_{est}}{\mu_{exp}} \right|$$

Mean Absolute Percentage Error (MAPE)

$$E_r = \frac{1}{n} \sum_{i=1}^{n} E_i$$