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4-total mean cordial labeling of special graphs

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ABSTRACT

Let G be a graph. Let $f : V(G) \to \{0, 1, 2, \ldots, k-1\}$ be a function where $k \in \mathbb{N}$ and $k > 1$. For each edge uv, assign the label $f(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. f is called k-total mean cordial labeling of G if $|t_{mf}(i) - t_{mf}(j)| \leq 1$, for all $i, j \in \{0, 1, \ldots, k-1\}$, where $t_{mf}(x)$ denotes the total number of vertices and edges labelled with x, $x \in \{0, 1, 2, \ldots, k-1\}$. A graph with admit a k-total mean cordial labeling is called k-total mean cordial graph.

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1 Introduction

Graphs in this paper are finite, simple and undirected. Ponraj et al. [3] have been introduced the concept of k-total mean cordial labeling and invesigate the 4-total mean cordial labeling of certain graphs path, cycle,star, bistar, comb, crown, square of path, double

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comb, double crown, double fan, subdivision of star, subdivision of comb, subdivision of ladder, helm, flower graph, gear graph and web graph in [3, 4, 5, 6, 7] . In this paper we investigate the 4-total mean cordial labeling behaviour of dumbbell graph, dragon graph, armed crown etc. Let x be any real number. Then $\lceil x \rceil$ stands for the smallest integer greater than or equal to x. Terms are not defined here follow from Harary [2] and Gallian [1].

2 k-total mean cordial graph

Definition 2.1. Let G be a graph. Let $f : V(G) \rightarrow \{0, 1, 2, \ldots, k-1\}$ be a function where $k \in \mathbb{N}$ and $k > 1$. For each edge uv, assign the label $f(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. f is called k-total mean cordial labeling of G if $|t_{mf}(i) - t_{mf}(j)| \leq 1$, for all $i, j \in \{0, 1, \ldots, k-1\}$, where $t_{mf}(x)$ denotes the total number of vertices and edges labelled with x, $x \in \{0, 1, 2, \ldots, k-1\}$. A graph with admit a k-total mean cordial labeling is called k-total mean cordial graph.

3 Preliminary

Definition 3.1. The *complement* $\overline{G}$ of a graph G also has $V(G)$ as its vertex set, but two vertices are adjacent in $\overline{G}$ if and only if they are not adjacent in G.

Definition 3.2. The *Cartesian product* of two graphs G_1 and G_2 is the graph $G_1 \times G_2$ with vertex set $V_1 \times V_2$ and two vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ are adjacent whenever $[u_1 = v_1$ and u_2 adj $v_2]$ or $[u_2 = v_2$ and u_1 adj $v_1]$.

Definition 3.3. Let $C_n^{(t)}$ denote the one point union of t cycles of length n.

Definition 3.4. The *armed crown* AC_n is obtained from the cycle $C_n : u_1u_2 \ldots u_nu_1$ with $V(AC_n) = V(C_n) \cup \{v_i, w_i : 1 \leq i \leq n\}$ and $E(AC_n) = E(C_n) \cup \{u_iv_i, v_iw_i : 1 \leq i \leq n\}$.

Definition 3.5. The graph obtained by joining two disjoint cycles $u_1\ u_2\ \ldots\ u_n\ u_1$ and $v_1\ v_2\ \ldots\ v_n\ v_1$ of same length with an edge u_1v_1 is called dumbbell graph Db_n.

Definition 3.6. The graph obtained by joining cycle $C_n : u_1u_2 \ldots u_nu_1$ and path $P_n : v_1v_2 \ldots v_n$ of same length with $u_1 = v_1$ is called dragon graph $C_n@P_n$.

Definition 3.7. The *shadow graph* $D_2(G)$ of a connected graph G is obtained by taking two copies of G, say G_1 and G_2. Join each vertex u_1 in G_1 to the the neighbours of corresponding vertex u_2 in G_2.

Definition 3.8. The graph $C_n \odot S_n$ is obtained from the cycle $C_n : u_1u_2 \dots u_nu_1$ with the vertex set $V(C_n \odot S_n) = V(C_n) \cup \{v_i : 1 \le i \le n\}$ and $E(C_n \odot S_n) = E(C_n) \cup \{u_1v_i : 1 \le i \le n\}$.

Definition 3.9. The graph $C_n \otimes S_n$ is obtained from the cycle $C_n : u_1u_2 \dots u_nu_1$ with the vertex set $V(C_n \otimes S_n) = V(C_n) \cup \{v, v_i : 1 \le i \le n\}$, where $u_1 = v_1$ and $E(C_n \otimes S_n) = E(C_n) \cup \{vu_1\} \cup \{vv_i : 2 \le i \le n\}$.

4 Main results

Theorem 4.1. The book with rectangular pages $K_{1,n} \times K_2$ is 4-total mean cordial for all n.

Proof. Let $V(K_{1,n} \times K_2) = \{u, v, u_i, v_i : 1 \le i \le n\}$ and

$$E(K_{1,n} \times K_2) = \{uv, uu_i, vv_i, u_iv_i : 1 \le i \le n\}.$$

Clearly $|V(K_{1,n} \times K_2)| + |E(K_{1,n} \times K_2)| = 5n + 3$.
Assign the labels 0 and 3 respectively to the vertices u and v.

Case 1. $n \equiv 0 \pmod 4$.
Let $n = 4r$, $r \in N$. Assign the label 0 to the r vertices $u_1, u_2, \dots, u_r$. Then we assign the label 1 to the $2r$ vertices $u_{r+1}, u_{r+2}, \dots, u_{3r}$. Next we assign the label 3 to the r vertices $u_{3r+1}, u_{3r+2}, \dots, u_{4r}$. Now we assign the label 0 to the $2r$ vertices $v_1, v_2, \dots, v_{2r}$. Next we assign the label 2 to the r vertices $v_{2r+1}, v_{2r+2}, \dots, v_{3r}$. Finally we assign the label 3 to the r vertices $v_{3r+1}, v_{3r+2}, \dots, v_{4r}$.

Case 2. $n \equiv 1 \pmod 4$.
Let $n = 4r + 1$, $r \ge 1$. Assign the label to the vertices u_i $(1 \le i \le 4r)$ as in Case 1. We now assign the label 0 to the vertex u_{4r+1}. Now we assign the label 0 to the $2r - 1$ vertices $v_1, v_2, \dots, v_{2r-1}$. Next we assign the label 1 to the vertex v_{2r}. Now we assign the label 2 to the r vertices $v_{2r+1}, v_{2r+2}, \dots, v_{3r}$. Then we assign the label 3 to the r vertices $v_{3r+1}, v_{3r+2}, \dots, v_{4r}$. Finally we assign the label 2 to the vertex v_{4r+1}.

Case 3. $n \equiv 2 \pmod 4$.
Let $n = 4r + 2$, $r \ge 1$. Label the vertices u_i, v_i $(1 \le i \le 4r + 1)$ as in Case 2. Now we assign the labels 0,2 to the vertices u_{4r+2}, v_{4r+2}.

Case 4. $n \equiv 3 \pmod 4$.
Let $n = 4r + 3$, $r \ge 1$. As in Case 3, assign the label the vertices u_i, v_i $(1 \le i \le 4r + 2)$. Finally we assign the labels 1,3 to the vertices u_{4r+3}, v_{4r+3}.

This vertex labeling f is a 4-total mean cordial labeling follows from the Table 1

Nature of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n = 4r$	$5r+1$	$5r$	$5r+1$	$5r+1$
$n = 4r+1$	$5r+2$	$5r+2$	$5r+2$	$5r+2$
$n = 4r+2$	$5r+4$	$5r+3$	$5r+3$	$5r+3$
$n = 4r+3$	$5r+4$	$5r+5$	$5r+4$	$5r+5$

Table 1:

Case 5. $n = 2, 3$.
A 4-total mean cordial labeling is given in Tabel 2 □

n	u	v	u_1	u_2	u_3	v_1	v_2	v_3
2	0	3	0	1		2	2	
3	0	3	0	1	2	0	1	3

Table 2:

Theorem 4.2. The armed crown AC_n is 4-total mean cordial for all $n \geq 3$.

Proof. Take the vertex set and edge set of AC_n as in definition 3. Clearly $|V(AC_n)| + |E(AC_n)| = 6n$.

Case 1. $n \equiv 0 \pmod 4$.
Let $n = 4r$, $r \in N$. Assign the label 2 to the $2r$ vertices $u_1, u_2, \ldots, u_{2r}$. Then we assign the label 3 to the $2r$ vertices $u_{2r+1}, u_{2r+2}, \ldots, u_{4r}$. Next we assign the label 0 to the $2r$ vertices $v_1, v_2, \ldots, v_{2r}$. Now we assign the label 1 to the r vertices $v_{2r+1}, v_{2r+2}, \ldots, v_{3r}$. Then we assign the label 2 to the vertex v_{3r+1}. Next we assign the label 3 to the $r-1$ vertices $v_{3r+2}, v_{3r+3}, \ldots, v_{4r}$. Assign the label 0 to the $2r$ vertices $w_1, w_2, \ldots, w_{2r}$. Then we assign the label 1 to the $2r$ vertices $w_{2r+1}, w_{2r+2}, \ldots, w_{4r}$.

Case 2. $n \equiv 1 \pmod 4$.
Let $n = 4r+1$, $r \geq 1$. Assign the label to the vertices u_i, v_i, w_i $(1 \leq i \leq 4r)$ as in Case 1. We now assign the labels 3,1,0 to the vertices $u_{4r+1}, v_{4r+1}, w_{4r+1}$.

Case 3. $n \equiv 2 \pmod 4$.
Let $n = 4r+2$, $r \geq 1$. Assign the label 3 to the $3r+1$ vertices $u_1, u_2, \ldots, u_{3r+1}$. Next we assign the label 2 to the $r+1$ vertices $u_{3r+2}, u_{3r+3}, \ldots, u_{4r+2}$. Then we assign the label 0 to the $2r+1$ vertices $v_1, v_2, \ldots, v_{2r+1}$. Now we assign the label 1 to the $2r+1$ vertices $v_{2r+2}, v_{2r+3}, \ldots, v_{4r+2}$. Now we assign the label 0 to the $2r+1$ vertices $w_1, w_2, \ldots, w_{2r+1}$. Then we assign the label 1 to the $2r+1$ vertices $w_{2r+2}, w_{2r+3}, \ldots, w_{4r+2}$.

Case 4. $n \equiv 3 \pmod 4$.
Let $n = 4r+3$, $r \geq 1$. As in Case 3, assign the label the vertices u_i, v_i, w_i $(1 \leq i \leq 4r+2)$.

Finally we assign the labels 3, 1, 0 to the vertices u_{4r+3}, v_{4r+3}, w_{4r+3}.

This vertex labeling f is a 4-total mean cordial labeling follows from the Table 3

Nature of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n=4r$	$6r$	$6r$	$6r$	$6r$
$n=4r+1$	$6r+1$	$6r+2$	$6r+1$	$6r+2$
$n=4r+2$	$6r+3$	$6r+3$	$6r+3$	$6r+3$
$n=4r+3$	$6r+4$	$6r+5$	$6r+4$	$6r+5$

Table 3:

Case 5. $n=2,3$.
A 4-total mean cordial labeling is given in Tabel 4 □

n	u_1	u_2	u_3	v_1	v_2	v_3	w_1	w_2	w_3
3	3	2	3	0	1	1	0	1	0

Table 4:

Theorem 4.3. The graph $P_n \odot \overline{K_3}$ is 4-total mean cordial for all n.

Proof. Let P_n be the path $u_1\ u_2\ \ldots\ u_n$. Let x_i,y_i,z_i be the pendent vertices adjacent with u_i where $1 \le i \le n$.
Clearly $\left|V\left(P_n \odot \overline{K_3}\right)\right| + \left|E\left(P_n \odot \overline{K_3}\right)\right| = 8n-1$.

Consider the vertices u_1,u_2,...,u_n. Assign the label 0 to the n vertices u_1,u_2,...,u_n. Next we assign the label 1 to the n vertices $x_1, x_2, \ldots, x_n$. Assign the label 3 to the $2n$ vertices y_1,y_2,...,y_n,z_1,z_2,...,z_n.
Clearly $t_{mf}(0) = 2n-1$, $t_{mf}(1) = 2n$, $t_{mf}(2) = 2n$ and $t_{mf}(3) = 2n$. □

Corollary 4.3.1. The graph $C_n \odot \overline{K_3}$ is 4-total mean cordial for all $n \ge 3$.

Proof. Obviously the vertex labeling of Theorem 4 is also a 4-total mean cordial labeling of $C_n \odot \overline{K_3}$. □

Theorem 4.4. The dumbbell graph Db_n is 4-total mean cordial for all $n \ge 3$.

Proof. Take the vertex set and edge set of dumbbell graph as in definition 3. Note that $|V(Db_n)| + |E(Db_n)| = 4n+1$.

Case 1. n is odd.
Consider the vertices u_1,u_2,...,u_n. Assign the label 0 to the $\frac{n+1}{2}$ vertices u_1,u_2,...,$u_{\frac{n+1}{2}}$.

Next we assign the label 1 to the $\frac{n-1}{2}$ vertices $u_{\frac{n+3}{2}}, u_{\frac{n+5}{2}}, \ldots, u_n$. Now we consider the vertices $v_1, v_2, \ldots, v_n$. Next we assign the label 2 to the $\frac{n+1}{2}$ vertices $v_1, v_2, \ldots, v_{\frac{n+1}{2}}$. Finally we assign the label 3 to the $\frac{n-1}{2}$ vertices $v_{\frac{n+3}{2}}, v_{\frac{n+5}{2}}, \ldots, v_n$.

Case 2. n is even.
Assign the label 1 to the $\frac{n-2}{2}$ vertices $u_1, u_2, \ldots, u_{\frac{n-2}{2}}$. Then we assign the label 0 to the $\frac{n+2}{2}$ vertices $u_{\frac{n}{2}}, u_{\frac{n+2}{2}}, \ldots, u_n$. Next we assign the label 2 to the $\frac{n-2}{2}$ vertices $v_1, v_2, \ldots, v_{\frac{n-2}{2}}$. Now we assign the label 1 to the vertex $v_{\frac{n}{2}}$. Finally we assign the label 3 to the $\frac{n}{2}$ vertices $v_{\frac{n+2}{2}}, v_{\frac{n+4}{2}}, \ldots, v_n$.

The Table 5 shows that this vertex labeling f is a 4-total mean cordial labeling □

Nature of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
n is odd	n	$n+1$	n	n
n is even	$n+1$	n	n	n

Table 5:

Theorem 4.5. The dragon graph $C_n@P_n$ is 4-total mean cordial for $n \geq 3$.

Proof. Let C_n be the cycle $u_1\, u_2 \ldots u_n\, u_1$ and path $v_1\, v_2 \ldots v_n$. Note that $|V(C_n@P_n)| + |E(C_n@P_n)| = 4n-2$.

Case 1. n is odd.
Assign the label 1 to the vertex $u_1 = v_1$. Then we assign the label 0 to the $\frac{n+1}{2}$ vertices $u_2, u_3, \ldots, u_{\frac{n+3}{2}}$. Next we assign the label 1 to the $\frac{n-3}{2}$ vertices $u_{\frac{n+5}{2}}, u_{\frac{n+7}{2}}, \ldots, u_n$. Now we assign the label 2 to the $\frac{n+1}{2}$ vertices $v_2, v_3, \ldots, v_{\frac{n+1}{2}}$. Finally we assign the label 3 to the $\frac{n-1}{2}$ vertices $v_{\frac{n+3}{2}}, v_{\frac{n+5}{2}}, \ldots, v_n$.

Case 2. n is even.
Assign the label 2 to the vertex $u_1 = v_1$. Next we asign the label 0 to the $\frac{n}{2}$ vertices $u_2, u_3, \ldots, u_{\frac{n+2}{2}}$. Then we assign the label 1 to the $\frac{n-2}{2}$ vertices $u_{\frac{n+4}{2}}, u_{\frac{n+6}{2}}, \ldots, u_n$. Now we assign the label 2 to the $\frac{n-2}{2}$ vertices $v_2, v_3, \ldots, v_{\frac{n}{2}}$. Finally we assign the label 3 to the $\frac{n}{2}$ vertices $v_{\frac{n+2}{2}}, v_{\frac{n+4}{2}}, \ldots, v_n$.

The Table 6 shows that this vertex labeling f is a 4-total mean cordial labeling □

Nature of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
n is odd	n	n	$n-1$	$n-1$
n is even	$n-1$	$n-1$	n	n

Table 6:

Theorem 4.6. The graph G obtained by subdividing the pendent edges of the bistar $B_{n,n}$ is 4-total mean cordial for all n.

Proof. Let $V(G) = \{u, v, u_i, v_i, x_i, y_i : 1 \leq i \leq n\}$ and
$E(G) = \{uv, ux_i, x_iu_i, vy_i, y_iv_i : 1 \leq i \leq n\}$. Clearly $|V(G)| + |E(G)| = 8n + 3$.
Assign the labels 0,2 to the vertices u,v respectively. Now we assign the label 2 to the n vertices x_1,x_2,...,x_n. Next we assign the label 0 to the n vertices u_1,$u2$,...,u_n. We now assign the label 3 to the n vertices y_1,y_2,...,y_n. Finally we assign the label 0 to the n vertices v_1,v_2,...,v_n.
Obviously $t_{mf}(0) = 2n + 1$, $t_{mf}(1) = 2n + 1$, $t_{mf}(2) = 2n + 1$ and $t_{mf}(3) = 2n$. □

Theorem 4.7 The graph $B_{n,n} \odot K_1$ is 4-total mean cordial for all n.

Proof. Let $V(B_{n,n} \odot K_1) = \{u, v, x, y, u_i, v_i, x_i, y_i : 1 \leq i \leq n\}$ and
$E(B_{n,n} \odot K_1) = \{uv, xu, yv, uu_i, u_ix_i, vv_i, v_iy_i : 1 \leq i \leq n\}$.
Clearly $|V(B_{n,n} \odot K_1)| + |E(B_{n,n} \odot K_1)| = 8n + 7$.
Assign the labels 0,2,2,3 to the vertices u,v,x,y respectively. Now we assign the label 0 to the n vertices u_1,u_2,...,u_n. Next we assign the label 1 to the n vertices x_1,x_2,...,x_n. We now assign the label 2 to the n vertices x_1,x_2,...,x_n. Finally we assign the label 3 to the n vertices y_1,y_2,...,y_n.
Clearly $t_{mf}(0) = 2n + 1$, $t_{mf}(1) = t_{mf}(2) = t_{mf}(3) = 2n + 2$. □

Ttheorem 4.8. The graph $C_n^{(2)}$ is 4-total mean cordial for all $n \geq 3$.

Proof. Let $u_1\ u_2\ \ldots\ u_n\ u_1$ and $v_1\ v_2\ \ldots\ v_nv_1$ be the two cycles and $u_1 = v_1$. Clearly $\left|V\left(C_n^{(2)}\right)\right| + \left|E\left(C_n^{(2)}\right)\right| = 4n - 1$.

Case 1. n is odd.
Consider the vertices u_1,u_2,...,u_n. Assign the label 2 to the vertex $u_1 = v_1$. Now we assign label 0 to the $\frac{n+1}{2}$ vertices u_2,u_3,...,$u_{\frac{n+3}{2}}$. Next we assign the label 1 to the $\frac{n-3}{2}$ vertices $u_{\frac{n+5}{2}}$,$u_{\frac{n+7}{2}}$,...,u_n. Assign the label 1 to the vertex v_2. Now we assign the label 2 to the $\frac{n-3}{2}$ vertices v_3,v_4,...,$v_{\frac{n+1}{2}}$. Finally we assign the label 3 to the $\frac{n-1}{2}$ vertices $v_{\frac{n+3}{2}}$,$v_{\frac{n+5}{2}}$,...,v_n.

Case 2. n is even.
Assign the label 2 to the vertex u_1. Now we assign the label 0 to the $\frac{n}{2}$ vertices u_2,u_3,...,$u_{\frac{n+2}{2}}$. Then we assign the label 1 to the $\frac{n-2}{2}$ vertices $u_{\frac{n+4}{2}}$,$u_{\frac{n+6}{2}}$,...,u_n. Now we assign the label 0 to the vertex v_2. We now assign the label 3 to the $\frac{n}{2}$ vertices v_3,v_4,...,$v_{\frac{n+4}{2}}$. Finally we assign the label 2 to the $\frac{n-4}{2}$ vertices $v_{\frac{n+6}{2}}$,$v_{\frac{n+8}{2}}$,...,v_n.

The Table 7 shows that this vertex labeling f is a 4-total mean cordial labeling

□

Nature of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
n is odd	n	$n-1$	n	n
n is even	n	n	$n-1$	n

Table 7:

Theorem 4.9. The graph $C_n \odot S_n$ is 4-total mean cordial for all $n \geq 3$.

Proof. Take the vertex set and edge set of $C_n \odot S_n$ as in definition 3.8.
Note that $|V(C_n \odot S_n)| + |E(C_n \odot S_n)| = 4n$.

Case 1. n is odd.
Consider the vertices $u_1, u_2, \ldots, u_n$. Assign the label 2 to the $\frac{n+1}{2}$ vertices $u_1, u_2, \ldots, u_{\frac{n+1}{2}}$. Next we assign the label 3 to the $\frac{n-1}{2}$ vertices $u_{\frac{n+3}{2}}, u_{\frac{n+5}{2}}, \ldots, u_n$. Now we move the vertices $v_1, v_2, \ldots, v_n$. Next we assign the label 0 to the n vertices $v_1, v_2, \ldots, v_n$.

Case 2. n is even.
Assign the label 2 to the vertex u_1. Now we assign the label 1 to the $\frac{n-2}{2}$ vertices $u_2, u_3, \ldots, u_{\frac{n}{2}}$. Then we assign the label 0 to the $\frac{n}{2}$ vertices $u_{\frac{n+2}{2}}, u_{\frac{n+4}{2}}, \ldots, u_n$. Next we assign the label 0 to the vertex v_1. We now assign the label 2 to the $\frac{n-2}{2}$ vertices $v_2, v_3, \ldots, v_{\frac{n}{2}}$. Finally we assign the label 3 to the $\frac{n}{2}$ vertices $v_{\frac{n+2}{2}}, v_{\frac{n+4}{2}}, \ldots, v_n$.

The Table 8 shows that this vertex labeling f is a 4-total mean cordial labeling □

Nature of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
n is odd	n	n	n	n
n is even	n	n	n	n

Table 8:

Theorem 4.10. The graph $C_n \otimes S_n$ is 4-total mean cordial for all $n \geq 3$.

Proof. Take the vertex set and edge set of $C_n \otimes S_n$ as in definition 3. Clearly $|V(C_n \otimes S_n)| + |E(C_n \otimes S_n)| = 4n$.

Case 1. n is odd.
Assign the label 0 to the $\frac{n+1}{2}$ vertices $u_1, u_2, \ldots, u_{\frac{n+1}{2}}$. Next we assign the label 1 to the $\frac{n-1}{2}$ vertices $u_{\frac{n+3}{2}}, u_{\frac{n+5}{2}}, \ldots, u_n$. Now we assign the label 3 to the vertex v. Finally we assign the label 2 to the $n-1$ vertices $v_2, v_3, \ldots, v_n$.

Case 2. n is even.
Now we asign the label 2 to the $\frac{n-2}{2}$ vertices $u_1, u_2, \ldots, u_{\frac{n-2}{2}}$. Then we assign the label

3 to the $\frac{n}{2}$ vertices $u_{\frac{n}{2}}, u_{\frac{n+2}{2}}, \ldots, u_{n-1}$. We now assign the label 2 to the vertex u_n. Now we assign the label 2 to the vertex v. Finally we assign the label 0 to the $n-1$ vertices $v_2, v_3, \ldots, v_n$.

This vertex labeling f is a 4-total mean cordial labeling follows from the Table 9 □

Nature of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
n is odd	n	n	n	n
n is even	n	n	n	n

Table 9:

Theorem 4.11. The graph $D_2(P_n)$ is 4-total mean cordial for all $n \geq 3$.

Proof. Let $V(D_2(P_n)) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(D_2(P_n)) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i v_{i+1}, v_i u_{i+1} : 1 \leq i \leq n-1\}$.
Clearly $|V(D_2(P_n))| + |E(D_2(P_n))| = 6n - 4$.

Case 1. $n \equiv 0 \pmod 4$.
Let $n = 4r$, $r \in N$. Assign the label 1 to the r vertices $u_1, u_2, \ldots, u_r$. Then we assign the label 2 to the r vertices $u_{r+1}, u_{r+2}, \ldots, u_{2r}$. Next we assign the label 3 to the $2r$ vertices $u_{2r+1}, u_{2r+2}, \ldots, u_{4r}$. Now we assign the label 0 to the $3r$ vertices $v_1, v_2, \ldots, v_{3r}$. We assign the label 2 to the r vertices $v_{3r+1}, v_{3r+2}, \ldots, v_{4r}$.

Case 2. $n \equiv 1 \pmod 4$.
Let $n = 4r+1$, $r \geq 1$. Now assign the label 1 to the r vertices $u_1, u_2, \ldots, u_r$. Next we assign the label 2 to the r vertices $u_{r+1}, u_{r+2}, \ldots, u_{2r}$. Then we assign the label 3 to the $2r$ vertices $u_{2r+1}, u_{2r+2}, \ldots, u_{4r}$. We assign the label 1 to the vertex u_{4r+1}. Now we assign the label 0 to the $3r+1$ vertices $v_1, v_2, \ldots, v_{3r+1}$. We assign the label 2 to the $r-1$ vertices $v_{3r+2}, v_{3r+3}, \ldots, v_{4r}$. Now we assign the label 3 to the vertex v_{4r+1}.

Case 3. $n \equiv 2 \pmod 4$.
Let $n = 4r+2$, $r \geq 1$. Assign the label 3 to the r vertices $u_1, u_2, \ldots, u_r$. Next we assign the label 2 to the $r+1$ vertices $u_{r+1}, u_{r+2}, \ldots, u_{2r+1}$. Then we assign the label 3 to the $2r+1$ vertices $u_{2r+2}, u_{2r+3}, \ldots, u_{4r+2}$. Now we assign the label 0 to the $3r+1$ vertices $v_1, v_2, \ldots, v_{3r+1}$. Now we assign the label 2 to the r vertices $v_{3r+2}, v_{3r+3}, \ldots, v_{4r+1}$. Next we assign the label 0 to the vertex v_{4r+2}.

Case 4. $n \equiv 3 \pmod 4$.
Let $n = 4r+3$, $r \geq 3$. Assign the label 3 to the r vertices $u_1, u_2, \ldots, u_r$. Next we assign the label 2 to the $r+2$ vertices $u_{r+1}, u_{r+2}, \ldots, u_{2r+2}$. Then we assign the label 3 to the $2r+1$ vertices $u_{2r+3}, u_{2r+4}, \ldots, u_{4r+3}$. Now we assign the label 0 to the $3r+2$ vertices

v_1,v_2,. . .,v_{3r+2}. Now we assign the label 2 to the $r-1$ vertices v_{3r+3},v_{3r+4},. . .,v_{4r+1}. Next we assign the labels 3,0 to the vertices v_{4r+2},v_{4r+3}.

This vertex labeling f is a 4-total mean cordial labeling follows from the Table 10

Nature of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n=4r$	$6r-1$	$6r-1$	$6r-1$	$6r-1$
$n=4r+1$	$6r+1$	$6r$	$6r+1$	$6r$
$n=4r+2$	$6r+2$	$6r+2$	$6r+2$	$6r+2$
$n=4r+3$	$6r+4$	$6r+3$	$6r+3$	$6r+4$

Table 10:

Case 5. $n=2,3$.
A 4-total mean cordial labeling is given in Tabel 11 □

n	u_1	u_2	u_3	u_4	u_5	u_6	u_7	v_1	v_2	v_3	v_4	v_5	v_6	v_7
3	1	3	3					0	0	1				
7	1	1	2	2	3	3	3	0	0	0	0	0	3	0

Table 11:

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