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RESEARCH PAPER

A new higher-order theory for the static and dynamic responses of sandwich FG plates

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Abstract

In this study, a static and free vibration analysis of single layer FG and sandwich FG plates is carried out using a fifth order shear and normal deformation theory. The displacement field of the present theory includes the terms considering the effect of transverse shear and normal deformation. Also, the terms of the thickness co-ordinate are expanded upto fifth order to predict the accurate bending behavior of the plates. The equations of motion are derived based on Hamilton's principle, and further solved using Navier's solution scheme. The present results of displacement, stresses and natural frequencies in sandwich FG plates are obtained and compared with other higher order theories available in literature to check the validity and efficacy of the theory.

Keywords: Sandwich FG plates, Shear and normal deformation, Static, Dynamic.

Introduction

Functionally graded material is a new class of material which is having a wide applications in the field of aerospace, aircraft, marine, offshore, energy sector, industrial, construction etc. FGM is made up of ceramic and metal in which the material properties are varying through the thickness. High temperature resistance, lightweight, good strength-to-weight ratio, high stiffness are the significant features of FGM material over other composite material. Therefore, the use of FGM material is highly demanded. The FGM sandwich plate is made up isotropic homogeneous core and FGM face sheet or FGM core and homogeneous face sheets. Therefore, many researchers have presented a different analytical and mathematical models to study the static and dynamic analysis of FGM sandwich plates.

Kirchhoff [1] and Mindlin [2] developed CPT and FSDT respectively for the static and free vibration analysis of beams, plates and shells. As CPT and FSDT are the assumption based theories, are not applicable for the analysis of thick beams and plates. Therefore, many researchers have developed a higher order shear deformation theories for the analysis of laminated composite and FGM thick plates. Sayyad and Ghugal [3] presented the non-linear hygro-thermo-mechanical analysis of FGM plates resting using four unknown theory. Shinde and Sayyad [4] presented a quasi-3D polynomial shear and normal deformation theory for laminated composite, sandwich, and functionally graded beams. Thai et al. [5] presented an analysis of functionally graded sandwich plates using the FSDT. Thai and Kim [6] employed a four variable shear deformation theory for the bending and free vibration analysis of functionally graded sandwich plates. Li et al.[7] studied free vibration analysis of functionally graded material sandwich plates, based on three-dimensional linear theory of elasticity. Daszkiewicz et al. [8] presents geometrical nonlinear analysis of functionally graded shells using 2-D constitutive model. A large deformation analysis of functionally graded shell based on the first order shear deformation theory is presented by Aciniega and Reddy [9]. Demirhan and Taskin [10] applied a four variable shear deformation theory based on Levy solution, for bending analysis of functionally graded sandwich plates. Abdelaziz et al. [11] have presented a static analysis of functionally graded sandwich plates using a four variable theory. Zenkour [12] presented the bending and free vibration analysis of functionally graded sandwich plates using the sinusoidal plate theory. Zenkour and Alghamdi [13, 14] presented the effect of thermal and mechanical load on the bending analysis of FGM sandwich plates. Free vibration of functionally graded shallows shells with complex planforms is studied by Kurpa et al. [15] using the R-function theory and Ritz approach. Dong and Dung [16] investigated the governing equations for nonlinear vibration of FGM sandwich doubly curved shallow shells reinforced by FGM stiffners, based on FSDT. A four variable refined plate theory is applied for free vibration analysis of functionally graded sandwich plates made up of soft and hard cores by Hadji et al. [17]. A refined Zigzag theory based on Ritz method is applied for the bending and free vibration analysis of FG sandwich plates by Sciuva and Sorrenti [18]. Rouzegar and Gholami [19] presented thermo-elastic bending analysis of functionally graded sandwich plates using the hyperbolic shear deformation theory. Belabed et al.[20] developed a new 3-unknown

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hyperbolic shear deformation theory for the free vibration analysis of FG sandwich plates. Attia et al. [21] has presented a four variable refined plate theories, accounts parabolic, sinusoidal, hyperbolic and exponential distributions of transverse shear strain for the free vibration analysis of FG sandwich plates. A static response of FG plates and shells using the optimized sinusoidal higher order shear deformation theory is presented by Mantari and Soares [22]. Thai and Kim [23] presented a review article on modeling and analysis of FG plates and shells. Recently, Irfan and Siddiqui [24] reviews recent advancements in finite element formulation for sandwich plates. Tornabene et al. [25] studied the dynamic behavior of FG conical, cylindrical shells and annular plate structures using FSDT. Do and Thai [26] presented a modified Kirchhoff theory for the free vibration analysis of FGM plates. Wu et al. [27] presented RMVT-based meshless collocation and element-free Galerkin methods for the quasi-3D analysis of multilayered composite and FGM plates.

Mohammadi et al. [28], Mohammadi et al. [29], Mohammadi et al. [30], Mohammadi et al. [31], Mohammadi et al. [32], Mohammadi et al. [33], Mohammadi et al. [34], Mohammadi et al. [35], Mohammadi et al. [36], Farajpour et al. [37] presented free vibration and shear bukling analysis of orthotropic rectangular graphene sheets in elastic and thermal environment. Mohammadi et al. [38], Mohammadi et al. [39], Moosavi et al. [40], Asemi at al. [41], Asemi at al. [42], Asemi at al. [43], Asemi et al. [44], Danesh et al. [45], Farajpour et al. [46], Farajpour et al. [47], [48], Goodarzi et al. [49] presented non-linear free vibration ananlysis of piezoelectric nano-plates using nonlocal elasticity theory. Mohammadi and Rastgoo [50], Mohammadi and Rastgoo [51], Mohammadi et al. [52] studied the primary and secondary resonance analysis of porous FG nanoplate and nanobeam in non-linear elastic medium. Mohammdi et al. [53], Safarabadi and Mohammdi [54], Baghani et al. [55] studied the vibration analysis of rotating nanobeam considering the surface energy effect. Farajpour and Rastgoo [56], Farajpour and Rastgoo [57], Farajpour et al. [58], Ghayour et al. [59] studied the vibration and buckling analysis of microtubules in nanoshells and plates in elastic and thermal environment.

A review of FG thick cylindrical and thick shells is presented by Zamani et al. [60]. Hosseini et al. [61], Hosseini et al. [62], Nejad et al. [63], Nejad et al. [64], Gharibi et al. [65] presented a thermoelastic analysis of FG rotating pressure vessels. A torsional vibration of FG nanobeam under magnetic field based on the nonlocal elasticity theory is presented by Zarezadeh et al. [66], Noroozi et al. [67], Barati et al. [68], [69], Khoram et al. [70]. Hadi et al. [71], Shishesaz et al. [72], Mazarei et al. [73], Zamani et al. [74], investigates the termo-elasto-plastic analysis of FG spherical shells.

Shortcomings of other studies

1) In the other studies, the effect of transverse normal strain is not fully explored while predicting the static and dynamic analysis of laminated composite and FG plates, due to more complex mathematics and to avoid more number of unknown parameters. But, the inclusion of the effect of transverse normal strain and higher order expansion of polynomial shape function in terms of thickness co-ordinate is highly recommended by Carrera et al. [75, 76] and Koiter [77] in his study to predict the accurate bending behavior of thick plates and shells.
2) Most of the recently developed higher order theories involve four unknowns which are not sufficient and accurate to predict the correct global response (bending, buckling, and vibration) of the structure.

Novelty of the Present Work

Hence, with reference to Carrera's and Koiter's recommendation a new fifth order shear and normal deformation theory is developed by Naik and Sayyad [78], Sayyad and Naik [79] and Ghumare and Sayyad [80] for the static and dynamic analysis of laminated composite and functionally graded plates. The features of the present theory are summarized as follows,

1. Through the literature review it has been observed that, the studies on static and dynamic analysis of FGM sandwich plates is limited. Therefore in the present study, static and dynamic analysis of single layer and FGM sandwich plates are presented.
2. In this study, the fifth order shear and normal deformation theory is applied to obtain the displacement, stresses and frequencies in the plates.
3. The theory includes, the effect of both transverse normal and transverse shear deformation to predict the accurate the bending behavior of the FGM sandwich plates/shells, as recommended by Carrera.
4. Also, as the polynomial shape function is extended upto fifth term in the present study, it predicts the bending behavior more precisely with less percentage of error.
5. To find the non-dimensional numerical results of displacements and stresses the nine variationally consistent governing equations are derived using Hamiltons principle and solved using the Navier solution technique.
6. To validate the accuracy and efficacy of the present theory the displacement, stresses and natural frequencies for plates, are compared with other theories available in literature.
7. The results obtained are presented in tabular and graphical formats to understand the bending behavior of plates/shells through thickness.

Methodology

In the present study, a simply supported single layer FG and FG sandwich plates are considered. A plate having width a along x- direction, breadth b in y- direction, thickness h in z- direction and radii of curvature R_1 and R_2 is considered. FG sandwich plate, top and bottom face sheets are made up of functionally graded material and the core is assumed to be homogeneous isotropic material. The variation of material properties in FG sandwich plate along the thickness h is as shown in Fig. 1. The upper face sheet section is between h_1 to h_2, the homogeneous core section is between h_2 to h_3 and lower face sheet section is between h_3 to h_4.

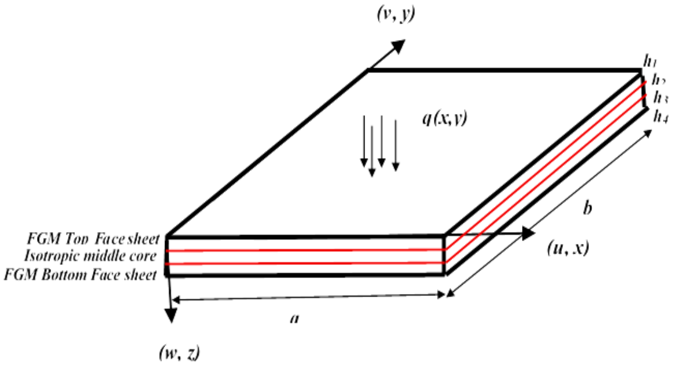

Figure 1. Material gradation of FG sandwich plate.

The modulus of elasticity in the FG shells varying through the thickness, and expressed as,

$$E^{(N)}(z) = E_m + \left(E_c - E_m\right)V_c^{(N)}, \tag{1}$$

where, E_m, E_c are the modulus of elasticity of metal and ceramic respectively. $V_c^{(N)}$ is the volume fraction in N^{th} layer, and expressed as,

$$\begin{aligned} V^1 &= \left(\frac{z-h_1}{h_2-h_1}\right)^p && \text{for} \quad z \in [h_1, h_2] \\ V^2 &= 1 && \text{for} \quad z \in [h_2, h_3] \\ V^3 &= \left(\frac{z-h_4}{h_3-h_4}\right)^p && \text{for} \quad z \in [h_3, h_4] \end{aligned} \tag{2}$$

where, p denotes the power-law index. When the value of p =0 shell is fully ceramic and when p=∞ shell is fully metallic. In the present study various lamination schemes of FG sandwich shells are considered as 1-0-1, 1-1-1, 1-2-1, 2-1-2, and 2-2-1. The thickness of each layer is given as below.

1. For 1-0-1 sandwich scheme: $h_1 = -h/2$, $h_2 = 0$, $h_3 = 0$ and $h_4 = h/2$

2. For 1-1-1 sandwich scheme: $h_1 = -h/2$, $h_2 = -h/6$, $h_3 = h/6$, and $h_4 = h/2$

3. For 1-2-1 sandwich scheme: $h_1 = -h/2$, $h_2 = -h/4$ $h_3 = h/4$ and $h_4 = h/2$

4. For 2-1-2 sandwich scheme: $h_1 = -h/2$ $h_2 = -h/10$ $h_3 = h/10$ and $h_4 = h/2$

5. For 2-2-1 sandwich scheme: $h_1 = -h/2$ $h_2 = -h/10$ $h_3 = 3h/10$ and $h_4 = h/2$

Development of theory

Displacement Field

Based on the assumptions of classical shell theory and the displacement field for a fifth order shear and normal deformation theory is written as,

$$
\begin{aligned}
u &= u_0 - z\frac{\partial w_0}{\partial x} + \left(z - \frac{4z^3}{3h^2}\right)\phi_x + \left(z - \frac{16z^5}{5h^4}\right)\psi_x \\
v &= v_0 - z\frac{\partial w_0}{\partial y} + \left(z - \frac{4z^3}{3h^2}\right)\phi_y + \left(z - \frac{16z^5}{5h^4}\right)\psi_y \\
w &= w_0 + \left(1 - \frac{4z^2}{h^2}\right)\phi_z + \left(1 - \frac{16z^4}{h^4}\right)\psi_z
\end{aligned} \tag{3}
$$

where, u, v, are the in-plane displacements in x-, y- directions and w is the transverse displacement in z- direction at any point. $\phi_x, \phi_y, \phi_z, \psi_x, \psi_y, \psi_z$ are the shear slopes in x-, y- and z- direction respectively.

Strain-Displacement Relationship

The normal and shear strains associated with the displacement field can be obtained as,

$$
\begin{aligned}
\varepsilon_x &= \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w_0}{\partial x^2} + f_1(z)\frac{\partial \phi_x}{\partial x} + f_2(z)\frac{\partial \psi_x}{\partial x} + \frac{f_1^{'}(z)}{R_1}\phi_z + \frac{f_2^{'}(z)}{R_1}\psi_z \\
\varepsilon_y &= \frac{\partial v_0}{\partial y} - z\frac{\partial^2 w_0}{\partial y^2} + f_1(z)\frac{\partial \phi_y}{\partial y} + f_2(z)\frac{\partial \psi_y}{\partial y} + \frac{f_1^{'}(z)}{R_2}\phi_z + \frac{f_2^{'}(z)}{R_2}\psi_z \\
\varepsilon_z &= f_1^{''}(z)\phi_z + f_2^{''}(z)\psi_z \\
\gamma_{xy} &= \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z\frac{\partial^2 w_0}{\partial x \partial y} + f_1(z)\left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x}\right) + f_2(z)\left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x}\right) \\
\gamma_{xz} &= f_1^{'}(z)\phi_x + f_2^{'}(z)\psi_x + f_1^{'}(z)\frac{\partial \phi_z}{\partial x} + f_2^{'}(z)\frac{\partial \psi_z}{\partial x} \\
\gamma_{yz} &= f_1^{'}(z)\phi_y + f_2^{'}(z)\psi_y + f_1^{'}(z)\frac{\partial \phi_z}{\partial y} + f_2^{'}(z)\frac{\partial \psi_z}{\partial y}
\end{aligned} \tag{4}
$$

where,

$$
\begin{aligned}
& f_1(z) = \left(z - \frac{4z^3}{3h^2}\right),\ f_2(z) = \left(z - \frac{16z^5}{5h^4}\right),\ f_1^{'}(z) = \left(1 - \frac{4z^2}{h^2}\right), \\
& f_2^{'}(z) = \left(1 - \frac{16z^4}{h^4}\right),\ f_1^{''}(z) = -\frac{8z}{h^2},\ f_2^{''}(z) = -\frac{64z^3}{h^4}
\end{aligned} \tag{5}
$$

Stress- Strain Relationship

The stresses occurred can be obtained using the Hooke's law and expressed as,

$$
\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix}^N = \frac{E(z)}{(1+\mu)}
\begin{bmatrix}
\frac{(1-\mu)}{(1-2\mu)} & \frac{\mu}{(1-2\mu)} & \frac{\mu}{(1-2\mu)} & 0 & 0 & 0 \\
\frac{\mu}{(1-2\mu)} & \frac{(1-\mu)}{(1-2\mu)} & \frac{\mu}{(1-2\mu)} & 0 & 0 & 0 \\
\frac{\mu}{(1-2\mu)} & \frac{\mu}{(1-2\mu)} & \frac{(1-\mu)}{(1-2\mu)} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2}
\end{bmatrix}^N
\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}^N \tag{6}
$$

where, $\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{xz}, \tau_{yz}$ represents the normal and shear stress components. $E(z)$ is the modulus of elasticity and μ is Poisson ratio.

3.4 The equations of motion

Hamilton's Principle is used to derive the equations of motion, as,

$$\int_{t_1}^{t_2} \left(\delta U - \delta V + \delta K\right) dt = 0 \tag{7}$$

where δ is the variational operator, t_1 and t_2 is the initial and final time respectively, (δ*U*, δ*V*, δ*K*) represents the various forms of energies as strain, potential and kinetic. Substituting values of these energies in Eq. (7), one can rewrite the Eq. (7) as

$$\int_0^a\int_0^b\int_{-h/2}^{+h/2} \left(\sigma_x\delta\varepsilon_x + \sigma_y\delta\varepsilon_y + \sigma_z\delta\varepsilon_z + \tau_{xy}\delta\gamma_{xy} + \tau_{xz}\delta\gamma_{xz} + \tau_{yz}\delta\gamma_{yz}\right) dz\,dy\,dx - \int_0^a\int_0^b q(x,y)\,\delta w\,dy\,dx + \rho\int_{dv}\left(\frac{\partial^2 u}{\partial t^2}\delta u + \frac{\partial^2 v}{\partial t^2}\delta v + \frac{\partial^2 w}{\partial t^2}\delta w\right) dv = 0 \tag{8}$$

Substituting the values of stresses and strains from eq. (4) to eq.(6) in eq.(8) and collecting the terms, the nine equations of motion are derived as below,

$$\begin{aligned}
\delta u_0 :\ & A_{11}\frac{\partial^2 u_0}{\partial x^2} - B_{11}\frac{\partial^3 w_0}{\partial x^3} + A_{S1_{11}}\frac{\partial^2\phi_x}{\partial x^2} + A_{S2_{11}}\frac{\partial^2\psi_x}{\partial x^2} + A_{12}\frac{\partial^2 v_0}{\partial x\partial y} - B_{12}\frac{\partial^3 w_0}{\partial x\partial y^2} + A_{S1_{12}}\frac{\partial^2\phi_y}{\partial x\partial y} + A_{S2_{12}}\frac{\partial^2\psi_y}{\partial x\partial y} + E_{13}\frac{\partial\phi_z}{\partial x} + F_{13}\frac{\partial\psi_z}{\partial x} + A_{66}\frac{\partial^2 u_0}{\partial y^2} \\
& + A_{66}\frac{\partial^2 v_0}{\partial x\partial y} - 2B_{66}\frac{\partial^3 w_0}{\partial x\partial y^2} + A_{S1_{66}}\frac{\partial^2\phi_x}{\partial y^2} + A_{S1_{66}}\frac{\partial^2\phi_y}{\partial x\partial y} + A_{S2_{66}}\frac{\partial^2\psi_x}{\partial y^2} + A_{S2_{66}}\frac{\partial^2\psi_y}{\partial x\partial y} = \left(I_1\right)\frac{\partial^2 u_0}{\partial t^2} - \left(I_2\right)\frac{\partial^3 w_0}{\partial x\partial t^2} + \left(I_4\right)\frac{\partial^2\phi_x}{\partial t^2} + \left(I_5\right)\frac{\partial^2\psi_x}{\partial t^2}
\end{aligned} \tag{9}$$

$$\begin{aligned}
\delta v_0 :\ & A_{12}\frac{\partial^2 u_0}{\partial x\partial y} - B_{12}\frac{\partial^3 w_0}{\partial x^2\partial y} + A_{S1_{12}}\frac{\partial^2\phi_x}{\partial x\partial y} + A_{S2_{12}}\frac{\partial^2\psi_x}{\partial x\partial y} + A_{22}\frac{\partial^2 v_0}{\partial y^2} - B_{22}\frac{\partial^3 w_0}{\partial y^3} + A_{S1_{22}}\frac{\partial^2\phi_y}{\partial y^2} + A_{S2_{22}}\frac{\partial^2\psi_y}{\partial y^2} + E_{23}\frac{\partial\phi_z}{\partial y} + F_{23}\frac{\partial\psi_z}{\partial y} + A_{66}\frac{\partial^2 u_0}{\partial x\partial y} \\
& + A_{66}\frac{\partial^2 v_0}{\partial x^2} - 2B_{66}\frac{\partial^3 w_0}{\partial x^2\partial y} + A_{S1_{66}}\frac{\partial^2\phi_x}{\partial x\partial y} + A_{S1_{66}}\frac{\partial^2\phi_y}{\partial x^2} + A_{S2_{66}}\frac{\partial^2\psi_x}{\partial x\partial y} + A_{S2_{66}}\frac{\partial^2\psi_y}{\partial x^2} = \left(I_1\right)\frac{\partial^2 v_0}{\partial t^2} - \left(I_2\right)\frac{\partial^3 w_0}{\partial y\partial t^2} + \left(I_4\right)\frac{\partial^2\phi_y}{\partial t^2} + \left(I_5\right)\frac{\partial^2\psi_y}{\partial t^2}
\end{aligned} \tag{10}$$

$$\begin{aligned}
\delta w_0 :\ & B_{11}\frac{\partial^3 u_0}{\partial x^3} - D_{11}\frac{\partial^4 w_0}{\partial x^4} + B_{S1_{11}}\frac{\partial^3\phi_x}{\partial x^3} + B_{S2_{11}}\frac{\partial^3\psi_x}{\partial x^3} + B_{12}\frac{\partial^3 v_0}{\partial x^2\partial y} - D_{12}\frac{\partial^3 w_0}{\partial x^2\partial y^2} + B_{S1_{12}}\left(\frac{\partial^3\phi_y}{\partial x^2\partial y} + \frac{\partial^3\psi_y}{\partial x^2\partial y}\right) \\
& + J_{13}\frac{\partial^2\phi_z}{\partial x^2} + O_{13}\frac{\partial^2\psi_z}{\partial x^2} + B_{12}\frac{\partial^3 u_0}{\partial x\partial y^2} - D_{12}\frac{\partial^3 w_0}{\partial x^2\partial y^2} + B_{S1_{12}}\left(\frac{\partial^3\phi_x}{\partial x\partial y^2} + \frac{\partial^3\psi_x}{\partial x\partial y^2}\right) + B_{22}\frac{\partial^3 v_0}{\partial y^3} - D_{22}\frac{\partial^4 w_0}{\partial y^4} \\
& + B_{S1_{22}}\frac{\partial^3\phi_y}{\partial y^3} + B_{S2_{22}}\frac{\partial^3\psi_y}{\partial y^3} + J_{23}\frac{\partial^2\phi_z}{\partial y^2} + O_{23}\frac{\partial^2\psi_z}{\partial y^2} + 2B_{66}\left(\frac{\partial^3 u_0}{\partial x\partial y^2} + \frac{\partial^3 v_0}{\partial x^2\partial y}\right) - 4B_{66}\frac{\partial^4 w_0}{\partial x^2\partial y^2} \\
& + 2B_{S1_{66}}\left(\frac{\partial^3\phi_x}{\partial x\partial y^2} + \frac{\partial^3\phi_y}{\partial x^2\partial y}\right) + 2B_{S2_{66}}\left(\frac{\partial^3\psi_x}{\partial x\partial y^2} + \frac{\partial^3\psi_y}{\partial x^2\partial y}\right) = -q + \left(I_2\right)\frac{\partial^3 u_0}{\partial x\partial t^2} - I_3\frac{\partial^4 w_0}{\partial x^2\partial t^2} \\
& + I_8\frac{\partial^3\phi_x}{\partial x\partial t^2} + I_9\frac{\partial^3\psi_x}{\partial x\partial t^2} + \left(I_2\right)\frac{\partial^3 v_0}{\partial y\partial t^2} - I_3\frac{\partial^4 w_0}{\partial y^2\partial t^2} + I_8\frac{\partial^3\phi_y}{\partial y\partial t^2} + I_9\frac{\partial^3\psi_y}{\partial y\partial t^2} + I_1\frac{\partial^2 w_0}{\partial t^2} + I_{10}\frac{\partial^2\phi_z}{\partial t^2} + I_{11}\frac{\partial^2\psi_z}{\partial t^2}
\end{aligned} \tag{11}$$

$$\begin{aligned}
\delta\phi_x :\ & A_{S1_{11}}\frac{\partial^2 u_0}{\partial x^2} - B_{S1_{11}}\frac{\partial^3 w_0}{\partial x^3} + A_{SS1_{11}}\frac{\partial^2\phi_x}{\partial x^2} + C_{11}\frac{\partial^2\psi_x}{\partial x^2} + A_{S1_{12}}\frac{\partial^2 v_0}{\partial x\partial y} - B_{S1_{12}}\frac{\partial^3 w_0}{\partial x\partial y^2} + A_{SS1_{12}}\frac{\partial^2\phi_y}{\partial x\partial y} + C_{12}\frac{\partial^2\psi_y}{\partial x\partial y} + L_{1_{13}}\frac{\partial\phi_z}{\partial x} + L_{2_{13}}\frac{\partial\psi_z}{\partial x} \\
& + A_{S1_{66}}\frac{\partial^2 u_0}{\partial y^2} + A_{S1_{66}}\frac{\partial^2 v_0}{\partial x\partial y} - 2B_{S1_{66}}\frac{\partial^3 w_0}{\partial x\partial y^2} + A_{SS1_{66}}\frac{\partial^2\phi_x}{\partial y^2} + A_{SS1_{66}}\frac{\partial^2\phi_y}{\partial x\partial y} + C_{66}\frac{\partial^2\psi_x}{\partial y^2} + C_{66}\frac{\partial^2\psi_y}{\partial x\partial y} - G_{44}\phi_x - I_{44}\psi_x - G_{44}\frac{\partial\phi_z}{\partial x} - I_{44}\frac{\partial\psi_z}{\partial x} \\
& = I_4\frac{\partial^2 u_0}{\partial t^2} - I_8\frac{\partial^3 w_0}{\partial x\partial t^2} + I_6\frac{\partial^2\phi_x}{\partial t^2} + I_{15}\frac{\partial^2\psi_x}{\partial t^2}
\end{aligned} \tag{12}$$

$$\begin{aligned}
\delta\psi_x :\ & A_{S2_{11}}\frac{\partial^2 u_0}{\partial x^2} - B_{S2_{11}}\frac{\partial^3 w_0}{\partial x^3} + C_{11}\frac{\partial^2\phi_x}{\partial x^2} + A_{SS2_{11}}\frac{\partial^2\psi_x}{\partial x^2} + A_{S2_{12}}\frac{\partial^2 v_0}{\partial x\partial y} - B_{S2_{12}}\frac{\partial^3 w_0}{\partial x\partial y^2} + C_{12}\frac{\partial^2\phi_y}{\partial x\partial y} + A_{SS2_{12}}\frac{\partial^2\psi_y}{\partial x\partial y} + M_{1_{13}}\frac{\partial\phi_z}{\partial x} + M_{2_{13}}\frac{\partial\psi_z}{\partial x} \\
& + A_{S2_{66}}\frac{\partial^2 u_0}{\partial y^2} + A_{S2_{66}}\frac{\partial^2 v_0}{\partial x\partial y} - 2B_{S2_{66}}\frac{\partial^3 w_0}{\partial x\partial y^2} + C_{66}\frac{\partial^2\phi_x}{\partial y^2} + C_{66}\frac{\partial^2\phi_y}{\partial x\partial y} + A_{SS2_{66}}\frac{\partial^2\psi_x}{\partial y^2} + A_{SS2_{66}}\frac{\partial^2\psi_y}{\partial x\partial y} - I_{44}\phi_x - H_{44}\psi_x - I_{44}\frac{\partial\phi_z}{\partial x} - H_{44}\frac{\partial\psi_z}{\partial x} \\
& = I_5\frac{\partial^2 u_0}{\partial t^2} - I_9\frac{\partial^3 w_0}{\partial x\partial t^2} + I_{15}\frac{\partial^2\phi_x}{\partial t^2} + I_7\frac{\partial^2\psi_x}{\partial t^2}
\end{aligned} \tag{13}$$

$$
\begin{aligned}
&\delta\phi_y : A_{S1_{12}}\frac{\partial^2 u_0}{\partial x\partial y}+\frac{A_{S1_{12}}}{R_1}\frac{\partial w_0}{\partial y}-B_{S1_{12}}\frac{\partial^3 w_0}{\partial x^2\partial y}+A_{SS1_{12}}\frac{\partial^2\phi_x}{\partial x\partial y}+C_{12}\frac{\partial^2\psi_x}{\partial x\partial y}+A_{S1_{22}}\frac{\partial^2 v_0}{\partial y^2}+\frac{A_{S1_{22}}}{R_2}\frac{\partial w_0}{\partial y}-B_{S1_{22}}\frac{\partial^3 w_0}{\partial y^3}+A_{SS1_{22}}\frac{\partial^2\phi_y}{\partial y^2}\\
&+C_{22}\frac{\partial^2\psi_y}{\partial y^2}+L_{1_{23}}\frac{\partial\phi_z}{\partial y}+L_{2_{23}}\frac{\partial\psi_z}{\partial y}+A_{S1_{66}}\frac{\partial^2 u_0}{\partial x\partial y}+A_{S1_{66}}\frac{\partial^2 v_0}{\partial x^2}-2B_{S1_{66}}\frac{\partial^3 w_0}{\partial x^2\partial y}+A_{SS1_{66}}\frac{\partial^2\phi_x}{\partial x\partial y}+A_{SS1_{66}}\frac{\partial^2\phi_y}{\partial x^2}+C_{66}\frac{\partial^2\psi_x}{\partial x\partial y}+C_{66}\frac{\partial^2\psi_y}{\partial x^2}\\
&-G_{55}\phi_y-I_{55}\psi_y-G_{55}\frac{\partial\phi_z}{\partial y}-I_{55}\frac{\partial\psi_z}{\partial y}+\left(\frac{Q_{512}}{R_1}+\frac{Q_{522}}{R_2}\right)\phi_z+\left(\frac{Q_{612}}{R_1}+\frac{Q_{622}}{R_2}\right)\psi_z=\left(I_4+\frac{I_8}{R_2}\right)\frac{\partial^2 v_0}{\partial t^2}-I_8\frac{\partial^3 w_0}{\partial y\partial t^2}+I_6\frac{\partial^2\phi_y}{\partial t^2}+I_{15}\frac{\partial^2\psi_y}{\partial t^2}
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
&\delta\psi_y : A_{S2_{12}}\frac{\partial^2 u_0}{\partial x\partial y}-B_{S2_{12}}\frac{\partial^3 w_0}{\partial x^2\partial y}+C_{12}\frac{\partial^2\phi_x}{\partial x\partial y}+A_{SS2_{12}}\frac{\partial^2\psi_x}{\partial x\partial y}+A_{S2_{22}}\frac{\partial^2 v_0}{\partial y^2}-B_{S2_{22}}\frac{\partial^3 w_0}{\partial y^3}+C_{22}\frac{\partial^2\phi_y}{\partial y^2}+A_{SS2_{22}}\frac{\partial^2\psi_y}{\partial y^2}+M_{1_{23}}\frac{\partial\phi_z}{\partial y}\\
&+M_{2_{23}}\frac{\partial\psi_z}{\partial y}+A_{S2_{66}}\frac{\partial^2 u_0}{\partial x\partial y}+A_{S2_{66}}\frac{\partial^2 v_0}{\partial x^2}-2B_{S2_{66}}\frac{\partial^3 w_0}{\partial x^2\partial y}+C_{66}\frac{\partial^2\phi_x}{\partial x\partial y}+C_{66}\frac{\partial^2\phi_y}{\partial x^2}+A_{SS2_{66}}\frac{\partial^2\psi_x}{\partial x\partial y}+A_{SS2_{66}}\frac{\partial^2\psi_y}{\partial x^2}-I_{55}\phi_y-H_{55}\psi_y-I_{55}\frac{\partial\phi_z}{\partial y}\\
&-H_{55}\frac{\partial\psi_z}{\partial y}=I_5\frac{\partial^2 v_0}{\partial t^2}-I_9\frac{\partial^3 w_0}{\partial y\partial t^2}+I_{15}\frac{\partial^2\phi_y}{\partial t^2}+I_7\frac{\partial^2\psi_y}{\partial t^2}
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
&\delta\phi_z : G_{44}\frac{\partial\phi_x}{\partial x}+I_{44}\frac{\partial\psi_x}{\partial x}+G_{44}\frac{\partial^2\phi_z}{\partial x^2}+I_{44}\frac{\partial^2\psi_z}{\partial x^2}+G_{55}\frac{\partial\phi_y}{\partial y}+I_{55}\frac{\partial\psi_y}{\partial y}+G_{55}\frac{\partial^2\phi_z}{\partial y^2}+I_{55}\frac{\partial^2\psi_z}{\partial y^2}-E_{13}\frac{\partial u_0}{\partial x}+J_{13}\frac{\partial^2 w_0}{\partial x^2}-L_{1_{13}}\frac{\partial\phi_x}{\partial x}\\
&-M_{1_{13}}\frac{\partial\psi_x}{\partial x}-E_{23}\frac{\partial v_0}{\partial y}+J_{23}\frac{\partial^2 w_0}{\partial y^2}-L_{1_{23}}\frac{\partial\phi_y}{\partial y}-M_{1_{23}}\frac{\partial\psi_y}{\partial y}-N_{1_{33}}\phi_z-N_{3_{33}}\psi_z=I_{10}\frac{\partial^2 w_0}{\partial t^2}+I_{13}\frac{\partial^2\phi_z}{\partial t^2}+I_{12}\frac{\partial^2\psi_z}{\partial t^2}
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
&\delta\psi_z : I_{55}\frac{\partial\phi_x}{\partial x}+H_{55}\frac{\partial\psi_x}{\partial x}+I_{55}\frac{\partial^2\phi_z}{\partial x^2}+H_{55}\frac{\partial^2\psi_z}{\partial x^2}+I_{44}\frac{\partial\phi_y}{\partial y}+H_{44}\frac{\partial\psi_y}{\partial y}+I_{44}\frac{\partial^2\phi_z}{\partial y^2}+H_{44}\frac{\partial^2\psi_z}{\partial y^2}-F_{13}\frac{\partial u_0}{\partial x}+O_{13}\frac{\partial^2 w_0}{\partial x^2}\\
&-L_{2_{13}}\frac{\partial\phi_x}{\partial x}-M_{2_{13}}\frac{\partial\psi_x}{\partial x}-F_{23}\frac{\partial v_0}{\partial y}+O_{23}\frac{\partial^2 w_0}{\partial y^2}-L_{2_{23}}\frac{\partial\phi_y}{\partial y}-M_{2_{23}}\frac{\partial\psi_y}{\partial y}-N_{3_{33}}\phi_z-N_{2_{33}}\psi_z=I_{11}\frac{\partial^2 w_0}{\partial t^2}+I_{12}\frac{\partial^2\phi_z}{\partial t^2}+I_{14}\frac{\partial^2\psi_z}{\partial t^2}
\end{aligned}
\tag{17}
$$

where, vibration and mechanical integration constants are expressed as,

$$
\begin{aligned}
&\left(I_1,I_2,I_3,I_4,I_5\right)=\int_{-h/2}^{h/2}\left[1,z,z^2,\varphi_1(z),\varphi_2(z)\right]dz\\
&\left(I_6,I_8,I_{15}\right)=\int_{-h/2}^{h/2}\varphi_1(z)\left[\varphi_1(z),z,\varphi_2(z)\right]dz\\
&\left(I_7,I_9\right)=\int_{-h/2}^{h/2}\varphi_2(z)\left[\varphi_2(z),z\right]dz\\
&\left(I_{10},I_{12},I_{13}\right)=\int_{-h/2}^{h/2}\varphi_1^{'}(z)\left[1,\varphi_2^{'}(z),\varphi_1^{'}(z)\right]dz\\
&\left(I_{11},I_{14}\right)=\int_{-h/2}^{h/2}\varphi_2^{'}(z)\left[1,\varphi_2^{'}(z)\right]dz
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
&\left(A_{ij},B_{ij},D_{ij},A_{S1_{ij}}\right)=Q_{ij}\int_{-h/2}^{h/2}\left[1.0,\ z,z^2,\ f_1(z)\right]dz\\
&\left(A_{S2_{ij}},B_{S1_{ij}},B_{S2_{ij}}\right)=Q_{ij}\int_{-h/2}^{h/2}\left[f_2(z),zf_1(z),\ zf_2(z)\right]dz\\
&\left(A_{SS1_{ij}},A_{SS2_{ij}},C_{ij}\right)=Q_{ij}\int_{-h/2}^{h/2}\left\{\left[f_1(z)\right]^2,\left[f_2(z)\right]^2,\left[f_1(z)\,f_2(z)\right]\right\}dz\\
&\left(G_{ij},H_{ij},I_{ij}\right)=Q_{ij}\int_{-h/2}^{h/2}\left\{\left[f_1^{'}(z)\right]^2,\left[f_2^{'}(z)\right]^2,\left[f_1^{'}(z)\,f_2^{'}(z)\right]\right\}dz\\
&\left(E_{ij},F_{ij}\right)=Q_{ij}\int_{-h/2}^{h/2}\left\{\left[f_1^{''}(z)\right],\left[f_2^{''}(z)\right]\right\}dz\\
&\left(N_{1_{ij}},\ N_{2_{ij}},\ N_{3_{ij}}\right)=Q_{ij}\int_{-h/2}^{h/2}\left\{\left[f_1^{''}(z)\right]^2,\ \left[f_2^{''}(z)\right]^2,\ \left[f_1^{''}(z)\,f_2^{'}(z)\right]\right\}dz\\
&\left(J_{ij},\ L_{1_{ij}},\ L_{2_{ij}}\right)=Q_{ij}\int_{-h/2}^{h/2}f_1^{''}(z)\left[z,\ f_1(z),\ f_2(z)\right]dz\\
&\left(O_{ij},\ M_{1_{ij}},\ M_{2_{ij}}\right)=Q_{ij}\int_{-h/2}^{h/2}f_2^{''}(z)\left[z,\ f_1(z),\ f_2(z)\right]dz
\end{aligned}
\tag{19}
$$

The boundary conditions satisfying the top and bottom conditions associated with the present theory are expressed as, Along the edges x =0 and x =a,

$$
\begin{aligned}
&\text{either} \quad u_0 = 0 \quad \text{or} \quad N_x \quad \text{is prescribed} \\
&\text{either} \quad v_0 = 0 \quad \text{or} \quad N_{xy} \quad \text{is prescribed} \\
&\text{either} \quad w_0 = 0 \quad \text{or} \quad \frac{\partial M_x^b}{\partial x} + \frac{2\partial M_{xy}^b}{\partial y} \quad \text{is prescribed} \\
&\text{either} \quad \frac{\partial w_0}{\partial x} = 0 \quad \text{or} \quad M_{xy}^b \quad \text{is prescribed} \\
&\text{either} \quad \phi_x = 0 \quad \text{or} \quad M_x^{s_1} \quad \text{is prescribed} \\
&\text{either} \quad \psi_x = 0 \quad \text{or} \quad M_x^{s_2} \quad \text{is prescribed} \\
&\text{either} \quad \phi_y = 0 \quad \text{or} \quad M_{xy}^{s_1} \quad \text{is prescribed} \\
&\text{either} \quad \psi_y = 0 \quad \text{or} \quad M_{xy}^{s_2} \quad \text{is prescribed} \\
&\text{either} \quad \phi_z = 0 \quad \text{or} \quad Q_{xz}^{S1} \quad \text{is prescribed} \\
&\text{either} \quad \psi_z = 0 \quad \text{or} \quad Q_{xz}^{S2} \quad \text{is prescribed}
\end{aligned} \tag{20}
$$

Along the edges y =0 and y =b,

$$
\begin{aligned}
&\text{either} \quad u_o = 0 \quad \text{or} \quad N_{xy} \quad \text{is prescribed} \\
&\text{either} \quad v_0 = 0 \quad \text{or} \quad N_y \quad \text{is prescribed} \\
&\text{either} \quad w_0 = 0 \quad \text{or} \quad \frac{\partial M_x^b}{\partial y} + \frac{2\partial M_{xy}^b}{\partial x} \quad \text{is prescribed} \\
&\text{either} \quad \frac{\partial w_0}{\partial y} = 0 \quad \text{or} \quad M_{xy}^b \quad \text{is prescribed} \\
&\text{either} \quad \phi_x = 0 \quad \text{or} \quad M_{xy}^{s_1} \quad \text{is prescribed} \\
&\text{either} \quad \psi_x = 0 \quad \text{or} \quad M_{xy}^{s_2} \quad \text{is prescribed} \\
&\text{either} \quad \phi_y = 0 \quad \text{or} \quad M_y^{s_1} \quad \text{is prescribed} \\
&\text{either} \quad \psi_y = 0 \quad \text{or} \quad M_y^{s_2} \quad \text{is prescribed} \\
&\text{either} \quad \phi_z = 0 \quad \text{or} \quad Q_{yz}^{S1} \quad \text{is prescribed} \\
&\text{either} \quad \psi_z = 0 \quad \text{or} \quad Q_{yz}^{S2} \quad \text{is prescribed}
\end{aligned} \tag{21}
$$

Navier's Closed Form Solution

The double trigonometric form,the Navier solution technique is employed to solve the nine equations of motion for the simply supported FG sandwich plate.

$$
\begin{aligned}
\begin{Bmatrix} u_0 \\ \phi_x \\ \psi_x \end{Bmatrix} &= \sum_{m,n=1,3,5..}^{\infty} \begin{Bmatrix} u_{mn} \\ \phi_{xmn} \\ \psi_{xmn} \end{Bmatrix} \cos\alpha x \sin\beta y \, e^{i\omega_{mn}t} \\
\begin{Bmatrix} v_0 \\ \phi_y \\ \psi_y \end{Bmatrix} &= \sum_{m,n=1,3,5..}^{\infty} \begin{Bmatrix} v_{mn} \\ \phi_{ymn} \\ \psi_{ymn} \end{Bmatrix} \sin\alpha x \cos\beta y \, e^{i\omega_{mn}t} \\
\begin{Bmatrix} w_0 \\ \phi_z \\ \psi_z \end{Bmatrix} &= \sum_{m,n=1,3,5..}^{\infty} \begin{Bmatrix} w_{mn} \\ \phi_{zmn} \\ \psi_{zmn} \end{Bmatrix} \sin\alpha x \sin\beta y \, e^{i\omega_{mn}t}
\end{aligned} \tag{22}
$$

where, $\{u_{mn}, \phi_{xmn}, \psi_{xmn}\}, \{v_{mn}, \phi_{ymn}, \psi_{ymn},\}, \{w_{mn}, \phi_{zmn}, \psi_{zmn}\}$ are the unknown coefficients of displacements and shear slopes. $\alpha = m\pi/a,\ \beta = n\pi/b$; $i = \sqrt{-1}$; ωis the natural frequency. The expression for the transverse load is also expressed in double trigonometric form as,

$$
q(x, y) = \sum_{m,n=1,3,5..}^{\infty} q_{mn} \sin\alpha x \sin\beta y \tag{23}
$$

where, q_{mn} is the unknown coefficient of the transverse load, taken as $q_{mn} = q_0 \;\; (m = n{=}1)$ for sinusoidal load and $q_{mn} = \dfrac{16\, q_0}{mn\pi^2}\,(m,\; n{=}1,\,3,\,5......)$ for uniformly distributed load.

Therefore, substituting Eq. (22-23) into the Eq. (9-17), the resultant equations can be expressed in matrix form. The transverse load is taken as zero for free vibration analysis and the time dependent terms are discarded for static analysis.

For static analysis the resultant equation is expressed as,

$$[K]\{\Delta\} = \{f\} \tag{24}$$

whereas for free vibration analysis the resultant equation expressed as,

$$\{[K] - \omega^2[M]\}\{\Delta\} = \{0\} \tag{25}$$

where, $[K]$ represents the stiffness matrix, $\{f\}$ represents the force vector and $\{\Delta\}$ represents the vector of unknowns. Appendix shows the elements of stiffness matrix, force vector and vector of unknowns.

Numerical Results

The static and free vibration analysis of single layer and sandwich functionally graded plate are presented in the present study. The present results are compared with results available in the literature to validate the accuracy and efficacy of the present theory. For the comparison purpose the numerical results are presented in the following non-dimensional form.

$$\begin{gathered}\bar{w}\left(\frac{a}{2},\frac{b}{2},0\right) = \frac{10h^3E_c}{q_0a^4}w,\;\; \hat{w}\left(\frac{a}{2},\frac{b}{2},0\right) = \frac{1000h^3E_0}{q_0a^4}w, \\ \bar{\sigma}_x\left(\frac{a}{2},\frac{b}{2},-\frac{h}{2}\right) = \frac{h}{q_0a}\sigma_x,\;\; \bar{\tau}_{xz}\left(0,\frac{b}{2},\frac{z}{h}\right) = \frac{h}{q_0a}\tau_{xz},\quad \hat{\omega} = \omega_{mn}h\sqrt{\frac{\rho_c}{E_c}}\end{gathered} \tag{26}$$

where, ρ_0 =1.0 and E_0 =1.0

Table 1. The material properties of the functionally graded material used are given as below,

Material	Properties	Metal : Aluminum(Al)	Ceramic: Alumina(Al_2O_3)
1	Modulus of Elasticity (E)	*Em*=70 GPa	*Ec*=380 GPa
	Poisson Ratio (μ)	μ=0.3	μ=0.3
	Density (ρ)	ρ_m= 2707 kg/m^3	ρ_c= 3800 kg/m^3
2	Modulus of Elasticity (E)	*Em*=70 GPa	*Ec*=151 GPa
	Poisson Ratio (μ)	μ=0.3	μ=0.3
	Density (ρ)	ρ_m= 2707 kg/m^3	ρ_c= 3800 kg/m^3

Table 2. Non-dimensional transverse displacement and stresses in single layer FG plate at various power law index (*a/h*=10)

(Material 1)

p	**Theory**	**SSL**			**UDL**		
		$\bar{w}$	$\bar{\sigma}_x$ **(*h*/3)**	$\bar{\tau}_{xz}$ **(*h*/6)**	$\bar{w}$	$\bar{\sigma}_x$ **(*h*/2)**	$\bar{\tau}_{xz}$ **(0)**
1	Present	0.5695	1.4588	0.2607	0.8985	4.4385	0.5369
	Thai et al. [5]	0.5875	1.5062	0.2510	--	--	--
	Demirhan and Taskin [10]	0.5889	1.4894	0.2622	0.9287	4.4745	0.5446
	Demirhan and Taskin [10]	--	--	--	0.9288	4.0131	0.5454
	Thai et al. [5]	0.5890	1.4898	0.2599	--	--	--
	Thai et al. [5]	0.5890	1.4898	0.2608	--	--	--
2	Present	0.7225	1.3688	0.2763	1.1393	5.1083	0.5682
	Thai et al. [5]	0.7570	1.4147	0.2496	--	--	--
	Demirhan and Taskin [10]	0.7573	1.3954	0.2763	1.1940	5.2296	0.5734
	Demirhan and Taskin [10]	--	--	--	1.1940	5.1376	0.5725
	Thai et al. [5]	0.7573	1.3960	0.2721	--	--	--
	Thai et al. [5]	0.7573	1.3960	0.2737	--	--	--
4	Present	0.8429	1.1456	0.2630	1.3275	5.7773	0.5395
	Thai et al. [5]	0.8823	1.1985	0.2362	--	--	--
	Demirhan and Taskin [10]	0.8819	1.1783	0.2580	1.3890	5.8915	0.5346
	Demirhan and Taskin [10]	--	--	--	1.3884	5.5911	0.5307
	Thai et al. [5]	0.8815	1.1794	0.2519	--	--	--
	Thai et al. [5]	0.8815	1.1794	0.2537	--	--	--
8	Present	0.9446	0.9088	0.2145	1.4868	6.8889	0.4402
	Thai et al. [5]	0.9738	0.9687	0.2262	--	--	--
	Demirhan and Taskin [10]	0.9750	0.9466	0.2121	1.5343	6.8999	0.4392
	Demirhan and Taskin [10]	--	--	--	1.5337	6.4234	0.4367
	Thai et al. [5]	0.9747	0.9477	0.2087	--	--	--
	Thai et al. [5]	0.9746	0.9477	0.2088	--	--	--

Table 3. Non-dimensional transverse displacement and stresses in FG sandwich plate subjected to sinusoidal load at various power law index (*a/h*=10) (Material 2)

	p	Theory	Scheme				
			1-0-1	2-1-2	1-1-1	2-2-1	1-2-1
$\hat{w}$	0	Present	0.1948	0.1948	0.1950	0.1948	0.1948
		Zenkour [12]	0.1961	0.1961	0.1961	0.1961	0.1961
		Thai and Kim [6]	0.1961	0.1961	0.1961	0.1961	0.1961
	1	Present	0.3215	0.3043	0.2900	0.2787	0.2615
		Zenkour [12]	0.3235	0.3062	0.2919	0.2808	0.2709
		Thai and Kim [6]	0.3237	0.3064	0.2920	0.2809	0.2710
	2	Present	0.3712	0.3502	0.3308	0.3135	0.3006
		Zenkour [12]	0.3732	0.3522	0.3328	0.3161	0.3026
		Thai and Kim [6]	0.3737	0.3526	0.3330	0.3163	0.3027
	5	Present	0.4072	0.3900	0.3694	0.3466	0.3326
		Zenkour [12]	0.4091	0.3916	0.3713	0.3495	0.3347
		Thai and Kim [6]	0.4101	0.3927	0.3720	0.3501	0.3350
	10	Present	0.4154	0.4025	0.3837	0.3590	0.3460
		Zenkour [12]	0.4175	0.4037	0.3849	0.3492	0.3412
		Thai and Kim [6]	0.3988	0.3894	0.3724	0.3492	0.3361
$\bar{\sigma}_x$	0	Present	1.9963	1.9963	1.9963	1.9963	1.9963

		Zenkour [12]	2.0545	2.0545	2.0545	2.0545	2.0545
		Thai and Kim [6]	1.9758	1.9758	1.9758	1.9758	1.9758
	1	Present	1.5490	1.4683	1.4002	1.2891	1.2987
		Zenkour [12]	1.5820	1.4986	1.4289	1.3234	1.3259
		Thai and Kim [6]	1.5324	1.4517	1.3830	1.2775	1.2810
	2	Present	1.7865	1.6911	1.6000	1.4336	1.4555
		Zenkour [12]	1.8245	1.7241	1.6303	1.4739	1.4828
		Thai and Kim [6]	1.7709	1.6750	1.5824	1.4253	1.4358
	5	Present	1.9501	1.8780	1.7849	1.5642	1.6124
		Zenkour [12]	1.9957	1.9155	1.8184	1.6148	1.6411
		Thai and Kim [6]	1.9358	1.8648	1.7699	1.5640	1.5931
	10	Present	1.9832	1.9337	1.8510	1.6117	1.6768
		Zenkour [12]	2.0336	1.9731	1.8815	1.6198	1.6485
		Thai and Kim [6]	1.9678	1.9216	1.8375	1.6160	1.6587
$\bar{\tau}_{xz}$	0	Present	0.2383	0.2383	0.2383	0.2383	0.2383
		Zenkour [12]	0.2462	0.2462	0.2462	0.2462	0.2462
		Thai and Kim [6]	0.2387	0.2387	0.2387	0.2387	0.2387
	1	Present	0.2722	0.2551	0.2517	0.2547	0.2528
		Zenkour [12]	0.2991	0.2777	0.2681	0.2668	0.2600
		Thai and Kim [6]	0.2566	0.2593	0.2602	0.2582	0.2593
	2	Present	0.2894	0.2563	0.2502	0.2572	0.2539
		Zenkour [12]	0.3329	0.2942	0.2781	0.2763	0.2654
		Thai and Kim [6]	0.2552	0.2617	0.2650	0.2624	0.2655
	5	Present	0.3369	0.2613	0.2448	0.2583	0.2494
		Zenkour [12]	0.3937	0.3193	0.2915	0.2890	0.2715
		Thai and Kim [6]	0.2468	0.2576	0.2649	0.2627	0.2694
	10	Present	0.3892	0.2723	0.2445	0.2606	0.2454
		Zenkour [12]	0.4415	0.3364	0.2953	0.2967	0.2768
		Thai and Kim [6]	0.2419	0.2534	0.2627	0.2611	0.2698

Table 4 Non-dimensional transverse displacement and stresses in FG sandwich plate subjected to uniformly distributed load at various power law. (*a*/*h*=10) (Material 1)

p	**Theory**	**Scheme**				
		1-0-1	**2-1-2**	**1-1-1**	**2-2-1**	**1-2-1**
		$\bar{w}$				
0	Present	0.4639	0.4639	0.4639	0.4639	0.4639
	Demirhan and Taskin [10]	0.4666	0.4666	0.4666	0.4666	0.4666
1	Present	1.1694	1.0344	0.9341	0.8662	0.8043
	Demirhan and Taskin [10]	1.1765	1.0409	0.9402	0.8745	0.8093
2	Present	1.7018	1.4524	1.2565	1.1222	1.0086
	Demirhan and Taskin [10]	1.7100	1.4606	1.2644	1.1372	1.0153
5	Present	2.2844	1.9950	1.6909	1.4529	1.2757
	Demirhan and Taskin [10]	2.2877	1.9996	1.6999	1.4792	1.2845
10	Present	2.4399	2.2160	1.8981	1.6090	1.4116
	Demirhan and Taskin [10]	2.4438	2.2154	1.9045	1.6408	1.4296
		$\bar{\sigma}_x$				
0	Present	2.8960	2.8960	2.8960	2.8960	2.8960
1	Present	1.3787	1.2212	1.1030	0.9522	0.9482
2	Present	2.0080	1.7210	1.4924	1.2064	1.1986
5	Present	2.6741	2.3534	2.0063	1.5070	1.5216
10	Present	2.8406	2.6036	2.2456	1.6350	1.6832
		$\bar{\tau}_{xz}$				
0	Present	0.4867	0.4867	0.4867	0.4867	0.4867
1	Present	0.6209	0.5614	0.5495	0.5561	0.5495
2	Present	0.6843	0.5714	0.5591	0.5780	0.5710
5	Present	0.8704	0.5560	0.5271	0.5848	0.5740
10	Present	1.1889	0.5810	0.5024	0.5853	0.5609

Table 5 .Non-dimensional natural frequencies in single layer FG plate at various power law. (Material 1)

a/h	**Mode**	**Theory**	***p***				
			0	**0.5**	**1**	**4**	**10**
5	1	Present	0.2121	0.1824	0.1658	0.1407	0.1316
		Thai and Kim [6]	0.2113	0.1807	0.1631	0.1378	0.1301
		Li et al. [7]	0.2112	0.1805	0.1631	0.1397	0.1324
		Thai and Kim [6]	0.2113	0.1807	0.1631	0.1378	0.1301
	2	Present	0.4658	0.4039	0.3674	0.3038	0.2807
		Thai and Kim [6]	0.4623	0.3989	0.3607	0.2980	0.2771
		Li et al. [7]	0.4618	0.3978	0.3604	0.3049	0.2856
		Thai and Kim [6]	0.4623	0.3989	0.3607	0.2980	0.2771
	3	Present	0.6752	0.6556	0.5357	0.4365	0.4000
		Thai and Kim [6]	0.6688	0.5803	0.5254	0.4284	0.3948
		Li et al. [7]	0.6676	0.5779	0.5245	0.4405	0.4097
		Thai and Kim [6]	0.6688	0.5803	0.5254	0.4284	0.3948
10	1	Present	0.0577	0.0493	0.0448	0.0389	0.0368
		Thai and Kim [6]	0.0577	0.0490	0.0442	0.0381	0.0364
		Li et al. [7]	0.0577	0.0490	0.0442	0.0382	0.0366
		Thai and Kim [6]	0.0577	0.0490	0.0442	0.0381	0.0364
	2	Present	0.1380	0.1184	0.1076	0.0921	0.0867
		Thai and Kim [6]	0.1377	0.1174	0.1059	0.0903	0.0856
		Li et al. [7]	0.1376	0.1173	0.1059	0.0911	0.0867
		Thai and Kim [6]	0.1377	0.1174	0.1059	0.0903	0.0856
	3	Present	0.2121	0.1825	0.1659	0.1407	0.1317
		Thai and Kim [6]	0.2113	0.1807	0.1631	0.1378	0.1301
		Li et al. [7]	0.2112	0.1805	0.1631	0.1397	0.1324
		Thai and Kim [6]	0.2113	0.1807	0.1631	0.1378	0.1301

Table 6. Non-dimensional natural frequencies in FG sandwich plate at various power law (Material 1)

p	**Theory**	**Scheme**									
		1-0-1		**2-1-2**		**1-1-1**		**2-2-1**		**1-2-1**	
	a/h	5	10	5	10	5	10	5	10	5	10
0	Present	1.6771	1.8268	1.6771	1.8268	1.6771	1.8268	1.6771	1.8268	1.6771	1.8268
	Li et al.[7]	1.6771	1.8268	1.6771	1.8268	1.6771	1.8268	1.6771	1.8268	1.6771	1.8268
	Thai et al.[5]	1.6974	1.8244	1.6697	1.8244	1.6697	1.8244	1.6697	1.8244	1.6697	1.8244
0.5	Present	1.3536	1.4461	1.3905	1.4860	1.4217	1.5213	1.4461	1.5501	1.4694	1.5766
	Li et al.[7]	1.3536	1.4461	1.3905	1.4861	1.4218	1.5213	1.4454	1.5493	1.4694	1.5767
	Thai et al.[5]	1.3473	1.4442	1.3841	1.4841	1.4152	1.5192	1.4386	1.5471	1.4626	1.5745
1	Present	1.1748	1.2447	1.2291	1.3018	1.2777	1.3553	1.3162	1.3998	1.3534	1.4413
	Li et al.[7]	1.1749	1.2447	1.2292	1.3018	1.2777	1.3533	1.3143	1.3956	1.3524	1.4394
	Thai et al.[5]	1.1691	1.2429	1.2232	1.3000	1.2414	1.3533	1.3078	1.3956	1.3467	1.4393
5	Present	0.8913	0.9449	0.9337	0.9810	0.9980	1.0453	1.0635	1.1169	1.1193	1.1757
	Li et al.[7]	0.8909	0.9448	0.9336	0.9810	0.9980	1.0453	1.0561	1.1088	1.1190	1.1757
	Thai et al.[5]	0.8853	0.9431	0.9286	0.9796	0.9916	1.0435	1.0488	1.1077	1.1056	1.1735
10	Present	0.8690	0.9275	0.8928	0.9409	0.9498	0.9952	1.0194	1.0695	1.0733	1.1247
	Li et al.[7]	0.8683	0.9273	0.8923	0.9408	0.9498	0.9952	1.0095	1.0610	1.0729	1.1247
	Thai et al.[5]	0.8599	0.9246	0.8860	0.9390	0.9428	0.9932	1.0012	1.0587	1.0648	1.1223

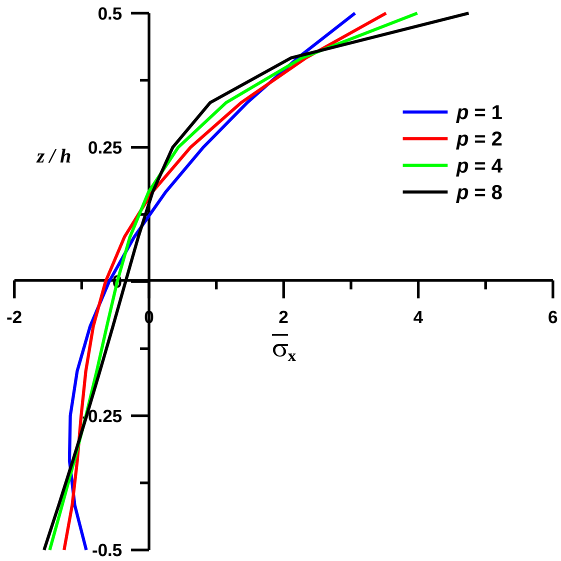

Figure 2. Through thickness variation of in-plane stress in single layer FG plate subjected to sinusoidal load.

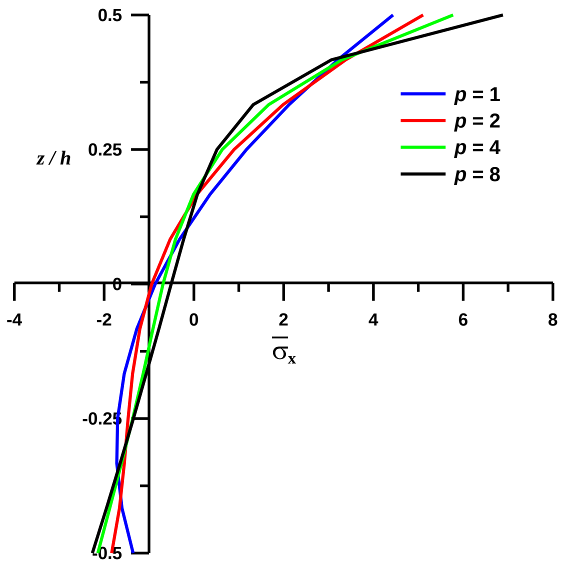

Figure 3. Through thickness variation of in-plane stress in single layer FG plate subjected to uniformly distributed load.

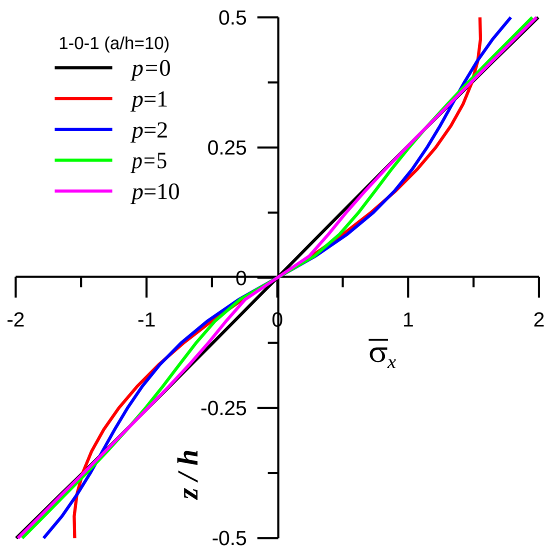

Figure 4. Through thickness variation of in-plane stress in 1-0-1 FG sandwich plate subjected to sinusoidal load

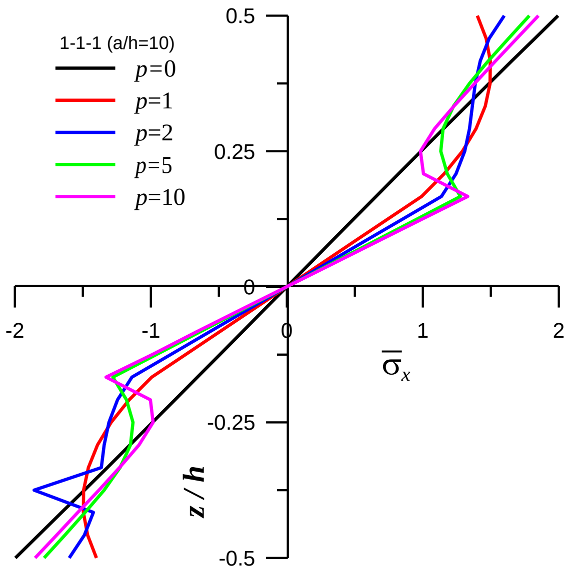

Figure 5. Through thickness variation of in-plane stress in 1-1-1 FG sandwich plate subjected to sinusoidal load

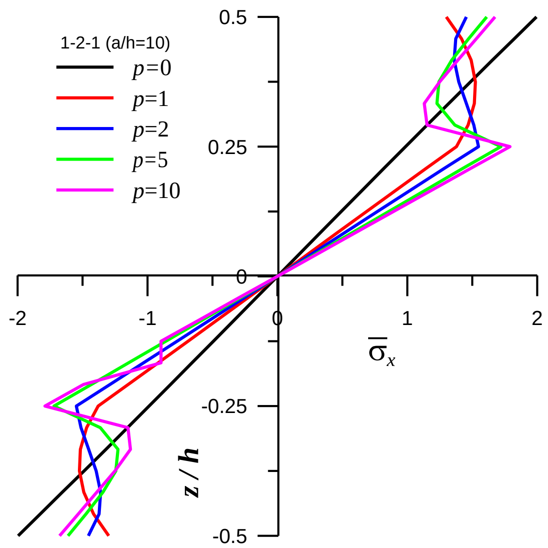

Figure 6. Through thickness variation of in-plane stress in 1-2-1 FG sandwich plate subjected to sinusoidal load

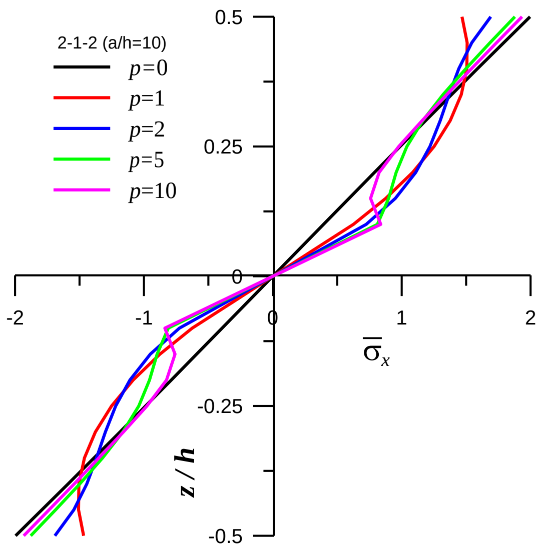

Figure 7. Through thickness variation of in-plane stress in 2-1-2 FG sandwich plate subjected to sinusoidal load

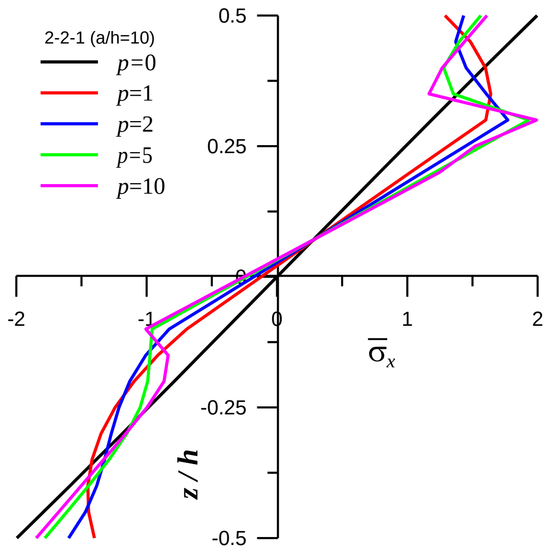

Figure 8. Through thickness variation of in-plane stress in 2-2-1 FG sandwich plate subjected to sinusoidal load

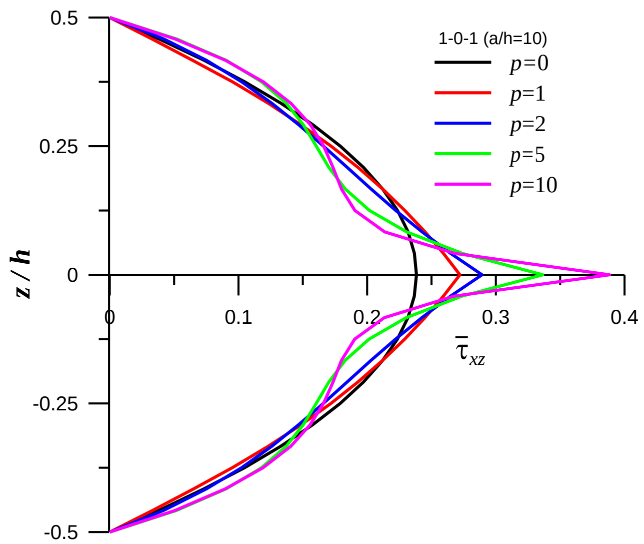

Figure 9. Through thickness variation of transverse shear stress in 1-0-1 FG sandwich plate subjected to sinusoidal load

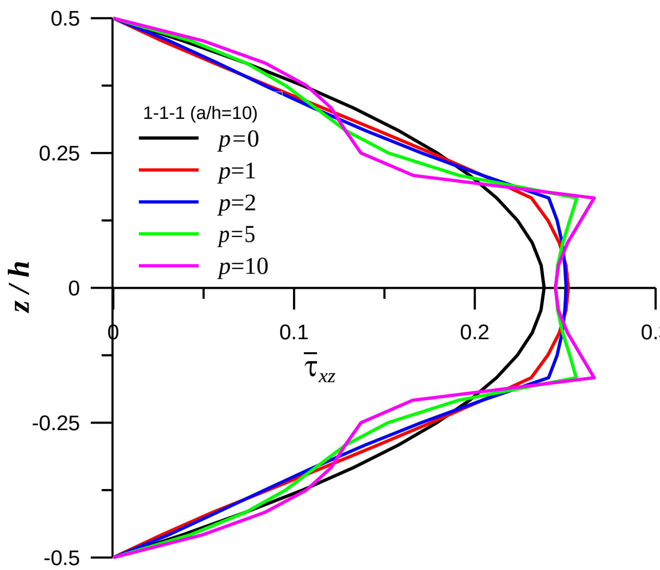

Figure 10. Through thickness variation of transverse shear stress in 1-1-1 FG sandwich plate subjected to sinusoidal load

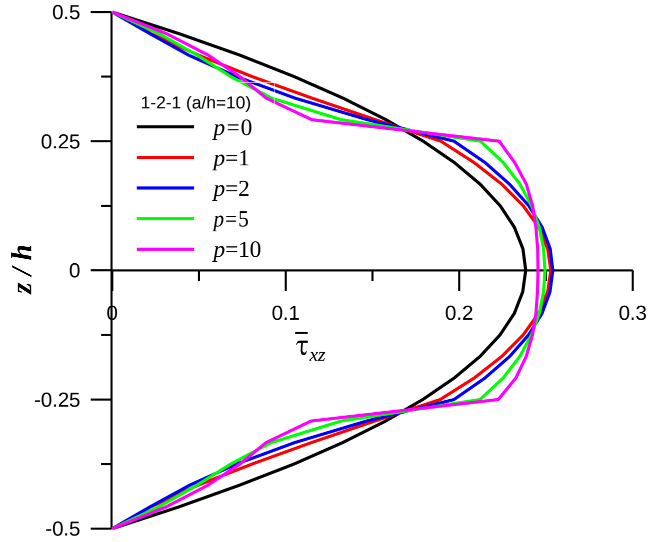

Figure 11. Through thickness variation of transverse shear stress in 1-2-1 FG sandwich plate subjected to sinusoidal load

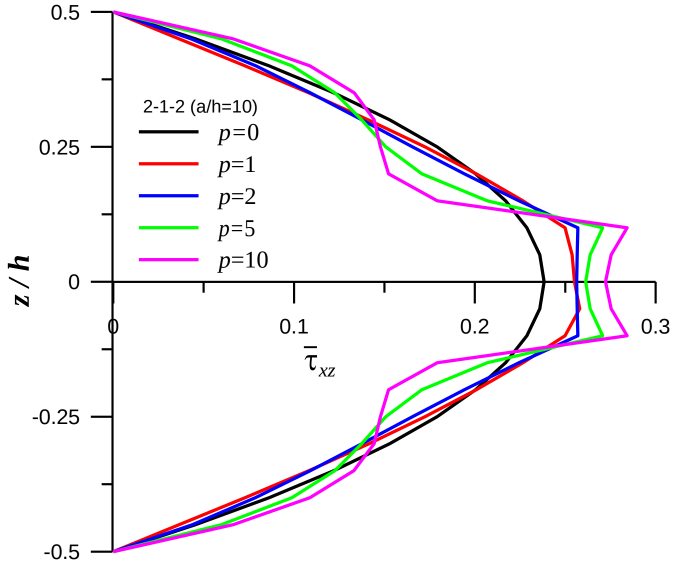

Figure 12. Through thickness variation of transverse shear stress in 2-1-2 FG sandwich plate subjected to sinusoidal load

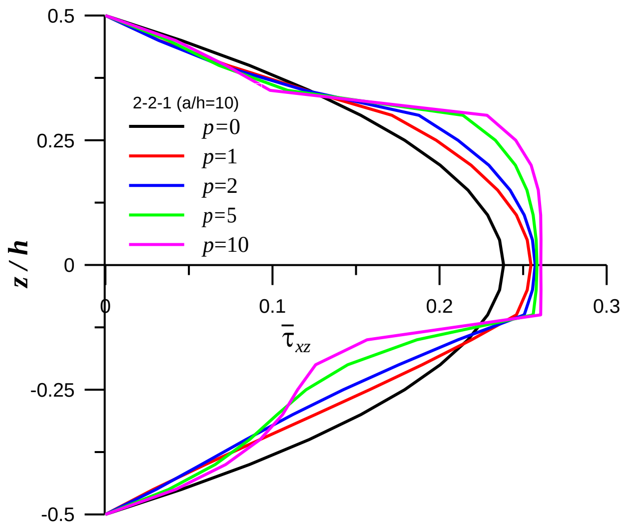

Figure 13. Through thickness variation of transverse shear stress in 2-2-1 FG sandwich plate subjected to sinusoidal load.

Discussion

Static Analysis

Table 2 through 4 shows the transverse displacement and stresses in single layer FG and sandwich FG plate subjected to sinusoidal and uniformly distributed load for various power law index. The present results are compared and found in close agreement with Thai et al. [5], Thai and Kim [6], Li et al. [7], Demirhan and Taskin [10], Zenkour [12]. Table 2 shows the transverse displacement and stresses for various power law index (p = 1, 2, 4, 8) at aspect ratio a/h=10. From Table 1 it is clearly observed that the transverse displacement increases with increase in power law index value for sinusoidal and uniformly distributed load , whereas the in-plane stresses decreases with increase in power law index for sinusoidal load and increases for uniformly distributed load. For both type of loading the transverse shear stresses are decreases with increase in power law index value. Table 3 and Table 4 shows the transverse displacement and stresses in sandwich FG plate subjected to sinusoidal and uniformly distributed load respectively. In case of sandwich FG plate the transverse displacement and stresses are found to be maximum in 1-0-1 scheme and minimum in 1-2-2 or 2-2-1 scheme which shows that the transverse displacement and stresses increases with increase in the thickness of middle core. The results for the stresses in sandwich FG plate subjected to uniformly distributed load is presented first time in the present study which is the major contribution of the present study. Fig. 2 and 3 shows the through thickness variation of in-plane stresses in single layer FG plate under sinusoidal and uniformly distributed load respectively at various values of power law index. Fig. 4 through Fig. 8 shows the in-plane stress variation in sandwich FG plate and Fig. 9 through Fig. 13 shows the transverse shear stress variation of sandwich FG plate through the thickness.

Free Vibration

The numerical results for the free vibration analysis of single layer FG and Sandwich FG plate are presented in Table 5 and Table 6 respectively for various power law index (p= 0, 0.5, 1, 4, 10). In Table 5 the frequencies are obtained for different mode i.e. 1, 2, 3. The present results are compared and found in good agreement with the results presented by Thai and Kim [6], Li at al.[7] and Thai et al.[5]. Also, from Table 4 it is observed that the frequencies are increases with increase in mode of frequency and decreases with increase in the power law index. Table 5 shows natural frequencies in sandwich FG plate for a/h=5, 10. The natural frequencies in sandwich FG plate are found to be maximum in 1-2-1 scheme and minimum in 1-0-1 which shows that the natural frequency decreases with decrease in the thickness of middle core.

Conclusions

In the present study, a static and free vibration analysis of single layer FG and sandwich FG plate is presented using a new fifth order shear and normal deformation theory. The present results are compared with those available in literature and found to be in excellent agreement. The major contribution of the present theory is that it presents displacement and stresses results for sandwich FG plate subjected to uniformly distributed load which can be treated as benchmark for future research work.

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Appendix

$$K_{11}=-A_{11}\alpha^2-A_{66}\beta^2, \quad K_{12}=K_{21}=-A_{12}\alpha\beta-A_{66}\alpha\beta,$$
$$K_{13}=K_{31}=\frac{A_{11}}{R_1}\alpha+\frac{A_{12}}{R_R}\beta+B_{11}\alpha^3+B_{12}\alpha\beta^2+2B_{66}\alpha\beta^2,$$
$$K_{14}=K_{41}=-A_{S1_{11}}\alpha^2-A_{S2_{66}}\beta^2,\ K_{15}=K_{51}=-A_{S2_{11}}\alpha^2-A_{S2_{66}}\beta^2,$$
$$K_{16}=K_{61}=-A_{S1_{12}}\alpha\beta-A_{S1_{66}}\alpha\beta, \quad K_{17}=K_{71}=-A_{S2_{12}}\alpha\beta-A_{S2_{66}}\alpha\beta,$$
$$K_{18}=K_{81}=\left(E_{13}-\frac{Q_{111}}{R_1}-\frac{Q_{112}}{R_2}\right)\alpha,\ K_{19}=K_{91}=\left(F_{13}-\frac{Q_{211}}{R_1}-\frac{Q_{212}}{R_2}\right)\alpha,$$
$$K_{22}=-A_{22}\beta^2-A_{66}\alpha^2,$$
$$K_{23}=K_{32}=\left(\frac{A_{12}}{R_1}+\frac{A_{22}}{R_2}\right)\beta+B_{12}\alpha^2\beta+B_{22}\beta^3+2B_{66}\alpha^2\beta,$$
$$K_{24}=K_{42}=-A_{S1_{12}}\alpha\beta-A_{S1_{66}}\alpha\beta,\ K_{25}=K_{52}=-A_{S2_{12}}\alpha\beta-A_{S2_{66}}\alpha\beta,$$
$$K_{26}=K_{62}=-A_{S1_{22}}\beta^2-A_{S1_{66}}\alpha^2,\ K_{27}=K_{72}=-A_{S2_{22}}\beta^2-A_{S2_{66}}\alpha^2,$$
$$K_{28}=K_{82}=\left(E_{23}-\frac{Q_{112}}{R_1}-\frac{Q_{122}}{R_2}\right)\beta,\ K_{29}=K_{92}=\left(F_{23}-\frac{Q_{212}}{R_1}-\frac{Q_{222}}{R_2}\right)\beta$$
$$K_{33}=-\left(D_{11}\alpha^4+D_{22}\beta^4\right)-2\alpha^2\beta^2\left(D_{12}+2D_{66}\right)-2\alpha^2\left(\frac{B_{11}}{R_1}+\frac{B_{12}}{R_2}\right)$$
$$-2\beta^2\left(\frac{B_{12}}{R_1}+\frac{B_{22}}{R_2}\right)-\left(\frac{A_{11}}{R_1^2}+\frac{A_{22}}{R_2^2}+\frac{2A_{12}}{R_1R_2}\right),$$
$$K_{34}=K_{43}=B_{S1_{11}}\alpha^3+B_{S1_{12}}\alpha\beta^2+2B_{S1_{66}}\alpha\beta^2+\frac{A_{S1_{11}}}{R_1}\alpha+\frac{A_{S1_{12}}}{R_2}\alpha,$$
$$K_{35}=K_{53}=B_{S2_{11}}\alpha^3+B_{S2_{12}}\alpha\beta^2+2B_{S2_{66}}\alpha\beta^2+\frac{A_{S2_{11}}}{R_1}\alpha+\frac{A_{S2_{12}}}{R_2}\alpha,$$
$$K_{36}=K_{63}=B_{S1_{12}}\alpha^2\beta+B_{S1_{22}}\beta^3+2B_{S1_{66}}\alpha^2\beta+\frac{A_{S1_{12}}}{R_1}\beta+\frac{A_{S1_{22}}}{R_2}\beta,$$
$$K_{37}=K_{73}=B_{S2_{12}}\alpha^2\beta+B_{S2_{22}}\beta^3+2B_{S2_{66}}\alpha^2\beta+\frac{A_{S2_{12}}}{R_1}\beta+\frac{A_{S2_{22}}}{R_2}\beta,$$
$$K_{38}=K_{83}=-J_{13}\alpha^2-J_{23}\beta^2-\frac{E_{13}}{R_1}-\frac{E_{23}}{R_2}+\left(\frac{Q_{311}}{R_1}+\frac{Q_{312}}{R_2}\right)\alpha^2$$
$$+\left(\frac{Q_{312}}{R_1}+\frac{Q_{322}}{R_2}\right)\beta^2+\left(\frac{Q_{111}}{R_1^2}+\frac{Q_{112}}{R_1R_2}\right)+\left(\frac{Q_{112}}{R_1R_2}+\frac{Q_{122}}{R_2^2}\right),$$
$$K_{39}=K_{93}=-O_{13}\alpha^2-O_{23}\beta^2-\frac{F_{13}}{R_1}-\frac{F_{23}}{R_2}+\left(\frac{Q_{411}}{R_1}+\frac{Q_{412}}{R_2}\right)\alpha^2$$
$$+\left(\frac{Q_{412}}{R_1}+\frac{Q_{422}}{R_2}\right)\beta^2+\left(\frac{Q_{211}}{R_1^2}+\frac{Q_{212}}{R_1R_2}\right)+\left(\frac{Q_{212}}{R_1R_2}+\frac{Q_{222}}{R_2^2}\right),$$

$$
\begin{aligned}
&K_{44}=-A_{SS1_{11}}\alpha^{2}-A_{SS1_{66}}\beta^{2}-G_{44},\\
&K_{45}=K_{54}=-C_{11}\alpha^{2}-C_{66}\beta^{2}-I_{44},\\
&K_{46}=K_{64}=-A_{SS1_{12}}\alpha\beta-A_{SS1_{66}}\alpha\beta,\\
&K_{47}=K_{74}=-C_{12}\alpha\beta-C_{66}\alpha\beta,\\
&K_{48}=K_{84}=\left(L_{1_{13}}\alpha-G_{44}\alpha-\frac{Q_{511}}{R_{1}}-\frac{Q_{512}}{R_{2}}\right),\\
&K_{49}=K_{94}=\left(L_{2_{13}}\alpha-I_{44}\alpha-\frac{Q_{611}}{R_{1}}-\frac{Q_{612}}{R_{2}},\right)\\
&K_{55}=-A_{SS2_{11}}\alpha^{2}-A_{SS2_{66}}\beta^{2}-H_{44},\\
&K_{56}=K_{65}=-C_{12}\alpha\beta-C_{66}\alpha\beta,\\
&K_{57}=K_{75}=-A_{S2_{12}}\alpha\beta-A_{SS2_{66}}\alpha\beta,\\
&K_{58}=K_{85}=-I_{44}\alpha+M_{1_{13}}\alpha-\frac{Q_{711}}{R_{1}}-\frac{Q_{712}}{R_{2}},\\
&K_{59}=K_{95}=-H_{44}\alpha+M_{2_{13}}\alpha-\frac{Q_{811}}{R_{1}}-\frac{Q_{812}}{R_{2}},\\
&K_{66}=-A_{SS1_{22}}\beta^{2}-A_{SS1_{66}}\alpha^{2}-G_{55},\\
&K_{67}=K_{76}=-C_{22}\beta^{2}-C_{66}\alpha^{2}-I_{55},\\
&K_{68}=K_{86}=L_{1_{23}}\beta-G_{55}\beta-\frac{Q_{512}}{R_{1}}-\frac{Q_{522}}{R_{2}},\\
&K_{69}=K_{96}=L_{2_{23}}\beta-I_{55}\beta-\frac{Q_{612}}{R_{1}}-\frac{Q_{622}}{R_{2}},\\
&K_{77}=-A_{SS2_{22}}\beta^{2}-A_{SS2_{66}}\alpha^{2}-H_{55},\\
&K_{78}=K_{87}=M_{1_{23}}\beta-I_{55}\beta-\frac{Q_{712}}{R_{1}}-\frac{Q_{722}}{R_{2}},\\
&K_{79}=K_{97}=M_{2_{23}}\beta-H_{55}\beta-\frac{Q_{812}}{R_{1}}-\frac{Q_{822}}{R_{2}},\\
&K_{88}=-G_{44}\alpha^{2}-G_{55}\beta^{2}-N_{1_{33}}+2\frac{Q_{1613}}{R_{1}}+2\frac{Q_{1623}}{R_{2}}\\
&+\frac{Q_{1311}}{R_{1}^{2}}+2\frac{Q_{1312}}{R_{1}R_{2}}+\frac{Q_{1322}}{R_{2}^{2}},\\
&K_{89}=K_{98}=-I_{44}\alpha^{2}-I_{55}\beta^{2}-N_{3_{33}}+2\frac{Q_{1613}}{R_{1}}\\
&+2\frac{Q_{1623}}{R_{2}}+\frac{Q_{1311}}{R_{1}^{2}}+2\frac{Q_{1312}}{R_{1}R_{2}}+\frac{Q_{1322}}{R_{2}^{2}},\\
&K_{99}=-L_{1_{44}}\alpha^{2}-H_{55}\beta^{2}-N_{2_{33}}
\end{aligned}
\tag{A.1}
$$

$$
F=\left\{0,0,-q_{0},0,0,0,0,0,0\right\}^{T} \tag{A.2}
$$

$$
\Delta=\left\{u_{mn},v_{mn},w_{mn},\phi_{xmn},\psi_{xmn},\phi_{ymn},\psi_{ymn},\phi_{zmn},\psi_{zmn}\right\}^{T} \tag{A.3}
$$