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Ultimate Pit Limit Determination Using Flashlight Algorithm

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ABSTRACT

In this paper, the flashlight (FL) algorithm, which is categorized as a heuristic method, has been suggested to determine the ultimate pit limit (UPL). In order to apply the suggested algorithm and other common algorithms, such as the dynamic programming, the Korobov, and the floating cone, and to validate the capability of the proposed method, the ultimate pit limit was determined in a cross-section of the Korkora reserve, which is located in Kurdistan province, northwestern of Iran and consists of 3080 blocks. The comparison of the FL algorithm and other methods revealed that same as high accuracy dynamic programming methods, the proposed algorithm could find the optimum value, while the Korobov and the floating cone algorithms failed to determine the optimum limit.

Keywords : *Heuristic algorithm; Ultimate pit limit; Optimization; Flashlight algorithm*

1. Introduction

Determining the ultimate pit limit (UPL) is one of the most challenging topics in surface mining that should be investigated in the preliminary stages of mining operations. It should be determined when some parameters, such as the profitability of the mining project, are proven after the exploration stage. Moreover, some critical procedures in open-pit planning, including designing, locating, and performing the feasibility study, will be conducted once an economic block model has been proven by considering its financial aspects, such as determining the grade of minerals, their prices, and operating costs.

The objective function in the designing stage is to maximize the final value of the pit by observing the technical and economic considerations. Due to the importance of setting the ultimate pit limit, many researchers have tried to present various methods. Generally, these methods can be categorized into the three following groups:

1. Rigorous algorithms are applied to find the optimum solution to the problem. For instance, the dynamic programming method presented by Lerchs and Grossmann algorithm is considered as one of the most important algorithms in this category [1].
2. Heuristic algorithms, such as the floating cone and Korobov algorithms, are used for finding an approximate solution by applying the block search regardless of being close to the true solution or not [2, 3].
3. A meta-heuristic algorithm is a higher-level procedure or heuristic designed to find, generate, or select a heuristic (partial search algorithm) that may provide a sufficiently good solution to an optimization problem, especially with incomplete or imperfect information or limited computation capacity. The ant colony algorithm is one of these algorithms that has been applied in this field [4].

Although rigorous algorithms can achieve the optimal solution in 3D block modelings, they are usually time-consuming and difficult to implement, especially when the problem dimensions are large. Unlike rigorous algorithms, the heuristic and meta-heuristic algorithms are very fast. The most important drawback of the algorithms, as mentioned before, is the uncertainty of the optimal solution.

Despite the great importance of the ultimate pit limit, there is limited research on how exactly it is determined. As mentioned, Pana developed the floating cone algorithm in 1965 [2]. Then, in 2012, Elahi et al. developed the floating cone II and III algorithms for eliminating the floating cone problems [5]. Although the presented algorithms modified the floating cone method, the optimal solution could not achieve with them. Akbari et al. (2008) determined the ultimate pit limit considering the metal price uncertainties and using the real option valuation approach [6]. Also, Underwood and Tolwinski (1998) determined the ultimate pit limit using a network flow algorithm [7].

Unlike the mentioned research works, most researchers have examined the effects of changes in economic and environmental parameters on the final pit limit. For instance, Askari-Nasab and Awuah-Offei, in 2009, determined the ultimate pit limit with discounted block values by using the intelligent open-pit simulator (IOPS) [8]. In 2011, Sayadi et al. used ANNs to determine the ultimate pit limit [9]. Dimitrakopoulos (2011) explained the importance of uncertainties in open pit design [10]. The ecological costs of optimizing the ultimate pit outline in open-pit metal mines have been considered by Xu et al. (2014) [11]. Chatterjee et al. (2016) determined the production phase and the ultimate pit limit under the uncertainty of commodity price [12]. Richmond (2018), considering the importance of uncertainty in open-pit design, integrated a Monte Carlo-based simulation and the heuristic optimization techniques into a global system that directly provides NPV optimal pit outlines [13]. In 2018, Burgarelli et al. developed a new approach based on Brownian motion for direct block scheduling under marketing uncertainties [14]. Rahimi et al. (2018) presented a new algorithm to optimize mine production planning, mined material destination, and ultimate pit limit [15]. Adibi et al. (2018) used the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method to design the ultimate pit limit by considering the sustainable development parameters [16]. Saleki et al. (2019) found a mathematical relationship between the ultimate pit limits, which are generated by discounted and undiscounted block value maximization in open-pit mining [17].

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In this paper, a heuristic flashlight algorithm has been developed to determine the ultimate pit limit. This developed algorithm has a high speed and accuracy and achieves the optimum solution in all problems based on programming logic. Unlike other traditional heuristic algorithms that only consider the ultimate limit value in the last step, this algorithm optimizes the solution at all steps by checking the ultimate limit value.

2. Ultimate limit determination methods

Due to the importance of the ultimate pit limit, several most common methods have been introduced and discussed in this section.

2.1. Dynamic programming

This method was first introduced in 1965 by Lerchs and Grossmann as the optimal design of open-pit mines. This is a rigorous method that finds the optimum solution only in 2D mode. This method examines the final solution using numerical calculations and graphical solutions. This algorithm is mainly based on determining the critical path.

2.2. Floating cone

Pana introduced this algorithm in 1965. The algorithm includes a cone with the blocks of positive economic values in a heuristic manner. Since each mineral block should pay back the mining costs of the uppermost level of tailing blocks, thus only the cones consisting of mineral blocks and tailing of positive values can be mined. The floating cone method is one of the easiest and fastest algorithms that can only determine the simple, optimum solution, non-overlying examples, and in most cases, it contains errors, but it is still used for the rules of thumb due to the simplicity. One of the drawbacks of this method is the overlying and formation of larger cones with fewer profits. To eliminate the defects of this method, the floating cone II and floating cone III algorithms were presented [5].

Floating cone II works in a way that a cone is formed in each row for each positive number, and the cones are arranged in a descending order. It then mines the cones from large to small and plots a diagram of the changes in the ultimate limit value. Then, according to the diagram, it mines the cones until the value of the ultimate limit is increasing, and this process is repeated for all the rows. Floating cone III has the same steps as floating cone II, except that all positive numbers are checked at once instead of searching in a row. Despite the improvement in the final solution, these methods still fail to introduce the optimum solution.

2.3. Korobov algorithm

Korobov first introduced this method in 1974. This algorithm also acts in a heuristic manner, and by finding the positive blocks, assigns the value of the blocks to the upper negative blocks to finally determine where the optimum limit is located. Since the assignment of positive values to negative blocks does not follow a specific trend, the Korobov algorithm is also stuck in the local optimum. As a result, in some cases, the solution obtained by this algorithm is far away from the optimum solution.

3. Flashlight algorithm

Due to the importance of the ultimate pit limit designing and the mentioned problems in different algorithms, the flashlight (FL) algorithm has been developed in the present paper as a new method. This algorithm is based on the fact that the designer determines the parameters in the pit bottom blocks, and other parts of the pit are determined based on the conditions of the wall slope and the topography of the area.

Therefore, the single-bottom or multi-bottom pit should always place in the orebody, because further wastes extraction is not economical. Now, the question arises as to what positive block can be used as the bottom or within the optimum ultimate limit?

The solution to this question largely depends on the objective function used in the problem. In other words, the pit should be designed to maximize the final value of the existing blocks. The mining of each block requires the extracting of a cone from other blocks on top of the block, and the block can place within the optimal limit only when its mining can help improve the final value and does not reduce the value. Generally, the decision should be made according to the flowchart shown in figure 1 for the feasibility study of block mining.

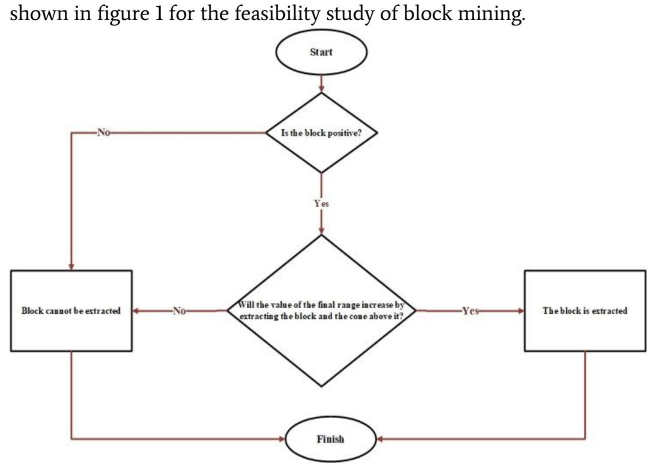

Figure 1: Decision making on extracting the block

The FL algorithm considers that mining a positive block and its upper cone is only economical when it ultimately improves the solution. The negative blocks cannot be mined on their own unless they are in the cone, which is above a positive block, and the value of the positive block is high enough to continue to be economical despite the presence of upper negative blocks. Since FL is a holistic algorithm, the probability of getting stuck in the local optimum is greatly reduced. Besides, checking the final solution in each run of the algorithm will make the optimal solution accessible. However, in the component-based algorithms, such as floating cone or Korobov, either the mining of blocks with positive value is ignored because of the overlying of the cones or the final value will be greater than the actual value due to the failure to detect effective negative blocks. For solving this problem, all positive blocks in this algorithm are found in the first step and placed in the ultimate limit. Then, each block will be determined from bottom to top and from right to left (or left to right) whether its mining helps improve the final solution.

In this method, the optimum limit is determined using a hypothetical flashlight. At the end of the process, all blocks that have been lit up by the flashlight are considered as the ultimate pit limit. The positive blocks in this algorithm can be placed in two groups, i.e., blacklist and white list. The blacklist blocks cannot take either the main or replaced flashlight. The white list blocks can take the main or replaced flashlights, but for reducing the running time, they will not operate during the process.

3.1. Steps of the Flashlight algorithm

The steps to determine the ultimate limit in open-pit mines by the flashlight algorithm are as follows.

1. Start
2. From bottom to top and from right to left (or left to right), the flashlights on all positive blocks are placed in the dark.
3. While the algorithm does not reach the last block in the top right (or top left), steps 4 to 16 will be continued:
4. All numbers in light illumination are summed and recorded in the UPL *total*.
5. Move from bottom to top and from right to left (or left to right) to find the positive block.
6. If the block has the main flashlight, go to 7 otherwise go to 8.
7. Turn off the main flashlight and from the bottom to top and from right to left (or left to right) and put the replaced flashlight on

the blocks, which are in the dark and are not on the blacklist, then go to 9.

8. If the block is in the white list, go to 3; otherwise, turn off all the flashlights that have illuminated the mentioned block; give the replace flashlights to the all positive blocks which are in the dark and are not in the blacklist from right to left (or left to right).
9. The sum of the value in the light is placed in the UPL rep1, and the counter i=0 and j is the number of replacing flashlights.
10. if i<j, go to 11; otherwise, go to 3.
11. if UPL total<UPL rep1, turn off the main flashlight and put it in the blacklist. Change the replace flashlights to the main and go to 3.
12. From the bottom to top and from the right to left move to the next replaced flashlight that is not on the white list.
13. Turn off the replaced flashlight and put the sum of values in the light area in the UPL rep2.
14. If the UPL rep1 <= the UPL rep2, place the replaced flashlight in the blacklist; otherwise, placed it in the white list.
15. Turn on the replaced flashlight again.
16. i=i+1 and go to 10.
17. The sum of the illuminated area value is the ultimate pit value.
18. End

In general, two constraints should be considered to obtain the appropriate results in the FL algorithm.

1. Include the positive blocks that increase the value of the ultimate limit.
2. Exclude the positive blocks that, if fall in the ultimate limit, decrease the limit value and may even make the pit uneconomical.

If an algorithm comprehensively covers the two objectives of the mentioned constraints, this algorithm determines the optimum ultimate limit. The FL algorithm considers objective 1 in step 2, as it starts by examining all positive numbers and considers objective 2 in the steps 5 and 6, so that if a positive block decreases the ultimate limit value, it is removed from the calculation.

The advantage of the FL algorithm over other common mentioned algorithms is that it examines the ultimate limit value at each step and improves the final solution at each step with an overall look, but other algorithms such as Korobov and floating cone calculate the final value after going through the defined steps.

A hypothetical example is presented for the implementation steps of the FL algorithm in figure 2. It should be noted that the vertical movement in this algorithm must be from bottom to top while there is no restriction on moving horizontally, i.e., from right to left or left to right. Although the movements in this example were supposed to be from bottom to top and from left to right, if the horizontal movement had been done in the opposite direction, from right to left, the final results would not have changed.

Figure (2.A) shows a hypothetical economic block model. In the first step, to find the UPL, the algorithm moves from the bottom left to the top right and illuminates a major flashlight in the block if it encounters a positive block in the dark.

As shown in figure (2.B), once a flashlight is placed in the block (4,4), there is no other positive block in the dark, and thus, the illuminated area is a cone whose apex is the block (4,4), and the value of this cone is +12. This value is set to the UPL total. The algorithm then moves again from the bottom left to the top right and turns off the block (4,4), which is the first major flashlight after reaching the block and provides an alternative flashlight to all positive numbers going into the dark.

As depicted in figure (2.C), two replaced flashlights are placed on the blocks (3,3) and (3,5). As there are two replaced flashlights, the algorithm can just check these flashlights twice. The illuminated area has a value of -5, which is set to UPL rep1, as presented in stage 1 in Table 1. By comparing the numerical values of UPL total and UPL rep1, it was found that the illuminated area B has a more economical value than area C. Therefore, the algorithm will check the replaced flashlights individually.

The algorithm moves from the bottom left to the top right, and as it is shown in figure (2.D), the flashlight on block (3,3) is turned off. As it is presented in Table 2 stage1, UPL rep2 is calculated as 0. After comparing UPL rep1 and UPL rep2, it is obvious that turning off this flashlight is better, and block (3,3) is moved to the blacklist. The replaced flashlight is turned on again and the algorithm moves to the next positive block. As it is shown in Figure 2.E, replaced flashlight on block (3,5) is turned off. In Table (1) stage 2, the value of UPL rep2 is calculated as +9. By comparing this amount with UPL rep1, it was found out that turning off this flashlight would increase the value, and therefore this block moves to the blacklist.

After checking all possible positive blocks, the algorithm again surveys block (4,4) and turns it on, as shown in figure (2.F). In the next step, the algorithm turns off flashlight on the block (4,4), and because of that, blocks (3,3) and (3,5) are in the blacklist and cannot accept the replaced flashlight.

As found in figure (2.G), the only block that can accept the flashlight is (2,4). Based on the algorithm principles, this block can only check one time. With comparing UPL rep1 (+14) and UPL total (+12) in this stage, it is concluded that the value of the pit is increased (Table 1 stage 3). Therefore, the main flashlight is turned off and added to the blacklist. The replaced flashlight is changed to the main flashlight, and UPL rep1 changed to UPL total. As seen in figure (2.H), the main flashlight is turned off again. It is concluded from Table 1 stage 4 that UPL rep1 is +1, and it is less than UPL total. For surveying the replaced flashlight, if this flashlight is turned off, the value of UPL rep2 is 0, and therefore, this block moves to the white list (Table 1 stage 4).

As block (1,5) is in the white list, the condition of this block will not be checked again. As represented in figure (2.I), the best result is +14, and the apex of the cone is placed on the block (2,5).

Table 1: Stages of the FL algorithm

Parameter	UPL_{total}	UPL_{rep1}	Replace FL Counter	UPL_{rep2}	BlackList Separated By Comma	White List Separated By Comma
Stage 1	+12	-5	+2	0	(3,3)	-
Stage 2	+12	-5	+2	+9	(3,3),(3,5)	-
Stage 3	+14	-	-	-	(3,3),(3,5),(4,4)	-
Stage 4	+14	+1	+1	0	(3,3),(3,5),(4,4)	(1,5)

4. Numerical analysis

In order to compare the FL algorithm with other heuristic algorithms, different scenarios are discussed in this section, where previous algorithms are to be unable to compute the optimum limit.

4.1. Floating cone algorithm

This algorithm has two major drawbacks: overlying and the formation of a larger cone with less profit. Figure (2.A) shows the hypothetical block model. Along with the FL algorithm, some various algorithms, including dynamic programming, floating cone, and Korobov, have also been used to determine the UPL in the hypothetical block model. The results have been given in Table 2.

A) Overlying

Overlying occurs when a group of negative blocks is shared between two cones, so that none of the cones have a positive value on their own, but if the two cones are formed at the same time, the total value of limit will be positive. According to figures (3.B) and (3.C), the floating cone algorithm forms a cone after reaching block (3.3) whose final value is -1 and does not mine the cone. Like conditions occur for the block (3.4) and this cone is not mined. However, as shown in figure (3.D), the optimum ultimate limit is the combination of the two cones, and the floating cone algorithm cannot achieve this result.

The results from the study of this example by the floating cone, Korobov, dynamic programming, and the FL algorithms are presented in Table 2.

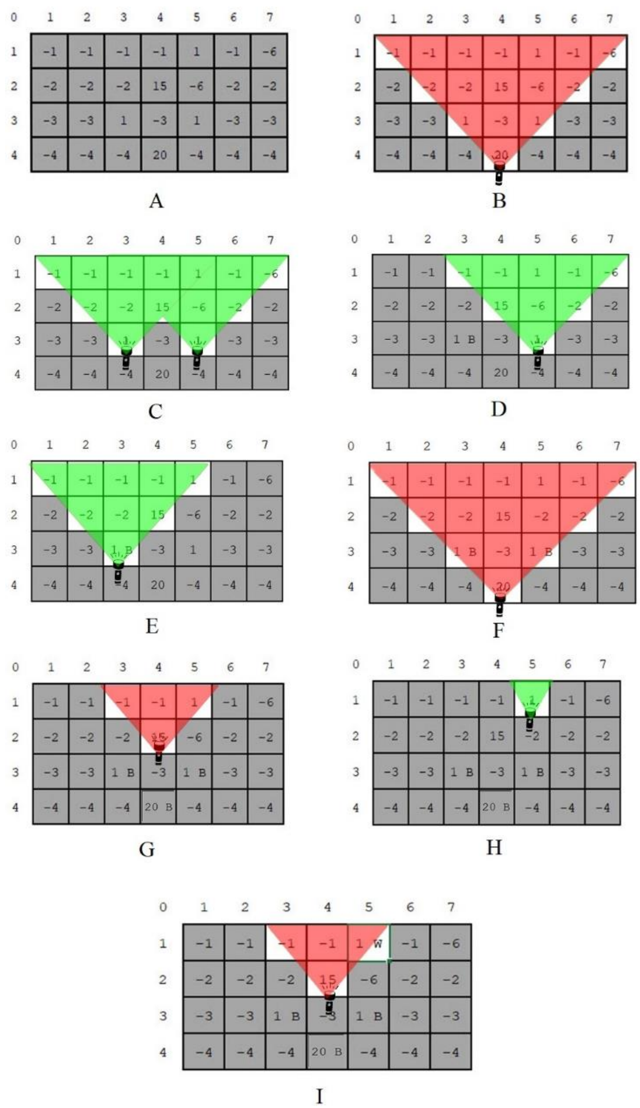

Figure 2: Example steps of the FL algorithm

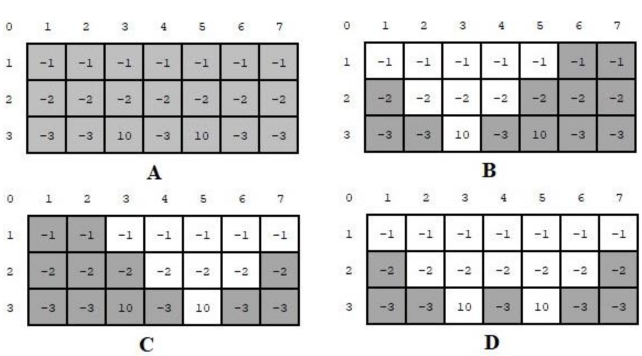

Figure 3: Overlying problem in the floating cone algorithm

Table 2: Results of the UPL from different algorithms in overlying problem

Algorithm	Korbov	Floating cone	Dynamic programming	Flashlight
Pit value	+3	0	+3	+3

B) Extending the ultimate pit beyond the optimal pit limits

According to figure 4, the economic block model specified in section A is solved by the floating cone algorithm as the first cone is mined according to figure (4.B), whose value is + 2, and then the second cone in figure (4.C) is mined, and the final value of the pit is +1. As seen, the floating cone introduces a larger limit with a lower final value as the ultimate limit, which is the main disadvantage of this method. The results of the evaluation of this example with different algorithms are presented in Table 3.

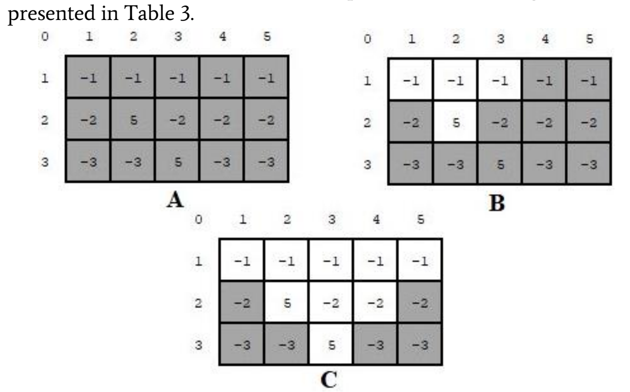

Figure 4: Formation of a larger cone with lower profit by floating cone algorithm

Table 3: Results of different algorithms in extending the ultimate pit beyond the optimal pit limits problem

Algorithm	Korbov	Floating cone	Dynamic programming	Flashlight
Pit value	+2	+1	+2	+2

4.2. The Korobov algorithm

Since the process of assigning positive blocks to negative blocks does not follow a particular trend, the Korobov algorithm is unable to determine the optimum limit in some cases. For instance, the results revealed that the Korobov algorithm could not find the ultimate limit in the hypothetical block model of Figure (5.A).

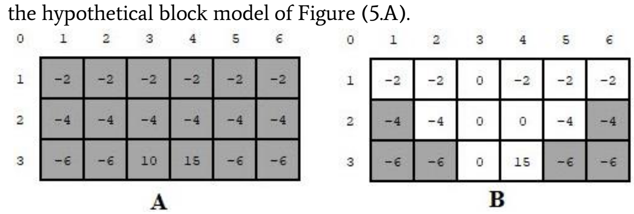

Figure 5: The Korobov algorithm error

As depicted in Figure (5.A), there is no optimum limit in this block model. However, after reaching block (3.3), the value of this block was given with the Korobov algorithm to blocks (2.3), (2.4), and (1.3). Eventually, the specified limit in Figure (5.B) is defined as the ultimate limit with a value of +3. The results from the study of this example with different algorithms are presented in Table 4.

Table 4: Results from different algorithms in the Korbov problem

Algorithm	Korbov	Floating cone	Dynamic programming	Flashlight
Pit value	+3	-	-	-

4.3. Case study

The related data of the Korkora skarn iron ore mine, which is located in the Kurdistan province, northwestern Iran, has been used to evaluate the performance of the FL algorithm. The geographical location of the mine is shown in Figure 6.

The mine is a part of the Sanandaj-Sirjan belt and is located in a mountainous area, about 2540 m above the sea level. The Korkora ore body has a length of 758 m, with a width ranging from 64 to 349 m, and a thickness ranging from 7 to 37 m. The slope of the iron horizon is about 10-11° to the south. The mineral deposit is hosted by a rhyolite unit, located in the center, with a fault of a steep slope. The oldest rocks in the Korkora mine are the Cretaceous limestones. Magnetite and hematite are the main products of the mineralization. The total reserve of the mine is about 7 million tons, with an average grade of 52% Magnetite. The ore body is mined through open-pit mining. The height and slope of the benches are 10 m and 90°, respectively.

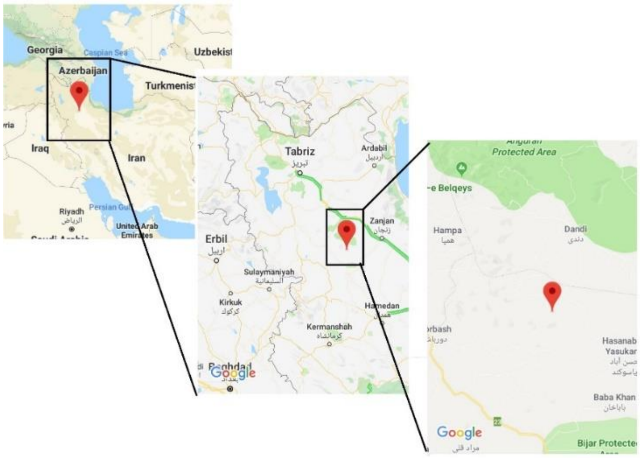

Figure 6: Geographical location of Korkora mine

In this paper, the cross-section consisting of 3080 blocks of this limit was explored using different methods to evaluate the accuracy and validity of the FL algorithm compared to other heuristic and rigorous algorithms. A Matlab code was developed for the implementation of the FL algorithm. Table 4 contains four different values that represent the UPL obtained by applying the floating cone, Korobov, dynamic programming, and FL algorithms, respectively.

Since the dynamic programming algorithm always shows the optimum ultimate limit in 2D mode and comparing the sections A, B, C, and D, it is found that the Korobov and the floating cone algorithms were unable to find the optimum two-bottom limit obtained in section C by the dynamic programming algorithm.The FL algorithm, however, managed to escape the local optima and reach the optimum ultimate limit. The values of the limits are specified in Table 5.

Table 5: Results from different algorithms in Korkora cross-section (Value × 10000 Rials)

Algorithm	Korbov	Floating cone	Dynamic programming	Flashlight
Pit value	+38260	+36254	+48354	+48354

As seen, the numerical value obtained from the floating cone algorithm is much lower than the value obtained by the dynamic programming algorithm. However, the FL algorithm determines a value exactly equal to the optimum value obtained by the dynamic programming algorithm.

5. Conclusions

The methods applied to determine the UPL are divided into three general groups: rigorous, heuristic, and meta-heuristic. The disadvantages of rigorous methods include low calculation speed and long data processing time. Although the heuristic methods have faster calculation speeds, they are getting stuck in local optimums and cannot specify the optimum limit. In this paper, the FL algorithm was introduced to overcome these shortcomings, and the following results were obtained:

1. The FL algorithm is capable of solving the problems of the well-known heuristic algorithms such as the floating cone and the Korobov, and in the case where these algorithms were unable to determine the optimal ultimate limit, the FL algorithm would give the optimum solution.

2. The FL is based on the movement from bottom to top, and unlike other heuristic algorithms, it examines the total values of the limit in each step of the algorithm to finally obtain the optimum value.

3. Various methods determined the ultimate limit of Korkora mine, and ultimately it was found that the value obtained from the FL algorithm is equal to the optimum value obtained from the dynamic programming method, which indicated the high accuracy of this algorithm. However, the Korobov and floating cone algorithms failed to determine the optimum limit.

4. The proposed algorithm can be used in a variety of software for designing of the UPL.

REFERENCES

[1] Lerchs, H., Grossmann., I.F. (1964). Optimum Design of Open-Pit Mines. *Joint C.O.R.S. and O.R.S.A. Conference, Montreal.*

[2] Pana, M.T. (1965). The simulation approach to open-pit design. *5th APCOM, Tucson Arizona, Tucson Arizona.*

[3] Korobov, S. (1974). Method for determining optimal open pit limits (Montreal: Ecole Polytechnique de l'Université de Montréal. Technical report EP74.

[4] Shishvan, M. S., & Sattarvand, J. (2015). Long term production planning of open pit mines by ant colony optimization. European Journal of Operational Research, 240(3), 825-836.

[5] Elahi, E., Kakaie, R., & Yusefi, A. (2012). A new algorithm for optimum open pit design: Floating cone method III. Journal of Mining and environment, 2(2), 118-125.

[6]. Akbari, A. D., OSANLOU, M., & Shirazi, M. A. (2008). Determination of ultimate pit limits in open mines using real option approach. INTERNATIONAL JOURNAL OF ENGINEERING SCIENCE, 19, 23-38.

[7]. Underwood, R., & Tolwinski, B. (1998). A mathematical programming viewpoint for solving the ultimate pit problem. European Journal of Operational Research, 107(1), 96-107.

[8] Askari-Nasab, H., & Awuah-Offei, K. (2009). Open pit optimisation using discounted economic block values. Mining Technology, 118(1), 1-12.

[9] Sayadi, A. R., Fathianpour, N., & Mousavi, A. A. (2011). Open pit optimization in 3D using a new artificial neural network. Archives of Mining Sciences, 56(3), 389-403.

[10] Dimitrakopoulos, R. (2011). Stochastic optimization for strategic mine planning: a decade of developments. Journal of Mining Science, 47(2), 138-150.

[11] Xu, X. C., Gu, X. W., Qing, W. A. N. G., Liu, J. P., & Jun, W. A. N. G. (2014). Ultimate pit optimization with ecological cost for open pit metal mines. Transactions of Nonferrous Metals Society of China, 24(5), 1531-1537

[12] Chatterjee, S., Sethi, M. R., & Asad, M. W. A. (2016). Production phase and ultimate pit limit design under commodity price uncertainty. European Journal of Operational Research, 248(2), 658-667.

[13] Richmond, A. (2018). Direct net present value open pit optimization with probabilistic models. In Advances in Applied Strategic Mine Planning (pp. 217-228). Springer, Cham.

[14] Burgarelli, H. R., Souza, F. R., Nader, A. S., Torres, V. F. N., Câmara, T. R., Ortiz, C. E. A., & Galery, R. (2018). Direct block scheduling under marketing uncertainties. REM-International Engineering Journal, 71(2), 275-280.

[15] Rahimi, E., Moosavi, E., Shirinabadi, R. & Gholinejad, M. (2018). Optimized algorithm in mine production planning, mined material destination, and ultimate pit limit. Journal of Central South University. 25(6), 1475-1488.

[16] Adibi, N., Ataee-pour, M., Rahmanpour, M. (2015). Integration of sustainable development concepts in open pit mine design. Journal of Cleaner Production, 108, 1037-1049.

[17] Saleki, M., Kakaie, R., & Ataei, M. (2019). Mathematical relationship between ultimate pit limits generated by discounted and undiscounted block value maximization in open pit mining. Journal of Sustainable Mining, 18(2), 94-99.