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RESEARCH PAPER

Bank Efficiency Forecasting Model Based on the Modern Banking Indicators Using a Hybrid Approach of Dynamic Stochastic DEA and Meta-Heuristic Algorithms

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Abstract
Evaluating the efficiency of banks is crucial to orient their future decisions. In this regard, this paper proposes a new model based on dynamic stochastic data envelopment analysis in a fuzzy environment by considering the modern banking indicators to predict the efficiency of banks, which belongs to the category of NP-hard problems. To deal with the uncertainty in efficiency forecasting, the mean chance theory was used to express the constraints of the model and the expected value in its objective function to forecast the expected efficiency of banks. To solve the proposed model, two hybrid algorithms were designed by combining Monte Carlo (MC) simulation technique with Genetic Algorithm (GA) and Imperialist Competitive Algorithm (ICA). In order to improve the performances of MC-GA and MC-ICA parameters, the Response Surface Methodology (RSM) was applied to set their proper values. In addition, a case study in the modern banking industry was presented to evaluate the performance of the proposed model and the effectiveness of the hybrid algorithms. The results showed that the proposed model had high accuracy in predicting efficiency. Finally, to validate the designed hybrid algorithms, their results were compared with each other in terms of accuracy and convergence speed to the solution.

Keywords: Dynamic stochastic data envelopment analysis, Fuzzy programming, Hybrid meta-heuristic algorithm, Modern banking, Monte Carlo simulation.

1. Introduction

Efficiency is the amount of divisions ordered from a task. In more mathematical or scientific terms, it is a measure of the extent to which input is well used for an intended task or function (output). It often specifically entails the capability of a specific application of effort to produce a specific outcome with a minimum amount or quantity of waste, expense, or unnecessary effort. Improving efficiency leads to growth and development, and most developed countries have invested heavily in promoting the attitude toward efficiency and generalizing the techniques and methods used to promote it. A review of the performance of countries that have experienced significant economic growth over the past few decades indicates that most of these countries have achieved growth through efficiency gains (Papi et al., 2018). As a result, the need to use efficiency measurement systems becomes increasingly apparent. In recent years, the use of data envelopment analysis (DEA) technique has been

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rapidly expanding to evaluate the efficiency of organizations and has been used in the evaluation of various industries such as banking, post, hospitals, training centers, power plants, and refineries (Arteaga et al., 2019). In general, the concept of efficiency is divided into five categories, which include pure technical efficiency, scale efficiency, allocative efficiency, cost efficiency, and scope efficiency. Banks are considered to be the backbone of the financial system, whose performance depends more on technical efficiency. In a complex economic system such as that of Iran, little knowledge exists regarding efficiency in the service industry (Tajeddini, 2011). In the modern banking industry, efficiency is interpreted as the maximum potential ratio between the output and the input of the banking services process, which shows the optimal distribution of available resources that would allow achieving the maximum potential (Cvilikas & Jurkonyte-Dumbliauskiene, 2016).

Due to the uncertainty in the environmental factors governing the branches in the modern banking industry, the inputs and outputs of the branches are uncertain and forecasting efficiency and planning to improve their performance is a serious need of managers. In addition, in real-world problems with imprecise data, the accuracy of prediction is very important. According to the above description, this study presents a new model to forecast the efficiency of banks based on the modern banking indicators. In addition, two hybrid algorithms will be designed to solve the proposed model in this paper.

The rest structure of this paper is as follows. Section 2 provides a systematic literature review of the previous studies on stochastic DEA models in different environments and the modern banking indicators. In section 3, we present a new model based of dynamic stochastic DEA by considering the modern banking indicators to predict the efficiency of bank branches. In section 4, the hybrid algorithms are designed to solve the proposed model. The computational analysis is proposed in section 5. Finally, the conclusions and suggestions are provided in section 6.

2. Literature Review

In this section, first the literature on DEA in fuzzy, stochastic, and dynamic environments and then the studies on the modern banking indicators will be reviewed.

2.1. Related Literature on DEA in Fuzzy, Stochastic, and Dynamic Environments

Chance constraint planning is one of the subdivisions of stochastic programming. Charnes and Cooper (1959) entered the chance-constrained programming into the operation research literature for the first time. Along with Rhodes (1978), they surveyed the discussion of DEA for calculating efficiency, with their methodology commonly being called the CCR model. The CCR model evaluates the efficiency of decision making units (DMUs) via the optimal value of the ratio of weighted outputs to weighted inputs. The envelopment in CCR is constant returns to scale, meaning that a proportional increase in inputs results in a proportionate increase in outputs. Banker et al. (1984) proposed the BCC model. Unlike the CCR model, the BCC model allows for variable returns to scale. They show that the addition of a convexity constraint to the CCR model results in a DEA model that allows increasing, constant, and decreasing returns to scale. Sengupta et al. (1982) developed the stochastic DEA (SDEA) model. These researchers combined DEA models with chance constraint programming for the first time and used stochastic models to estimate efficiency. Cooper et al. (1998) proposed the stochastic BCC model by applying the presented concepts in the BCC model and assuming that the inputs and outputs are random variables with normal distribution. Punyangarm et al. (2006) presented a new model based on fuzzy stochastic DEA in which the inputs and outputs were considered as trapezoidal fuzzy numbers. Qin and Liu (2010) proposed a new model in stochastic DEA model in which the inputs and outputs are trapezoidal fuzzy numbers, the constraints are probabilistic inequalities, and the objective function is the expected value of the ratio of outputs to inputs. Dai et al. (2010) presented a new model for DEA in a fuzzy environment in which the expected value for the objective function and the credibility theory for the constraints are used. Yaghoubi and Amiri (2015) presented a new dynamic random fuzzy DEA model with common weights (using multi objective DEA approach) to predict the efficiency of DMUs under mean chance constraints.

Wanke and Barros et al. (2016) used fuzzy DEA model and regression technique to evaluate the efficiency of the Nigerian airports. The results showed that the simultaneous use of fuzzy DEA model and regression method leads to accurate results in productivity analysis. Wanke and Azad et al. (2016) presented an integrated multi-criteria decision making (MCDM) approach to predict the efficiency of Islamic banks. They used neural networks with TOPSIS in a two-stage approach to predict the efficiency of 114 Islamic banks from 24 countries. Foroughi and Shureshjani (2017) converted the generalized fuzzy DEA model to a parametric model in which inputs and outputs selection depends on the manager’s ideas. They used two numerical examples to express the effectiveness of their approach. Hatami-Marbini et al. (2017) proposed a new fully fuzzy DEA approach where all inputs and outputs are considered as fuzzy numbers. They used a lexicographic multi-objective linear programming technique to solve the proposed model in their study and compared it with existing models, using a dataset from the literature. Hu et al. (2017) proposed a new fuzzy DEA model with common weights and multiple fuzzy inputs and outputs. In addition, they developed an algorithm to solve the proposed model and find the compromise of its solution. The results showed that their proposed approach causes flexibility in calculating efficiency and distinguishes efficient units from each other. Papi et al. (2018) determined the health productivity of multiple provinces using the inaccurate DEA technique with fuzzy data. Namakin et al. (2018) proposed a new method based on crisp linear programming for solving the fully fuzzy DEA model where all parameters are Z-numbers. In addition, they proved that the previous method for solving fully fuzzy DEA models with z-numbers is not valid.

Yu et al. (2019) proposed a new way of constructing the non-convex meta-framework of the dynamic network DEA. They investigated and compared the dynamic performance of financial holding company and non-financial holding company banks in Taiwan. Peykani et al. (2019) presented a novel fuzzy DEA model based on a general fuzzy measurement. They developed an adjustable and flexible fuzzy DEA model to consider DMUs’ preferences and applied it for measuring the efficiency of hospitals in the USA. Arteaga et al. (2019) incorporated two distinct complementary types of sequentially cumulative processes within a dynamic, slacks-based measure DEA model. The results accounted for the evolution of the knowledge accumulation of DMUs. Hatami-Marbini (2019) presented a new fuzzy network DEA model with an imprecise network benchmarking to compute the technical efficiency of the airport and travel sector with the fuzzy values for all the inputs and outputs to reflect the human judgments. Finally, he offered a classification scheme based on fuzzy efficiencies with the aim of classifying airport sectors. Gaganis et al. (2020) used a sample of 2413 banks from 79 countries to predict their efficiencies based on fuzzy dynamic DEA. Their results revealed that the more the regulatory requirements were, the less the bank’s efficiency became. Avkiran and Morita (2020) predicted the Japanese banks stock performance from relative efficiency scores with DEA and simulated annealing. By designating short and long portfolios in a profitable investment policy, their method showed the commercial value of it. Amirteimoori et al. (2020) proposed a two-stage fuzzy DEA approach in which the overall efficiencies were calculated as the total weight of stage efficiencies. Their approach evaluated the efficiency of DMUs from both optimistic and pessimistic viewpoints.

2.2. Related Literature on Modern Banking Industry

The previous studies show the increasing growth of modern banking in the world, and learning this technology has led to the development of infrastructure, the growth of related technologies, and the development of standards at the international level. Moreover, the benefits of modern banking in better serving customers and improving productivity indicators in banks have attracted a lot of attention in the world. Banks need to find a way to asset efficiency, consider some important financial ratios, and find the strengths and weaknesses. "CAMELS" model is a new model to evaluate banks based on modern banking indicators. Management soundness, asset quality, capital adequacy, liquidity and sensitivity, and earnings and profitability are the focal points of this model (Rostami, 2015). Cole and Gunther (1995) for the first time examined the speed of information of CAMELS indicators using the econometric method, benchmark method, and banks' financial statements in relation to predicting the efficiency of banks in the modern banking industry. The results showed the success of CAMELS indicators in predicting the efficiency of banks. Seçme et al. (2009) evaluated fuzzy efficiency in the Turkish banking sector using CAMELS indicators, Analytic Hierarchy Process (AHP), and TOPSIS method. The results showed that in addition to financial performance, the non-financial performance of banks should be considered in the competitive environment in the modern banking industry. Kenneth and Adeniyi (2014) examined the efficiency of Nigerian banks based on modern banking indicators for the years 2006 to 2010. The results showed that the weakening of CAMELS indicators reduces the efficiency of banks and increases the resulting high risk.

Ghosh and Rakshit (2017) evaluated the banking system of India using modern banking indicators. The results showed that capital adequacy and profitability are the most important indicators, and management index has the most impact. Pekkaya and Demir (2018) determined the priority of CAMELS indicators based on banking efficiency. In this study that was conducted among Turkish state-owned banks, the AHP technique was used to rank banks. Vives (2019) identified the relationship between competition and stability in modern banking with respect to the impact of digital technologies and competition policies in the new banking industry. Keffala (2020) used the CAMELS approach to evaluate the efficiency of Islamic banks with dynamic panel data econometrics. The results showed that using forwards decrease the efficiency and the futures have a marginal effect on the efficiency of sample banks.

3. Problem Description

Today, the importance of predicting and understanding the future efficiency of DMUs and measuring it to improve performance and ultimately sustain the organization due to changing environmental conditions, complexity of technology, scarcity of resources, fierce competition, and diversification of services is an inevitable necessity. Promoting efficiency leads to growth and development, and most developed countries have invested heavily in promoting efficiency. In this regard, evaluating the efficiency of banks as one of the largest effective organizations in the economy of any country is very important to guide their future decisions. As a result, the efficiency of banks needs to be predicted so that their economic growth can be monitored in future decisions. A review of the performance of banks that have experienced significant economic growth over the past few decades indicates that most of them have achieved growth through increased efficiency. According to a review of previous research, a model that predicts the efficiency of banks in the real world – via the simultaneous consideration of the fuzzy, stochastic, and dynamic dimensions of their environment– had not been observed and was considered as a research opportunity for this article.

Based on the above description, this study designs a new model based on dynamic stochastic DEA in fuzzy environment by considering variable returns to scale and modern banking indicators to predict the efficiency of banks. In order to deal with the uncertainty in efficiency forecasting, the expected value and the mean chance theory has been used to express the objective function and the constraints of the proposed model, respectively. The proposed model is hard to compute and cannot be solved in polynomial time, thus it is in the NP-hard problems category. Therefore, two hybrid algorithms are designed by combining the MC simulation technique with the GA and ICA algorithms to solve the proposed model. In the following sections, each of them is fully described. In order to improve the performances of MC-GA and MC-ICA parameters, the RSM is applied to set their proper values. Finally, to evaluate the performance of the proposed model and the effectiveness of the designed hybrid algorithms, a case study on the modern banking industry is analyzed using MATLAB R 2015a software, and the results will be discussed.

3.1. Model Assumption

The proposed model in this research is designed based on the following assumptions:

- The modeling is based on the dynamic BCC model to consider variable returns to scale for bank branches.
- Due to the uncertainty in the environmental factors governing the branches in the banking industry, the inputs and outputs of them are considered as triangular fuzzy random variables.
- The dependence of the inputs and outputs of the branches in different time periods is considered in the model (dynamic DEA).
- Each period is analyzed separately with its inputs and outputs.
- Each branch uses fixed and quasi-fixed inputs in each period to generate fixed and quasi-fixed outputs.

3.2. Model Formulation

In the following section, the notation, parameters, sets, and decision variables of the proposed mathematical model are presented.

3.2.1. Problem Parameters

i	Index of the inputs (i= 1,…, m)
r	Index of the outputs (r= 1,…, s)
l	Index of the quasi-fixed inputs (outputs) (l= 1,…, L)
j	Index of the DMUs (j= 1,…, n)
t	Index of the periods (t= 1,…, T)
X_j^t	The column vector of fuzzy random inputs of DMU_j at period t
Y_j^t	The column vector of fuzzy random outputs of DMU_j at period t
K_j^t	The column vector of fuzzy random quasi–fix inputs (outputs) of DMU_j at period t
x_{ij}^t	The normal random variable of input i of DMU_j at period t

y_{rj}^{t} The normal random variable of output r of DMU_j at period t

k_{lj}^{t} The normal random variable of quasi–fix input (output) l of DMU_j at period t

μ_{ij}^{t} The mean of the normal random variable x_{ij}^{t}

$\sigma_{ij}^{2^t}$ The variation of the normal random variable x_{ij}^{t}

$\overline{\mu}_{rj}^{t}$ The mean of the normal random variable y_{rj}^{t}

$\overline{\sigma}_{rj}^{2^t}$ The variation of the normal random variable y_{rj}^{t}

$\overline{\overline{\mu}}_{lj}^{t}$ The mean of the normal random variable k_{lj}^{t}

$\overline{\overline{\sigma}}_{lj}^{2^t}$ The variation of the normal random variable k_{lj}^{t}

α_{j}^{t} The predetermined mean chance levels corresponding to the constraint j at period t

3.2.2. Problem Variables

V^t The weights vector of inputs at period t

U^t The weights vector of outputs at period t

β^{t-1} The weights vector of quasi–fix inputs at period t

ρ^t The weights vector of quasi–fix outputs at period t

v_i^t The weight of the input i at period t

u_r^t The weight of the output r at period t

β_l^{t-1} The weight of quasi–fix input l at period t

ρ_l^t The weight of quasi–fix output l at period t

Z_j^t The efficiency of DMU_j at period t

3.2.3. Dynamic Fuzzy Stochastic BCC Model Formulation

The traditional BCC model was proposed by Banker et al. (1984). This model can be used to measure the efficiency of each DMU for a specific period when the returns to scale of DMUs are variable and the outputs and inputs are constant. The BCC model for DMU_0 is as follows:

$$
\begin{aligned}
&Max: \quad Z_0 = \frac{U Y_0 + u_0}{V X_0} \\
&st: \\
&\quad \frac{U Y_j + u_0}{V X_j} \le 1 \qquad j=1,2,...,n \\
&\quad U, V \ge \varepsilon \qquad u_0 : free \quad in \quad sign
\end{aligned}
\tag{1}
$$

where ε is non-Archimedean infinitesimal value for forestalling weights to be equal to zero. Adding free variable u_o enhances the capability of model to situations in which DMUs act in variable return to scale. In addition, DMU_0 is efficient if the optimal value of the objective function is equal to 1. Otherwise, DMU_0 is inefficient (Toloo & Nalchigar, 2009). In a dynamic environment, it is assumed that inputs and outputs are changing sequentially in each period. Each DMU uses fixed and quasi-fixed inputs (X^t and K^{t-1}) to generate fixed and quasi-fixed outputs (Y^t and K^t) in the period t. The dynamic BCC model is as follows:

$$Max: \quad Z_0^t = \frac{U^t Y_0^t + \rho^t K_0^t + u_0}{V^t X_0^t + \beta^t K_0^{t-1}}$$

$$st:$$

$$\frac{U^t Y_j^t + \rho^t K_j^t + u_0}{V^t X_j^t + \beta^{t-1} K_j^{t-1}} \le 1 \quad j=1,2,...,n \tag{2}$$

$$U^t, V^t, \rho^t, \beta^{t-1} \ge \varepsilon \quad t=1,2,...,T \quad u_0: free \quad in \quad sign$$

However, in the real world, inputs and outputs are often not conclusive and we can only obtain the possibility distributions of them, so fuzziness and randomness may exist simultaneously in this data. Thus, in this paper, the inputs and outputs are considered as triangular fuzzy random variables with normal distributions, following as:

$$X_j^t = (x_{ij}^t - a_{ij}^t, x_{ij}^t, x_{ij}^t + b_{ij}^t) \qquad x_{ij}^t \sim N(\mu_{ij}^t, \sigma_{ij}^{2t}) \tag{3}$$

$$Y_j^t = (y_{rj}^t - c_{rj}^t, y_{rj}^t, y_{rj}^t + d_{rj}^t) \qquad y_{rj}^t \sim N(\bar{\mu}_{rj}^t, \bar{\sigma}_{rj}^{2t}) \tag{4}$$

$$K_j^t = (k_{lj}^t - e_{lj}^t, k_{lj}^t, k_{lj}^t + f_{lj}^t) \qquad k_{lj}^t \sim N(\bar{\bar{\mu}}_{lj}^t, \bar{\bar{\sigma}}_{lj}^{2t}) \tag{5}$$

where a_{ij}^t, b_{ij}^t, c_{rj}^t, d_{rj}^t, e_{lj}^t and f_{lj}^t are positive numbers for each i *(i=1,..,m)*, r *(r=1,..,s)* and l (l=1,..,L) which are predicted by the decision maker for the next financial period to predict the efficiency of DMU_j. To deal with uncertainty when predicting efficiency in the real world, we use the expected value of a fuzzy random variable in the objective function and the mean chance theory in the constraints. Therefore, the new model based on dynamic stochastic BCC in fuzzy environment (DFS-BCC) with the consideration of variable returns to scale to predict the efficiency of DMU_0 can be formulated as follows:

$$Max: \quad Z_0^t = E\left(\frac{U^t Y_0^t + \rho^t K_0^t + u_0}{V^t X_0^t + \beta^{t-1} K_0^{t-1}}\right) \tag{6}$$

Subject to:

$$Ch\left\{(V^t X_j^t + \beta^{t-1} K_j^{t-1}) - (U^t Y_j^t + \rho^t K_j^t + u_o) \ge 0\right\} \ge 1 - \alpha_j^t \quad j=1,2,...,n \tag{7}$$

$$U^t, V^t, \rho^t, \beta^{t-1} \ge \varepsilon \qquad u_o: free \quad t=1,..,T \quad j=1,..,n \tag{8}$$

The objective function shown in Eq. (6) aims to maximize the α_0-expected efficient value to predict the efficiency of DMU_0. The optimal values of U^t, V^t, ρ^t and β^t are referred to as the expectation efficient value of DMU_0. Constraints Eq. (7) contain the mean chance of fuzzy random events, and must be satisfied their values during the solution process at least with mean chance level α_j. Eq. (8) indicates the range of variations of the model variables.

3.2.4. Equivalent Stochastic Representation of DFS-BCC Model

The proposed DFS-BCC model is difficult to solve. To reduce the computational complexity, we discuss the equivalent stochastic representation of constraints and objective in this section.

3.2.4.1. Equivalent Stochastic Representation of the Mean Chance Constraints

When the inputs and outputs of the DMUs are considered as triangular fuzzy random variables with normal distributions, the constraints of DFS-BCC model (Eq. (7)) can be transformed to their equivalent stochastic representation by theorem (1).

Theorem (1). Suppose $\varsigma=(X(\theta)-a,\ X(\theta),\ X(\theta)+b)$ and $\gamma=(Y(\theta)-c,\ Y(\theta),\ Y(\theta)+d)$ be two independent triangular fuzzy random variables for each θ. If $X\sim N(\mu_1,\sigma_1^2)$, $Y\sim N(\mu_2,\sigma_2^2)$ and $a,b,c,d>0$, we have:

$$
\begin{aligned}
Ch\left\{x_1\varsigma-x_2\gamma\geq r\right\}=&\frac{\sqrt{x_1^2\sigma_1^2+x_2^2\sigma_2^2}\,(x_1(b-a)+x_2(c-d))}{2(x_1a+x_2d)(x_1b+x_2c)\sqrt{2\pi}}exp\left(-\frac{(r-(x_1\mu_1-x_2\mu_2))^2}{2(x_1^2\sigma_1^2+x_2^2\sigma_2^2)}\right)\\
&+\frac{\sqrt{x_1^2\sigma_1^2+x_2^2\sigma_2^2}}{2\sqrt{2\pi}(x_1b+x_2c)}exp\left(-\frac{(x_1(b+\mu_1)+x_2(c-\mu_2)-r)^2}{2(x_1^2\sigma_1^2+x_2^2\sigma_2^2)}\right)-\frac{\sqrt{x_1^2\sigma_1^2+x_2^2\sigma_2^2}}{2\sqrt{2\pi}(x_1a+x_2d)}\\
&*exp\left(-\frac{(r+x_1(a-\mu_1)+x_2(d+\mu_2))^2}{2(x_1^2\sigma_1^2+x_2^2\sigma_2^2)}\right)+\frac{x_1(\mu_1-a)-x_2(\mu_2+d)-r}{2(x_1a+x_2d)}\\
&*\varphi\left(\frac{r+x_1(a-\mu_1)+x_2(d+\mu_2)}{\sqrt{x_1^2\sigma_1^2+x_2^2\sigma_2^2}}\right)+\frac{((x_1\mu_1-x_2\mu_2)-r)(x_1(b-a)+x_2(c-d))}{2(x_1a+x_2d)(x_1b+x_2c)}\\
&*\varphi\left(\frac{r-(x_1\mu_1-x_2\mu_2)}{\sqrt{x_1^2\sigma_1^2+x_2^2\sigma_2^2}}\right)+\frac{x_1(b+\mu_1)-x_2(\mu_2-c)-r}{2(x_1b+x_2c)}\\
&*\varphi\left(\frac{x_1(b+\mu_1)+x_2(c-\mu_2)-r}{\sqrt{x_1^2\sigma_1^2+x_2^2\sigma_2^2}}\right)+\frac{r+x_1(b-\mu_1)+x_2(c+\mu_2)}{2(x_1b+x_2c)}
\end{aligned}
\tag{9}
$$

where x_1 and x_2 are nonnegative real numbers and at least one of them is nonzero, and $\varphi(0)$ is the probability distribution of standard normal distribution function (Qin and Liu, 2010). Then, by theorem (1), the constraints of DFS-BCC model (Eq. (7)) can be transformed to the following equivalent stochastic one:

$$
\begin{aligned}
&Ch\left\{(V^{t^T}X_j^t+B^{t-1^T}K_j^{t-1})-(U^{t^T}Y_j^t+\rho^{t^T}K_j^t+u_0)\geq 0\right\}=\left\{\sqrt{\sum_{i=1}^{m}v_i^{2t}\sigma_{ij}^{2t}+\sum_{l=1}^{L}\beta_l^{2t-1}\bar{\bar{\sigma}}_{lj}^{2t-1}+\sum_{r=1}^{s}u_r^{2t}\bar{\sigma}_{rj}^{2t}+\sum_{l=1}^{L}\rho_l^{2t}\bar{\bar{\sigma}}_{lj}^{2t}}\right.\\
&*\left[\frac{(\sum_{i=1}^{m}v_i^t(b_{ij}^t-a_{ij}^t)^2+\sum_{l=1}^{L}\beta_l^{t-1}(f_{lj}^{t-1}-e_{lj}^{t-1})-\sum_{r=1}^{s}u_r^t(d_{rj}^t-c_{rj}^t)-\sum_{l=1}^{L}\rho_l^t(f_{lj}^t-e_{lj}^t)-u_0)}{2\sqrt{2\pi}(\sum_{i=1}^{m}v_i^ta_{ij}^t+\sum_{l=1}^{L}\beta_l^{t-1}e_{lj}^{t-1}+\sum_{r=1}^{s}u_r^td_{rj}^t+\sum_{l=1}^{L}\rho_l^tf_{lj}^t)(\sum_{i=1}^{m}v_i^tb_{ij}^t+\sum_{l=1}^{L}\beta_l^{t-1}f_{lj}^{t-1}+\sum_{r=1}^{s}u_r^tc_{rj}^t+\sum_{l=1}^{L}\rho_l^te_{lj}^t)}\right]
\end{aligned}
$$

$$
*\exp\left(-\frac{\left(\sum_{i=1}^{m}v_i^t\,\mu_{ij}^t+\sum_{l=1}^{L}\beta_l^{t-1}\,\bar{\bar{\mu}}_{lj}^{t-1}-\sum_{r=1}^{s}u_r^t\,\bar{\mu}_{rj}^t-\sum_{l=1}^{L}\rho_l^t\,\bar{\bar{\mu}}_{lj}^t\right)^2}{2\left(\sum_{i=1}^{m}v_i^{2t}\sigma_{ij}^{2t}+\sum_{l=1}^{L}\beta_l^{2t-1}\bar{\bar{\sigma}}_{lj}^{2t-1}+\sum_{r=1}^{s}u_r^{2t}\bar{\sigma}_{rj}^{2t}+\sum_{l=1}^{L}\rho_l^{2t}\bar{\bar{\sigma}}_{lj}^{2t}\right)}\right)\Bigg\}
$$

$$
+\Bigg\{\frac{\sqrt{\sum_{i=1}^{m}v_i^{2t}\sigma_{ij}^{2t}+\sum_{l=1}^{L}\beta_l^{2t-1}\bar{\bar{\sigma}}_{lj}^{2t-1}+\sum_{r=1}^{s}u_r^{2t}\bar{\sigma}_{rj}^{2t}+\sum_{l=1}^{L}\rho_l^{2t}\bar{\bar{\sigma}}_{lj}^{2t}}}{2\sqrt{2\pi}\left(\sum_{i=1}^{m}v_i^t\,b_{ij}^t+\sum_{l=1}^{L}\beta_l^{t-1}f_{lj}^{t-1}+\sum_{r=1}^{s}u_r^t\,c_{rj}^t+\sum_{l=1}^{L}\rho_l^t\,e_{lj}^t\right)}
$$

$$
*\exp\left(-\frac{\left(\sum_{i=1}^{m}v_i^t(b_{ij}^t+\mu_{ij}^t)+\sum_{l=1}^{L}\beta_l^{t-1}(f_{lj}^{t-1}+\bar{\bar{\mu}}_{lj}^{t-1})+\sum_{r=1}^{s}u_r^t(c_{rj}^t-\bar{\mu}_{rj}^t)+\sum_{l=1}^{L}\rho_l^t(e_{lj}^t-\bar{\bar{\mu}}_{lj}^t)\right)^2}{2\left(\sum_{i=1}^{m}v_i^{2t}\sigma_{ij}^{2t}+\sum_{l=1}^{L}\beta_l^{2t-1}\bar{\bar{\sigma}}_{lj}^{2t-1}+\sum_{r=1}^{s}u_r^{2t}\bar{\sigma}_{rj}^{2t}+\sum_{l=1}^{L}\rho_l^{2t}\bar{\bar{\sigma}}_{lj}^{2t}\right)}\right)\Bigg\}
$$

$$
-\Bigg\{\exp\left(-\frac{\left(\sum_{i=1}^{m}v_i^t(a_{ij}^t-\mu_{ij}^t)+\sum_{l=1}^{L}\beta_l^{t-1}(e_{lj}^{t-1}-\bar{\bar{\mu}}_{lj}^{t-1})+\sum_{r=1}^{s}u_r^t(d_{rj}^t+\bar{\mu}_{rj}^t)+\sum_{l=1}^{L}\rho_l^t(f_{lj}^t+\bar{\bar{\mu}}_{lj}^t)\right)^2+\sum_{l=1}^{L}\rho_l^t(f_{lj}^t+\bar{\bar{\mu}}_{lj}^t))^2}{2\left(\sum_{i=1}^{m}v_i^{2t}\sigma_{ij}^{2t}+\sum_{l=1}^{L}\beta_l^{2t-1}\bar{\bar{\sigma}}_{lj}^{2t-1}+\sum_{r=1}^{s}u_r^{2t}\bar{\sigma}_{rj}^{2t}+\sum_{l=1}^{L}\rho_l^{2t}\bar{\bar{\sigma}}_{lj}^{2t}\right)}\right)
$$

$$
*\frac{\sqrt{\sum_{i=1}^{m}v_i^{2t}\sigma_{ij}^{2t}+\sum_{l=1}^{L}\beta_l^{2t-1}\bar{\bar{\sigma}}_{lj}^{2t-1}+\sum_{r=1}^{s}u_r^{2t}\bar{\sigma}_{rj}^{2t}+\sum_{l=1}^{L}\rho_l^{2t}\bar{\bar{\sigma}}_{lj}^{2t}}}{2\sqrt{2\pi}\left(\sum_{i=1}^{m}v_i^t\,a_{ij}^t+\sum_{l=1}^{L}\beta_l^{t-1}e_{lj}^{t-1}+\sum_{r=1}^{s}u_r^t\,d_{rj}^t+\sum_{l=1}^{L}\rho_l^t\,f_{lj}^t\right)}\Bigg\}
$$

$$
+\Bigg\{\varphi\left(\frac{\sum_{i=1}^{m}v_i^t(a_{ij}^t-\mu_{ij}^t)+\sum_{l=1}^{L}\beta_l^{t-1}(e_{lj}^{t-1}-\bar{\bar{\mu}}_{lj}^{t-1})+\sum_{r=1}^{s}u_r^t(d_{rj}^t+\bar{\mu}_{rj}^t)+\sum_{l=1}^{L}\rho_l^t(f_{lj}^t+\bar{\bar{\mu}}_{lj}^t)}{\sqrt{\sum_{i=1}^{m}v_i^{2t}\sigma_{ij}^{2t}+\sum_{l=1}^{L}\beta_l^{2t-1}\bar{\bar{\sigma}}_{lj}^{2t-1}+\sum_{r=1}^{s}u_r^{2t}\bar{\sigma}_{rj}^{2t}+\sum_{l=1}^{L}\rho_l^{2t}\bar{\bar{\sigma}}_{lj}^{2t}}}\right.
$$

$$
*\frac{\sum_{i=1}^{m}v_i^t(\mu_{ij}^t-a_{ij}^t)+\sum_{l=1}^{L}\beta_l^{t-1}(\bar{\bar{\mu}}_{lj}^{t-1}-e_{lj}^{t-1})-\sum_{r=1}^{s}u_r^t(\bar{\mu}_{rj}^t+d_{rj}^t)-\sum_{l=1}^{L}\rho_l^t(\bar{\bar{\mu}}_{lj}^t+f_{lj}^t)+u_0}{2\left(\sum_{i=1}^{m}v_i^t\,a_{ij}^t+\sum_{l=1}^{L}\beta_l^{t-1}e_{lj}^{t-1}+\sum_{r=1}^{s}u_r^t\,d_{rj}^t+\sum_{l=1}^{L}\rho_l^t\,f_{lj}^t\right)}\Bigg\} \tag{10}
$$

$$
-\Bigg\{\varphi\left(\frac{-\sum_{i=1}^{m}v_i^t\,\mu_{ij}^t-\sum_{l=1}^{L}\beta_l^{t-1}\bar{\bar{\mu}}_{lj}^{t-1}+\sum_{r=1}^{s}u_r^t\,\bar{\mu}_{rj}^t+\sum_{l=1}^{L}\rho_l^t\,\bar{\bar{\mu}}_{lj}^t}{\sqrt{\sum_{i=1}^{m}v_i^{2t}\sigma_{ij}^{2t}+\sum_{l=1}^{L}\beta_l^{2t-1}\bar{\bar{\sigma}}_{lj}^{2t-1}+\sum_{r=1}^{s}u_r^{2t}\bar{\sigma}_{rj}^{2t}+\sum_{l=1}^{L}\rho_l^{2t}\bar{\bar{\sigma}}_{lj}^{2t}}}\right)
$$

$$
*\left[\frac{\left(\sum_{i=1}^{m}v_i^t\mu_{ij}^t+\sum_{l=1}^{L}\beta_l^{t-1}\bar{\bar{\mu}}_{lj}^{t-1}-\sum_{r=1}^{s}u_r^t\bar{\mu}_{rj}^t-\sum_{l=1}^{L}\rho_l^t\bar{\bar{\mu}}_{lj}^t\right)\left(\sum_{i=1}^{m}v_i^t(b_{ij}^t-a_{ij}^t)+\sum_{l=1}^{L}\beta_l^{t-1}(f_{lj}^{t-1}-e_{lj}^{t-1})-\sum_{r=1}^{s}u_r^t(d_{rj}^t-c_{rj}^t)\ldots\right.}{2\left(\sum_{i=1}^{m}v_i^t a_{ij}^t+\sum_{l=1}^{L}\beta_l^{t-1}e_{lj}^{t-1}+\sum_{r=1}^{s}u_r^t d_{rj}^t+\sum_{l=1}^{L}\rho_l^t f_{lj}^t\right)\left(\sum_{i=1}^{m}v_i^t b_{ij}^t+\sum_{l=1}^{L}\beta_l^{t-1}f_{lj}^{t-1}+\sum_{r=1}^{s}u_r^t c_{rj}^t\ldots\right.}\ldots
$$

$$
\ldots\frac{\left.-\sum_{l=1}^{L}\rho_l^t(f_{lj}^t-e_{lj}^t)\right)-u_0}{\left.+\sum_{l=1}^{L}\rho_l^t e_{lj}^t\right)}\Bigg]\Bigg\}+\frac{\sum_{i=1}^{m}v_i^t(b_{ij}^t-\mu_{ij}^t)+\sum_{l=1}^{L}\beta_l^{t-1}(f_{lj}^{t-1}-\bar{\bar{\mu}}_{lj}^{t-1})+\sum_{r=1}^{s}u_r^t(\bar{\mu}_{rj}^t+c_{rj}^t)+\sum_{l=1}^{L}\rho_l^t(\bar{\bar{\mu}}_{lj}^t+e_{lj}^t)+u_0}{2\left(\sum_{i=1}^{m}v_i^t b_{ij}^t+\sum_{l=1}^{L}\beta_l^{t-1}f_{lj}^{t-1}+\sum_{r=1}^{s}u_r^t c_{rj}^t+\sum_{l=1}^{L}\rho_l^t e_{lj}^t\right)}
$$

$$
+\Bigg\{\varphi\left(\frac{\sum_{i=1}^{m}v_i^t(b_{ij}^t+\mu_{ij}^t)+\sum_{l=1}^{L}\beta_l^{t-1}(f_{lj}^{t-1}+\bar{\bar{\mu}}_{lj}^{t-1})+\sum_{r=1}^{s}u_r^t(c_{rj}^t-\bar{\mu}_{rj}^t)+\sum_{l=1}^{L}\rho_l^t(e_{lj}^t-\bar{\bar{\mu}}_{lj}^t)}{\sqrt{\sum_{i=1}^{m}v_i^{2t}\sigma_{ij}^{2t}+\sum_{l=1}^{L}\beta_l^{2t-1}\bar{\bar{\sigma}}_{lj}^{2t-1}+\sum_{r=1}^{s}u_r^{2t}\bar{\sigma}_{rj}^{2t}+\sum_{l=1}^{L}\rho_l^{2t}\bar{\bar{\sigma}}_{lj}^{2t}}}\right)
$$

$$
*\frac{\sum_{i=1}^{m}v_i^t(b_{ij}^t+\mu_{ij}^t)+\sum_{l=1}^{L}\beta_l^{t-1}(f_{lj}^{t-1}+\bar{\bar{\mu}}_{lj}^{t-1})-\sum_{r=1}^{s}u_r^t(\bar{\mu}_{rj}^t-c_{rj}^t)-\sum_{l=1}^{L}\rho_l^t(\bar{\bar{\mu}}_{lj}^t-e_{lj}^t)-u_0}{2\left(\sum_{i=1}^{m}v_i^t b_{ij}^t+\sum_{l=1}^{L}\beta_l^{t-1}f_{lj}^{t-1}+\sum_{r=1}^{s}u_r^t c_{rj}^t+\sum_{l=1}^{L}\rho_l^t e_{lj}^t\right)}\Bigg\}
$$

3.2.4.2. Equivalent Stochastic Representation of the Expectation Objective Function

When the inputs and outputs of the DMUs are considered as triangular fuzzy random variables with normal distributions, the objective function of DFS-BCC model (Eq. (6)) can be transformed to their equivalent stochastic representation by theorem (2).

Theorem (2). Suppose X=(x-a, x, x+b), Y=(y-c, y, y+d), K= (k-e, k, k+f), and Z=(z-g, z, z+h) be four triangular fuzzy random variables, where a, b, c, d, e, f, g, h $\in$ R and $x \sim N(\mu_1,\sigma_1^2)$, $y \sim N(\mu_2,\sigma_2^2)$, $k \sim N(\mu_3,\sigma_3^2)$, $z \sim N(\mu_4,\sigma_4^2)$, if $((Y+K)/(X+Z))_\alpha$ is the α-cut of the fuzzy variable (Y+K)/(X+Z), then we have (Qin and Liu, 2010):

$$E\left[\frac{Y+K}{X+Z}\right]=\frac{1}{2}\int_0^1 [\frac{y-(1-\alpha)c+k-(1-\alpha)e}{x+(1-\alpha)b+z+(1-\alpha)h}+\frac{y+(1-\alpha)d+k+(1-\alpha)f}{x-(1-\alpha)a+z-(1-\alpha)g}]\quad d\alpha$$
$$=-\frac{1}{2}(\frac{c+e}{b+h})-\frac{1}{2(b+h)}[y+k-c-e+x+z+b+h(\frac{c+e}{b+h})]*Ln(\frac{x+z}{x+z+b+h}) \tag{11}$$
$$-\frac{1}{2}(\frac{d+f}{a+g})+\frac{1}{2(a+g)}[y+k+d+f+x+z-a-g(\frac{d+f}{a+g})]*Ln(\frac{x+z}{x+z-a-g})$$

Suppose X_j^t, Y_j^t, K_j^t and K_j^{t-1} are triangular fuzzy random vectors of DMU_j as (3), (4), and (5), also:

$$\begin{array}{llll}
\mu_j^t=\sum_{i=1}^m v_i^t\mu_{ij}^t & \bar{\mu}_j^t=\sum_{r=1}^s u_r^t\bar{\mu}_{rj}^t & \bar{\bar{\mu}}_j^t=\sum_{l=1}^L \rho_l^t\bar{\bar{\mu}}_{lj}^t & \bar{\bar{\mu}}_j^{t-1}=\sum_{l=1}^L \beta_l^{t-1}\bar{\bar{\mu}}_{lj}^{t-1}\\
\sigma_j^{2t}=\sum_{i=1}^m v_i^t\delta_{ij}^{2t} & \bar{\sigma}_j^{2t}=\sum_{r=1}^s u_r^t\bar{\delta}_{rj}^{2t} & \bar{\bar{\sigma}}_j^{2t}=\sum_{l=1}^L \rho_l^t\bar{\bar{\delta}}_{lj}^{2t} & \bar{\bar{\sigma}}_j^{2t-1}=\sum_{l=1}^L \beta_l^{t-1}\bar{\bar{\delta}}_{lj}^{2t-1}\\
a_j^t=\sum_{i=1}^m v_i^t a_{ij}^t & b_j^t=\sum_{i=1}^m v_i^t b_{ij}^t & c_j^t=\sum_{r=1}^s u_r^t c_{rj}^t & d_j^t=\sum_{r=1}^s u_r^t d_{rj}^t\\
e_j^t=\sum_{l=1}^L \rho_l^t e_{lj}^t & f_j^t=\sum_{l=1}^L \rho_l^t f_{lj}^t & e_j^{t-1}=\sum_{l=1}^L B_l^{t-1}e_{lj}^{t-1} & f_j^{t-1}=\sum_{l=1}^L \beta_l^{t-1}f_{lj}^{t-1}
\end{array} \tag{12}$$

Then, by Theorem (2), the objective function of DFS-BCC model (Eq. (6)) can be transformed to the following equivalent stochastic one:

$$Z_0^t=E(\frac{U^{tT}Y_j^t+\rho^{tT}K_j^t+u_0}{V^{tT}X_j^t+B^{t-1T}K_j^{t-1}})=$$
$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\Bigg\{-\frac{1}{2}(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}})-\frac{1}{2}(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})+Ln(\frac{x_j^t+k_j^{t-1}}{x_j^t+k_j^{t-1}+b_j^t+f_j^{t-1}})*\frac{1}{2(b_j^t+f_j^{t-1})}(y_j^t+k_j^t$$
$$+c_j^t+e_j^t+x_j^t(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})+k_j^{t-1}(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})+b_j^t(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})+f_j^{t-1}(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})+u_0)$$
$$+\frac{1}{2(a_j^t+e_j^{t-1})}(y_j^t+d_j^t+k_j^t+f_j^t+x_j^t(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}})+k_j^{t-1}(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}})-a_j^t(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}}) \tag{13}$$
$$-e_j^{t-1}(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}})-u_0)*Ln(\frac{x_j^t+k_j^{t-1}}{x_j^t+k_j^{t-1}-a_j^t-e_j^{t-1}})\Bigg\}*\frac{1}{4\pi^2\sigma_j^t\bar{\sigma}_j^t\bar{\bar{\sigma}}_j^t\bar{\bar{\sigma}}_j^{t-1}}$$
$$*exp(-\frac{(x_j^t-\mu_j^t)^2}{2\sigma_j^{2t}}-\frac{(y_j^t-\bar{\mu}_j^t)^2}{2\bar{\sigma}_j^{2t}}-\frac{(k_j^t-\bar{\bar{\mu}}_j^t)^2}{2\bar{\bar{\sigma}}_j^{2t}}-\frac{(k_j^{t-1}-\bar{\bar{\mu}}_j^{t-1})^2}{2\bar{\bar{\sigma}}_j^{2t-1}})\,dk_j^{t-1}dk_j^t dy_j^t dx_j^t$$
$$=\frac{1}{\sigma_j^t\sqrt{2\pi}}\int_{-\infty}^{+\infty}f(x_j^t)\quad dX_j^t-\frac{1}{2}(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}})-\frac{1}{2}(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})$$

where:

$$
\begin{aligned}
f(x_j^t) = exp(-\frac{(x_j^t-\mu_j^t)^2}{2\sigma_j^{2t}})*\Bigg\{ & \frac{1}{2(b_j^t+f_j^{t-1})}[\bar{\mu}_j^t+\bar{\bar{\mu}}_j^t+c_j^t+e_j^t+x_j^t(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})+\bar{\bar{\mu}}_j^{t-1}(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})+b_j^t(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}}) \\
& +f_j^{t-1}(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})+u_0]*Ln(\frac{x_j^t+\bar{\bar{\mu}}_j^{t-1}}{x_j^t+\bar{\bar{\mu}}_j^{t-1}+b_j^t+f_j^{t-1}})+\frac{1}{2(a_j^t+e_j^{t-1})}[\bar{\mu}_j^t+d_j^t+\bar{\bar{\mu}}_j^t+f_j^t+x_j^t(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}})+\bar{\bar{\mu}}_j^{t-1}(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}}) \\
& -a_j^t(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}})-e_j^{t-1}(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}})-u_0]*Ln(\frac{x_j^t+\bar{\bar{\mu}}_j^{t-1}}{x_j^t+\bar{\bar{\mu}}_j^{t-1}-a_j^t-e_j^{t-1}})\Bigg\}
\end{aligned} \tag{14}
$$

Since the Eq. (13) and Eq. (10) contain integration and standard normal distribution φ, respectively, the equivalent stochastic representation of DFS-BCC model cannot be solved via the conventional optimization algorithms. Thus, in the next section, by considering the proposed model that belongs to the category of NP-hard optimization problems, two hybrid algorithms will be designed by combining Monte Carlo (MC) simulation technique with GA and ICA Algorithms to solve it.

4. Solution Approach

4.1. MC-GA

The meta-heuristic algorithms are the best methods to solve complex models. The GA was proposed by Holland (1975) and developed by Goldberg (1989). This meta-heuristic algorithm increases the chance of obtaining the global optimal solution in complex problems. Therefore, it was used in this paper to solve the proposed DFS-BCC model. The main steps of the designed MC-GA are as follows.

4.1.1. Chromosome Representation

The nonnegative vector as $R^t = [u_r^t, v_i^t, \rho_l^t, \beta_l^{t-1}]$ is characterized as a chromosome to represent a solution for each i ($i=1,..,m$), r ($r=1,..,s$) and l ($l=1,..,L$) at period t.

4.1.2. Initialization Process

Firstly, we define an integer number (*pop-size*) of chromosomes, and then initialize pop-size chromosomes randomly with uniform distribution between 0 to 1.

4.1.3. Evaluation Process

The evaluation of each chromosome is computed according to the objective function of the model via Eq. (13).

4.1.4. Calculate the Objective Function by Monte-Carlo Simulation

The MC simulation is employed to compute the objective function of model (Eq. (13)). MC simulation is a computer-based analytical technique which uses statistical sampling methods for finding a probabilistic approximation to the solution of an integral equation or model by using sequences of random numbers which leads results in complex systems (Kuah et al., 2012). In this paper, the MC simulation technique is used to solve the integrals applied in the objective function in the proposed model ($\int_{-\infty}^{\infty} f(x_j^t)dx_j^t$). The MC simulation procedure for approximating the integral used in Eq. (13) is as follows:

begin
n←number of simulation (n-simulation)
for i =1 to n do
hi←Generate a uniform distributed random number between -1 to +1
obtain the mean value of the function: $\hat{f} = \frac{1}{n} * \sum_{i=1}^{n} f\left(\frac{h_i}{1-h_i^2}\right)\left(\frac{1+h_i^2}{(1-h_i^2)^2}\right)$
calculate the approximation of integral: $\int_{-\infty}^{+\infty} f(x_j^t)dx_j^t \approx 2\hat{f}$
end

4.1.5. Selection and Crossover Processes

In order to conduct the selection process, firstly the probability of crossover (p_c) is defined ($0< p_c<1$). Then, a random real number (r) is generated between 0 to 1. If $r<p_c$ then chromosome *i* is selected as a parent (*i=1,.., pop_size*). This process is repeated until all parents are selected. Then, crossover process on each pair of parent chromosomes is performed to generate child chromosomes (offspring) as follows:

$$Offspring_1 = r*parent_1 + (1-r)*parent_2$$
$$Offspring_2 = (1-r)*parent_1 + r*parent_2 \quad (15)$$

4.1.6. Mutation Process

In order to conduct the mutation process, firstly the probability of mutation (p_m) is defined ($0< p_m<0.05$). Then, a random real number (r) is generated for chromosome *i* between 0 to 0.05. If $r<p_m$, chromosome *i* is selected (*i=1,.., pop-size*). Then, two genes are randomly selected from it and their values are changed. Then, the feasibility of offspring is examined through model constraints.

4.1.7. Population Update

The chromosomes that have the best fitness function value in the previous population are added to the offspring that has been produced by crossover and mutation processes to generate the new population.

4.1.8. Stop Criteria

The MC-GA algorithm will be stopped after the predetermined iterations.

4.2. MC-ICA

The ICA was proposed by Atashpaz-Gargari and Lucas (2007). The ICA is a new meta-heuristic algorithm in evolutionary calculations founded on socio-political evolution of human for solving mathematical models associated with optimization. Therefore, ICA can be applied as a very practical algorithm to solve the complex problems such as the DFS-BCC model. The procedure of developed MC-ICA is summarized as follows.

Each solution in the ICA is an array which is called a country (Atashpaz & Lucas, 2007). The representation of countries, initialization and evaluation process in the MC-ICA algorithm is similar to the MC-GA algorithm. After generating countries, the MC simulation is employed to compute the objective function of model (13), which was discussed in the previous section.

In the ICA, N_{imp} of the countries with the best value of objective function in a population are selected as the imperialists and the N_{col} rest of the countries are colonies belonging to an empire. Then, the normalized value of the objective function for each imperialist must be calculated as follows:

$$C_n = \frac{W_n^t - W_{best}^t}{W_{\max}^t - W_{\min}^t} \tag{16}$$

where C_n is the normalized value of the objective function for imperialist n. W_{best}^t, W_{max}^t, and W_{min}^t are the best, maximum, and minimum values of the objective function in each iteration, respectively. The normalized power of each imperialist is defined by:

$$P_n = \left| C_n / \sum_{i=1}^{N_{imp}} C_i \right| \tag{17}$$

Then, the initial number of colonies of an empire will be as follows:

$$NC_n = round\left\{P_n . N_{col}\right\} \tag{18}$$

where NC_n is the initial number of colonies of the imperialist n. Next, the imperialist countries start to improve their colonies with attracting them to themselves. This movement is shown in Figure (1), in which d is the distance between the imperialist and the colony (Atashpaz & Lucas, 2007).

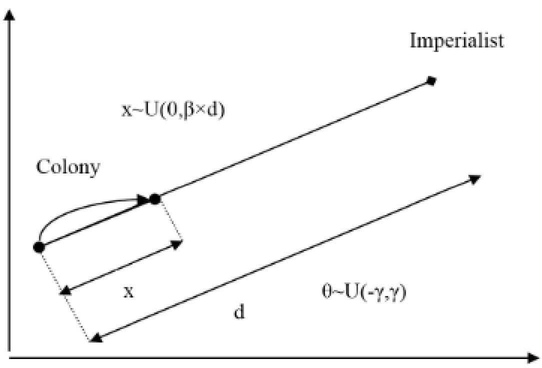

Figure 1. The Movement of Colonies Toward the Imperialist

where x is a random variable with uniform distribution and β is real anumber between 1 to 2. Also, θ is a random number with uniform distribution.

$$X \sim U(0, \beta \times d) \tag{19}$$

During the movement of colonies toward the imperialist country in each empire, some of them may reach to an objective function better than the imperialist. In this case, the imperialist and the colony change their situations with each other. Then, the total power of an empire is calculated as below:

$$TC_n = cost(imperialist_n) + \zeta \times mean(colonies \quad of \quad empire_n) \tag{20}$$

where TC_n is the fitness value of the empire n and ζ is a real number between 0 to 1.

Every empire that cannot enhance its power will be omitted from the competition. The possession probability of each empire is calculated as follows:

$$P_{pn} = \left| max\{TC_i\} - TC_n \Big/ \sum_{i=1}^{N_{imp}} (max\{TC_i\} - TC_n) \right| \tag{21}$$

where TC_n is the total cost of empire n. The powerless empires will decline and their colonies are shared between other empires in the competition. Finally, the time when there is only one empire between all countries is considered as the stopping criterion (Atashpaz & Lucas, 2007).

5. Computational Results

5.1. Parameters Tuning

The results of all meta-heuristics are significantly influenced by their parameter setting. In this research, the RSM is used for tuning the parameters of the MC-GA and MC-ICA algorithms. The RSM is an optimization methodology that looks for optimization responses in two phases. In the first phase, this technique seeks the determination of the optimal area by using semi regression models, and the determination of optimization response is accomplished in the second phase. To obtain the optimal values of the effective parameters of meta-heuristic algorithms in RSM, three levels for each parameter – including low level (L), medium level (M) and high level (H) – are considered. Each parameter is coded as −1 when it is at low level, 0 when it is at medium level, and +1 when it is at high level. The coded parameters are defined as follows (Kaymaz & McMahon, 2005):

$$o_i = \frac{p_i - (\frac{h+l}{2})}{(\frac{h-l}{2})} \tag{22}$$

where P_i and O_i are real and coded parameters, respectively. The parameters of the proposed hybrid algorithms and their levels are shown in Table 1.

Table 1. Parameters and Initial Levels for the Proposed Algorithms

The MC-GA parameters					
Level	P_C	P_m	**Pop-size**	**Iteration**	**n-simulation**
-1	0.5	0.01	5	10	40
0	0.7	0.05	12	20	70
+1	0.9	0.1	20	30	100
The MC-ICA parameters					
Level	**β**	**ζ**	**θ**	N_{imp}	**n-simulation**
-1	0.1	0.1	10	10	40
0	0.5	0.5	90	20	70
+1	1	1	180	30	100

In addition, the tuned parameters of MC-GA and MC-ICA algorithms are presented in Table 2.

Table 2. Optimal Values of Parameters Obtained Through RSM for the Proposed Algorithms

The MC-GA parameters				
P_C	P_m	Pop-size	Iteration	n-simulation
0.75	0.04	12	25	74
The MC-ICA parameters				
β	ζ	θ	N_{imp}	n-simulation
0.45	062	45	15	61

5.2. Numerical Calculations

To ensure the validity of modeling idea and the effectiveness of the proposed hybrid algorithms, a case study was conducted on 10 branches of an Iranian bank in order to predict their efficiency for the next financial period in this section. Based on the modern banking indicators, this paper considered four inputs in predicting the bank branches efficiency, namely total cost, capital adequacy, asset quality, and liquidity.

Table 3. The Inputs (Outputs) of Bank Branches for the Next Period (Unit: 1000 Million Rials per Month)

Branch	**Inputs**			**Quasi-fix input**	**Outputs**		**Quasi-fix output**
	Total cost	**Capital adequacy**	**Asset quality**	**Liquidity**	**Bank granting facilities**	**Electronic transactions**	**Liquidity**
1	$(x^1_{11}-88, x^1_{11}, x^1_{11}+127)$ $x^1_{11} \sim N(56.7.7, 53.87)$	$(x^1_{21}-15, x^1_{21}, x^1_{21}+50)$ $x^1_{21} \sim N(81.87, 1.36)$	$(x^1_{31}-30, x^1_{31}, x^1_{31}+60)$ $x^1_{31} \sim N(162.7, 21.18)$	$(k^0_{11}-155, k^0_{11}, k^0_{11}+150)$ $k^0_{11} \sim N(30856, 9156)$	$(y^1_{11}-300, y^1_{11}, y^1_{11}+880)$ $y^1_{11} \sim N(21551, 5232)$	$(y^1_{21}-550, y^1_{21}, y^1_{21}+800)$ $y^1_{21} \sim N(14303, 4272)$	$(k^1_{11}-170, k^1_{11}, k^1_{11}+150)$ $k^1_{11} \sim N(32606, 9904)$
2	$(x^1_{12}-100, x^1_{12}, x^1_{12}+90)$ $x^1_{12} \sim N(5832, 49.16)$	$(x^1_{22}-15, x^1_{22}, x^1_{22}+50)$ $x^1_{22} \sim N(67.45, 1.81)$	$(x^1_{32}-30, x^1_{32}, x^1_{32}+60)$ $x^1_{32} \sim N(101.7, 27.65)$	$(k^0_{12}-150, k^0_{12}, k^0_{12}+150)$ $k^0_{12} \sim N(27737, 5466)$	$(y^1_{12}-300, y^1_{12}, y^1_{12}+100)$ $y^1_{12} \sim N(17956, 6784)$	$(y^1_{22}-600, y^1_{22}, y^1_{22}+800)$ $y^1_{22} \sim N(13985, 3478)$	$(k^1_{12}-150, k^1_{12}, k^1_{12}+150)$ $k^1_{12} \sim N(28537, 6566)$
3	$(x^1_{13}-100, x^1_{13}, x^1_{13}+90)$ $x^1_{13} \sim N(621.7, 41.6)$	$(x^1_{23}-15, x^1_{23}, x^1_{23}+50)$ $x^1_{23} \sim N(75.45, 1.44)$	$(x^1_{33}-30, x^1_{33}, x^1_{33}+60)$ $x^1_{33} \sim N(95.79, 14.79)$	$(k^0_{13}-150, k^0_{13}, k^0_{13}+150)$ $k^0_{13} \sim N(29784, 8745)$	$(y^1_{13}-300, y^1_{13}, y^1_{13}+100)$ $y^1_{13} \sim N(17854, 4517)$	$(y^1_{23}-600, y^1_{23}, y^1_{23}+800)$ $y^1_{23} \sim N(13456, 3541)$	$(k^1_{13}-150, k^1_{13}, k^1_{13}+150)$ $k^1_{13} \sim N(30782, 8905)$
4	$(x^1_{14}-100, x^1_{14}, x^1_{14}+90)$ $x^1_{14} \sim N(6243, 71.13)$	$(x^1_{24}-15, x^1_{24}, x^1_{24}+50)$ $x^1_{24} \sim N(47.41, 1.1)$	$(x^1_{34}-30, x^1_{34}, x^1_{34}+60)$ $x^1_{34} \sim N(132.7, 39.18)$	$(k^0_{14}-150, k^0_{14}, k^0_{14}+150)$ $k^0_{14} \sim N(27437, 5457)$	$(y^1_{14}-300, y^1_{14}, y^1_{14}+100)$ $y^1_{14} \sim N(17354, 4198)$	$(y^1_{24}-600, y^1_{24}, y^1_{24}+800)$ $y^1_{24} \sim N(15214, 6021)$	$(k^1_{14}-150, k^1_{14}, k^1_{14}+150)$ $k^1_{14} \sim N(29436, 5750)$
5	$(x^1_{15}-100, x^1_{15}, x^1_{15}+90)$ $x^1_{15} \sim N(542.1, 61.13)$	$(x^1_{24}-15, x^1_{24}, x^1_{24}+50)$ $x^1_{25} \sim N(78.95, 1.24)$	$(x^1_{35}-30, x^1_{35}, x^1_{35}+60)$ $x^1_{35} \sim N(85.74, 19.70)$	$(k^0_{15}-150, k^0_{15}, k^0_{15}+150)$ $k^0_{15} \sim N(25737, 8494)$	$(y^1_{15}-300, y^1_{15}, y^1_{15}+100)$ $y^1_{15} \sim N(18045, 6478)$	$(y^1_{25}-600, y^1_{25}, y^1_{25}+800)$ $y^1_{25} \sim N(14765, 4795)$	$(k^1_{15}-150, k^1_{15}, k^1_{15}+150)$ $k^1_{15} \sim N(27747, 8597)$
6	$(x^1_{16}-100, x^1_{16}, x^1_{16}+177)$ $x^1_{16} \sim N(519.7, 35.7)$	$(x^1_{26}-15, x^1_{26}, x^1_{26}+50)$ $x^1_{26} \sim N(79.87, 1.34)$	$(x^1_{36}-30, x^1_{36}, x^1_{36}+60)$ $x^1_{36} \sim N(38.54, 34.1)$	$(k^0_{16}-150, k^0_{16}, k^0_{16}+150)$ $k^0_{16} \sim N(28417, 6457)$	$(y^1_{16}-300, y^1_{16}, y^1_{16}+100)$ $y^1_{16} \sim N(18343, 5413)$	$(y^1_{26}-600, y^1_{26}, y^1_{26}+800)$ $y^1_{26} \sim N(12874, 2954)$	$(k^1_{16}-150, k^1_{16}, k^1_{16}+150)$ $k^1_{16} \sim N(29418, 6658)$
7	$(x^1_{17}-100, x^1_{17}, x^1_{17}+90)$ $x^1_{17} \sim N(563.1, 51.3)$	$(x^1_{27}-15, x^1_{27}, x^1_{27}+50)$ $x^1_{27} \sim N(15.91, 1.96)$	$(x^1_{37}-30, x^1_{37}, x^1_{37}+60)$ $x^1_{37} \sim N(74.74, 36.7)$	$(k^0_{17}-110, k^0_{17}, k^0_{17}+150)$ $k^0_{17} \sim N(21871, 4128)$	$(y^1_{17}-300, y^1_{17}, y^1_{17}+100)$ $y^1_{17} \sim N(17652, 4123)$	$(y^1_{27}-600, y^1_{27}, y^1_{27}+800)$ $y^1_{27} \sim N(15014, 5784)$	$(k^1_{17}-124, k^1_{17}, k^1_{17}+150)$ $k^1_{17} \sim N(23875, 4629)$
8	$(x^1_{18}-100, x^1_{18}, x^1_{18}+90)$ $x^1_{18} \sim N(490.7, 36.7)$	$(x^1_{28}-15, x^1_{28}, x^1_{28}+50)$ $x^1_{28} \sim N(39.41, 1.22)$	$(x^1_{38}-30, x^1_{38}, x^1_{38}+60)$ $x^1_{38} \sim N(192.7, 31.18)$	$(k^0_{18}-80, k^0_{18}, k^0_{18}+150)$ $k^0_{18} \sim N(17171, 4120)$	$(y^1_{18}-300, y^1_{18}, y^1_{18}+100)$ $y^1_{11} \sim N(56.7.7, 53.87)$	$(y^1_{28}-600, y^1_{28}, y^1_{28}+800)$ $y^1_{18} \sim N(16324, 1478)$	$(k^1_{18}-800, k^1_{18}, k^1_{18}+150)$ $k^1_{18} \sim N(19171, 4529)$
9	$(x^1_{19}-100, x^1_{19}, x^1_{19}+90)$ $x^1_{19} \sim N(452.7, 42.1)$	$(x^1_{29}-15, x^1_{29}, x^1_{29}+50)$ $x^1_{29} \sim N(71.45, 1.91)$	$(x^1_{39}-30, x^1_{39}, x^1_{39}+60)$ $x^1_{39} \sim N(62.74, 28.41)$	$(k^0_{19}-80, k^0_{19}, k^0_{19}+150)$ $k^0_{19} \sim N(18181, 7412)$	$(y^1_{19}-300, y^1_{19}, y^1_{19}+100)$ $y^1_{19} \sim N(17216, 2169)$	$(y^1_{29}-600, y^1_{29}, y^1_{29}+800)$ $y^1_{29} \sim N(14852, 5789)$	$(k^1_{19}-800, k^1_{19}, k^1_{19}+150)$ $k^1_{19} \sim N(21180, 7819)$
10	$(x^1_{1,10}-100, x^1_{1,10}, x^1_{1,10}+90)$ $x^1_{1,10} \sim N(470.6, 41.3)$	$(x^1_{2,10}-15, x^1_{2,10}, x^1_{2,10}+50)$ $x^1_{2,10} \sim N(30.41, 1.54)$	$(x^1_{3,10}-30, x^1_{3,10}, x^1_{3,10}+60)$ $x^1_{3,10} \sim N(114.7, 32.1)$	$(k^0_{1,10}-150, k^0_{1,10}, k^0_{1,10}+150)$ $k^0_{1,10} \sim N(26737, 7454)$	$(y^1_{1,10}-300, y^1_{1,10}, y^1_{1,10}+100)$ $y^1_{1,10} \sim N(16127, 1654)$	$(y^1_{2,10}-600, y^1_{2,10}, y^1_{2,10}+800)$ $y^1_{2,10} \sim N(13753, 3745)$	$(k^1_{1,10}-150, k^1_{1,10}, k^1_{1,10}+150)$ $k^1_{1,10} \sim N(28730, 7658)$

Total cost is composed of operating and non-operating costs. Capital adequacy means the ability of banks to cope with potential problems and provide benefits of depositors. Quality of assets consists of claims from other banks and credit institutions, claims from government, and investments. Liquidity is the funds that natural or legal persons provide to the bank. Regarding the modern banking indicators in predicting bank branches efficiency, there are also three outputs, namely bank granting facilities, electronic transactions, and liquidity. Bank granting facilities that make up the bulk of the bank's expenses are an important part of any bank's operations. Electronic transactions include the sums of purchase transactions, money transfer transactions, internet bank transactions, and bank mobile transactions. Net profit is quasi-fix input (output) or link data, because it is the output of one financial period and the input of the next period for each branch. In addition, the inputs and outputs of bank branches were selected as triangular fuzzy random variables with normal distribution as shown in Tables 3.

Table 4 shows the predicted efficiency values of the bank branches for the next financial period under different risk levels with the equivalent stochastic representation of the DFS-BCC Model.

Table 4. The Predicted Efficiency Values Under Various Risk Levels With MC-GA and MC-ICA

Algorithm	Bank branch	α=0.05	α=0.2	α=0.3	α=0.5	α=0.7
MC-GA	1	0.875	0.868	0.861	0.798	0.756
	2	0.864	0.857	0.851	0.787	0.745
	3	0.817	0.810	0.780	0.740	0.698
	4	0.886	0.879	0.851	0.809	0.767
	5	0.893	0.886	0.823	0.816	0.774
	6	0.903	0.896	0.892	0.826	0.784
	7	0.895	0.888	0.835	0.818	0.776
	8	0.932	0.925	0.889	0.855	0.813
	9	0.909	0.902	0.879	0.832	0.798
	10	0.796	0.789	0.756	0.719	0.667
MC-ICA	1	0.877	0.871	0.866	0.857	0.849
	2	0.836	0.830	0.825	0.816	0.808
	3	0.848	0.842	0.837	0.828	0.820
	4	0.919	0.913	0.908	0.899	0.891
	5	0.923	0.917	0.912	0.903	0.895
	6	0.949	0.943	0.938	0.929	0.921
	7	0.928	0.922	0.917	0.908	0.900
	8	0.969	0.963	0.958	0.949	0.941
	9	0.956	0.950	0.945	0.936	0.928
	10	0.816	0.810	0.805	0.796	0.788

All computational analyses were performed on a Linux-based workstation with a 2.4 GHz processor and 2 GB RAM with using the tuned parameters of MC-GA and MC-ICA algorithms as presented in Table 2. As shown in Table 4, as the risk level increases, the predicted efficiency values of the MC-GA and MC-ICA algorithms for each branch are decreased.

Table 5. The Comparison Between the Real and Predicted Efficiencies at α=0.5

Branch	Inputs			Quasi-fix input	Outputs		Quasi-fix output	Real efficiency (Dynamic BCC)	Predicted efficiency (MC-GA)	Predicted efficiency (MC-ICA)
	Total cost	Capital adequacy	Asset quality	Liquidity	Bank granting facilities	Electronic transaction	Liquidity			
1	637.7	106.87	192.7	40856	25951	18303	42606	0.886	0.798	0.857
2	628.2	92.45	131.7	37737	22956	17985	38537	0.868	0.787	0.816
3	666.7	100.45	125.8	39784	22854	17456	40782	0.872	0.740	0.828
4	669.3	72.41	162.7	37437	22354	19214	39436	0.898	0.809	0.899
5	587.1	103.95	115.74	35737	23045	18765	37747	0.899	0.816	0.903
6	609.7	104.87	68.54	38417	23343	16874	39418	0.921	0.826	0.929
7	608.1	40.91	104.74	31871	22652	19014	33875	0.913	0.818	0.908
8	535.7	64.41	222.7	27171	21324	16256	29171	0.941	0.855	0.949
9	497.7	96.45	92.74	28181	22216	18852	31180	0.932	0.832	0.936
10	515.6	55.41	144.7	36737	21127	17753	38730	0.864	0.719	0.796

To evaluate the accuracy of the DFS-BCC model and the effectiveness of proposed hybrid algorithms in predicting bank efficiencies, a comparison was conducted between the predicted efficiencies and the real efficiencies. The real efficiency values of the bank branches were conducted using the dynamic BCC model (Eq. 2) based on the actual inputs and outputs of the branches that have been completed upon completion of the forecast period in the banking system. Basic models in DEA have moderate risk levels. Therefore, the above comparison was made at the risk level of 0.5, and the results are shown in Table 5. The results indicate the high accuracy of the proposed model and algorithms in predicting efficiency.

In this paper, in order to evaluate the performance of the proposed DFS-BCC model and the hybrid algorithms, the coefficient of determination criteria (R^2) and the root mean square error (RMSE) were used, and the results are shown in Table 6.

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (Z_r - Z_p)^2}{\sum_{i=1}^{n} (Z_r - \bar{Z})^2} \tag{23}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (Z_r - Z_p)^2}{n}} \tag{24}$$

where Z_r , $Z_{p,}$ and $\bar{Z}$ are the real efficiency, predicted efficiency, and the mean of real efficiency values of bank branches, respectively.

Table 6. The Results of R^2 and RMSE

Evaluation criteria	Predicted efficiency via MC-GA	Predicted efficiency via MC-ICA
R2	0.80	0.64
RMSE	0.11	0.032

As it can be seen in Table 6, the MC-ICA algorithm performed better than the MC-GA algorithm in both R^2 and *RMSE* criteria in predicting the efficiency of bank branches.

Figures 2 and 3 show the average and the best fitness curves for the MC-GA and MC-ICA algorithms, respectively. These figures display the maximum total predicted efficiency using MC-GA and MC-ICA. In figures 2 and 3, horizontal axes represent subsequent iterations and the vertical axes represent fitness value. As can be seen from these figures, it can be said that the MC-ICA is greater than the MC-GA on average and the best fitness rate in their iterations. On the other hand, the result of MC-ICA algorithm was quite efficient in terms of the number of iterations and fitness value.

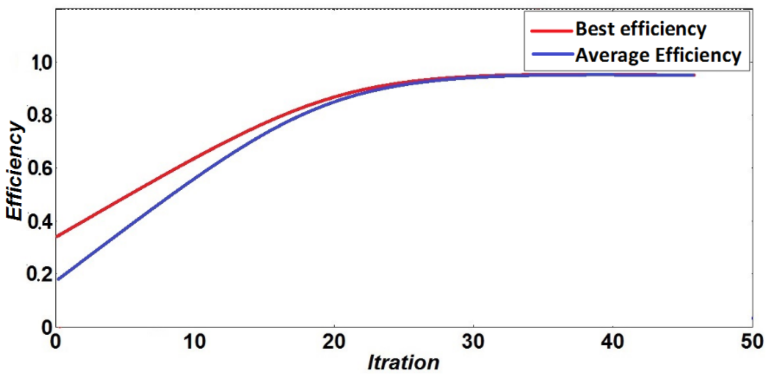

Figure 2. The Best and Average Fitness of MC-GA

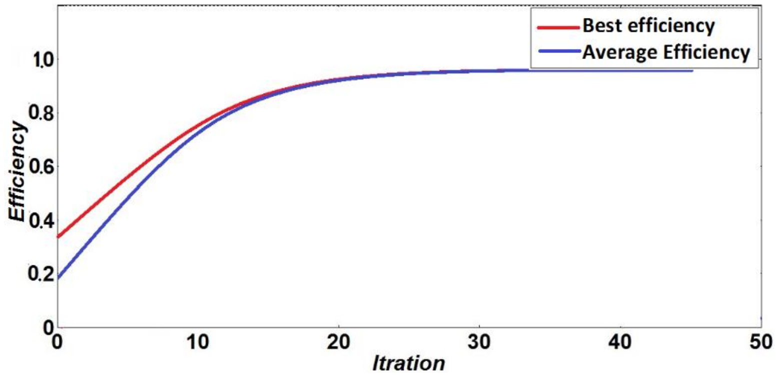

Figure 3. The Best and Average Fitness of MC-ICA

6. Discussions, Conclusions, and Recommendations

In recent years, the empirical modeling of modern banking technology and the evaluation of bank efficiency have begun to incorporate these theoretical developments and yield interesting insights that reflect the unique nature and role of modern banking systems in modern economies. As a result, the need to use efficiency measurement systems becomes increasingly apparent in the banking industry. In this regard, it is necessary to predict the efficiency of banks in the modern banking industry to provide more accurate planning for the allocation of resources to their inputs and greater efficiency of their outputs. Therefore, this paper designed a new model based on dynamic stochastic DEA in fuzzy environment (DFS-BCC model) by considering variable returns to scale and modern banking indicators to predict the efficiency of banks. In order to deal with the uncertainty in efficiency forecasting, the mean chance theory has been used to express the constraints of the model and the expected value in its objective function in order to forecast the expected efficiency of the banks. Taking into account the point that the proposed model belonged to the category of NP-hard combinatorial optimization problems, two hybrid algorithms were designed by combining MC simulation technique with GA algorithm and ICA algorithm to solve it. Moreover, in order to improve performances of MC-GA and MC-ICA parameters, RSM technique was applied to set their proper values. To prove the feasibility of proposed model and validity of the designed algorithms, a case study of the modern banking industry was given in this paper.

Consequently, the numerical results of the proposed model were analyzed. To evaluate the accuracy of the proposed DFS-BCC model in predicting banks efficiencies, a comparison was done between the predicted efficiencies and the real efficiencies were conducted using the dynamic BCC model. This comparison showed the high accuracy of the proposed model in predicting efficiency, which was useful for managers to determine future strategies to improve their bank branches. In addition, in order to assess the reliability of the solution, the results of the proposed algorithms were compared together based on R^2 and *RMSE* criteria. The results showed the MC-ICA algorithm performed better than the MC-GA algorithm in both criteria in predicting the efficiency of bank branches. Furthermore, the results of the proposed algorithms were compared together based on fitness curves. The results showed that MC-ICA algorithm performed better than MC-GA algorithm in terms of accuracy, speed convergence rate, and the number of solutions iterations.

In comparison to previous similar studies such as Yaghoubi and Amiri (2015), Wanke and Azad et al. (2016), Gaganis et al. (2020), and Avkiran and Morita (2020), the strength of the proposed model is that this model simultaneously considers the characteristics of stochastic, fuzzy, and dynamic environments in data analysis by considering variable returns to scale and modern banking indicators to predict the efficiency of banks, which can make the model more practical and adaptable to real world conditions. For example, our model is superior to that of Jafarian-Moghaddam and Ghoseiri (2011) because it is capable of dealing with uncertainty in efficiency forecasting by applying the mean chance and expected value theories. In addition, contrary to the argument of Qin and Liu (2010), our model considers the dynamic conditions prevailing in the branches in the modern banking industry.

Based on the results, we found that bank managers must foster the cost management (such as the cost of opening an account, creating a loan document package, or handling a specific type of transaction) if they intend to improve their efficiency in the modern banking industry. In addition, in order to predict efficiency more precisely, bank managers should have a more accurate planning of the liquidity allocated to branches as a quasi-fixed input (output). Finally, in order to better predict performance, bank managers must accurately measure the capital adequacy and asset quality of their branches at the end of each financial period. The following items are recommended for future studies:

- Developing the proposed model by considering the non-normal distribution for inputs and outputs of banks;
- Developing the proposed model by considering network dependence between bank branches;
- Developing the proposed model with probabilistic constraints, and
- Investigating other effective indicators in the modern banking industry in order to predict the efficiency of banks.

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