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RESEARCH PAPER

Size-dependent nano-spherical pressure vessels based on strain gradient theory

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Abstract

This study investigates the effect of size scale material parameters on stress distribution and radial displacement of nanosphere based on strain gradient theory. This model is more capable of studying mechanical behavior than classical elasticity theory as the size scale effect of the nanosphere is also considered. Minimum total potential energy is used to derive governing differential equation of nanosphere under internal hydrostatic pressure. Using the efficient numerical generalized differential quadrature (GDQ) method, the governing equation and corresponding boundary conditions are solved. The classical elasticity equation is obtained by setting the value of size scale material parameters to zero. With the comparison of these theories, the importance of the size scale material parameters is achieved. It is found that the radial displacement of nanosphere predicted by strain gradient theory is less than those predicted by classical elasticity theory but comparing the distribution of stress components along radius is more complex. The effect of the size of the nanosphere on the radial stress components is also studied. With an increasing outer radius of the nanosphere, the mechanical behavior predicted by strain gradient theory tends toward those in classical elasticity theory.

Keywords: Nanosphere; stress analysis; strain gradient theory; size-dependent; generalized differential quadrature (GDQ)

1. Introduction

In recent years, nanotechnology has revolutionized different scientific fields such as Chemistry, Engineering, and Biology, which has received widespread attention from many researchers [1-7]. The material properties at the nanoscale are different from those at the macro scale. The study of scientific phenomena is significant in the nanoscale because mechanical properties have been observed to change in less than 100 nanometers [6-16]. Nanospheres such as carbon nanostructures have comprehensive utilization of chemistry. Among those, Fluorine has a high-pressure tolerance [17, 18]. Also, liposomes are Spherical particles that are utilized in the food industry, cellular studies, and medical science that can hold and carry chemical solutions, so they are helpful for drug [19-24]. Due to the application of these spherical nanostructures, they may be subjected to hydrostatic pressure, so it is essential to investigate their mechanical behavior under hydrostatic pressure [25-27]. Since the classical elasticity theory could not predict the effect of size scale material parameters, various methods have been proposed to study nanomaterials' properties and mechanical behavior, such as experimental investigation, molecular dynamics simulation, and continuum mechanic's approach [28-30]. Due to the high cost and complexity of both experimental and molecular dynamics simulation, continuum mechanic's approaches such as nonlocal elasticity modified couple stress and strain gradient theories are used [30-35]. Among those, strain gradient theory initiated by Mindlin has been utilized to study beams, plates, and shells on nano/micro-scales. For example, stress analysis of rotating nanodisk made of functionally graded materials is studied by Hosseini et al. [36] which the nanodisk is assumed to be variable thickness. Based on strain gradient

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theory, a functionally graded nanodisk under thermoelastic loading is analyzed by Shishesaz et al. [34], in which it is assumed that any variation in temperature occurs only in the radial direction. Analysis of stress distribution and radial displacement of rotating functionally graded micro/nano-disks of variable thickness is performed by Hosseini et al. [15]. In this study, strain gradient theory is applied. It is assumed that nanodisk is subjected to thermal and mechanical loads while rotating with a constant angular velocity. The effect of small scale on the mechanical behavior of functionally graded nano-cylinders under radial pressure is investigated by Shishesaz et al. [37] in this paper. The governing equation is derived based on strain gradient theory, and the variations in material properties along the thickness direction are included based on three different models. In order to investigate the mechanical behavior of the living cell, an analytical solution based on strain gradient theory is performed by Hadi et al. [2]. In this paper, living cell is modeled in form of a sphere, and hydrostatic pressure on the distribution of stress and displacement of living cell is studied.

All size-dependent models mentioned above only investigate the mechanical behavior of tubes and disks in nanoscale. This study deals with the effect of material size on stress distribution and the displacement of the nanosphere under internal hydrostatic pressure based on strain gradient theory. Some results are also compared with classical theory.

2. Formulation

As shown in Figure 1, consider a nanosphere in spherical coordinate subjected to internal hydrostatic pressure.

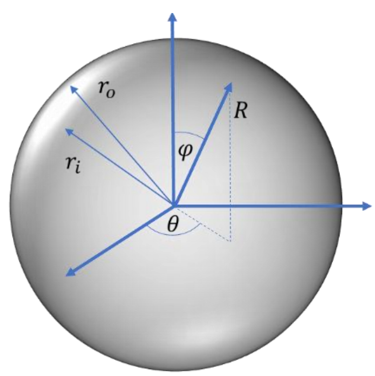

Fig 1. Nanosphere under hydrostatic pressure.

The governing equation of nanosphere can be derived using the principle of minimum potential energy as follow:

$$\delta U - \delta W = 0 \tag{1}$$

where δU and δW are a variation of total strain energy and external work, respectively.

In the strain gradient elasticity theory, the second gradient of the displacement field also appears in the total strain energy relation.

$$\delta U = \int_V \left(\sigma_{ij} \delta\varepsilon_{ij} + \tau_{klm} \delta\xi_{klm} \right) dV \tag{2}$$

Where σ, ε, τ and ξ are stress tensor, strain tensor, high-order stress tensor, and gradient of the strain tensor, respectively. While the displacement field is small, the strain tensor can be expressed as follow:

$$\varepsilon=\begin{bmatrix}\frac{du}{dr} & 0 & 0\\ 0 & \frac{u}{r} & 0\\ 0 & 0 & \frac{u}{r}\end{bmatrix} \tag{3}$$

In above equation u is radial displacement in spherical coordinate, which is the only function of radius. So, the nonzero components of the gradient of strain tensor are derived as follow:

$$\xi_{rrr}=\frac{d^2u}{dr^2},\qquad \xi_{r\theta\theta}=\xi_{r\varphi\varphi}=\xi_{\theta\theta r}=\xi_{\varphi\varphi r}=\xi_{\varphi r\varphi}=\frac{d}{dr}\left(\frac{u}{r}\right)=\frac{1}{r}\frac{du}{dr}-\frac{u}{r^2} \tag{4}$$

By considering the symmetry of sphere and radially hydrostatic pressure, stress and strain relations are found by Hook's low as below:

$$\begin{Bmatrix}\sigma_r\\ \sigma_\theta\end{Bmatrix}=E\begin{bmatrix}A & 2B\\ B & A+B\end{bmatrix}\begin{bmatrix}du/dr\\ u/r\end{bmatrix},\quad A=\frac{1-\upsilon}{(1+\upsilon)(1-2\upsilon)}\ ,\quad B=\frac{\upsilon}{(1+\upsilon)(1-2\upsilon)} \tag{5}$$
$$\sigma_r=\sigma_\theta\ ,\ \varepsilon_\theta=\varepsilon_\varphi$$

The components of high-order stress tensor are defined as follow:

$$\tau_{ijk}=\frac{1}{2}a_1\left(\delta_{ij}\xi_{kpp}+2\delta_{jk}\xi_{ppi}+\delta_{ik}\xi_{jpp}\right)+2a_2\delta_{jk}\xi_{ipp}+a_3\left(\delta_{ij}\xi_{ppk}+\delta_{ik}\xi_{ppi}\right)$$
$$+2a_4\xi_{ijk}+a_5\left(\xi_{jki}+\xi_{kji}\right) \tag{6}$$

where δ_{ij} is Kronecker's delta and a_i is the size scale material parameter, respectively. Substituting Eq. (4) into Eq. (6), components of high-order stress tensor are derived as follow:

$$\begin{Bmatrix}\tau_{rrr}\\ \tau_{r\theta\theta}\\ \tau_{\theta\theta r}\end{Bmatrix}=\begin{bmatrix}2(a_1+a_2+a_3+a_4+a_5) & 4(a_1+a_2+a_3)\\ a_1+2a_2 & 2(a_1+a_4+a_5)+3a_2\\ 0.5a_1+a_3 & 2(a_3+a_4+a_5)+a_1\end{bmatrix}\begin{Bmatrix}d^2u/dr^2\\ \frac{d}{dr}\left(\frac{u}{r}\right)\end{Bmatrix} \tag{7}$$
$$\tau_{r\theta\theta}=\tau_{r\varphi\varphi}\ ,\qquad \tau_{\theta\theta r}=\tau_{\varphi\varphi r}=\tau_{\varphi r\varphi}=\tau_{\theta r\theta}$$

Using Eq. (2), variation of the total strain energy is derived as follow:

$$\delta U=\int_V\left(\sigma_{ij}\delta\varepsilon_{ij}+\tau_{ijk}\delta\xi_{ijk}\right)dv$$
$$=\int_0^{2\pi}\int_0^{\pi}\int_{r_i}^{r_o}\left(\sigma_r\delta\varepsilon_r+2\sigma_\theta\delta\varepsilon_\theta+\tau_{rrr}\delta\xi_{rrr}+2\tau_{r\theta\theta}\delta\xi_{r\theta\theta}+4\tau_{\theta\theta r}\delta\xi_{\theta\theta r}\right)\left(r^2\sin\varphi\ dr\ d\varphi\ d\theta\right) \tag{8}$$

By substituting Eq. (3) and Eq. (4) into Eq. (2), variation of total strain energy is expanded as:

$$\delta U = 4\pi \int_{r_i}^{r_o} \begin{pmatrix} \sigma_r \delta\left(\dfrac{du}{dr}\right) + 2\sigma_\theta \delta\left(\dfrac{u}{r}\right) + \tau_{rrr}\delta\left(\dfrac{d^2u}{dr^2}\right) \\ +2\tau_{r\theta\theta}\delta\left(\dfrac{d}{dr}\left(\dfrac{u}{r}\right)\right) + 4\tau_{\theta\theta r}\delta\left(\dfrac{d}{dr}\left(\dfrac{u}{r}\right)\right) \end{pmatrix} r^2 dr \tag{9}$$

Integrating part by part over the above equation, variation of the total strain energy is calculated as follow:

$$\begin{aligned} \delta U &= 4\pi \int_{r_i}^{r_o} \begin{pmatrix} -\dfrac{d}{dr}\left(r^2\sigma_r\right) + 2r\sigma_\theta + \dfrac{d}{dr^2}\left(r^2\tau_{rrr}\right) - 2\dfrac{d}{dr}\left(r\tau_{r\theta\theta}\right) - 2\tau_{r\theta\theta} \\ -4\dfrac{d}{dr}\left(r\tau_{\theta\theta r}\right) - 4\tau_{\theta\theta r} \end{pmatrix} \delta(u)\, dr \\ &+ \left[\left(r^2\sigma_r - \dfrac{d}{dr}\left(r^2\tau_{rrr}\right) + 2r\tau_{r\theta\theta} + 4r\tau_{\theta\theta r}\right)\delta u\right]_{r_i}^{r_o} + \left[\left(r^2\tau_{rrr}\right)\dfrac{d}{dr}(\delta u)\right]_{r_i}^{r_o} \end{aligned} \tag{10}$$

Variation of the external work done by hydrostatic pressure p on the inner and outer surface of the nanosphere is determined as follow:

$$\begin{aligned} \delta W &= 4\pi r^2 \left[p\delta u + \tau_{rrr}^{ext}\delta\varepsilon_{rr} + \tau_{\theta\theta r}^{ext}\delta\varepsilon_{\theta\theta} + \tau_{\varphi\varphi r}^{ext} \right]_{r_o}^{r_i} \\ &= 4\pi r^2 \left[p\delta u + \tau_{rrr}^{ext}\dfrac{d}{dr}(\delta u) + \dfrac{2\tau_{\theta\theta r}^{ext}}{r}\delta u \right]_{r_o}^{r_i} \end{aligned} \tag{11}$$

Substituting Eq. (10) and (11) into Eq. (1), the governing equation is derived as below:

$$\begin{aligned} &\left(-2\left(a_1+a_2+a_3+a_4+a_5\right)r^4\right)\dfrac{d^4u}{dr^4} + \left(-8\left(a_1+a_2+a_3+a_4+a_5\right)r^3\right)\dfrac{d^3u}{dr^3} \\ &+ \left(EAr^4 + 8\left(a_1+a_2+a_3+a_4+a_5\right)r^2\right)\dfrac{d^2u}{dr^2} + \left(2EAr^3\right)\dfrac{du}{dr} + \left(-2EAr^2\right)u = 0 \end{aligned} \tag{12}$$

where relative boundary conditions are expressed as:

$$\begin{aligned} &\left[-2\left(a_1+a_2+a_3+a_4+a_5\right)r^3\right]\dfrac{d^3u}{dr^3} + \left[-4\left(a_1+a_2+a_3+a_4+a_5\right)r^2\right]\dfrac{d^2u}{dr^2} \\ &+ \left[EAr^3 + \left(8\left(a_1+a_2+a_3\right) + 12\left(a_4+a_5\right)\right)r\right]\dfrac{du}{dr} \qquad @\, r = r_i \,\&\, r_o \\ &+ \left[2EBr^2 - \left(8\left(a_1+a_2+a_3\right) + 12\left(a_4+a_5\right)\right)\right]u = \left(p + \dfrac{2\tau_{\theta\theta r}^{ext}}{r}\right)r^3, \end{aligned} \tag{13}$$

$$\left[2\left(a_1+a_2+a_3+a_4+a_5\right)r^2\right]\frac{d^2u}{dr^2}+\left[4\left(a_1+a_2+a_3\right)r\right]\frac{du}{dr} \qquad @r=r_i \,\&\, r_o$$
$$+\left[-4\left(a_1+a_2+a_3\right)\right]u=\left(\tau_{rrr}^{ext}\right)r^2$$

Below non-dimensional parameters are used to derive non-dimensional governing equation and associated boundary conditions.

$$\begin{array}{lll} \bar{r}=\dfrac{r}{r_0} & \bar{u}=\dfrac{u}{r_o} & \bar{a}_k=\dfrac{a_k}{Er_o^2}, \quad k=1,2,...,6 \\ \bar{p}=\dfrac{p}{E} & \bar{\tau}_{rrr}^{ext}=\dfrac{\tau_{rrrE}^{ext}}{Er_o} & \bar{\tau}_{\theta\theta r}^{ext}=\dfrac{\tau_{\theta\theta r}^{ext}}{Er_o} \end{array} \tag{14}$$

So, the non-dimensional governing equation and relative boundary conditions are defined as follow:

$$\begin{array}{ll} f_4\dfrac{d^4\bar{u}}{d\bar{r}^4}+f_3\dfrac{d^3\bar{u}}{d\bar{r}^3}+f_2\dfrac{d^2\bar{u}}{d\bar{r}^2}+f_1\dfrac{d\bar{u}}{d\bar{r}}+f_0\bar{u}=0 & \\ g_3\dfrac{d^3\bar{u}}{d\bar{r}^3}+g_2\dfrac{d^2\bar{u}}{d\bar{r}^2}+g_1\dfrac{d\bar{u}}{d\bar{r}}+g_0\bar{u}=g & @r=r_i \,\&\, r_o \\ h_2\dfrac{d^2\bar{u}}{d\bar{r}^2}+h_1\dfrac{d\bar{u}}{d\bar{r}}+h_0\bar{u}=h & @r=r_i \,\&\, r_o \end{array} \tag{15}$$

where:

$$\begin{array}{ll} f_4=-2\left(\bar{a}_1+\bar{a}_2+\bar{a}_3+\bar{a}_4+\bar{a}_5\right)\bar{r}^4 & g_3=-2\left(\bar{a}_1+\bar{a}_2+\bar{a}_3+\bar{a}_4+\bar{a}_5\right)\bar{r}^3 \\ f_3=-8\left(\bar{a}_1+\bar{a}_2+\bar{a}_3+\bar{a}_4+\bar{a}_5\right)\bar{r}^3 & g_2=-4\left(\bar{a}_1+\bar{a}_2+\bar{a}_3+\bar{a}_4+\bar{a}_5\right)\bar{r}^2 \\ f_2=A\bar{r}^4+8\left(\bar{a}_1+\bar{a}_2+\bar{a}_3+\bar{a}_4+\bar{a}_5\right)\bar{r}^2 & g_1=A\bar{r}^3+\left(8\left(\bar{a}_1+\bar{a}_2+\bar{a}_3\right)+12\left(\bar{a}_4+\bar{a}_5\right)\right)\bar{r} \\ f_1=2A\bar{r}^3 & g_0=2BA\bar{r}^2-\left(8\left(\bar{a}_1+\bar{a}_2+\bar{a}_3\right)+12\left(\bar{a}_4+\bar{a}_5\right)\right) \\ f_0=-2A\bar{r}^2 & g=\left(\bar{p}+\dfrac{2\bar{\tau}_{\theta\theta r}^{ext}}{\bar{r}}\right)\bar{r}^3 \end{array} \tag{16}$$

$$h_2=2\left(\bar{a}_1+\bar{a}_2+\bar{a}_3+\bar{a}_4+\bar{a}_5\right)\bar{r}^2$$
$$h_1=4\left(\bar{a}_1+\bar{a}_2+\bar{a}_3\right)\bar{r}$$
$$h_0=-4\left(\bar{a}_1+\bar{a}_2+\bar{a}_3\right)$$
$$h=\bar{\tau}_{rrr}^{ext}$$

3. Method of solution

the general differential quadrature (GDQ) method is employed to solve the governing equation and calculate displacement field or stress tensor of nanosphere. According to the GDQ method, the domain of the governing equation is discrete into N nodes, and the derivative of displacement filed at an arbitrary point is written as a linear combination of displacement at all points.

$$\frac{d^n\bar{u}}{d\bar{r}^n}=\sum_{j=1}^{N}C_{ij}^{n}\bar{u}_j \qquad i=1,2,...,N \tag{17}$$

where c_{ij}^{n} denotes a weighted coefficient that is expressed as below.

$$\begin{aligned}
&C_{ij}^{(1)}=\frac{\prod_{k=1,k\neq i}^{N}\left(r_i-r_k\right)}{\left(r_i-r_j\right)\prod_{k=1,k\neq j}^{N}\left(r_j-r_k\right)}\ , \qquad i,j=1,2,...,N,\quad i\neq j\\
&C_{ii}^{(1)}=-\sum_{k=1,k\neq i}^{N}C_{ik}^{(1)}\\
&C_{ij}^{(n)}=n\left(C_{ii}^{(n-1)}C_{ij}^{(1)}-\frac{C_{ij}^{(n-1)}}{r_i-r_j}\right), \qquad i,j=1,2,...,N,\quad i\neq j\\
&C_{ii}^{(n)}=-\sum_{k=1,k\neq i}^{N}C_{ik}^{(n)}
\end{aligned} \tag{18}$$

Where $\prod$ denotes the product of a sequence of factors. Using the GDQ method, the governing equation can be written as follow:

$$f_4\big|_{\bar{r}_i}\sum_{j=1}^{N}C_{ij}^{4}\bar{u}_j+f_3\big|_{\bar{r}_i}\sum_{j=1}^{N}C_{ij}^{3}\bar{u}_j+f_2\big|_{\bar{r}_i}\sum_{j=1}^{N}C_{ij}^{2}\bar{u}_j+f_1\big|_{\bar{r}_i}\sum_{j=1}^{N}C_{ij}^{1}\bar{u}_j+f_0\big|_{\bar{r}_i}\bar{u}_i=0\ , \qquad i=1,2,...,N \tag{19}$$

As well as relative boundary conditions can be written as follow:

$$\begin{aligned}
&g_3\big|_{\bar{r}_1}\sum_{j=1}^{N}C_{1j}^{3}\bar{u}_j+g_2\big|_{\bar{r}_1}\sum_{j=1}^{N}C_{1j}^{2}\bar{u}_j+g_1\big|_{\bar{r}_1}\sum_{j=1}^{N}C_{1j}^{1}\bar{u}_j+g_0\big|_{\bar{r}_1}\bar{u}_1=g\big|_{\bar{r}_1}\\
&g_3\big|_{\bar{r}_N}\sum_{j=1}^{N}C_{Nj}^{3}\bar{u}_j+g_2\big|_{\bar{r}_N}\sum_{j=1}^{N}C_{Nj}^{2}\bar{u}_j+g_1\big|_{\bar{r}_N}\sum_{j=1}^{N}C_{Nj}^{1}\bar{u}_j+g_0\big|_{\bar{r}_N}\bar{u}_N=g\big|_{\bar{r}_N}\\
&h_2\big|_{\bar{r}_1}\sum_{j=1}^{N}C_{1j}^{2}\bar{u}_j+h_1\big|_{\bar{r}_1}\sum_{j=1}^{N}C_{1j}^{1}\bar{u}_j+h_0\big|_{\bar{r}_1}\bar{u}_1=h\big|_{\bar{r}_1}\\
&h_2\big|_{\bar{r}_N}\sum_{j=1}^{N}C_{Nj}^{2}\bar{u}_j+h_1\big|_{\bar{r}_N}\sum_{j=1}^{N}C_{Nj}^{1}\bar{u}_j+h_0\big|_{\bar{r}_N}\bar{u}_N=h\big|_{\bar{r}_N}
\end{aligned} \tag{20}$$

Substituting Eq. (20) into Eq. (19), the solution of the governing equation for displacement field, stress tensor, and high-order stress tensor are achieved.

4. Result and discussion

In previous sections, a non-dimensional equation governing the nanosphere under hydrostatic pressure was derived. The effect of size scale material parameters on displacement field and corresponding stresses are investigated using the mentioned method in section 3. Also, a compression between classical elasticity and strain gradient theory has been performed. The material and geometry properties of the nanosphere are given in table 1.

Table 1. The material and geometry properties of nanosphere.

$E(Gpa)$	υ	$r_o(nm)$	r_i	$p_i(Mpa)$
100	0.28	60	$0.8r_o$	60

To validate, a comparison between the result of this study and Ghannad et al. [38] has been performed using the following non-dimensional parameters.

$$u^* = 1000\frac{u}{r_i}, \qquad r^* = \frac{r}{r_i} \tag{21}$$

By setting all of size scale material parameters to zero, an excellent agreement between the result of this paper and those of classical theory presented by Ghannad et al. [38] is observed in figure 2

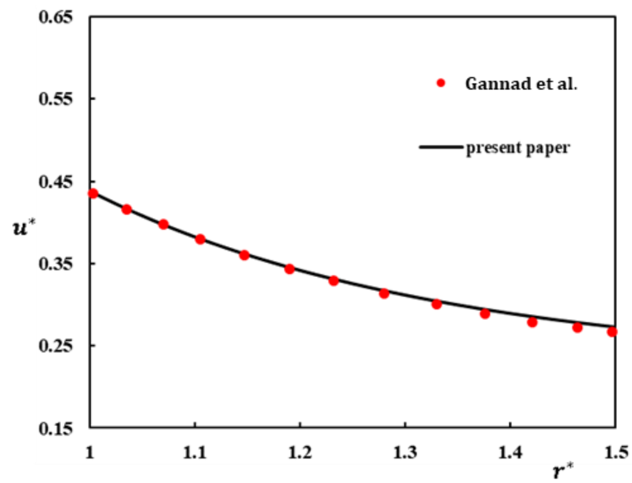

Fig 2. Comparison of the result between this study and those of classical theory presented by Ghannad et al. [38].

A multiplayer (L) is employed to investigate the effect of size scale material parameters.

$$[a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5] = [0.2386 \quad 0.0134 \quad 0.0013 \quad 0.0934 \quad 0.2462] \times L \times 10^{-4}$$

Whenever multiplayer parameter takes a value of zero, the mechanical behavior of nanosphere in the classical elasticity and strain gradient theories are identical. The effect of size scale material parameters on the radial displacement of the nanosphere is shown in figure 3. Obviously, by selecting $L=0$, the displacement of nanosphere based on classical elasticity and strain gradient theories are identical. On the other hand, by increasing size scale material parameters, non-dimensional radial displacement of nanosphere is decreased. As a result, the predicted radial displacement of nanosphere by strain gradient theory is smaller than those by classical elasticity theory.

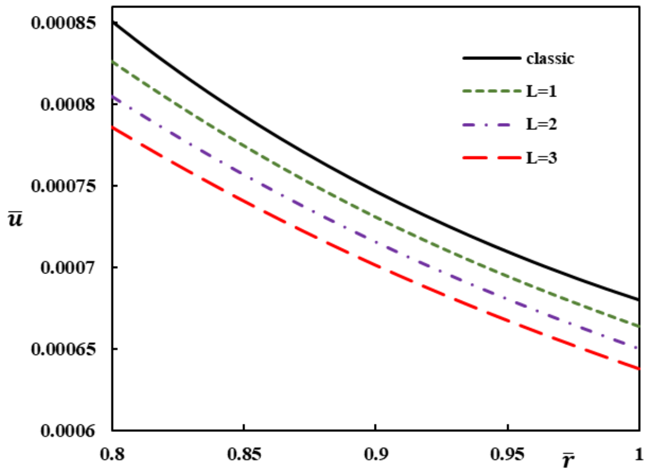

Fig 3. Radial displacement of nanosphere for various size scale material parameters

In figure 4, radial stress distribution along nanosphere radius is presented for various situations. For internal hydrostatic pressure loading, the radial stress is compressive stress based on both classical elasticity and strain gradient theories. Based on classical elasticity theory, the stress in the inner radius equal to internal hydrostatic pressure, while with increasing nanosphere radius, radial stress decreased to zero. Whereas any raise in size scale material parameters due to increasing multiplayer L caused radial stress becomes less than those based on strain gradient theory in inner radius, unlike classical elasticity theory, its value is not zero in outer radius. Consequently, all the radial stress distributions between the inner and outer radius have been intersected.

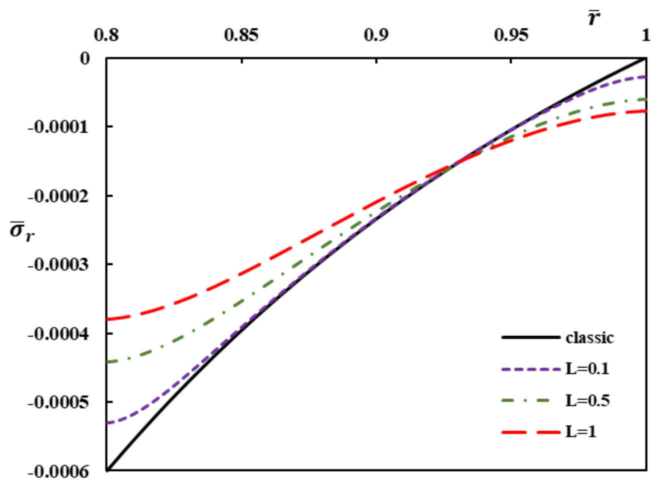

Fig 4. Radial stress distribution along nanosphere radius for various size scale material parameters

As shown in figure 4, unlike radial stress, the tangential stress is tensile. In both theories, the tangential stress is nonzero in any radius. As the nanosphere radius increases, the tangential stress decreases. The effect of size scale material parameters on tangential stress in the inner radius is more complex than the outer. By increasing L, initially tangential stress increases, then its value decreases in the inner radius. Whereas in the outer radius of the nanosphere, as it increases, the tangential stress decreases.

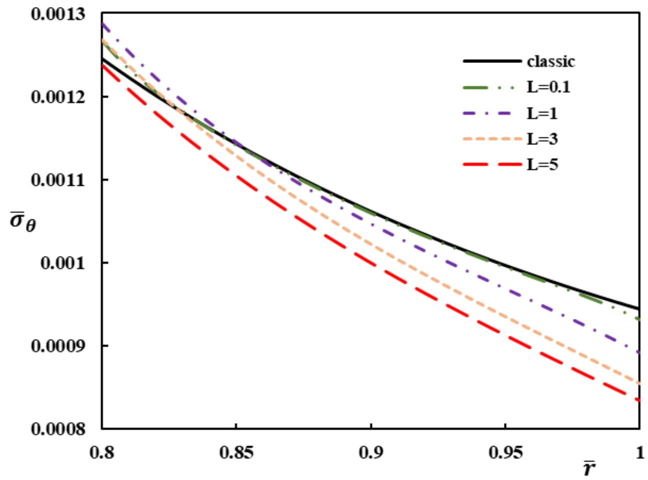

Fig 5. tangential stress distribution along nanosphere radius for various size scale material parameters

The distribution of hi-order stress $\overline{\tau}_{rrr}$ along radial direction can be seen in figure 6. The maximum value $\overline{\tau}_{rrr}$ is among the inner and outer radius of the nanosphere. With decreasing L, hi-order stress $\overline{\tau}_{rrr}$ decreases, and its maximum value tends toward the nanosphere's inner radius. However, the value of this hi-order stress is zero in both inner and outer nanosphere radius. As it decreases more, it seems the value $\overline{\tau}_{rrr}$ tends toward zero.

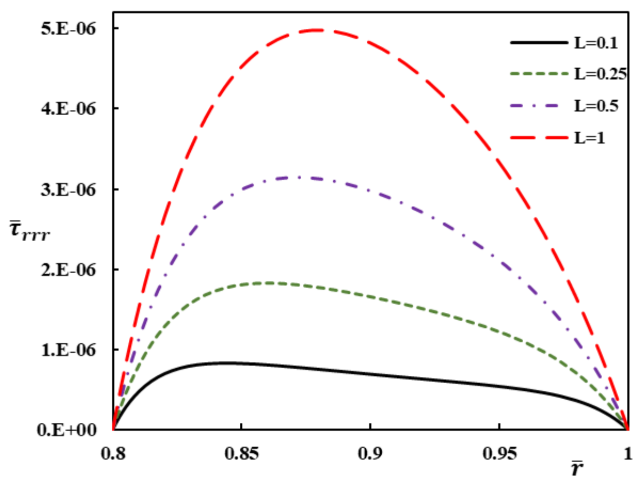

Fig 6. Hi-order stress $\overline{\tau}_{rrr}$ distribution along nanosphere radius for various size scale material parameters

The distribution of hi-order stress components $\overline{\tau}_{r\theta\theta}$ and $\overline{\tau}_{\theta\theta r}$ along the radial direction is shown in Figures 7 and 8, respectively, as L increases, both of them in a fixed radius of nanosphere decreases. The effect of the size scale material parameter is significant in the inner radius of the nanosphere. As it decreases more, it seems the value $\overline{\tau}_{r\theta\theta}$ and $\overline{\tau}_{\theta\theta r}$ tends toward zero.

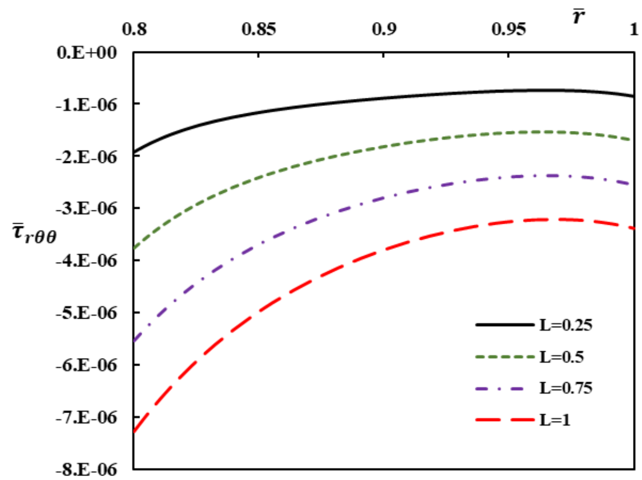

Fig 7. Hi-order stress $\overline{\tau}_{r\theta\theta}$ distribution along nanosphere radius for various size scale material parameters

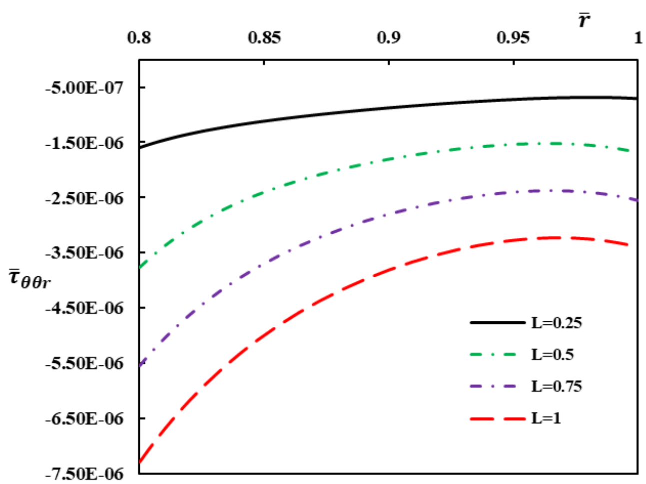

Fig 8. Hi-order stress $\overline{\tau}_{\theta\theta r}$ distribution along nanosphere radius for various size scale material parameters

Figure 8 is presented to capture the effect of the size of nanosphere radius on radial stress component for various size scale material parameters. As shown in this figure, the radial stress component does not depend on the outer radius of the nanosphere in classical elasticity theory. So, the value of the maximum radial stress is constant in the nanosphere. On the other hand effect of size is significant for a small radius of nanosphere-based strain gradient theory. So that with an increasing outer radius of nanosphere, mechanical behavior predicted by strain gradient theory tends toward those in classical elasticity theory.

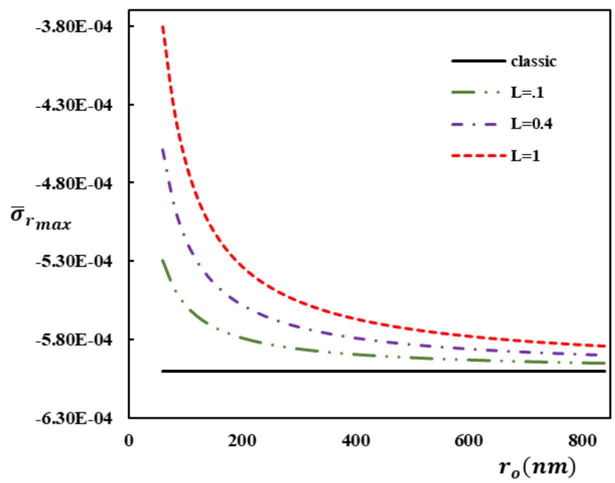

Fig 9. Variation of maximum radial stress component versus outer nanosphere radius.

5. conclusion

In this paper, we studied the mechanical behaviors of the nanosphere under internal hydrostatic pressure. The effect of size scale material parameters on the mechanical behavior of nanosphere such as radial displacement, tangential and radial stress components were studied. The results showed that the effect of size scale material parameters in nanoscale is significant, and classical elasticity theory could not predict these effects. By considering internal hydrostatic pressure, maximum radial or tangential stress occurred in the inner radius of the nanosphere. The effect of size scale material parameters on radial stress in the inner radius was significant than the outer radius, while for the tangential stress, the opposite is true. The strain gradient theory predicts materials more inflexible than those in the classical elasticity theory in the nanoscale.

Conflict of interest

All authors declare that they have no conflicts of interest.

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