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Generation of Entanglement in Qutrit Spin Coherent States by Nonlinear Hamiltonian

A. Naji*, N. Shadman

Department of Physics, Faculty of Science, Shahid Chamran University of Ahvaz, Ahvaz, Islamic Republic of Iran

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Abstract

We study entanglement in coherent spin states and several superpositions of multi-qutrit coherent states evolved under the one-axis counter-twisting Hamiltonian in the presence and absence of a magnetic field. Considering a non-entangled multi-qutrit spin coherent state as an initial one, it is found that the entanglement is instigated with an oscillatory behavior in time; however, its average is a decreasing function of the magnetic field. Also, we observe that under this Hamiltonian, the two-qutrit superposed state retains its maximum entanglement with no change, while, the negativity for the three-qutrit superposed state oscillates in time and its average increases in the presence of the magnetic field.

Keywords: Entanglement; Qutrit; Coherent states; One-axis counter-twisting Hamiltonian.

Introduction

Entanglement is an important resource with applications in quantum information theory and quantum computations [1-3]. The spin coherent states and their superpositions have many applications in quantum information [4]. Recently high dimensional systems, including multi-qutrit ones have been the focus of more attention due to their usefulness in achieving different tasks, for example dense coding [5-11]. The generation of entanglement to the coherent states have been an interesting topics in quantum information [11-13]. The one-axis counter-twisting Hamiltonian generates spin squeezing in multi qubit states [14, 15]. The close relationship between spin squeezing and quantum entanglement motivates us to apply this Hamiltonian for production of entanglement in multi qutrit systems.

The aim of this paper is to study the production of entanglement in multi-qutrit systems using a nonlinear Hamiltonian and application of a magnetic field. We organize our paper as follows. We define the qutrit coherent states and the superpositions of qutrit coherent states. Also, the entangling Hamiltonian is introduced. Next, we study the entanglement of the above mentioned quantum systems under the one-axis counter twisting Hamiltonian, in the presence and absence of a magnetic field. The negativity of the systems obtain analytically and numerically. Finally, the last section is devoted to discussions.

Materials and Methods

1. Qutirt Spin Coherent States

A qutrit is a three-level system in Hilbert space H^3. This state may be described in a three-dimensional Hilbert space by an orthonormal basis $|1,1\rangle$, $|1,0\rangle$, $|1,-1\rangle$.

The spin coherent state is defined by Radcliffe as [13]

* Corresponding author: Tel: +989166125496; Email: a.naji@scu.ac.ir

$$|\alpha\rangle=\frac{1}{\left(1+|\alpha|^2\right)^j}e^{\alpha J_+}|j,-j\rangle=\frac{1}{\left(1+|\alpha|^2\right)^j}\sum_{m=0}^{2j}\frac{\alpha^m}{m!}J_+^m|j,-j\rangle \qquad (1)$$

Using the definition of the ladder operators

$$J_+^m|j,-j\rangle=m!\binom{2j}{m}^{\frac{1}{2}}|j,-j+m\rangle \qquad (2)$$

We obtain

$$|\alpha,j\rangle=\frac{1}{\left(1+|\alpha|^2\right)^j}\sum_{m=-j}^{j}\binom{2j}{m+j}^{\frac{1}{2}}\alpha^{j+m}|j,m\rangle \qquad (3)$$

Where $|j,m\rangle$ are the eigenvectors of the angular momentum operators $\hat{J}^2$ and $\hat{J}_z$ with eigenvalues equal to $j(j+1)$ and m, respectively. Now, for $j=1$ we obtain

$$|\alpha,1\rangle=\frac{1}{1+|\alpha|^2}\left[|2\rangle+\sqrt{2}\alpha|1\rangle+\alpha^2|0\rangle\right] \qquad (4)$$

where

$$|1,1\rangle\equiv|0\rangle,\ |1,0\rangle\equiv|1\rangle,\ |1,-1\rangle\equiv|2\rangle. \qquad (5)$$

We consider a general superposition of two and three of these qutrit spin coherent states as follows

$$|\psi\rangle=\cos\theta\left(|\alpha\rangle\otimes|\beta\rangle\right)+e^{i\phi}\sin\theta\left(|\alpha'\rangle\otimes|\beta'\rangle\right) \qquad (6)$$

$$|\psi'\rangle=\cos\theta\left(|\alpha\rangle\otimes|\beta\rangle\otimes|\gamma\rangle\right)+e^{i\phi}\sin\theta\left(|\alpha'\rangle\otimes|\beta'\rangle\otimes|\gamma'\rangle\right) \qquad (7)$$

In the following sections the entanglement properties of these states will be considered.

2. Nonlinear Hamiltonian

We consider the following Hamiltonian introduced by Kitagawa and Ueda [14]

$$H=\chi J_x^2+\mu J_y^2+\gamma\left(J_xJ_y+J_yJ_x\right)+f\left(J_z\right) \qquad (8)$$

Where the collective spin operators may define for qutrit systems as

$$J_{\alpha x}=\sum_{i=1}^{N}j_{i\alpha}\ \left(\alpha=x,y,z\right) \qquad (9)$$

where $j_{i\alpha}$ are the angular momentum operators for the i^{th} qutrit. We consider the action of this Hamiltonian on the states (6) and (7) which turns out to lead to the production of squeezed states.

By substituting $\mu=\gamma=f(J_z)=0$ in equation (8), the one axis counter twisting Hamiltonian reads

$$H=\chi J_x^2$$
(10)

Also, in the presence of the magnetic fields, we have

$$H_1=\chi J_x^2+BJ_z \qquad (11)$$

These are called one axis counter twisting Hamiltonian and have applications in the study of quantum optical systems, quantum dots and Bose-Einstein condensates [15-21]. It is interesting to note that similar investigations have been done using the two-axis counter twisting Hamiltonian, with two nonlinear terms, in Reference [12].

The negativity is introduced as measure of entanglement for mixed bipartite state by Vidal [22]. The negativity is sufficient and necessary for 2×2 and 2×3 systems while for higher dimensional ones, including 3×3 necessary condition is not enough [22-23]. The negativity is defined as follows

$$N(\rho)=\frac{\left\|\rho^{T_A}\right\|-1}{d-1}, \qquad (12)$$

where d is dimension of qudit systems and $\left\|\rho^{T_A}\right\|$ is the trace norm of the partial transpose density matrix ρ^{T_A}. For the qutrit systems, it is equal to the absolute value of the sum of negative eigenvalues of ρ^{T_A} [22]. When $0<N(\rho)\le1$ the two qutrits are entangled.

3. Bipartite Entanglement in Multi-Qutrit Systems

We consider a coherent state of two qutrits, in which both are in the ground state and according to equation (5), we write

$$|\Psi\rangle_0=|2,2\rangle. \qquad (13)$$

Action of the Hamiltonian (10) on the state (13) leads to the partial transposed of density matrix of the system as follow

$$\rho^{T_A}=\begin{pmatrix} m & 0 & q & 0 & 0 & 0 & p & 0 & r \\ 0 & 0 & 0 & 2p & 0 & 2r & 0 & 0 & 0 \\ p & 0 & r & 0 & 0 & 0 & u & 0 & v \\ 0 & 2q & 0 & 0 & 0 & 0 & 0 & 2r & 0 \\ 0 & 0 & 0 & 0 & 4r & 0 & 0 & 0 & 0 \\ 0 & 2r & 0 & 0 & 0 & 0 & 0 & 2v & 0 \\ q & 0 & u^* & 0 & 0 & 0 & r & 0 & s \\ 0 & 0 & 0 & 2r & 0 & 2s & 0 & 0 & 0 \\ r & 0 & s & 0 & 0 & 0 & v & 0 & n \end{pmatrix} \qquad (14)$$

where

$$m=-\frac{1}{4}\cos^2\chi t\left(3\sin^2\chi t+2\cos\chi t\right)$$

$$p=\frac{1}{16}\left(-\cos^2 2\chi t+e^{i\chi t}-e^{3i\chi t}+e^{-4i\chi t}\right)$$

$$q=\frac{1}{16}\left(-\cos^2 2\chi t+e^{-i\chi t}-e^{3i\chi t}+e^{4i\chi t}\right)$$

$$r=\frac{1}{4}\sin^2\chi t\cos^2\chi t$$

$$s = \frac{1}{16}\left(-\cos^2 2\chi t - e^{i\chi t} + e^{3i\chi t} + e^{-4i\chi t}\right)$$
$$u = -\frac{1}{4}\sin^2 \chi t\left(3\cos^2 \chi t + 2i\sin \chi t\right)$$
$$v = \frac{1}{16}\left(-\cos^2 2\chi t - e^{-i\chi t} + e^{3i\chi t} + e^{4i\chi t}\right) \qquad (15)$$

Using equations (12), (14) and (15), we finally have an expression (which we do not write it down) for the time dependent negativity, for the two-qutrit system at hand. We have also considered the action of Hamiltonian (10) on the following three and four-qutrit systems

$$|\psi_0\rangle = |2,2,2\rangle,\ |\psi_0\rangle = |2,2,2,2\rangle \qquad (16)$$

We have presented the negativity as a function of time for the two, three and four-qutrit systems in Figure 1. It is observed that the negativity oscillates in time but its amplitude is independent of χ . Only in some points where negativity is zero, the entanglement dies down. The comparison of curves shows that the negativity reduces as N is increased.

Now we consider the two-, three- and four-qutrit systems under the action of the Hamiltonian H_1 in equation (11), in the presence of a magnetic field. We have plotted the negativity as a function of time for these systems and several values of the field in Figures 2, 3 and 4 respectively. It is observed that the magnetic field decreases the negativity of the systems irrespective of the number of qutrits.

4. Entanglement in Superposition of Two-Qutrit Coherent States

We consider a pure state superposed of coherent states according to equation (6) as follows

$$|\psi\rangle = \cos\theta\,|\alpha\rangle\otimes|\beta\rangle + \sin\theta\,|\alpha\rangle\otimes|\beta\rangle \qquad (17)$$

We assume that the coherent state parameters are real and also satisfy the following relations

$\alpha = -\alpha'$, $\beta = -\beta'$ and $\varphi = 0$ (18)

Now, we choose $\alpha = \beta = 1$, $\theta = \frac{\pi}{4}$ and use equations (4) and (17) to write

$$|\psi\rangle = \frac{1}{2\sqrt{2}}\left(|0,0\rangle + |0,2\rangle + 2|1,1\rangle + |2,0\rangle + |2,2\rangle\right) \qquad (19)$$

Using equation (12) the negativity obtained for the above state is 0.5 which implies that it is entangled; however, the maximum entanglement is not attained. Now, we consider the action of Hamiltonian (10) on the

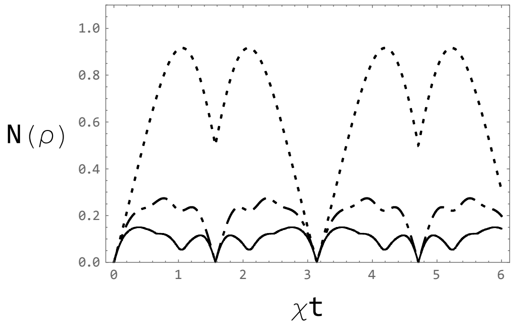

Figure 1. $N(\rho)$ as a function of χt for N=4 (solid line), N=3 (dash dotted line) and N=2 (dotted line).

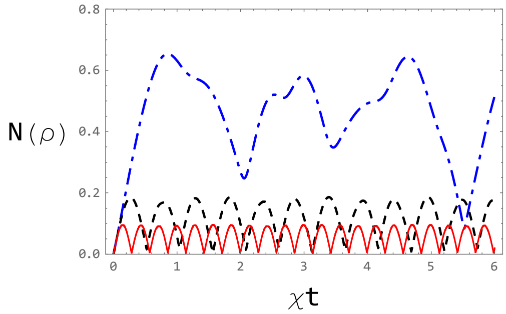

Figure 2. $N(\rho)$ as a function of χt for N=2; B=1 (dash dotted line), B=5 (dotted line) and B=10 (solid line).

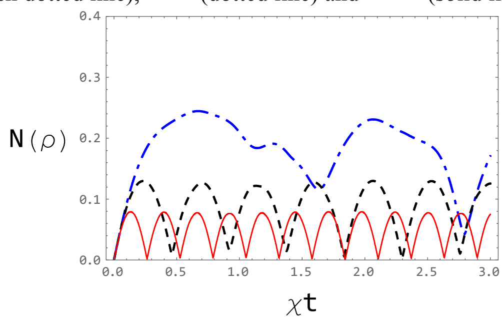

Figure 3. $N(\rho)$ as a function of χt for N=3; B=1 (dash dotted line), B=5 (dotted line) and B=10 (solid line).

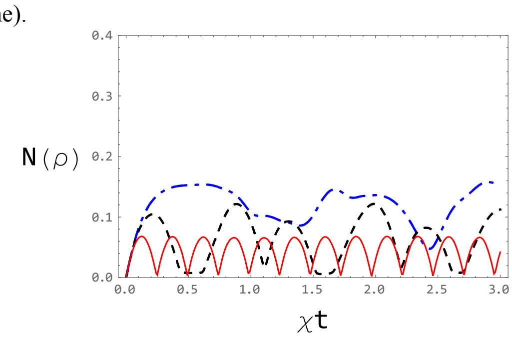

Figure 4. $N(\rho)$ as a function of χt for N=4; B=1 (dash dotted line), B=5 (dotted line) and B=10 (solid line).

coherent state (17), we observe that the reduced density matrix does not change in time and the negativity remains a constant equal to 0.5; that is the state (17) is the eigenvector of Hamiltonian (10).

Now, we consider the negativity of the state (17) in the presence of the field, evolved by the Hamiltonian H_1. Doing the calculations, we have plotted negativity for three values of the magnetic field in Figure 5. It is observed that the amplitude and also the time average of the oscillation are increasing functions of the magnetic field.

Considering Figure 6, we note that the negativity is an increasing function of θ.

5. Entanglement in Superposition of Two-Qutrit Coherent States (α=1, $\beta=2$, $\theta=\frac{\pi}{4}$)

Assuming α=1, $\beta=2$, $\theta=\frac{\pi}{4}$, Eq. (17) can be written as

$$|\psi\rangle=\frac{1}{\sqrt{50}}\left(4|0,0\rangle+|0,2\rangle+4|1,1\rangle+4|2,0\rangle+|2,2\rangle\right) \quad (20)$$

Considering the action of Hamiltonian (10) on this state, we obtain its partial transposed matrix state ρ^{T_A} as follows.

$$\rho^{T_A}=\begin{pmatrix} m & 0 & r & 0 & 0 & 0 & m & 0 & r \\ 0 & 0 & 0 & u & 0 & x & 0 & 0 & 0 \\ n & 0 & s & 0 & 0 & 0 & n & 0 & s \\ 0 & p & 0 & 0 & 0 & 0 & 0 & p & 0 \\ 0 & 0 & 0 & 0 & v & 0 & 0 & 0 & 0 \\ 0 & q & 0 & 0 & 0 & 0 & 0 & q & 0 \\ m & 0 & r & 0 & 0 & 0 & m & 0 & r \\ 0 & 0 & 0 & u & 0 & x & 0 & 0 & 0 \\ n & 0 & s & 0 & 0 & 0 & n & 0 & s \end{pmatrix} \quad (21)$$

The explicit expressions of non zero elements are skipped here for abbreviation.

Now, using equations (12) and (21) we obtain the time-dependent negativity as follows

$$N(\rho)=0.5[-1+0.01|59+9\cos 4\chi t|+0.014\left(|4757-324\cos 4\chi t-81\cos 8\chi t|\right)^{\frac{1}{2}}$$
$$+\left|0.41-0.09\cos 4\chi t-\left(5.5\times10^{-17}\right)i\sin 4\chi t\right|] \quad (22)$$

$N(\rho)$ is 0.46 for $\chi=0$; it checks that the initial state is entangled. We have plotted the negativity given by equation (22) in Figures 7 as a function of time. Figure 7 implies that negativity oscillates in time. Its amplitude is not a function of χ, but higher χ values, correspond to smaller time periods. Figure 7 shows that the entanglement is an increasing function of α.

Now, we consider the action of the Hamiltonian H_1 in equation (11) on the state (20). Performing similar

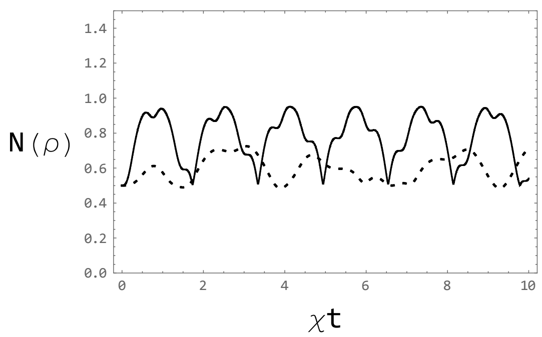

Figure 5. $N(\rho)$ as function of χt assuming $\alpha=\beta=1$ for $B=1$ (dotted line) and $B=5$ (solid line).

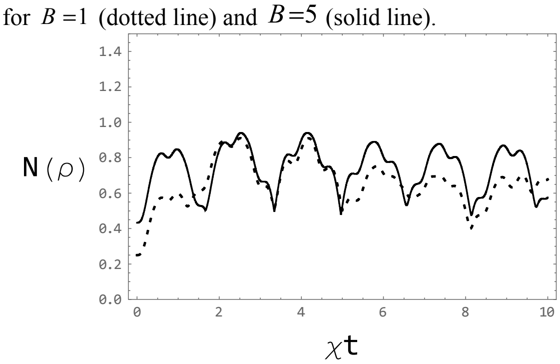

Figure 6. $N(\rho)$ as function of χt assuming $\alpha=\beta=1$, $B=1$ for $\theta=\frac{\pi}{12}$ (dotted line) and $\theta=\frac{\pi}{6}$ (solid line).

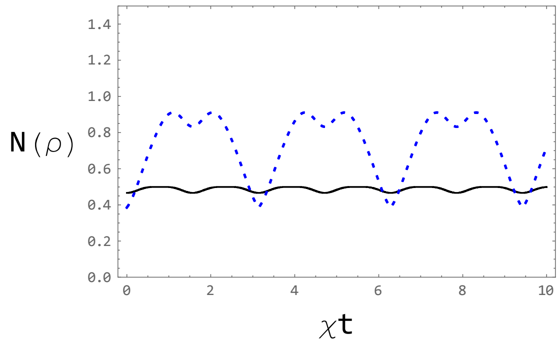

Figure 7. $N(\rho)$ as function of χt assuming $\beta=2$, $\varphi=0$, $\theta=\frac{\pi}{4}$ for $\alpha=2$ (dotted line) and $\alpha=1$ (solid line).

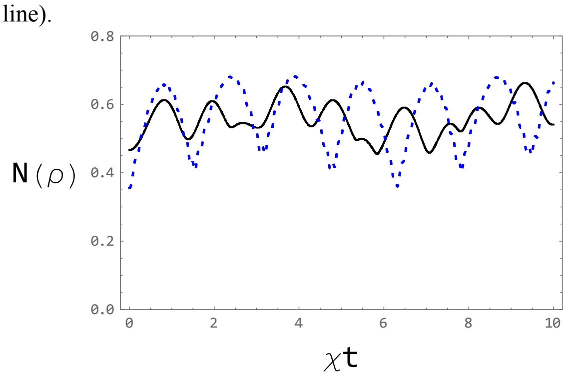

Figure 8. $N(\rho)$ as function of χt assuming $\alpha=1$, $\beta=2$, $\varphi=0$, $\theta=\frac{\pi}{4}$ for $B=20$ (dotted line) and $B=1$ (solid line).

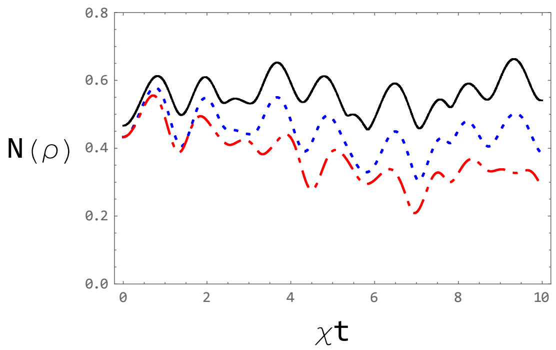

Figure 9. $N(\rho)$ as function of χt assuming $\alpha=1$, $\beta=2$, $B=1$, $\theta=\frac{\pi}{4}$ for $\varphi=0$ (solid line), $\varphi=\frac{\pi}{4}$ (dotted line) and $\varphi=\frac{\pi}{2}$ (dash dotted line).

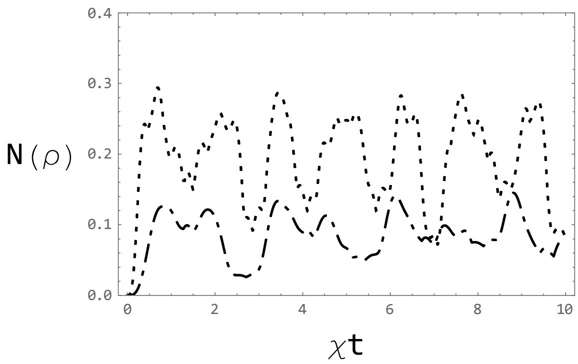

Figure 10. $N(\rho)$ as function of χt assuming $\alpha=\beta=\gamma=1$, $\varphi=0$, $\theta=\frac{\pi}{4}$ for $B=5$ (dotted line) and $B=1$ (dash dotted line).

calculations as above, we have plotted the negativity as a function of time for two values of field and several values of φ in Figures 8 and 9 respectively.

It is observed that the oscillation amplitude and the time average of the entanglement are increasing functions of the magnetic field. It is also noted that the time average of the negativity is a decreasing function of the parameter φ.

6. Entanglement in Superposition of Three-Qutrit Coherent States ($\alpha=\beta=\gamma=1$, $\theta=\frac{\pi}{4}$ *)*

We consider the three qutrits coherent state in equation (7). We assume that the coherent state parameters are real and $\alpha=-\alpha'$, $\beta=-\beta'$, $\gamma=-\gamma'$ and $\varphi=0$; therefore we have

$$|\psi\rangle=\frac{1}{4\sqrt{2}}(|0,0,0\rangle+|0,0,2\rangle+2|1,1,1\rangle+|0,2,0\rangle+|0,2,2\rangle+2|1,0,1\rangle+$$

$$2|1,1,0\rangle+2|1,1,2\rangle+2|1,2,1\rangle+|2,0,0\rangle+|2,0,2\rangle+2|2,1,1\rangle+|2,2,0\rangle+|2,2,2\rangle) \qquad (23)$$

Using equation (12), the negativity for this state turns out to be zero while its I-concerence is $\frac{\sqrt{5}}{2}$; that is this state is entangled.

Now, we consider the action of the Hamiltonian (10) on the state in equation (23). Performing some calculations, we obtain the negativity as a function of time. We have plotted negativity in Figures 10 and 11. Figure 10 shows that the negativity is an oscillating function of time and its time average is an increasing function of the magnetic field. Figure 11 implies that the negativity is a decreasing function of the parameter φ.

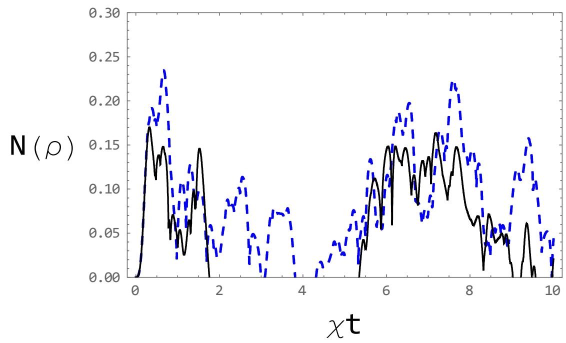

Figure 11. $N(\rho)$ as function of χt assuming $\alpha=\beta=\gamma=1$, $B=5$, $\theta=\frac{\pi}{4}$ for $\varphi=\frac{\pi}{4}$ (dotted line) and $\varphi=\frac{\pi}{2}$ (solid line).

7. Entanglement in Superposition of Three-Qutrit Coherent States ($\alpha=\beta=0$, $\gamma=1$, $\theta=\frac{\pi}{4}$ *)*

We consider the three qutrits coherent state in equation (7) with the parameter values $\alpha=-\alpha'$, $\beta=-\beta'$, $\gamma=-\gamma'$, $\varphi=0$, $\alpha=\beta=0$, $\gamma=1$ and $\theta=\frac{\pi}{4}$ to find

$$|\psi\rangle=\frac{1}{\sqrt{2}}(|2,2,0\rangle+|2,2,2\rangle)$$

(24)

We examine the entanglement properties of this state evolved by the Hamiltonian H given in equation (9). Performing some calculations as section 6, we obtain the partial transpose of the density matrix as follows

$$\rho^{T_A} = \begin{pmatrix} a & 0 & d & 0 & c & 0 & b & 0 & e \\ 0 & c & 0 & m & 0 & g & 0 & n & 0 \\ b & 0 & e & 0 & n & 0 & s & 0 & u \\ 0 & f & 0 & c & 0 & h & 0 & g & 0 \\ c & 0 & h & 0 & p & 0 & n & 0 & q \\ 0 & g & 0 & n & 0 & q & 0 & v & 0 \\ d & 0 & k & 0 & h & 0 & e & 0 & l \\ 0 & h & 0 & g & 0 & r & 0 & q & 0 \\ e & 0 & l & 0 & q & 0 & u & 0 & x \end{pmatrix} \quad (25)$$

The non zero elements of the density matrix of the system is too complicated to write out here, so we discuss the numerical results.

The negativity and I-concurrence are zero for state (25) at $\chi = 0$, as we expect. This state is not entangled at $t=0$ either. We have presented the negativity for the state in equation (25) as a function of time for two values of χ in Figure 12. It is observed that the negativity oscillates in time. Its amplitude is independent of χ, but the time periods are smaller for higher χ values. The state is entangled except at a set of discrete points, where the negativity is zero. Figure 12 also shows that the negativity is an increasing function of θ.

Finally, we consider the evolution of the Hamiltonian H_1 given by equation (11) on the state (24). Performing similar calculations as above, we have plotted the negativity as a function of time for two values of the field and several values of φ in Figures 13 and 14 respectively. It is observed that the amplitude is a decreasing function of the magnetic fields and φ.

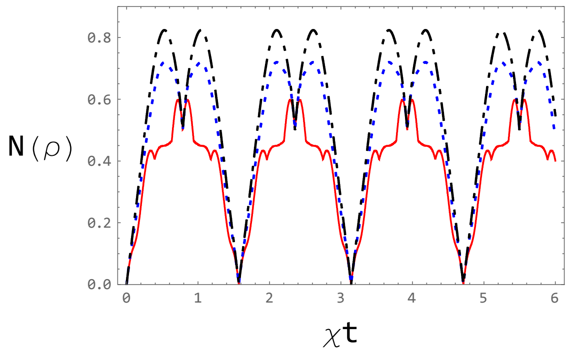

Figure 12. $N(\rho)$ as function of χt assuming $\alpha=\beta=0$, $\gamma=1$, $\varphi=0$ for $\theta=\frac{\pi}{4}$ (solid line). $\theta=\frac{\pi}{8}$ (dotted line) and $\theta=\frac{\pi}{12}$ (dash dotted line).

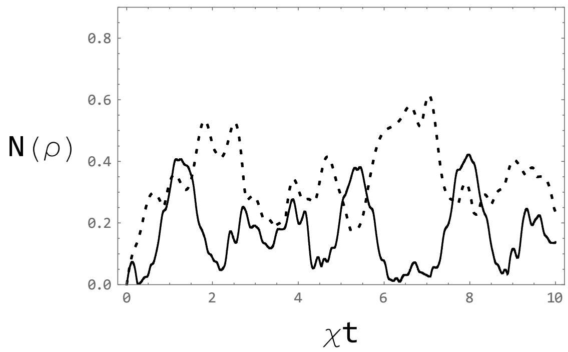

Figure 13. $N(\rho)$ as function of χt assuming $\alpha=\beta=0$, $\gamma=1$, $\varphi=0$, $\theta=\frac{\pi}{4}$ for $B=1$ (dotted line) and $B=10$ (solid line).

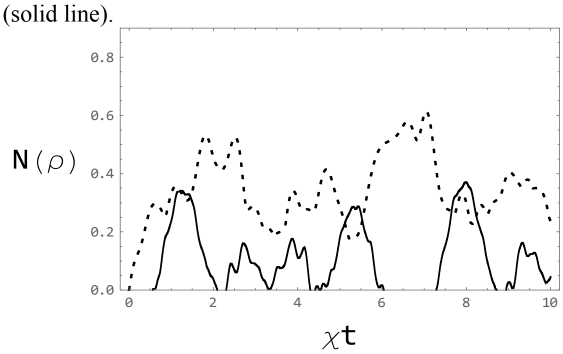

Figure 14. $N(\rho)$ as function of χt assuming $\alpha=\beta=0$, $\gamma=1$, $B=1$, $\theta=\frac{\pi}{4}$ for $\varphi=0$ (dotted line) and $\varphi=\frac{\pi}{6}$ (solid line).

Results and Discussion

We have studied the dynamics of entanglement of the coherent qutrit states and several superpositions of multi-qutrit coherent states under the influence of the one-axis counter twisting Hamiltonian in the presence and absence of magnetic fields. A summary of the important results follows:

a) The multi-qutrit coherent states are entangled under the one-axis counter twisting Hamiltonian H. Its entanglement is reduced under Hamiltonian H_1 in the presence of magnetic field. In fact the nonlinear Hamiltonian establishes some correlations among qutrit systems. While the magnetic field just rotates the qutrit systems; therefore, does nor increase and even decreases the entanglement. As negativity is a decreasing function of N, in large multi-qutrit systems, larger values of χ are required for stronger entanglement.

b) The superpositions of two-qutrit coherent states are entangled in all times under Hamiltonian H. The amplitude and also the time average of the oscillation of bipartite entanglement are increasing functions of the magnetic field; that is, one is able to generate the maximally entangled state in superpositions of two-qutrit coherent states by applying strong magnetic field. The amplitude of oscillation is an increasing function of α.

c) The entanglement of the superposition of three qutrits coherent states, is entangled under the influence of H except at a set of discrete points, where the negativity is zero. The magnetic field may have an adverse effect on the entanglement if H_1 is applied (for special case $\alpha = \beta = 0$, γ=1, $\varphi = 0$, $\theta = \frac{\pi}{4}$); therefore one can maximally entangle this state in all times by applying the small magnetic fields.

d) For all superposition of qutrit coherent states, the entanglement is a decreasing function of φ.

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