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Cylindrical Surface Wave Propagation on a Thin Plasma Layer

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Abstract

The surface waves propagating in a cylindrical thin plasma layer are studied. The cylindrical plasma layer is sandwiched between two regions of different dielectric constants. The linear dispersion relation is obtained by starting from the fluid model and Maxwell's equations. It is found that a hybridization occurred between the plasmonic oscillations and the acoustic excitation, which leads to a new surface mode in the present plasma system. Furthermore, it can be seen that the wave frequency is significantly tunable due to the optimization of the plasma parameters and the cylindrical geometry. Surface mode frequency can increase by increasing Fermi speed at low frequencies and approaching the light speed line. Using the present plasma layer model not only leads to a new coupling between the plasmonic oscillations and the acoustic excitation, but also provides a new mode that is more controllable due to the additional parameters of the model. The present results should be applicable for understanding the basic characteristics of plasma antenna, enantiomeric sensing devices and plasma-sensing based waveguides.

Keywords: Surface Waves; Plasma Layer; Plasmon-polariton.

Introduction

Two-dimensional (2D) plasma configurations deal with a growing intreset subject during the last years. By appearance and development of metamaterials, including the electron-ion plasma confined by special potentials within a two-dimensional configurations, the 2D plasma systems have attracted a great deal of attention for last decades; for instance in single layer graphene [1-3] and metallic layers described by a parabolic energy band [3-5].

In 2D electron gas, the surface waves have been studied by considering the Euler equation and the Maxwell's equations and linearizing them [7]. It has been shown that the excitation of plasmon oscillations can be possible in 2D gas configuration [8]. In 1973, Fetter employed the hydrodynamic description to study the basic features of Plasmon spectrum in a 2D electron gas with a uniform immobile neutralizing background [9]. The effect of an external magnetic field has also been examined on the spectrum of 2D Plasmons in electron gas to show how the physical feature changes under the influence of excerted magnetic field [10].

During the past four decades, the basic features of surface waves have been studied, especially in Cartesian geometries, along the interface of plasma-vacuum (or – dielectric medium) of bounded or semi-bounded systems owing to their potential applications in, eg.,

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plasma antennas, plasma waveguides, semiconductor science, laser physics and space science [11-19]. So, it may be expected that the basic features of surface modes in a cylindrical plasma configuration [18-22] are basically different that of a planar plasma [23-27] because of the harmonic aspects of the Bessel functions in the cylindrical configuration. On the other hand, Borg and Harris [28] have shown that the plasma columns can support the propagation of high-frequency surface waves and thus can be used instead of a metal antenna. Later, different physical properties of a plasma column (as a plasma antenna) have been investigated by several authors [28-30]. For example, current distribution, power pattern, directive radiation, and radiation performance. Propagation of slow waves in the metallic cylindrical waveguide including the current sources has beeb studied by the Jazi research group [31-33]. They investigated also the characteristics properties of THz mode in plasma waveguide [34,35]. The influence of surface waves on the acceleration of electrons in the plasma waveguide systems is studied in Ref. [35].

In this paper, we investigate the dispersion characteristics of surface waves in a 2D cylindrical plasma layer of radius R, sandwiched between two regions of different dielectric constants. It is found that a hybridization occurred between the plasmonic oscillations and acoustic excitation, which leads to a new surface mode in the present plasma system. The analytical expression of the dispersion relation of such hybrid surface modes is derived here for the first time. It can be seen that the wave frequency of the present surface mode is significantly tunable due to the optimization of the plasma parameters and the cylindrical geometry.

Mathematical Formulation

We consider a two-dimensional cylindrical electron plasma layer of radius R carrying an electric current, sandwiched between two different dielectric regions, with different dielectric constants ε_1 and ε_2. We suppose that the electron number density, the wave amplitude and, the wavenumber are constant along the azimuth direction. Next, we consider a TM surface wave with azimuth symmetry ($E \equiv (E_r, 0, E_z), B \equiv (0, B_\varphi, 0)$) along the z-axis (Fig. 1), to be governed by the following Maxwell's equations:

$$\frac{\partial}{\partial z}B_\varphi = \frac{i\omega}{c^2}\varepsilon_j E_r, \quad \frac{1}{r}\frac{\partial}{\partial r}(rB_\varphi) = \frac{-i\omega}{c^2}\varepsilon_j E_z, \quad \frac{\partial}{\partial z}E_r - \frac{\partial}{\partial r}E_z = i\omega B_\varphi \tag{1}$$

where ε_j, c, and ω denote the dielectric constant of

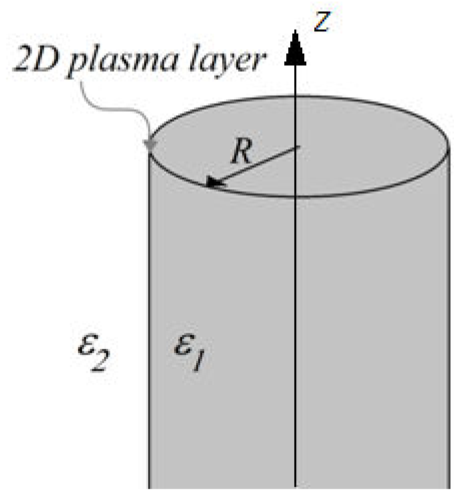

Figure 1. A schematic view of cylindrical plasma layer between two arbitrary dielectric mediums.

the jth medium, the light speed, and the wave frequency, respectively. By combining the components of equation (1), one can derive a set of independent differential equations for the magnetic and electric field components. Accordingly, it is easy to show that the z-component of the electric field obeys the following differential equation as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}E_z\right) + \frac{\partial^2}{\partial z^2}E_z + \frac{\omega^2}{c^2}\varepsilon_j E_z = 0 \tag{2}$$

We seek the solutions of Eq. (2), in the following form

$$\left(E_r^{(1.2)}, B_\varphi^{(1.2)}, E_z^{(1.2)}\right) \equiv \left(E_{r0}^{(1.2)}, B_{\varphi 0}^{(1.2)}, E_{z0}^{(1.2)}\right) e^{-i\omega t + iqz} \begin{cases} K_0(\kappa_2 r), & R < r \\ I_0(\kappa_1 r), & r < R \end{cases} \tag{3}$$

where the superscripts refer to the mediums 1 and 2, $\kappa_j^2 = q^2 - \varepsilon_j\,\omega^2/c^2$; also I_0 and K_0 are the zeroth-order modified Bessel's functions. The Maxwell's equations [1] together with the boundary conditions give us to write the solutions [3] in the following form.

$$\left(E_r^{(1.2)}, B_\varphi^{(1.2)}, E_z^{(1.2)}\right) \equiv E_0 \begin{pmatrix} \dfrac{qK_0'(\kappa_2 R)}{i\sqrt{q^2 - \varepsilon_2\frac{\omega^2}{c^2}}K_0(\kappa_2 R)}, \dfrac{\omega\varepsilon_2 K_0'(\kappa_2 R)}{ic^2\sqrt{q^2 - \varepsilon_2\frac{\omega^2}{c^2}}K_0(\kappa_2 R)}, 1 \\ \dfrac{qI_0'(\kappa_1 R)}{i\sqrt{q^2 - \varepsilon_2\frac{\omega^2}{c^2}}I_0(\kappa_1 R)}, \dfrac{\omega\varepsilon_1 I_0'(\kappa_1 R)}{ic^2\sqrt{q^2 - \varepsilon_2\frac{\omega^2}{c^2}}I_0(\kappa_1 R)}, 1 \end{pmatrix} e^{-i\omega t + iqz} \begin{cases} K_0(\kappa_2 r), & R < r \\ I_0(\kappa_1 r), & r < R \end{cases} \tag{4}$$

where $E_0 = E_{z0}^{(1)}$ and J_{ext} is the external current density exciting the plasma wave in the present physical system directed along the z-axis and is given by

$$J_{ext} = J_0 exp\,(-i\omega t + iqz) \quad (5)$$

In the present case, the z-component of the electric field can also appeared along with the cylindrical plasma layer as $E_z = E_0 exp\,(-i\omega t + iqz)$.

To derive the dispersion relation and the relevant energy relations, we employ the appropriate boundary conditions. The governing boundary condition on the tangential components of the magnetic field on either side of the plasma layer provide the following condition

$$B_\varphi^{(1)}\big|_{r=R} - B_\varphi^{(2)}\big|_{r=R} = \mu_0 J_{tot}|_z, \quad (6)$$

where $J_{tot} = J_{ext} + J_{on}$, with J_{on} being the polarization current density, given by [18]

$$J_{on} = i\omega\omega_{pe}^2\varepsilon_0 \frac{1}{\omega^2-\alpha q^2} E_z|_{r=R}. \quad (7)$$

Here, $\omega_{pe} = \sqrt{n_0\, e^2/m\,\varepsilon_0}$ is the plasma frequency, $\alpha = v_F^2/2$ with $v_F = \hbar\sqrt{2\pi n_0}/m$ denoting the electron Fermi speed. The boundary condition [6] together with Eq. [4] yield

$$D(\omega.q)E_0 = -iJ_0 \quad (8)$$

where

$$D(\omega.q) = \frac{\omega}{\mu_0 c^2}\left[\frac{\varepsilon_1 I_0'(\kappa_1 R)}{\kappa_1 I_0(\kappa_1 R)} - \frac{\varepsilon_2 K_0'(\kappa_2 R)}{\kappa_2 K_0(\kappa_2 R)} - \frac{\omega_{pe}^2}{\omega^2-\alpha q^2}\right] \quad (9)$$

In absence of the external current, one can obtain the dispersion relation by equating $D(\omega.q)$ to zero, i.e., $D(\omega.q) = 0$, which yields

$$\frac{I_0'(\kappa_1 R)}{I_0(\kappa_1 R)} - \frac{\varepsilon_2\kappa_1 K_0'(\kappa_2 R)}{\varepsilon_1\kappa_2 K_0(\kappa_2 R)} - \frac{\kappa_1}{\varepsilon_1}\frac{\omega_{pe}^2}{\omega^2-\alpha q^2} = 0 \quad (10)$$

The aforementioned equation represents the dispersion relation of surface plasmon polaritons coupled with electrostatic oscillations in a cylindrical plasma system. To the best of our knowledge, this is the first try to derive the dispersion relation [10]. In the case of large values of R (i.e., for planar interface), Eq. (10) reduces to

$$1 + \frac{\kappa_1\varepsilon_2}{\kappa_2\varepsilon_1} - \frac{\kappa_1}{\varepsilon_1}\frac{\omega_{pe}^2}{\omega^2-\alpha q^2} = 0 \quad (11)$$

In particular, if the ions are considered as immobile, the above dispersion relation agrees with that obtained by Moradi [19]. Next, we derive an expression for the attenuation length. For simplicity, let us restrict our attention to the special case of $\varepsilon_1 = \varepsilon_2$. In this case, by using the dispersion relation [10] we obtain

$$\kappa = \frac{\omega_{pe}^2}{2c^2}\frac{I_0(\kappa R)K_0(\kappa R)}{I_0'(\kappa R)K_0(\kappa R) - K_0'(\kappa R)I_0(\kappa R)} \pm \frac{1}{2}\sqrt{\left[\frac{\omega_{pe}^2}{c^2}\frac{I_0(\kappa R)K_0(\kappa R)}{I_0'(\kappa R)K_0(\kappa R) - K_0'(\kappa R)I_0(\kappa R)}\right]^2 - 4\left(\frac{\varepsilon\alpha}{c^2} - 1\right)q^2} \quad (12)$$

Thus, the attenuation length can be obtained from $l = 1/\kappa$, where the electromagnetic field reduces with coefficient $1/e$.

The axial component of the wave impedance of TM-modes ib absence of an external magnetic field can be defined as

In absence of an external magnetic field,

$$\eta = \frac{E_r}{H_\varphi} = \frac{\mu_\circ c^2}{\varepsilon}\frac{q}{\omega} \quad (13)$$

The above expression describes the impedance opposite to wave propagation in different mediums.

Results and Discussion

The propagation characteristics of surface waves in a cylindrical plasma layer sandwiched between two regions of different dielectric constants are studied. During the last years, the surface wave characteristics have been attracted a great deal of attention, especially in Cartesian geometries, owing to their potential applications in the different physical scenarios such as plasma waveguides, plasma antennas, laser physics, semiconductor science, and space science. Therefore, we expect that the physical characteristics of surface waves in a curved plasma configuration (Fig. 1) to be quite different from that of a planar geometry, because of the harmonic aspects of the Bessel functions in the curved geometry.

Figure 2 shows the behavior of the Plasmon surface wave frequency as a function of the normalized wave number, for different values of the Fermi speed. Here, we consider a thin plasma layer covered by the gold cylinder of radius R. The mode appearing in Fig. 2 exhibits the hybridization of the Plasmonic oscillations of the plasma layer by the acoustic excitation. The decreasing Fermi speed snoozes the saturation of the wave frequency of the surface waves. Figure 3 exhibits the dispersion relation of the plasma layer when it is free standing in vacuum. In this case, only the acoustic mode can be excited in the plasma layer, in which the wave frequency increases with the Fermi speed. It seems appropriate here to note that the quantum diffraction effect plays a contribution smaller than that of the Fermi pressure effect [37], and so we have neglected such small effects in the present investigation.

The influence of the curvature radius on the wave frequency is shown in Figure 4. It is seen that the wave frequency enhances with increasing the radius of the cylindrical plasma layer. It turns out that the planar

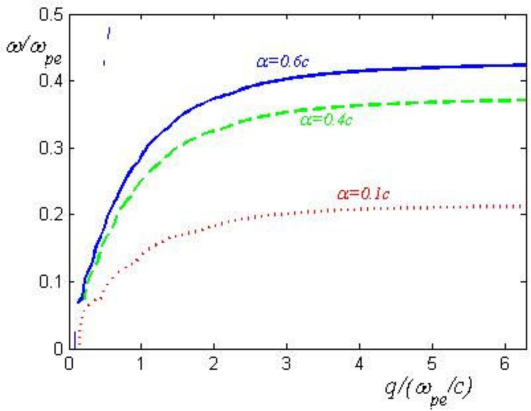

Figure 2. A thin plasma layer on a gold cylinder of radius R, for different values of the Fermi speed: $\alpha = 0.1c$ (dotted line), $\alpha = 0.4c$ (dashed line), and $\alpha = 0.6c$ (solid line).

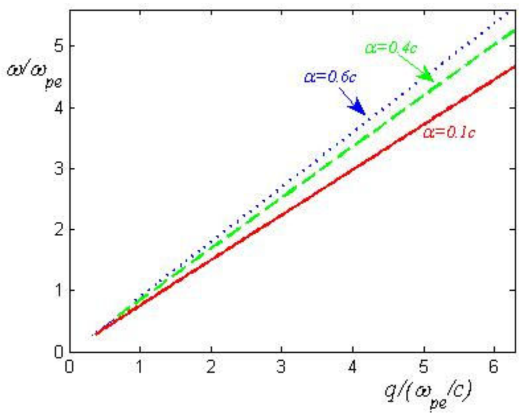

Figure 3. A cylinderical plasma layer of radius R free standing in vacuum, for different values of the Fermi speed: $\alpha = 0.6c$ (dotted line), $\alpha = 0.4c$ (dashed line), and $\alpha = 0.1c$ (solid line).

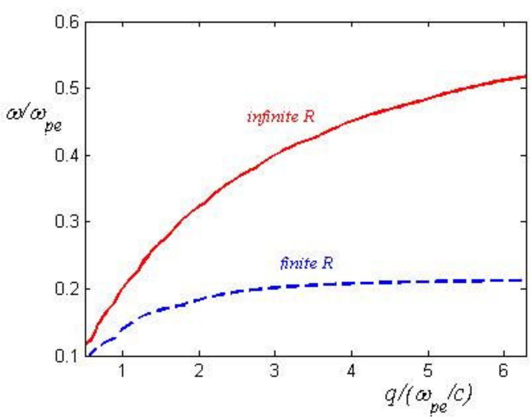

Figure 4. A thin plasma layer on a gold cylinder of radius R (dashed line) and planar limit ($R \to \infty$) (solid line).

plasma layer supports the surface waves with a higher

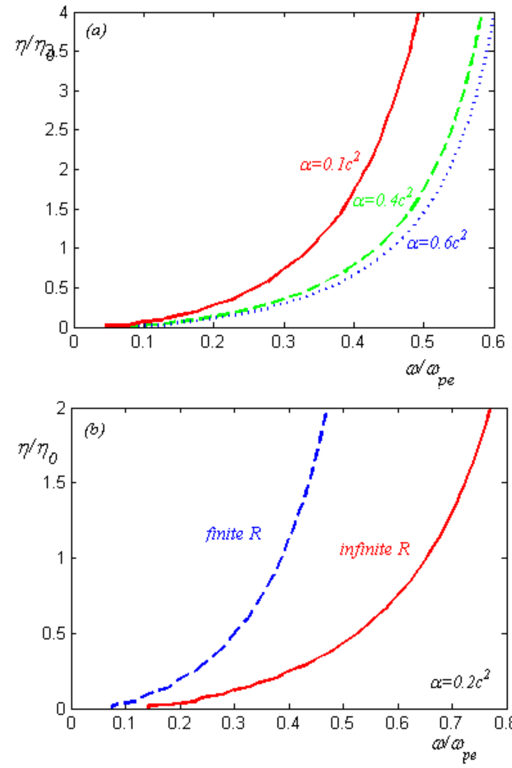

Figure 5. The behavior of normalized impedance of surface TM waves, with respect to the normalized wave frequency: (a) for different values of the Fermi speed: $\alpha = 0.1c$ (solid line), $\alpha = 0.4c$ (dashed line) and $\alpha = 0.6c$ (dotted line); (b) a comparison of cylindrical interface (dashed line) with the planar one (solid line).

frequency than that of the cylindrical geometry.

The wave impedance of TM surface waves with respect to the wave frequency is examined in Figure 5. We know that the impedance η is defined as the ratio of the perpendicular component of the electric and magnetic fields (i.e., $\eta = E_\perp / H_\perp$). Figure 5 shows that the impedance of surface TM wave decreases by increasing the Fermi velocity (panel a). Figure 5b exhibits that the impedance of surface waves in the planar plasma layer is smaller than that of the cylindrical geometry. This is in good agreement with the result in Figure 4.

To summarize, a theoretical investigation has been made on the physical characteristics of surface waves in a 2D cylindrical plasma layer. It is found that a hybridization occurred between the plasmonic oscillations and the acoustic excitation, which leads to a new surface mode in the present plasma system. The wave characteristics can be tuned by optimization of the physical and geometrical properties of the present plasma system. It is found that the cylindrical structure supports the wave propagation with lower velocity and higher impedance in comparison with the planar geometry. Since the development and optimization of physical properties of plasma antennas are increasing and the plasma properties play a crucial role for them, the effect of the plasma parameters on the basic features and operation of the plasma antenna can be a problem of interest, but is beyond the scope of the present investigation. Investigation of the hybrid surface waves propagating along with the cylindrical plasma layer shows that the physical characteristic of such waves is tunable by different paramerters, such as the plasma frequency, geometry, and medium properties. This turns out that the present results can give a better understanding of the principal concepts in plasma antenna, enantiomeric sensing devices, and plasma-sensing based waveguides.

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