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RESEARCH PAPER

A Mathematical Model for Solving Location-Routing Problem with Simultaneous Pickup and Delivery Using a Robust Optimization Approach

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Abstract

In this study, a robust optimization model is introduced, we propose a location-routing problem with simultaneous pickup and delivery under a hard time window that has a heterogeneous and limited depot and vehicle capacities and multi-variety of products and uncertain traveling time that considering all of these constraints together make the problem closer to real practical world's problems, that not been studied in previous papers. For this purpose, a mixed-integer linear programming (MILP) model is proposed for locating depots and scheduling vehicle routing with multiple depots. Then, the robust counterpart of the proposed MILP model is proposed. The results show that GA performs much better than the exact algorithm concerning time. GAMS software fails to solve the large-size problem, and the time to find a solution grows exponentially with increasing the size of the problem. However, the GA quite efficient for problems of large sizes, and can nearly find the optimal solution in a much shorter amount of time. Also, results in the Robust model show that increasing the confidence level has led to an increase in the value of the objective function of the robust counterpart model, this increase does not exhibit linear behavior. At 80% confidence level, the minimum changes in the objective function are observed, if we want to obtain a 90% confidence level, it requires more cost, but increasing the confidence level from 70% to 80% does not need more cost, so an 80% confidence level can be considered as an ideal solution for decision-makers.

Keywords:
Supply Chain;
Location-Routing Problem (LRP);
Simultaneous Pickup and Delivery;
Time Window;
Genetic Algorithm (GA);
Robust Optimization (RO) Approach

Introduction

In most industries, organizations have to compete with different internal and external competitors. Given the wide range of products offered in the market, customers have different options to choose from. Therefore, if an organization is willing to survive this competition, in addition to reducing prices and enhancing quality, should be able to recognize customers' demands and respond to these demands within the minimum possible time. Distribution systems play a key role here so that a good distribution system serves as a critical element in an organization's success. A good distribution system controls costs and customers' satisfaction at the same time. In a distribution system, depots and transportation networks are used to deliver the goods. As such, the distribution management should determine how many depots are required, where these depots should be established, and which transportation network can

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minimize costs and realize delivery objectives better. So, location and routing problems are two serious factors in an organization. One of the significant problems in the supply chain and logistics management is the design of the distribution network; an effectual distribution network decreases costs significantly. A vital dimension of the distribution network design is the consideration of all the practical restrictions. One of the most normally used problems in the distribution network area is the LRP. The LRP has been extensively studied in the literature. This problem deals with determining the location of facilities and the route of vehicles for serving several customers under some constraints, such as facility and vehicle capacities, route time, etc. The LRP aims to satisfy the demands of all customers and to minimize total costs, such as transportation costs, vehicle fixed costs, facility location fixed costs, and operating costs. In its general form, the LRP assumes that customers have only delivery demand, and it is interested in how to distribute the goods to customers with a fleet of vehicles. The location problem is at the strategic level while the routing problem is at a tactical level, but integrating them into a single problem will result in better non-sub-optimal solutions. LRP has followed a growing trend in recent years and different extensions have been proposed to this problem by various researchers (see for example Nagy and Salhi, Lopes et al., Prodhon and Prins, and Drexl and Schneider [1,2,3,4]). Time window constraint is one of these extensions. In LRPTW a time window is considered for each customer within which his/her products have to be delivered. Organizations have addressed this problem to raise their customer satisfaction, but their approach to time windows can be either hard (where time window can't be violated) or soft (time window can be violated by paying penalties). Simultaneous pickup and delivery and the existing arrivals supply chain are other extensions that can close this problem to a real-world problem. Uncertainty is another extension of LRP. To apply problems for real-world objectives, real data like real customers' demands and actual travel times should be applied; however, exact values of these real data are often not available. Indeed, the dynamic and complex nature of real-world problems has associated them with a large deal of uncertainty. Such problems may be modeled following various approaches including the use of random or fuzzy variables. The choice between using fuzzy or random variables in a problem depends on the problem's nature and data availability. For many problems where reliable data are available, random variables can be used to attain probability distributions. Besides, using scenario-based approaches commonly employed in stochastic methods may result in the use of a huge number of scenarios to represent uncertainty which can lead to computationally challenging problems. Therefore, using the Robustness approach is a better option for these scenarios. This research considered a distribution system design from plant to customers. It was assumed that product movement from plant to regional depots would be performed via long haul transportation and there is heterogeneous and capacitated depot and vehicle and multi-variety of products. Nowadays, the LRP robust model is in competition with fuzzy transportation and the number of transportation companies establishing robust model transportation services is increasing more and more. It has become an important policy for organizations because of its advantages in terms of cost and intra-modal coordination in large-scale freights. Last but not least, since 2012, the number of papers on the LRP robust model has followed a growing trend. The researcher's interest in hybrid problems (like location-routing, inventory-routing, location-inventory-routing, etc.) originates from the fact that hybrid problems provide better answers and help decision-makers to avoid sub-optimal answers.

In this research, the model determines the location of depots, selects, and allocates vehicles to warehouses, and allocates customers to selected vehicles (or warehouses). The mathematical model minimizes both fixed costs (such as the depot establishment and the purchase of a vehicle) and variable costs (including the cost of warehousing goods and carrying cost per unit of a product). Also, depot and vehicle capacities are heterogeneous and limited, and the variety of products in depots is considered multi-product. We propose a location-routing problem with simultaneous pickup and delivery under a hard time window that has a heterogeneous and limited depot and vehicle capacities and multi-variety of products and uncertain traveling time that considering all of these constraints together make the problem closer to real practical world's problems, that not been studied in previous papers. The results show that GA performs much better than the exact algorithm concerning time. Moreover, GAMS software fails to solve the large-size problem, and the time to find a solution grows exponentially with increasing the size of the problem. However, the GA quite efficient for problems of large sizes, and can nearly find the optimal solution in a much shorter amount of time. Also, results in the Robust model show that increasing the confidence level has led to an increase in the value of the objective function of the robust counterpart model, this increase does not exhibit linear behavior. At 80% confidence level, the minimum changes in the objective function are observed, if we want to obtain a 90% confidence level, it requires more cost, but increasing the confidence level from 70% to 80% does not need more cost, so an 80% confidence level can be considered as an ideal solution for decision-makers.

A location-routing problem with a hard time window and with delivery and pickup of the customers' orders at the right time is purposed in this research. In this paper, the Robust Optimization (RO) approach is used to control uncertain parameters. The model has been codded in GAMS and The CPLEX solver has been used to solve instances. Furthermore, a Genetic Algorithm (GA) is adapted to solve large-scale instances. The computation results time showed a slight difference between the performance of the GA and the exact method. Finally, a sensitivity analysis of traveling time and delivered demand by using the RO approach is performed. This paper is organized as follows. Section 2 presents a review of the literature on previous works. A mathematical model is proposed for the LRP in Section 3. Section 4 explains the GA adapted to solve large-scale instances. Section 5 analyzes the experiments and compares the exact method and the GA. Finally, conclusions and suggestions for future research are presented in Section 6.

Literature review

The idea of merging depot location and vehicle routing problems dates back to 50 years ago. Initial papers did not mix these two problems. Though the LRP idea has been presented in the 1960s, systematic researches can be found in the 1970s. Watson-Gandy and Dohrn [5], Jacobsen and Madsen [6], Or and Pierskalla [7], and Burness and White [8], were possibly the first researchers who considered client delivery while locating depots, through a non-linear profit function modeling where sales reduced with distance to the depot. Scientists paid an increasing deal of attention to LRP and established many methods of the problem since then. Nagi and Salhi [1], Prodhon and Prins [3], Lopes et al. [2], and Drexl and Schneider [4] reviewed LRP papers and classified this problem .Owing to their relevance to our study, LRPTW and LRP with uncertain data will be reviewed in this section. We momentarily review the associated papers to the LRPSPD. The LRPSPD contains two subproblems: the facility location problem (FLP) and the vehicle routing problem with simultaneous pickup and delivery (VRPSPD). Both the FLP and VRPSPD have been considered broadly in the literature.

Meanwhile, the LRPSPD can be considered as an extension of the traveling salesman location problem with pickup and delivery (TSPPD), presented by Mosheiov [9], in the relation to the number of depots to be located and the capacity of vehicles. Mosheiov [9] considers customer demands as stochastic variables and suggests a heuristic method based on ranking customers in addition to extending heuristics proposed in the literature for the TSPPD. The LRPSPD is as well an extension of the LRP in a relation to kinds of the customers' demand. LRP is one of the most researched topics in the literature. Because of the complication of the problem, diverse heuristic styles have been also proposed to solve bigger LRPs. Perl and Daskin

[10,11], Srivastava and Benton [12], Srivastava [13], and Hansen et al. [14], use typical heuristic methods for the problem. Meanwhile, meta-heuristic approaches have been effectively applied to the problem. Numerous instances for the application of metaheuristic methods can be assumed as tabu search (1999, 2005), simulated annealing (2002, 2010), greedy randomized adaptive search procedure (GRASP) (2006, 2010), memetic algorithms (2006), variable neighborhood search algorithms, and particle swarm optimization (2008). Inclusive reviews of the location-routing models and their applications are provided in Laporte [15], Min et al. [16], and Nagy and Salhi [17]. Lastly, the LRPSPD can be considered as a special situation of the many to many LRP (MMLRP) introduced by Nagy and Salhi [1] in which some customers request to direct cargos to others and flows among depots are allowed Wasner and Zapfel [18] also reflect one more section delivery problem, which is strictly linked to the MMLRP. In the problem, it is considered that vehicles accomplish both deliveries and pickups, and all inter-hub flows are carried out by a central hub. Therefore, the problem under consideration is cleared as determining the location of depots and hubs, allocating the customers and postal sectors to service areas of depots, and determining the delivery paths linking customers, depots, and hubs. The researchers show a nonlinear MIP formulation and a classified heuristic to solve the problem. Finally, it is reported a 14.7% cost savings over the current state by solving a real case problem with the proposed heuristic.

Jin et al. [19] proposed a mixed-integer linear mathematical model for the Multi-Depot Vehicle Routing Problem (MDVRP) with a two-stage solution approach. At the first stage of the solution approach, the problem is decomposed into two smaller problems, including allocation and routing problems; at the second stage, the problem is considered as an integrated one. Results showed that the second stage of the solution approach considerably improves the first stage. Liu et al. [20] proposed a Mixed Integer Linear Mathematical Model (MILMM) under uncertain conditions. They also presented a single-phase modeling approach with considering a single product and used an innovative approach to the allocation and routing of depots to demand points to solve the problem. Xiao et al. [21] introduced the Capacitated Vehicle Routing Problem (CVRP) in the distribution of goods aimed at minimizing fuel consumption.

Lalla-Ruiz et al. [22] proposed a new mathematical model for the multi-depot open vehicle routing problem (MDOVRP) by adding some new constraints as compared with previous studies. The computational results obtained from sample problems demonstrate the high efficiency of their proposed mathematical model. Du et al. [23] presented a fuzzy linear programming model aimed at minimizing the expected transportation risk when preparing hazardous materials and shipping from different warehouses to customers. Four meta-heuristic algorithms (such as particle swarm optimization (PSO), GA model, simulated annealing algorithm, and ant colony optimization (ACO)) are used to solve the problem and a numerical example has been presented for comparing the proposed algorithms. Alinaghian and Shokouhi [24] proposed a mathematical model for solving the reservoir routing problem. The objective function applied to this problem includes minimizing the number of vehicles and then minimizing the distance between all the traversed routes. In their study, the cargo space of each vehicle has several parts, and each reservoir is allocated to a product. They used a hybrid algorithm to solve the problem and compared the obtained results with those of the exact method and concluded that their proposed hybrid algorithm had high efficiency in problem-solving. Brandão [25] proposed the vehicle routing problem with time windows and used an iterated local search algorithm for solving the problem. This algorithm was used for large-size problems and a total of 418 sample problems were implemented. The results showed the effectiveness of the algorithm in solving large-size problems. In their study, Polyakovskiy et al. [26] addressed the two-dimensional product packing problem. Therefore, they proposed a mixed-integer linear programming model and solved the small-sized problems with the CPLEX solver. They also used heuristic algorithms to solve large-sized problems. YongboLi et al. [27] developed a multi-depot green vehicle routing problem (MDGVRP) by maximizing revenue and minimizing costs, time, and emission, and then, apply an improved ant colony optimization (IACO) algorithm that aims to efficiently solve the problem. Yong Shi et al. [28] formulated a model for a Home Health Care (HHC) Routing and Scheduling Problem by taking into account uncertain travel and service times, from the perspective of Robust Optimization (RO). After that, Gurobi Solver, Simulated Annealing, Tabu Search, and Variable Neighborhood Search are adapted to solve the model respectively. Finally, a series of experiments have been performed to validate the proposed models and algorithms. Yuan Wang et al. [29] addressed a Periodic Vehicle Routing Problem with Time Window and Service Choice problem. This problem is basically a combination of existing Periodic Vehicle Routing Problem with Time Window and Periodic Vehicle Routing Problem with Service Choice. They modeled it as a multi-objective problem. To solve this problem, they developed a heuristic algorithm based on Improved Ant Colony Optimization (IACO) and Simulate Annealing (SA) called Multi-Objective Simulate Annealing - Ant Colony Optimization (MOSA-ACO). Time window needs services to be transported to the customer within a wanted period (2001).

If a vehicle arrives early, it will induce waiting time and probably parking costs. If a vehicle made a delay, it reduces customer satisfaction, even breaks the contract in the worst cases. These added costs of early entrance and delay are named the penalty costs. In the case of the hard time window, the late entrance is severely forbidden setting the penalty cost equal to infinity, while the early entrance is permitted with no added charge (2016). The literature review showed a recent growing trend in LRPTW. Zarandi et al. [30] presented LRPTW under uncertainty. They considered demands and travel times as fuzzy variables, with fuzzy chance-constrained programming (CCP) model presented using credibility theory (2013). Gharavani and Setak [31] described an LRPTW onto which a semi-soft time window was imposed were delays in service delivery consequence penalties (cost) (2015). Gunduz [32] proposed single-stage LRP with time windows and presented a Tabu search heuristic to efficiently solve large-scale examples (2011). Mirzaei et al. [33] demonstrated the development of the LRPTW where energy minimization was considered: energy-efficient LRPTW. Govindan et al. [34] proposed a two-echelon LRPTW for sustainable supply chain network design and optimizing economic and environmental aims in a perishable food supply chain network. Time windows, specifically hard cases, add extra difficulty to LRP so none of the above-mentioned research tried to use hard time windows or realize optimal solutions by using exact solutions.

Also, adding other features to LRPTW such as uncertainty or merging it with other transportation problems to reach a more practical solution for the distribution network problem is missing in the literature. When previous data is lost or ambiguous, it is hard to define probabilistic distributions for uncertain parameters. In these cases, Robust optimization can be applied. Robust optimization is an extension to LRP to deal with uncertainty. Since the introduction of Robust optimization, many authors have used it for problem-solving. The need to use Robust optimization rises from ambiguity or uncertainty of parameters. In most cases, an inadequate amount of data is accessible to create a probability distribution upon the foundation of real data. In this regard. Ghaffari-Nasab et al. [35] proposed LRP with fuzzy demands and designed a fuzzy chance-constrained program to model it based on the fuzzy credibility theory. Lastly, a hybrid SA-based heuristic incorporated into stochastic simulations was presented and to solve the problem. Ismail Karaoglan et al. [36] introduce a heuristic approach to solve the LRPSPD problem, they consider that the variables are definite and aren’t under uncertainty and there is no time window for delivery cargoes to customers. Yong Shi et al. [37] developed a relatively robust optimization model for a vehicle routing problem with synchronized visits and uncertain scenarios considering greenhouse gas emissions. In this study, the greenhouse gas emissions are evaluated by the fuel consumption cost, and a hybrid tabu search and simulated annealing are proposed to solve it.

A summary of the most important papers published on the location and routing of vehicles is given in Table 1.

Table 1. A summary of the most used papers published on the multi-depot location-routing problem

Study	Model Type	Period	Product	Depot	Time Window	Vehicle Type	Solution Method
Derigs et al. (2011)	Deterministic	Single-period	Multi-product	Single-depot	-	Heterogeneous	Heuristic
Hanczar (2012)	Deterministic	Multi-period	Multi-product	Single-depot	-	Homogenous	Heuristic
Popović et al. (2012)	Probabilistic	Multi-period	Multi-product	Single-depot	-	Homogenous	Heuristic / meta-heuristic
Vidović et al. (2013)	Probabilistic	Multi-period	Multi-product	Single-depot	Hard	Homogenous	Heuristic / meta-heuristic
Popović et al. (2014)	Deterministic	Single-period	Single-product	Single-depot	-	Homogenous	Metaheuristic
Ray et al. (2014)	Deterministic	Single-period	Single-product	Multi-depot	-	Homogenous	Heuristic
Salehi et al. (2014)	Deterministic	Single-period	Single-product	Multi-depot	Hard	Heterogeneous	Metaheuristic
Lalla-Ruiz et al. (2015)	Deterministic	Single-period	Single -product	Single-depot	-	Homogenous	Metaheuristic
Rahimi-Vahed et al. (2015)	Deterministic	Multi-period	Single-product	Multi-depot	-	Homogenous	Heuristic
Abdulkader et al. (2015)	Deterministic	Single-period	Multi-product	Single-depot	-	Heterogeneous	Metaheuristic
Allahyari et al. (2015)	Deterministic	Single-period	Single product	Multi-depot	-	Heterogeneous	Heuristic /Metaheuristic
Lahyani et al. (2015)	Deterministic	Multi-period	Multi-product	Single-depot	Hard	Heterogeneous	Exact
Kaabi (2016)	Deterministic	Single-period	Multi-product	Single-depot	Hard	Heterogeneous	Metaheuristic
Sethanan et al. (2016)	Deterministic	Single-period	Multi-product	Single-depot	-	Homogenous	Metaheuristic
Du et al. (2017)	Fuzzy	Single-period	Single -product	Multi-depot	-	Heterogeneous	Metaheuristic
Alinaghian and Shokouhi (2018)	Deterministic	Single-period	Multi-product	Single-depot	Hard	Homogenous	Metaheuristic
Yongbo Li et al. (2019)	Deterministic	Single-period	Single -product	Multi-depot	-	Homogenous	Metaheuristic
Yong Shi et al. (2019)	Robust	Multi-period	Single -product	Single-depot	-	Homogenous	Metaheuristic
Yuan Wang et al. (2020)	Robust	Single-period	Multi-product	Single-depot	Hard	Homogenous	Metaheuristic
Yong Shi et al. (2020)	Robust	Single -period	Single -product	Single-depot	Hard	Heterogeneous	Metaheuristic
Current research (2020)	Robust	Single-period	Multi-product	Multi-depot	Hard	Heterogeneous	Exact /meta -heuristic

Considering the research gap in the literature listed in Table 1, in this study, a mathematical model for a multi-depot location routing problem (MDLRP) under conditions of uncertainty has been proposed and the location of distribution depots of diverse products and routing with heterogeneous vehicles have been simultaneously addressed. Customers with uncertain demands and uncertain transportation costs have led to the uncertain parameters in the proposed model. In this paper, the Bertsimas and Sim (B&S) Robust Optimization (RO) approach has been used to control uncertain parameters. Finally, meta-heuristic like GA has been applied to solve the problem with different sizes.

Problem formulation

In the part of this paper, the MDLRP with simultaneous pickup and delivery under conditions of uncertainty has been modeled. The proposed model simultaneously considers the location of depots and routing of the vehicle with a time window. For this purpose, vehicles travel from depots to customers and satisfy the customers' demand for each product. The vehicles are heterogeneous having different capacities and must satisfy the customers' demands for each product in a hard time window. Demand parameters and transportation costs are indefinite, so characterized as a fuzzy triangular number. The main objective of our work is to minimize the total cost of network design to determine the number and location of the depots and optimal vehicle routing. As shown in Fig. 1, the MDLRP is explained for 12 customers and 4 potential depots. According to Fig. 1, in the first phase, depots 1, 2, and 4 are selected, and the heterogeneous vehicles are used to deliver the products to customers by the routes, as shown in Fig. 1.

The assumptions of the MDLRP are as follows:

1. The MDLRP is considered as a single-period multi-product problem
2. The number and location of potential depots are indefinite.
3. Demands from consumers for delivery and transportation costs are considered uncertain.
4. The vehicles are heterogeneous having different capacities.

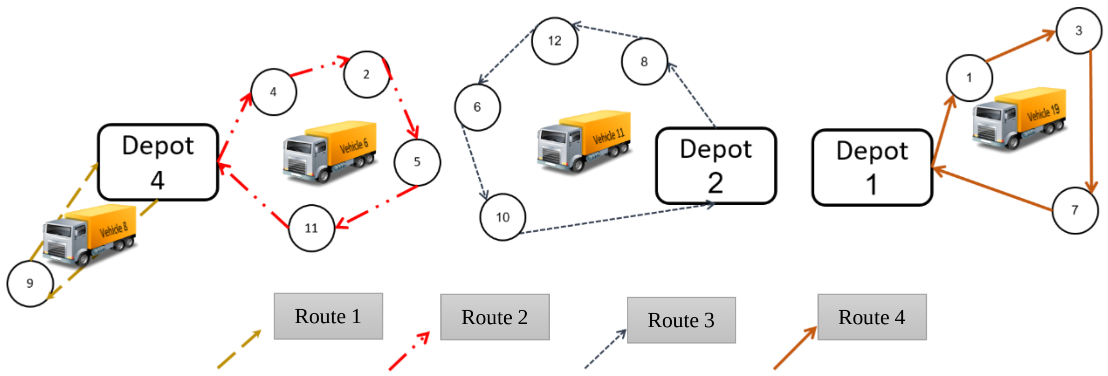


Fig. 1. A diagram of a multi-depot location routing problem

Multi-depot location routing problem under conditions of uncertainty

To model the MDLRP under conditions of uncertainty, the sets, parameters, and decision variables are presented as follows.

Sets

N_c=Set of customers($i, j = 1,2, \dots, N_c$)

N_k= Set of potential depots($k = 1,2, \dots, N_k$)

V =The index of vehicle($V = 1,2, \dots, v$)

P =The index of product type($p = 1,2, \dots, P$)

Index

i, j =Number of customer

k =Number of depots
v = Number of vehicle
$p = Number\ of\ product$

Parameters
$\widetilde{DD}_{ip}$=Delivery demand of customer *i* from product type *p*
$\widetilde{DD}_{ip}$= Pick-up demand of customer *i* from product type *p*
CD_k =Capacity of potential depot *k*
FD_k = the depot establishing cost at the potential location *k*
FV_v= Capacity of vehicle *v*
UV_p=Volume of per unit product *p*
C_v= Cost per unit travel time by vehicle *v*
$\tilde{T}_{kiv}$= Unit transportation time from depot k to customer i by vehicle v
$\tilde{T}_{ijv}$= Unit transportation time from customer *i* to customer *j* by vehicle v
M= A large constant
E_i =The maximum possible time required for serving at the customer node *i*
L_i=The minimum possible time required for serving at the customer node *i*
ST_i=Serving time to the customer I

Decision variables
α_k=If potential depot k =1; otherwise = 0
β_v=If the vehicle *v*=1; otherwise = 0
x_{kiv}=If vehicle v is traveled from depot *k* to costumer *i* = 1; otherwise = 0
y_{ijv}=If vehicle v is traveled from costumer *i* to costumer *j* = 1; otherwise = 0
z_{ikv}=If vehicle *v* is traveled from costumer i to depot *k* = 1; otherwise = 0
γ_{ik}= if customer i is assigned to depot k =1; otherwise = 0
f_{jvp}=The value of product *p* that be transported by vehicle v from customer demand i
at_i^v=arrival time of vehicle *v* to customer *i*
$u_{i,v}$=auxiliary variable for the sub tour elimination

Mathematical Model

$$\text{MinZ} = \sum_{k} \alpha_k FD_k + \sum_{v} \beta_v FV_v + \sum_{k,i,v} C_v \tilde{T}_{kiv} X_{kiv} + \sum_{i,j,v} C_v \tilde{T}_{ijv} Y_{ijv} + \sum_{k,i,v} C_v \tilde{T}_{ikv} Z_{ikv} \quad (1)$$

$s.t.$:

$$\sum_{k,i} X_{kiv} = \beta_v, \quad \forall v \quad (2)$$

$$\sum_{i} X_{kiv} = \sum_{i} Z_{ikv}, \quad \forall k, v \quad (3)$$

$$\sum_{k} X_{kiv} + \sum_{\substack{j \\ j \neq i}} Y_{jiv} = \sum_{\substack{j \\ j \neq i}} Y_{ijv} + \sum_{k} Z_{ikv}, \quad \forall i, v \quad (4)$$

$$\sum_{k,v} X_{kiv} + \sum_{\substack{j,v \\ j \neq i}} Y_{jiv} = 1, \quad \forall i \quad (5)$$

$$u_{iv} - u_{jv} + N_c * Y_{jiv} \leq N_c - 1, \quad \forall i, j, v, (i \neq j) \tag{6}$$

$$\sum_{k} \gamma_{ik} = 1, \quad \forall i \tag{7}$$

$$\sum_{v} X_{kiv} \leq \gamma_{ik}, \quad \forall i, k \tag{8}$$

$$\sum_{v} Z_{ikv} \leq \gamma_{ik}, \quad \forall i, k \tag{9}$$

$$\sum_{v} Y_{jiv} + \gamma_{ik} + \sum_{\substack{m \in N_0 \\ m \neq k}} \gamma_{jm} \leq 2, \quad \forall i, j, k, (i \neq j) \tag{10}$$

$$f_{jvp} \leq f_{ivp} - \widetilde{DD}_{ip} + PD_{ip} + M\left(1 - Y_{jiv}\right), \quad \forall i, j, v, p \tag{11}$$

$$f_{jvp} \geq f_{ivp} - \widetilde{DD}_{ip} + PD_{ip} - M\left(1 - Y_{jiv}\right), \quad \forall i, j, v, p \tag{12}$$

$$\sum_{p} f_{ivp} UV_p \leq CV_v \beta_v, \quad \forall i, v \tag{13}$$

$$\sum_{i,p} \gamma_{ik}\left(\widetilde{DD}_{ip} + PD_{ip}\right) UV_p \leq CD_k \alpha_k, \quad \forall k \tag{14}$$

$$at_i^v + ST_i + \tilde{T}_{ijv} - at_j^v \leq M\left(1 - Y_{jiv}\right), \quad \forall i, j, v \tag{15}$$

$$at_i^v \geq \tilde{T}_{kiv} - M(1 - X_{kiv}), \quad \forall i, k, v \tag{16}$$

$$E_i \leq at_i^v \leq L_i, \quad \forall i, v \tag{17}$$

$$at_i^v \geq 0, \quad \forall i, v \tag{18}$$

A mixed-integer linear programming model for the MDLRP under conditions of uncertainty is as follows. In Eq. 1, The objective function minimizes the total costs of the network designed. These costs include depot establishment costs, the cost of transporting vehicles, and uncertain transportation costs between customers and potential depots. Constraint (2) ensures that a vehicle is used to transfer the load from the depot to the customer. Constraint (3) ensures that each vehicle leaves the depot and returns to it. Constraints (4) and (5) guarantee that the equilibrium exists for each customer node, indicating the equality of the number of entries and exits to each customer node. Constraint (6) ensures that the sub tours are eliminated. Constraint (7) guarantees that each customer can only be assigned to one depot. Constraints (8) and (9) ensure that a customer is assigned to each depot, one of the vehicles assigned to the same depot is also assigned to the customer and all services are delivered to the customer by this vehicle. Constraint (10) guarantees that each customer can be assigned to at most one depot. Constraints (11) and (12) show the amount of demand to be delivered to each customer by each vehicle and ensure that each vehicle must have the product in its cabin tailored customers' demands before traveling from the depot. Constraint (13) ensures that each vehicle has a max-transferring for transferring the product to its customers. Constraint (14) guarantees that only depots having the capacity to deliver the product to customers can only be selected. Constraints (15) and (16) estimate the time of vehicle arrival to the customers and ensures that if the customers are assigned to the vehicle, arrival time can be achieved. Constraint (17) is related to the time window and ensures that the vehicle must satisfy the customers' demand for each product over a specified period of time. Constraint (18) shows different types of decision variables.

A robust optimization model for the MDLRP

In this section, Bertsimas and Sim [38] Robust Optimization (RO) approach has been used to control and analyze the effect of uncertain parameters. Considering the following optimization problem where the coefficients of the objective function are non-deterministic, the parameter Γ_0 controls the level of robustness in the objective function; therefore, we want to find the optimal value and optimal solution in cases where Γ_0 changes and has the highest effect on the solution.

P1): Min c^T x) *(21)*

Ax≤b *(20)*

l ≤ x ≤ u *(21)*

In general cases, values higher than Γ_0 increase the level of conservatism versus the higher cost to be paid for it in the objective function. Γ_0 is necessarily an integer and Γ_i can be either integers or non-integers.

$$(P_2): \min\ c^T x + z_0\Gamma_0 + \sum_{j\in J_0} r_{0j} \tag{22}$$

$$\sum_j a_{ij}x_j + z_i\Gamma_i + \sum_{j\in J_i} r_{ij} \le b_i \qquad \forall i \tag{23}$$

$$z_0 + r_{0j} \ge e_j y_j \qquad \forall j \in J_0 \tag{24}$$

$$z_i + r_{ij} \ge \hat{a}_{ij} y_j \qquad \forall i \ne 0, j \in J_i \tag{25}$$

$$r_{ij} \ge 0 \qquad \forall i, j \in J_i \tag{26}$$

$$y_j \ge 0 \qquad \forall j \tag{27}$$

$$z_i \ge 0 \qquad \forall i \tag{28}$$

$$-y_j \le x_j \le y_j \qquad \forall j \tag{29}$$

$$l_j \le x_j \le u_j \qquad \forall j \tag{30}$$

Accordingly, the robust counterpart is equivalent to a linear optimization problem which can be obtained as follows:

It should be mentioned that the variables added to the robust counterpart model ($z_i, y_j, r_{ij}, z_0, r_{0j}$) are used to adjust the robustness of the solution and apply the conservatism levels in the model, where *z* and r represent the vectors of the dual variables in the objective functions and constraints which are presented for the linearization of nonlinear formulas. Also, e_j denotes an uncertain parameter in the objective function.

After adjusting and converting the model into a robust optimization one, it is coded in GAMS software, then it is solved and its results are compared with those of the deterministic model.

As specified by the deterministic mathematical model, the uncertainty in the above parameters is related to the uncertainties of the objective function coefficients (c) and the technological coefficients (a).

$$\tilde{T}_{kiv} \in [T_{kiv} - \hat{T}_{kiv}, T_{kiv} + \hat{T}_{kiv}]$$
$$\tilde{T}_{ijv} \in [T_{ijv} - \hat{T}_{ijv}, T_{ijv} + \hat{T}_{ijv}]$$

$\tilde{T}_{ikv} \in [T_{ikv} - \hat{T}_{ikv}, T_{ikv} + \hat{T}_{ikv}]$
$\widetilde{DD}_{ip} \in [DD_{ip} - \widehat{DD}_{ip}, DD_{ip} + \widehat{DD}_{ip}]$

Therefore, by modeling Bertsimas and Sim's Robust Optimization (RO) approach for the uncertainties of the objective function coefficients and the technological coefficients, as shown in Eqs. 22-30, the robust counterpart formulation of the research model can be rewritten.

Then, based on the robust counterpart formulation in the study of Bertsimas and Sim [38], the mathematical model is transformed into its robust counterpart. Thus, Eq. 1 is converted to (31) and (32). Also, Eqs. 11 and 12 are converted to (33) and (34). Moreover, Eq. 14 is converted to (35), and the following relations are added to the mathematical model:

In the above mathematical model, there are 3 parameters Γ. Each Γ specifies the robustness level in the model. Γ^0 Controls the robustness level of the non-deterministic parameters in Equations 3-32. ${\Gamma^1}_{ip}$ Controls the robustness level of the non-deterministic parameters in Eqs. 33 and 34. ${\Gamma^2}_k$ Controls the robustness level of the non-deterministic parameters in Eq. 35. The control of the robustness level represents the worst and nominal values of the parameters.

$$\text{Min } Z \tag{31}$$

$$\sum_{k} \alpha_k FD_k + \sum_{v} \beta_v FV_v + \sum_{k,i,v} C_v T_{kiv} X_{kiv} + \sum_{i,j,v} C_v T_{ijv} Y_{ijv} + \sum_{k,i,v} C_v T_{ikv} Z_{ikv} + \lambda^0 \Gamma^0 + \sum_{k,i,v} \mu^0_{kiv} + \sum_{i,j,v} \mu^1_{i,j,v} + \sum_{k,i,v} \mu^2_{ikv} - Z \le 0 \tag{32}$$

$$f_{jvp} + DD_{ip} + {\lambda^1}_{ip}{\Gamma^1}_{ip} + \mu^3_{ip} \le f_{ivp} + PD_{ip} + M(1 - Y_{jiv}), \quad \forall i,j,v,p \tag{33}$$

$$f_{jvp} + DD_{ip} - {\lambda^1}_{ip}{\Gamma^1}_{ip} - \mu^3_{ip} \ge f_{ivp} + PD_{ip} - M(1 - Y_{jiv}), \quad \forall i,j,v,p \tag{34}$$

$$\sum_{i,p} (\gamma_{ik}(DD_{ip} + PD_{ip})UV_p + \mu^4_{ip}) + {\lambda^2}_k {\Gamma^2}_k \le CD_k \alpha_k, \quad \forall k \tag{35}$$

$$\lambda^0 + \mu^0_{kiv} \ge C_v \hat{T}_{kiv} X_{kiv} \qquad \forall i,k,v \tag{36}$$

$$\lambda^0 + \mu^1_{ijv} \ge C_v \hat{T}_{ijv} Y_{ijv} \qquad \forall i,j,v \tag{37}$$

$$\lambda^0 + \mu^2_{ikv} \ge C_v \hat{T}_{ikv} Z_{ikv} \qquad \forall i,k,v \tag{38}$$

$${\lambda^1}_{ip} + \mu^3_{ip} \ge \widehat{DD}_{ip} \qquad \forall i,p \tag{39}$$

$${\lambda^2}_k + \mu^4_{ip} \ge \gamma_{ik} \widehat{DD}_{ip}\, UV_p \qquad \forall i,k,p \tag{40}$$

$$\mu^0_{kiv}, \mu^1_{i,j,v}, \mu^2_{ikv}, \mu^3_{ip}, \mu^4_{ip} \ge 0 \tag{41}$$

The probability of constraint violation in the robust counterpart model

Bertsimas and Sim [38] prove that by the formulation of the corresponding robust counterpart under the value Γ (both for the objective functions and for the constraints), the probability of constraint violation is equal to $e^{\frac{-2\Gamma^2}{|J|}}$, where $|J|$ represents the number of non-deterministic parameters. Also, the confidence level (CL) (probability of the non-violation of a constraint) is equal to 1- $e^{\frac{-2\Gamma^2}{|J|}}$.

Meta-Heuristic solution method: The Proposed GA

This section discusses the proposed solution methods. As mentioned previously, in this study, an exact algorithm (using GAMS software with BARON solver) and a Meta-Heuristic algorithm have been used to solve the problem. Because the exact algorithms failed to solve larger-sized problems, a GA has been proposed. Therefore, first, the primary chromosome is designed to solve the MDLRP

Design of the primary chromosome structure

The design of the primary chromosome structure consists of two phases: one phase which determines the optimal location and number of depots and vehicles used, and another phase which determines the optimal vehicle routing to the customers' demand. As can be seen in Table 2, seven customers, three potential depots, and five vehicles are considered. The length of the designed chromosome is equal to the number of customers, and the first row (Table 2) determines the optimal location and number of potential depots and the vehicles used, and the second row specifies a sequence of traveling a vehicle.

Table 2. The structure of the chromosome designed for the problem

Customer	1	2	3	4	5	6	7
Vehicle-depot	13	11	5	11	11	5	9
Sequence	4	1	3	6	2	7	5

In the coding structure of the chromosome designed, random integers are generated between 1 and 15. Then, these random integers are categorized into three groups as shown in Table 3.

Table 3. How to code the primary chromosome designed for the MDLRP

$D1 = \{1,2,3,4,5\}$	$D2 = \{6,7,8,9,10\}$		$D3 = \{11,12,13,14,15\}$
	$D1$	$D2$	$D3$
Vehicle1	1	6	11
Vehicle 2	2	7	12
Vehicle 3	3	8	13
Vehicle 4	4	9	14
Vehicle 5	5	10	15

As shown in Tables 2 and 3, the structure of the chromosome is illustrated by the example below. For example, if a chromosome generates integers 1, 6, or 11, all three integers representing vehicle 1 are used for routing. However, if the generated integer is 1, the vehicle must start from depot 1, if the generated integer is 6, the vehicle must start from depot 2 and, if the generated integer is 11, the vehicle must start from depot 3. To fully illustrate the given initial solution, as shown in Table 2, so we have:

1. Vehicle 3 travels from depot 3 to customer 1
2. Vehicle 1 travels from depot 3 to customers 2,4 and 5
3. Vehicle 5 travels from depot 1 to customers 3 and 6
4. Vehicle 4 travels from depot 2to customer 7

Now, after assigning each customer to each depot and the vehicle, the vehicle routing (a sequence of traveling) must be specified. In the second row (Table 2), the generated chromosome shows a sequence of traveling the vehicles. Therefore, the vehicles visit customers from a lower priority to a higher priority. To illustrate the vehicle routing phase, as shown in Fig. 2, thus we have:

1. The vehicle travels from depot 3 to customer 1 and then returns to the same depot
2. The vehicle travels from depot 3 to customers 2,5 and 4 and then returns to the same depot
3. The vehicle travels from depot 1 to customers 3 and 6 and then returns to the same depot
4. The vehicle travels from depot 2 to customer 7 and then returns to depot 7

After designing the primary chromosome, in the following, the two most important operators of GA, namely, mutation and crossover (also called recombination) operators are described.

How to apply the crossover operator

In this paper, the two-point crossover operator is performed on both the first and second parts of the chromosome. After replacing the parts of the parent, offsprings need to be modified, if necessary, the first part must be modified so that no vehicle is used twice in each depot and permutation is maintained in the second part.

How to apply the mutation operator

The mutation operator is also performed in three steps:
In the first step, a gene (e.g., vehicle or depot) was chosen randomly and assigned to one of the customers randomly.

In the second step, the two genes from the chromosome are selected randomly and replaced with each other.

In the third step, a gene is selected randomly and the location of the depot for the same vehicle is changed.

Also, for the second part of the chromosome (sequencing), the mutation is only considered as a displacement of two gene sequences.

Before solving sample problems of different sizes, the primary parameters of the GA need to be adjusted. The Taguchi method is used for adjusting the primary parameters. The parameters of the GA used include: (1) weighted selection probability, (2) mutation probability, (3) Crossover probability, (4) the number of individuals in the population, and (5) a maximum number of repetitions. As displayed in Table 4, the three levels are considered for each of the five parameters in the Taguchi method. Following running 27 experiments with the Taguchi method, acceptable results have been obtained.

The highest value of S/N is a criterion for selecting the values of the parameters. By comparing the difference between the maximum and minimum values in S/N, the significant effect of the MaxIt parameter (maximum number of repetitions) on the improvement of the solution GA process is evident. The parameters of Pm (mutation probability) and nPop (the number of individuals in the population) and Pc (crossover probability) and beta (weighted selection probability pressure) are ranked in the next places, respectively, based on their effect.

Table 4. Levels of Parameters determined by the Taguchi method

Beta	Pm	Pc	nPop	MaxIt	Level
5	35/0	65/0	350	120	Low
5/6	55/0	75/0	300	200	Medium
8/7	65/0	88/0	670	330	High

Finally, Fig. 2 shows the mean S/N ratio plots and the mean values for adjusting the parameter of the GA by using the Taguchi method.

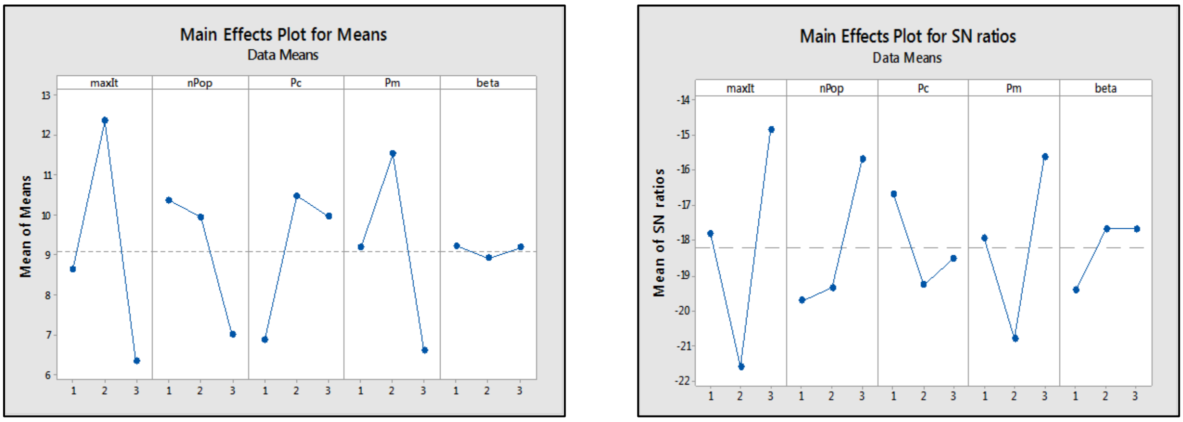


Fig. 2. The mean S/N ratio plots and mean values

According to the graphs obtained from Minitab, as shown in Fig 2, the best values for the parameter of MaxIt, nPop, Pc, Pm, and beta are as follows:

MaxIt = 330 (i.e., the third level)

nPop = 670 (i.e., the third level)

Pc = 0.65 (i.e., the first level)

Pm = 0.65 (i.e., the third level)

Beta = 6.5 (i.e., the second level)

Analysis of experiments

In this section, the exact method and GA are used in computing to find a solution and the results are analyzed. For this purpose, a sample problem of small size (Table 5 is designed based on the random data obtained (Table 6), and the results are presented by GAMS software and the GA. Tables 5 and 6 show the size of sample problems designed and interval bounds of the parameters generated based on uniform distribution, respectively.

Table 5. Small-size sample problems

Sample problem	$\|K\|$	$\|V\|$	$\|P\|$	$\|I\|$
1	3	5	3	8

Table 6. Interval bounds of the parameters generated

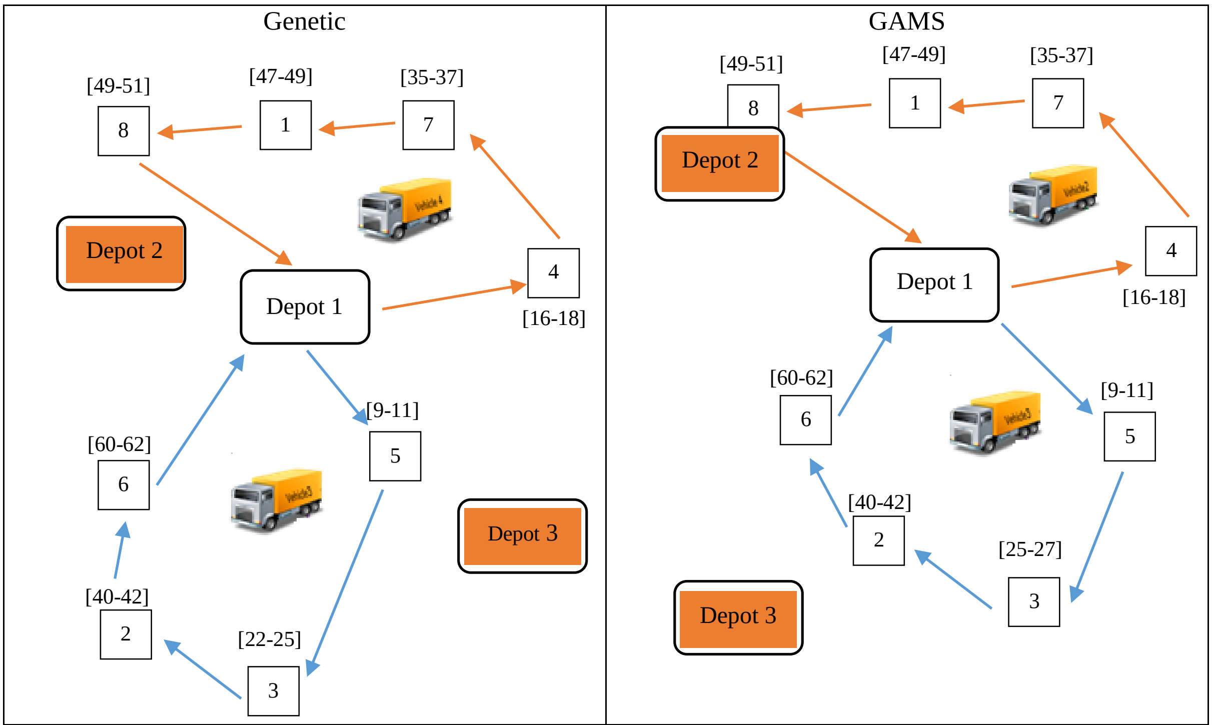
Parameter	Interval bound	parameter	Interval bound
PD_{ip}	[40,435]	UV_p	[10,30]
CD_k	[100000,180000]	C_v	[10,40]
FD_k	[100,200]	E_i	[10,200]
CV_v	[30000,80000]	L_i	[100,600]
FV_v	[30,100]	ST_i	[10,50]
Interval bounds		Parameter	
[5,100]		$\widetilde{DD}_{ip}$	
[10,50]		$\tilde{T}_{kiv}$	
[10,50]		$\tilde{T}_{ijv}$	

The results of the small-size problem and the values of the objective functions obtained from the exact method and the GA are presented in Table 7 and Fig. 3. Table 7 shows the values of objective functions and computational time, as well as the optimal location and number of depots by solution methods. Fig. 3 shows the optimal vehicle routing to deliver the customer's demand using the GAMS software and the GA. The numbers inside the brackets in each node indicate the arrival and departure times from each node by each vehicle, respectively, so that the departure time of the last customer represents the total time of the tour.

Table 7. The value of the objective function and the output variables of the small-size sample problem

Solution method	The value of the objective function	Computational time (S)	The optimal location and number of depots	The number of vehicles selected	Traveled route	Tour duration (min)
Exact	1323	57	1	2	8-1-7-4	51
				3	6-2-3-5	62
Metaheuristic (GA)	1325	92	1	4	8-1-7-4	51
				3	6-2-3-5	62

As can be seen from Table 7, the depots selected and vehicles used have the same number when using the two above-mentioned solution methods, but they differ only in the optimal type of the selected vehicles. Also, the relative percentage difference of the value of the objective function of GA is 1% as compared with the exact algorithm, indicating the GA has a higher efficiency in obtaining optimal solutions.



2 and 3	not selected facilities	2 and3	not selected facilities
1,2 and5	vehicles that are not used	1,4 and5	vehicles that are not used

Fig. 3. Optimal Vehicle Routing of small-size sample problem

To investigate and compare the solution methods of large-size problems, and evaluate the efficiency of the GA in solving the MDLRP, a total of 10 sample problems of different sizes (Table 8) are designed based on the data from Table 7.

Table 8. Small-size sample problems

sample problem	$\|K\|$	$\|V\|$	$\|P\|$	$\|I\|$
1	3	5	3	8
2	4	9	5	10
3	5	10	5	13
4	6	10	6	17
5	7	11	6	20
6	8	11	6	24
7	9	12	7	27
8	10	12	7	30
9	11	13	7	35
10	12	14	8	40

The numerical results of 10 sample problems are shown in Table 9. The results of this table include the optimal value of the objective function obtained from the GAMS software, as well as the values obtained from solving sample problems with the GA in 5 consecutive repetitions. The computational time for solving sample problems is also presented in Table 9.

Table 9. Comparison of results from the exact and GAs of solving sample problems

sample problems	Mean computational time (s)		The value of the objective function	
	GA	GAMS	GA	GAMS
1	92	57	1325	1323
2	128	73	1676	1670
3	205	238	1981	1964
4	318	789	2605	2573
5	465	1338	3074	2991
6	664	-	3983	-
7	989	-	4726	-
8	1211	-	5878	-
9	1658	-	7345	-
10	2148	-	9846	-

To evaluate the efficiency of the GA in obtaining optimal solutions, the relative percentage difference of each of the repetitions and the mean percentage difference of the total repetitions are given in Table 10.

Table 10. The relative percentage difference of sample problems 1-5

Sample problem	the relative percentage difference of five repetitions					the mean percentage difference
1	0.00001	0.00002	0.00001	0.000015	0.00001	0.000013
2	0.0036	0.0035	0.0034	0.0037	0.0035	0.0036
3	0.0085	0.0089	0.0086	0.0086	0.0086	0.00864
4	0.012	0.011	0.014	0.012	0.012	0.0122
5	0.028	0.027	0.029	0.026	0.027	0.0274

As can be seen from Table 10, with the increasing size of the problem, the relative percentage difference obtained from the GA is very small and in general, the solved sample problems are reported to be about 1%. Also, due to the complexity of the problem and model, GAMS software can only solve sample problems 1-5. Figs. 4 and 5 compare the mean values of the objective function and the computational time of solving sample problems with solution methods, respectively.

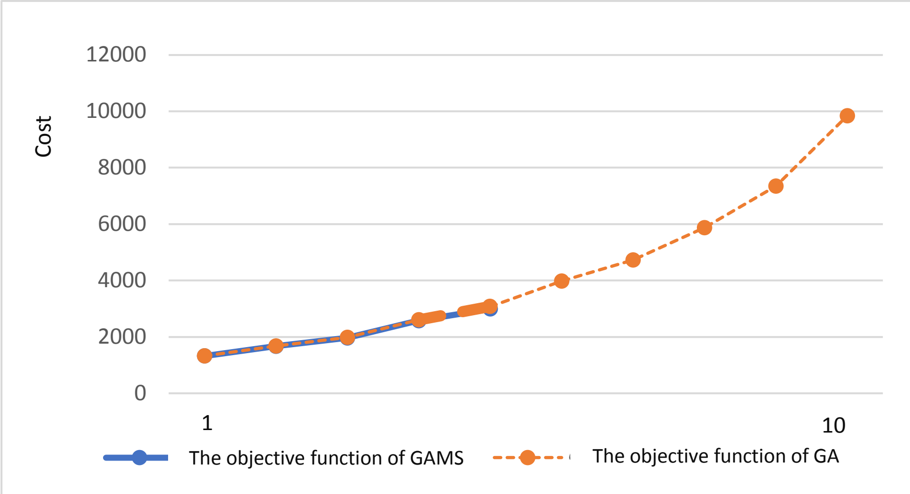


Fig. 4. Comparison of mean values of the objective function of different sample problems with solution methods

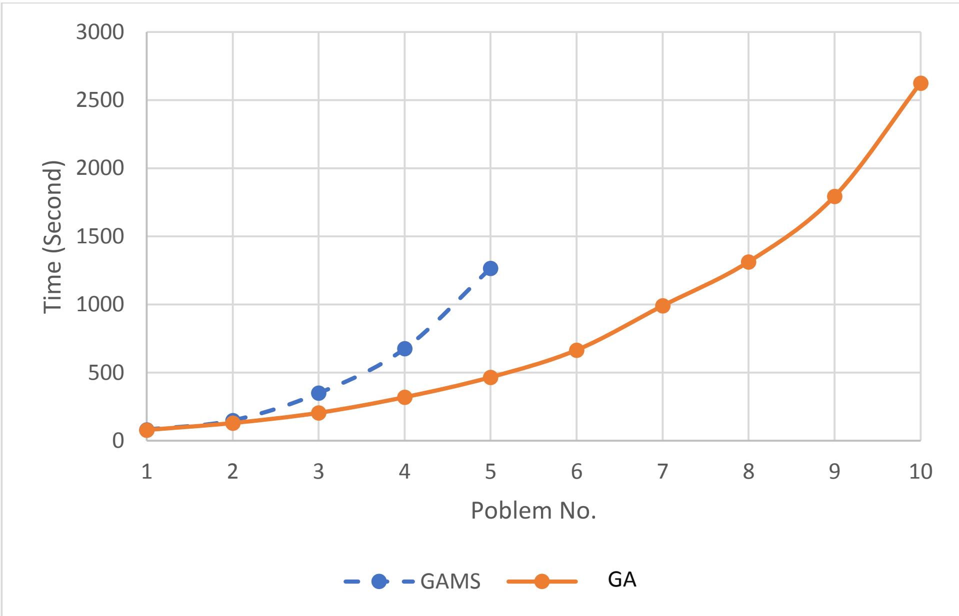


Fig. 5. Comparison of computational time of exact and GAs in sample problems

As is clear from the above, the GA solutions approximate to those of the exact algorithm, although they are slightly worse, they are close to optimal solutions. The GA performs much better than the exact algorithm concerning time. Moreover, GAMS software fails to solve the large-size problem, and the time to find a solution grows exponentially with increasing the size of the problem. However, the GA quite efficient for problems of large sizes, and can nearly find the optimal solution in a much shorter amount of time.

Due to uncertainties in parameters such as transportation cost and demand, the Bertsimas and Sim (B&S) Robust Optimization (RO) approach is used to model the problem. The parameters of the customers' demand for each product and transportation cost are considered to be robust. The B&S Robust Optimization (RO) approach has advantages over the probabilistic approach in terms of solving the problem, the lack of need for the probability distribution of uncertain data, and the use of historical data and decision-making experiences, leading to the selection of this method to control the uncertain parameters. For this reason, the B&S robust optimization model for MDLRP is as follows.

Analysis of the results of the robust mathematical model

Given the uncertainty in the parameters of the mathematical model, in Section 3, a robust mathematical model is presented to deal with uncertainty. In this mathematical model, $\Gamma 0, \Gamma^1{}_{ip}$, and $\Gamma^2{}_k$ are considered as control parameters of a robust optimization model. According to Bertsimas and Sim [38], the values of these parameters are selected based on the confidence level of decision-makers. In this section, different confidence levels are considered for sensitivity analysis, and accordingly, the control parameter values for the numerical example 2 are determined, as shown in Table 11.

Table 11. The control parameter values of the robust counterpart model

confidence level	The probability of constraint violation	Γ^0	$\Gamma^1{}_{ip}$	$\Gamma^2{}_k$
0.5	0.5	1.177	8.326	2.355
0.6	0.4	1.354	9.572	2.907
0.7	0.3	1.752	15.973	3.604
0.8	0.2	2.448	17.308	4.895
0.9	0.1	2.746	18.174	5.292
0.95	0.05	2.948	20.308	5.895
0.99	0.01	3.035	21.460	6.070

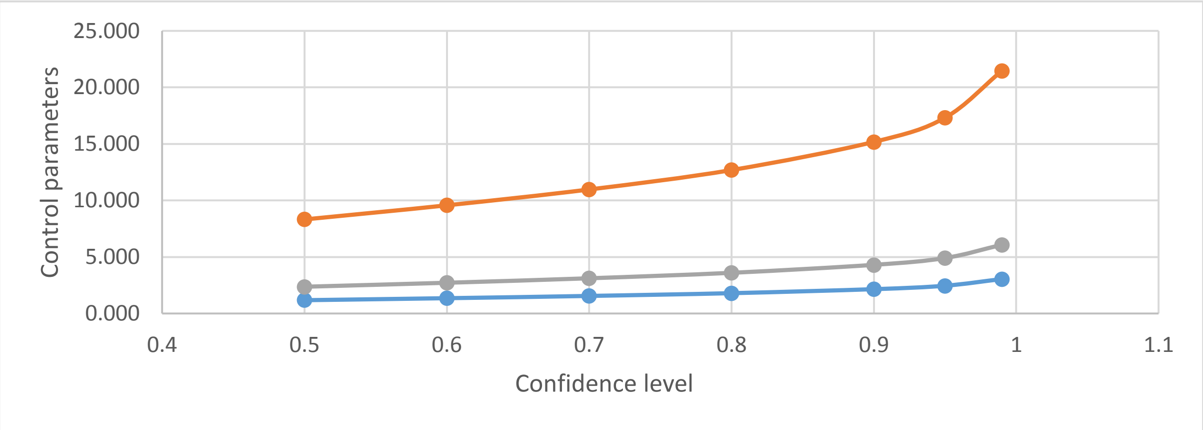


Fig. 6. Effect of confidence level on the control parameter values

As shown in Fig. 6, the control parameter values can be increased nonlinearly with increasing the confidence level. Increasing the control parameter values indicates that more parameters in their worst state are considered and then the model is optimized. Therefore, it can be argued that more uncertain parameters have deviated from their nominal value (the value used in an uncertain solution) as the confidence level increases, and thus the result is closer to real conditions.

Then, the mathematical model is optimized under the different confidence levels, and the value of its objective function is reported. The results are presented in Table 12.

As can be seen in Table 12, the percentage increase in the objective function compared to its best state (basic non-robust model) is represented by R% in which, the confidence level of the basic non-robust model is one and parameters assume no uncertainty and the nominal values given for the parameters are constant and exact (In fact, we know that and some parameters have tolerance).

Table 12. The results of optimizing the robust model

confidence level (CL)	Z (without uncertainty)	Z (with considering uncertainty and robust status)	R%	Percentage difference of each confidence level compared to the previous CL
0.5	1670	1718.5312	2.8%	2.8%
0.6	1670	2228.8720	33%	30.2%
0.7	1670	2399.3342	43%	10%
0.8	1670	2527.360	51%	8%
0.9	1670	2976.6527	78%	27%
0.95	1670	3167.249	89%	12%
0.99	1670	3293.7932	97%	8%

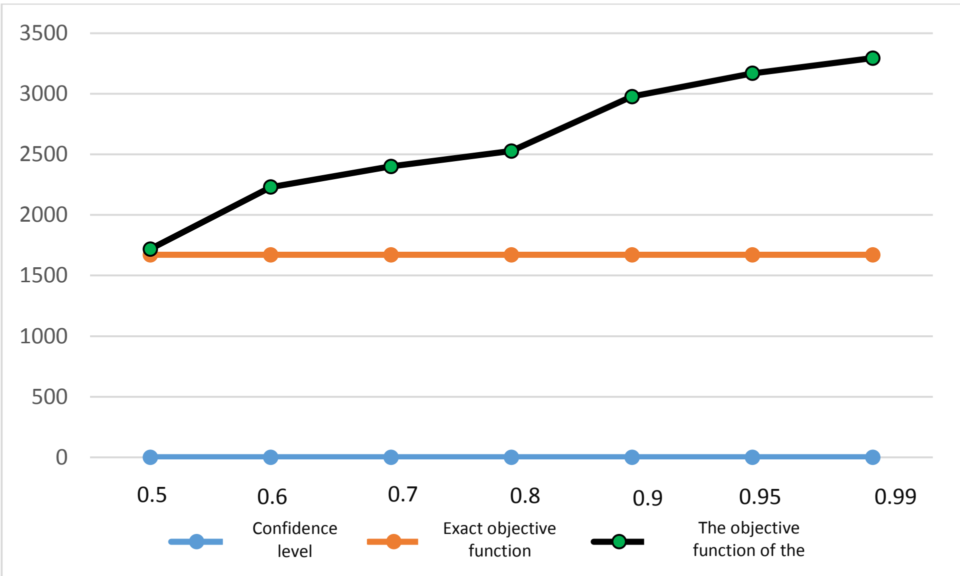


Fig. 7. Sensitivity analysis of the robust counterpart model

As shown in Fig. 7, increasing the confidence level has led to an increase in the value of the objective function of the robust counterpart model. “The reason for this is that the control parameter values are increased with increasing the confidence level, and as a result, more uncertain parameters would be required in the worst case, and thus the optimal solution of the objective function is increased. As shown in Fig. 7, this increase does not exhibit linear behavior, so the best possible state can be chosen among them. The best state is that has a higher degree of robustness, in other words, it has less variability compared to the confidence level. According to the above figure, at the 80% confidence level, the minimum changes in the objective function are observed as compared with the previous CL, while changes in the objective function are higher than those in the next CL, i.e., if we want to obtain a 90% confidence level, it requires more cost, but increasing the confidence level from 70% to 80% does not need a more cost, so an 80% confidence level can be considered as an ideal solution for the robust counterpart model. Analysis of the values of R% is performed in the same way. As is clear from the above, the higher the confidence level, the greater the percentage increase in the value of the objective function would be, indicating that the decision-maker must determine an equilibrium level between the confidence level and the R%, and the comparisons show that the equilibrium level can be obtained at an 80%confidence level.

Conclusions and Suggestions for Future Research

In this paper, a multi-depot location routing problem has been modeled under uncertainty in demand and transportation times to reduce total transportation costs. This study addresses the decision to select the optimal location and number of facilities as well as the heterogeneous vehicle routing problem (HVRP) for multiple products. Unlike other papers published, in this study, a robust optimization approach has been used to control uncertain parameters. Also, the GA has been applied to solve the problem using a heuristic chromosome, which shows the effectiveness of this chromosome in the results obtained from solving sample problems. To evaluate the efficiency of the GA with the designed chromosome, a total of 10 sample problems were designed. The results showed that GAMS software fails to solve large and medium-sized problems. However, by calculating the relative percentage difference, it is found that the maximum difference between the results of the objective function is about 1% concerning the GA and the GAMS software, while the GA has solved the problem in a much shorter amount of time as compared with the GAMS software.

Given the widespread use of the multi-depot vehicle routing problem (MDVRP), it is recommended that further research be undertaken in the following areas:

a) The GA is one of the meta-heuristic methods available and the study of a given problem with other meta-heuristic methods may lead to better or worse results. Therefore, this problem can be solved with other meta-heuristic algorithms and compared the results with those of the present study.
b) We propose a single-objective model, but a multi-objective model can present to minimize CO2 from the vehicle.
c) The proposed model can be implemented at real projects of petroleum products distribution systems, pharmaceuticals, food industry, and major industries in Iran to organize the transportation system such as oil and steel industries and shuttle services.

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