Journal of Algorithms and Computation

journal homepage: http://jac.ut.ac.ir

Edge pair sum labeling of spider graph

P. Jeyanthi*[1] and T. Saratha Devi†[2]

[1]Research Centre, Department of Mathematics, Govindammal Aditanar College for Women Tiruchendur, Tamil Nadu, India.

[2]Department of Mathematics, G. Venkataswamy Naidu College, Kovilpatti, Tamil Nadu, India.

ABSTRACT

An injective map $f : E(G) \to \{\pm 1, \pm 2, \cdots, \pm q\}$ is said to be an edge pair sum labeling of a graph $G(p, q)$ if the induced vertex function $f^* : V(G) \to Z - \{0\}$ defined by $f^*(v) = \sum_{e \in E_v} f(e)$ is one-one, where E_v denotes the set of edges in G that are incident with a vetex v and $f^*(V(G))$ is either of the form $\left\{\pm k_1, \pm k_2, \cdots, \pm k_{\frac{p}{2}}\right\}$ or $\left\{\pm k_1, \pm k_2, \cdots, \pm k_{\frac{p-1}{2}}\right\} \bigcup \left\{k_{\frac{p+1}{2}}\right\}$ according as p is even or odd. A graph which admits edge pair sum labeling is called an edge pair sum graph. In this paper we exhibit some spider graph.

Keyword: Edge pair sum labeling, edge pair sum graph, spider graph.

ARTICLE INFO

Article history:

Received 20, May, 2014

Received in revised form 14, September 2014

Accepted 12, November, 2014

Available online 20, November 2014

AMS subject Classification: 05C78.

*Corresponding author: P. Jeyanthi. Email: jeyajeyanthi@rediffmail.com

†rajanvino03@gmail.com

1 Introduction

All graphs in this paper are finite, simple and undirected. For standard terminology and notations we follow Gross and Yellen [1]. The symbols $V(G)$ and $E(G)$ denote the vertex set and edge set of a graph. R. Ponraj et. al. introduced the concept of pair sum labeling in [8] . An injective map $f : V(G) \to \{\pm 1, \pm 2, \ldots, \pm p\}$ is said to be a pair sum labeling of a graph $G(p, q)$ if the induced edge function $f_e : E(G) \to Z - \{0\}$ defined by $f_e(uv) = f(u) + f(v)$ is one-one and $f_e(E(G))$ is either of the form $\left\{\pm k_1, \pm k_2, \ldots, \pm k_{\frac{q}{2}}\right\}\}$ or $\left\{\pm k_1, \pm k_2, \ldots, \pm k_{\frac{q-1}{2}}\right\} \cup \left\{\pm k_{\frac{q+1}{2}}\right\}$ according as q is even or odd. A graph with a pair sum labeling, it is called a pair sum graph. Analogous to pair sum labeling we define a new labeling called an edge pair sum labeling in [3] and further studied in [4]-[7]. We proved that the path, cycle, star graph, $P_m \cup K_{1,n}, C_n \odot K_m^c$ if n is even, triangular snake, bistar, $K_{1,n} \cup K_{1,m}$, $C_n \cup C_n$ and complete bipartite graphs $K_{1,n}$ are edge pair sum graph.

Definition 1.1. *A tree is called a spider if it has a center vertex c of degree $R > 1$ and all the other vertex is either a leaf or with degree 2. Thus a spider is an amalgamation of k paths with various lengths. If it has x_1's path of length a_1, x_2's path of length $a_2, \ldots$. We shall denote the spider by $SP(a_1^{x_1}, a_2^{x_2}, \ldots, a_m^{x_m})$ where $a_1 < a_2 < \cdots < a_m$ and $x_1 + x_2 + \cdots + x_m = R$[9].*

Theorem 1.2. *The spider graph $SP(1^m, 2^t)$ is an edge pair sum graph.*

Proof. Let $V(SP(1^m, 2^t)) = \{u, v_i, u_j : 1 \leq i \leq m, 1 \leq j \leq 2t\}$ and $E(SP(1^m, 2^t)) = \{e_i = uv_i : 1 \leq i \leq m, e_i' = uu_i : 1 \leq i \leq t, e_i'' = u_i u_{t+i} : 1 \leq i \leq t\}$ be the vertices and edges of the graph $SP(1^m, 2^t)$.
Define the edge labeling $f : E(SP(1^m, 2^t)) \to \{\pm 1, \pm 2, \ldots, \pm(m+2t)\}$ by considering the following three cases.
Case (i) m is odd and t is odd.
$f(e_1) = -1$, for $1 \leq i \leq \frac{m-1}{2}$, $f(e_{1+i}) = (2i+1) = -f\left(e_{\frac{m+1}{2}+i}\right)$ for $1 \leq i \leq \frac{t-1}{2}$, $f(e_i') = (m+2i) = -f\left(e'_{\frac{t-1}{2}+i}\right)$ and $f(e_i'') = 2i = -f\left(e''_{\frac{t-1}{2}+i}\right)$, $f(e_t') = -(m+t+1)$ and $f(e_t'') = (m+t+2)$. The induced vertex labeling are as follows $f^*(v_1) = -1 = -f^*(u_t)$, for $1 \leq i \leq \frac{m-1}{2}$ $f^*(v_{1+i}) = (2i+1) = -f^*\left(v_{\frac{m+1}{2}+i}\right)$, for $1 \leq i \leq \frac{t-1}{2}$ $f^*(u_i) = (m+4i) = -f^*\left(u_{\frac{t-1}{2}+i}\right)$, $f^*(u_{t+i}) = 2i = -f^*\left(u_{\frac{3t-1}{2}+i}\right)$, and $f^*(u_{2t}) = (m+t+2) = -f^*(u)$. From the above vertex labeling $f^*(V(SP(1^m, 2^t))) = \{\pm 1, \pm 3, \pm 5, \ldots, \pm m, \pm 2, \pm 4, \pm 6, \ldots, \pm(t-$

$1), \pm(m+4), \pm(m+8), \ldots, \pm(m+2t-2), \pm(m+t+2)\}$. Hence f is an edge pair sum labeling of $SP(1^m, 2^t)$.
The example for the edge pair sum graph labeling of $SP(1^m, 2^t)$ for $m = 3$ and $t = 3$ is shown in Figure 1.

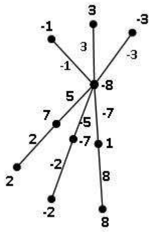

Figure 1.

Case (ii) m is even and t is odd.
For $1 \le i \le \frac{m}{2}$, $f(e_i) = (2i+1) = -f\left(e_{\frac{m}{2}+i}\right)$, $f(e'_1) = 1$, $f(e''_1) = -2$, for $1 \le i \le \frac{t-1}{2}$, $f(e'_{1+i}) = (m+2i+1) = -f\left(e'_{\frac{t+1}{2}+i}\right)$ and $f(e''_{1+i}) = 2(1+i) = -f\left(e''_{\frac{t+1}{2}+i}\right)$. The induced vertex labeling are as follows, for $1 \le i \le \frac{m}{2}$, $f^*(v_i) = (2i+1) = -f^*\left(v_{\frac{m}{2}+i}\right)$, $f^*(u_1) = -1 = -f^*(u)$, $f^*(u_{t+1}) = -2$, for $1 \le i \le \frac{t-1}{2}$, $f^*(u_{1+i}) = (m+4i+3) = -f^*\left(u_{\frac{t+1}{2}+i}\right)$ and $f^*(u_{t+1+i}) = 2(1+i) = -f^*\left(u_{\frac{3t+1}{2}+i}\right)$. From the above vertex labeling we get $f^*(V(SP(1^m, 2^t))) = \{\pm3, \pm5, \ldots, \pm(m+1), \pm1, \pm4, \pm6, \ldots, \pm(t+1), \pm(m+7), \pm(m+11), \ldots, \pm(m+2t+1)\} \cup \{-2\}$. Hence f is an edge pair sum labeling of $SP(1^m, 2^t)$.
Figure 2 illustrates the edge sum graph scheme for $SP(1^m, 2^t)$ where $m = 2$ and $t = 1$.

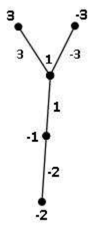

Figure 2.

Case (iii) m is even and t is even.
$f(e_1) = -1, f(e_2) = 2$, for $1 \le i \le \frac{m}{2} - 1$, $f(e_{2+i}) = (2i+1) = -f\left(e_{\frac{m}{2}+1+i}\right)$, for $1 \le i \le \frac{t}{2}$, $f(e'_i) = (m+2i-1) = -f\left(e'_{\frac{t}{2}+i}\right)$ and $f(e''_i) = 2(1+i) = -f\left(e''_{\frac{t}{2}+i}\right)$. The induced vertex labeling are $f^*(v_1) = -1 = -f^*(u)$, $f^*(v_2) = 2$, for $1 \le i \le \frac{m}{2} - 1$, $f^*(v_{2+i}) = (2i+1) = -f^*\left(v_{\frac{m}{2}+1+i}\right)$, for $1 \le i \le \frac{t}{2}$, $f^*(u_i) = (m+4i+1) = -f^*\left(u_{\frac{t}{2}+i}\right) =$ and $f^*(u_{t+i}) = 2(1+i) = -f^*\left(u_{\frac{3t}{2}+i}\right)$. From the above vertex labeling we get $f^*(V(SP(1^m, 2^t))) =$

$\{\pm 1, \pm 3, \pm 5, \ldots, \pm(m-1), \pm 4, \pm 6, \ldots, \pm(t+2), \pm(m+5), \pm(m+9), \ldots, \pm(m+2t+1)\} \cup \{2\}$.
Hence f is an edge pair sum labeling of $SP(1^m, 2^t)$. $\square$

The example for the edge pair sum graph labeling of $SP(1^m, 2^t)$ for $m = 2$ and $t = 2$ is shown in Figure 3.

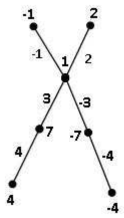

Figure 3.

Theorem 1.3. *The spider graph $SP(1^m, 2^t, 3)$ is an edge pair sum graph.*

Proof. Let $V(SP(1^m, 2^t, 3)) = \{u, v_i, u_j, v : 1 \le i \le m, 1 \le j \le 2t{+}2\}$ and $E(SP(1^m, 2^t, 3)) = \{e_i = uv_i : 1 \le i \le m, for 1 \le i \le t{+}1, e'_i = uu_i and e''_i = u_i u_{t+1+i}, e'''_1 = u_{2t+2}v\}$ be the vertices and edges of the graph $SP(1^m, 2^t, 3)$. Define the edge labeling $f : E(SP(1^m, 2^t, 3)) \to \{\pm 1, \pm, 2, \ldots, \pm(m+2t+3)\}$ by considering four cases.
Case (i) m is even and t is even.
for $1 \le i \le \frac{m}{2}$, $f(e_i) = (2i-1) = -f\left(e_{\frac{m}{2}+i}\right)$, for $1 \le i \le \frac{t}{2}$, $f(e'_i) = (m+2i-1) = -f\left(e'_{\frac{t}{2}+i}\right)$ and $f(e''_i) = (2i+6) = -f\left(e''_{\frac{t}{2}+i}\right)$, $f(e'_{t+1}) = 2$, $f(e''_{t+1}) = 4$ and $f(e''_1) = -6$.
The induced vertex labelings are for $1 \le i \le \frac{m}{2}$, $f^*(v_i) = (2i-1) = -f^*\left(v_{\frac{m}{2}+i}\right)$, for $1 \le i \le \frac{t}{2}$, $f^*(u_i) = (m+4i+5) = -f^*\left(u_{\frac{t}{2}+i}\right)$ and $f^*(u_{t+1+i}) = (2i+6) = -f^*\left(u_{\frac{3t+2}{2}+i}\right)$, $f^*(u_{t+1}) = 6 = -f^*(v)$, $f^*(u_{2t+2}) = -2 = -f^*(u)$.

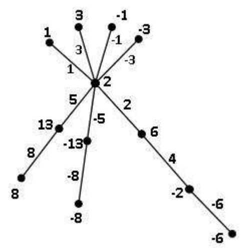

Figure 4.

From the above arguments we get $f^*(V(SP(1^m,2^t,3))) = \{\pm1,\pm3,\pm5,\ldots,\pm(m-1),\pm6,\pm8,\pm10,\ldots$ 6), $\pm2,\pm(m+9),\pm(m+13),\pm(m+17),\ldots,\pm(m+2t+5)\}$. Hence f is an edge pair sum labeling of $SP(1^m,2^t,3)$.

Figure 4 shows that $SP(1^m,2^t,3)$ is an edge pair sum graph for $m=4$ and $t=2$.

Case (ii) m is even and t is odd.

For $1 \le i \le \frac{m}{2}$, $f(e_i) = (5+2i) = -f\left(e_{\frac{m}{2}+i}\right)$, for $1 \le i \le \frac{t-1}{2}$, $f(e'_i) = (m+2i+5) = -f\left(e'_{\frac{t-1}{2}+i}\right)$ and $f(e''_i) = (2+2i) = -f\left(e''_{\frac{t-1}{2}+i}\right)$, $f(e'_t) = -2$, $f(e'_{t+1}) = -1$, $f(e''_{t+1}) = 3$ and $f(e''_t) = 5 = -f(e'''_1)$. The induced vertex labelings are as follows, for $1 \le i \le \frac{m}{2}$, $f^*(v_i) = (2i+5) = -f^*\left(v_{\frac{m}{2}+i}\right)$, for $1 \le i \le \frac{t-1}{2}$, $f^*(u_i) = (m+7+4i) = -f^*\left(u_{\frac{t-1}{2}+i}\right)$ and $f^*(u_{t+1+i}) = (2+2i) = -f^*\left(u_{\frac{3t+1}{2}+i}\right)$, $f^*(u_t) = 3 = -f^*(u)$, $f^*(u_{t+1}) = 2 = -f^*(u_{2t+2})$ and $f^*(u_{2t+1}) = 5 = -f^*(v)$. From the above vertex labeling we get $f^*(V(SP(1^m,2^t,3))) = \{\pm2,\pm3,\pm5,\pm7,\pm9,\ldots,\pm(m+5),\pm(m+11),\pm(m+15),\ldots,\pm(m+2t+5),\pm4,\pm6,\ldots,\pm(t+1)\}$. Hence f is an edge pair sum labeling of $SP(1^{2k},2^t,3)$.

Figure 5 shows that the spiders $SP(1^m,2^t,3)$ is an edge pair sum graph labeling for $m=4$ and $t=3$.

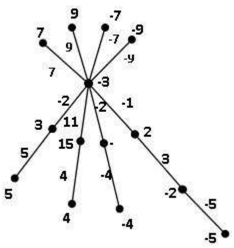

Figure 5.

$f(e_1) = 1$, $f(e'_1) = 2$, $f(e''_1) = (m+2) = -f(e'''_1)$, for $1 \le i \le \frac{m-1}{2}$, $f(e_{1+i}) = (2i+1) = -f\left(e_{\frac{m+1}{2}+i}\right)$, for $1 \le i \le \frac{t-1}{2}$, $f(e'_{1+i}) = (m+2i+2) = -f\left(e'_{\frac{t+1}{2}+i}\right)$ and $f(e''_{1+i}) = (m+3+2i) = -f\left(e''_{\frac{t+1}{2}+i}\right)$, $f(e'_{t+1}) = -4$, and $f(e''_{t+1}) = -2$. The induced vertex labelings are $f^*(v_1) = 1 = -f^*(u)$, $f^*(u_1) = (m+4) = -f^*(u_{2t+2})$, for $1 \le i \le \frac{m-1}{2}$, $f^*(v_{1+i}) = (2i+1) = -f^*\left(v_{\frac{m+1}{2}+i}\right)$, for $1 \le i \le \frac{t-1}{2}$, $f^*(u_{1+i}) = (2m+5+4i) = -f^*\left(u_{\frac{t+1}{2}+i}\right)$ and $f^*(u_{t+2+i}) = (m+3+2i) = -f^*\left(u_{\frac{3t+3}{2}+i}\right)$, $f^*(u_{t+2}) = (m+2) = -f^*(v) = f(e'''_1)$ and $f^*(u_{t+1}) = -6$. From the above vertex labeling we get $f^*(V(SP(1^m,2^t,3))) = \{\pm1,\pm3,\pm5,\ldots,\pm m,\pm(m+2),\pm(m+4),\pm(m+5),\pm(m+7),\ldots,\pm(m+t+2),\pm(2m+9),\pm(2m+13),\pm(2m+17),\ldots,\pm(2m+2t+3)\} \cup \{-6\}$. Hence f is an edge pair sum labeling of $SP(1^m,2^t,3)$.

Case (iv) m is odd and t is even.
$f(e_1) = -1$, $f(e'_{t+1}) = 2$, for $1 \leq i \leq \frac{m-1}{2}$, $f(e_{1+i}) = (2i+1) = -f\left(e_{\frac{m+1}{2}+i}\right)$, for $1 \leq i \leq \frac{t}{2}$, $f(e'_i) = (m+2i) = -f\left(e'_{\frac{t}{2}+i}\right)$ and $f(e''_i) = (2i+6) = -f\left(e''_{\frac{t}{2}+i}\right)$, $f(e''_{t+1}) = 4$ and $f(e'''_1) = -6$. The induced vertex labelings are $f^*(v_1) = -1 = -f^*(u)$, for $1 \leq i \leq \frac{m-1}{2}$, $f^*(v_{1+i}) = (2i+1) = -f^*\left(v_{\frac{m+1}{2}+i}\right)$, for $1 \leq i \leq \frac{t}{2}$, $f^*(u_i) = (m+4i+6) = -f^*\left(u_{\frac{t}{2}+i}\right)$, and $f^*(u_{t+1+i}) = (2i+6) = -f^*\left(u_{\frac{3t+2}{2}+i}\right)$, $f^*(v) = -6 = -f^*(u_{t+1})$, and $f^*(u_{2t+2}) = -2$. From the above arguments we get $f^*(V(SP(1^m, 2^t, 3))) = \{\pm1, \pm3, \pm5, \ldots, \pm(m-2), \pm6, \pm8, \pm10, \ldots, \pm(t+6), \pm(m+10), \pm(m+14), \pm(m+18), \ldots, \pm(m+2t+6)\} \cup \{-2\}$. Hence f is an edge pair sum labeling of $SP(1^m, 2^t, 3)$. $\square$

Figure 6 shows that the spiders $SP(1^m, 2^t, 3)$ is an edge pair sum graph labeling for $m = 3$ and $t = 4$.

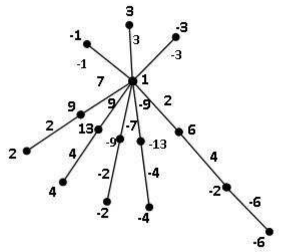

Figure 6.

Theorem 1.4. *The spider graph $SP(1^m, 2^t, 4)$ is an edge pair sum graph.*

Proof. Let $V(SP(1^m, 2^t, 4)) = \{u, v_i, u_j, v'_1, v'_2 : 1 \leq i \leq m, 1 \leq j \leq 2t+2\}$ and $E(SP(1^m, 2^t, 4)) = \{e_i = uv_i : 1 \leq i \leq m, for 1 \leq i \leq t+1, e'_i = uu_i and e'_{t+1+i} = u_iu_{t+1+i}, e''_1 = u_{2t+2}v'_1, e'''_1 = v'_1v'_2\}$ be the vertices and edges of the graph $SP(1^m, 2^t, 4)$. Define the edge labeling $f : E(SP(1^m, 2^t, 4)) \to \{\pm1, \pm2, \ldots, \pm(m+2t+4)\}$ as follows by four cases.

Case (i) m is odd and t is odd.
$f(e_1) = -1$, $f(e''_1) = -2$, $f(e'''_1) = -5$, $f(e'_1) = 2$, for $1 \leq i \leq \frac{m-1}{2}$, $f(e_{1+i}) = (2i+7) = -f\left(e_{\frac{m+1}{2}+i}\right)$, for $1 \leq i \leq \frac{t-1}{2}$, $f(e'_{1+i}) = (m+2i+6) = -f\left(e'_{\frac{t+1}{2}+i}\right)$ and $f(e'_{t+2+i}) = (6+2i) = -f\left(e'_{\frac{3t+3}{2}+i}\right)$, $f(e'_{t+1}) = -3$, $f(e'_{2t+2}) = 4$ and $f(e'_{t+2}) = 5$. The induced vertex labelings are $f^*(v_1) = -1 = -f^*(u_{t+1})$, for $1 \leq i \leq \frac{m-1}{2}$, $f^*(v_{1+i}) = (2i+7) = -f^*\left(v_{\frac{m+1}{2}+i}\right) =$, $f^*(u_1) = 7 = -f^*(v'_1) = -7$, for $1 \leq i \leq \frac{t-1}{2}$, $f^*(u_{1+i}) = (m+12+4i) = -f^*\left(u_{\frac{t+1}{2}+i}\right)$ and $f^*(u_{t+2+i}) = (6+2i) = -f^*\left(u_{\frac{3t+3}{2}+i}\right)$, $f^*(u_{t+2}) =$

$5 = -f^*(v_2') = -5$, $f^*(u_{2t+2}) = 2 = -f^*(u) = -2$. From the above vertex labeling we get $f^*(V(SP(1^m, 2^t, 4))) = \{\pm 1, \pm 2, \pm 5, \pm 7, \pm 9, \pm 11, \ldots, \pm(m+6), \pm(m+16), \pm(m+21), \pm(m+24), \ldots, \pm(m+2t+10), \pm 8, \pm 10, \ldots, \pm(t+5)\}$.
Hence f is an edge pair sum labeling of $SP(1^m, 2^t, 4)$.
Figure 7 shows that $SP(1^m, 2^t, 4)$ is an edge pair sum graph for $m = 3$ and $t = 1$.

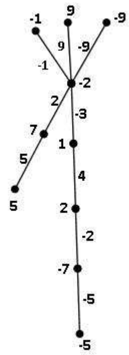

Figure 7.

Case (ii) m is odd and t is even.
$f(e_1) = 5$, $f(e_1'') = 3$, $f(e_1''') = -5$, for $1 \le i \le \frac{m-1}{2}$, $f(e_{1+i}) = (2i+5) = -f\left(e_{\frac{m+1}{2}+i}\right) = -(2i+5)$, for $1 \le i \le \frac{t}{2}$, $f(e_i') = (m+2i+4) = -f\left(e'_{\frac{t}{2}+i}\right)$, $f(e'_{t+1+i}) = (2i+2) = -f\left(e'_{\frac{3t+2}{2}+i}\right)$, $f(e'_{t+1}) = -2$, $f(e'_{2t+2}) = -1$. The induced vertex labelings are $f^*(v_1) = 5 = -f^*(v_2') = -5$, for $1 \le i \le \frac{m-1}{2}$, $f^*(v_{1+i}) = (2i+5) = -f^*\left(v_{\frac{m+1}{2}+i}\right)$, for $1 \le i \le \frac{t}{2}$, $f^*(u_i) = (m+4i+6) = -f^*\left(u_{\frac{t}{2}+i}\right)$, and $f^*(u_{t+1+i}) = (2i+2) = -f^*\left(u_{\frac{3t+2}{2}+i}\right)$, $f^*(u_{t+1}) = -3 = -f^*(u) = 3$ and $f^*(u_{2t+2}) = 2 = -f^*(v_1')$. From the above vertex labeling we get $f^*(V(SP(1^m, 2^t, 4))) = \{\pm 2, \pm 3, \pm 5, \pm 7, \pm 9, \ldots, \pm(m+4), \pm 4, \pm 6, \ldots, \pm(t+2), \pm(m+10), \pm(m+14), \pm(m+18), \ldots, \pm(m+2t+6)\}$.

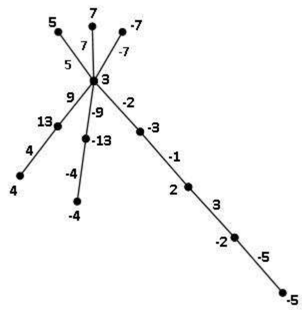

Figure 8.

Hence f is an edge pair sum labeling of $SP(1^m, 2^t, 4)$.
The example for the edge pair sum graph labeling of $SP(1^m, 2^t, 4)$ for $m = 3$ and $t = 2$ is shown in Figure 8.

Case (iii) m is even and t is odd.
For $1 \le i \le \frac{m}{2}$, $f(e_i) = (2i+1) = -f\left(e_{\frac{m}{2}+i}\right)$, $f(e'_1) = -1$, $f(e''_1) = -(m+3)$, $f(e'''_1) = -(m+2)$, for $1 \le i \le \frac{t-1}{2}$, $f(e'_{1+i}) = (m+2i+3) = -f\left(e'_{\frac{t+1}{2}+i}\right)$ and $f(e'_{t+2+i}) = (m+2+2i) = -f\left(e'_{\frac{3t+3}{2}+i}\right)$, $f(e'_{t+1}) = (m+3)$, $f(e'_{t+2}) = 2$, $f(e'_{2t+2}) = (m+2)$.
The induced vertex labeling are for $1 \le i \le \frac{m}{2}$, $f^*(v_i) = (2i+1) = -f^*\left(v_{\frac{m}{2}+i}\right)$, $f^*(u_1) = 1 = -f^*(u_{2t+2})$, for $1 \le i \le \frac{t-1}{2}$, $f^*(u_{1+i}) = (2m+5+4i) = -f^*\left(u_{\frac{t+1}{2}+i}\right)$ and $f^*(u_{t+2+i}) = (m+2+2i) = -f^*\left(u_{\frac{3t+3}{2}+i}\right)$, $f^*(u_{t+1}) = (2m+5) = -f^*(v'_1)$, $f^*(u_{t+2}) = 2$ and $f^*(v'_2) = -(m+2) = -f^*(u)$. From the above vertex labeling we get $f^*(V(SP(1^m, 2^t, 4))) = \{\pm 1, \pm 3, \pm 5, \pm 7, \ldots, \pm(m+1), \pm(2m+9), \pm(2m+13), \ldots, \pm(2m+2t+3), \pm(2m+5), \pm(m+2), \pm(m+4), \pm(m+6), \ldots, \pm(m+t+1)\} \cup \{2\}$. Hence f is an edge pair sum labeling of $SP(1^m, 2^t, 4)$.
The example for the edge pair sum graph labeling of $SP(1^m, 2^t, 4)$ for $m = 2$ and $t = 3$is shown in Figure 9.

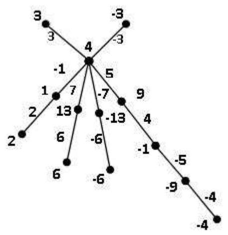

Figure 9.

Case (iv) m is even and t is even.
For $1 \le i \le \frac{m}{2}, f(e_i) = (2i+3) = -f\left(e_{\frac{m}{2}+i}\right)$, for $1 \le i \le \frac{t}{2}$, $f(e'_i) = (m+2i+3) = -f\left(e'_{\frac{t}{2}+i}\right)$, $f(e'_{t+1+i}) = (m+2i+4) = -f\left(e'_{\frac{3t+2}{2}+i}\right)$, $f(e'_{t+1}) = 2$, $f(e'_{2t+2}) = 4, f(e''_1) = -1$, $f(e'''_1) = -2$. The induced vertex labeling are for $1 \le i \le \frac{m}{2}, f^*(v_i) = (2i+3) = -f^*\left(v_{\frac{m}{2}+i}\right)$, for $1 \le i \le \frac{t}{2}$, $f^*(u_i) = (2m+4i+7) = -f^*\left(u_{\frac{t}{2}+i}\right)$ and $f^*(u_{t+1+i}) = (m+2i+4) = -f^*\left(u_{\frac{3t+2}{2}+i}\right)$, $f^*(u_{2t+2}) = 3 = -f(v'_1)$, $f^*(v'_2) = -2 = -f^*(u)$ and $f^*(u_{t+1}) = 6$. From the above vertex labeling we get $f^*(V(SP(1^m, 2^t, 4))) = \{\pm 2, \pm 3, \pm 5, \ldots, \pm(m+3), \pm(2m+11), \pm(2m+15), \ldots, \pm(2m+2t+7), \pm(m+6), \pm(m+8), \pm(m+10), \ldots, \pm(m+t+4)\} \cup \{6\}$.
Hence f is an edge pair sum labeling of $SP(1^m, 2^t, 4)$. $\square$

The example for the edge pair sum graph labeling of $SP(1^m, 2^t, 4)$ for $m = 2$ and $t = 4$ is shown in Figure 10.

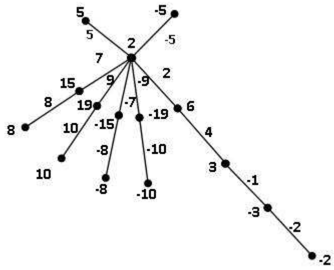

Figure 10.

Theorem 1.5. *The spider graph $SP(1^m, 3^t, 4)$ is an edge pair sum graph if t is even.*

Proof. Let $V(SP(1^m, 3^t, 4)) = \{u, v_i, u_j, v'_1 : 1 \leq i \leq m, 1 \leq j \leq 3t{+}3\}$ and $E(SP(1^m, 3^t, 4)) = \{e_i = uv_i : 1 \leq i \leq m$, for $1 \leq i \leq t+1, e'_i = uu_i, e'_{t+1+i} = u_i u_{t+1+i}$ and $e'_{2t+2+i} = u_{t+1+i}u_{2t+2+i}, e''_1 = u_{3t+3}v'_1\}$ be the vertices and edges of the graph $SP(1^m, 3^t, 4)$.
Define the edge labeling $f : E(SP(1^m, 3^t, 4)) \rightarrow \{\pm1, \pm2, \ldots, \pm(m + 3t + 4)\}$ as follows.
Case (i) m is odd.
$f(e_1) = 5$, for $1 \leq i \leq \frac{m-1}{2}$, $f(e_{1+i}) = (2i + 5) = -f\left(e_{\frac{m+1}{2}+i}\right)$, for $1 \leq i \leq \frac{t}{2}$ $f(e'_i) = (m + 2i + 4) = -f\left(e'_{\frac{t}{2}+i}\right)$, $f(e'_{t+1+i}) = (m + t + 4 + 2i) = -f\left(e'_{\frac{3t+2}{2}+i}\right)$, $f(e'_{2t+2+i}) = (6 + 2i) = -f\left(e'_{\frac{5t+4}{2}+i}\right)$, $f(e'_{2t+2}) = -1$, $f(e'_{3t+3}) = 3$, $f(e''_1) = -5$, $f(e'_{t+1}) = -2$.
The induced vertex labelings are $f^*(v_1) = 5 = -f^*(v'_1)$, for $1 \leq i \leq \frac{m-1}{2}$ $f^*(v_{1+i}) = (2i + 5) = -f^*\left(v_{\frac{m+1}{2}+i}\right)$, for $1 \leq i \leq \frac{t}{2}$, $f^*(u_i) = (2m + t + 8 + 4i) = -f^*\left(u_{\frac{t}{2}+i}\right)$, $f^*(u_{t+1+i}) = (m{+}t{+}10{+}4i) = -f^*\left(u_{\frac{3t+2}{2}+i}\right)$ and $f^*(u_{2t+2+i}) = (6{+}2i) = -f^*\left(u_{\frac{5t+4}{2}+i}\right)$, $f^*(u_{3t+3}) = -2 = -f^*(u_{2t+2})$, $f^*(u) = 3 = -f^*(u_{t+1})$. From the above arguments we get $f^*(V(SP(1^m, 3^t, 4))) = \{\pm2, \pm3, \pm5, \pm7, \pm9, \pm11, \ldots, \pm(m{+}4), \pm(2m{+}t{+}12), \pm(2m{+}t{+}16), \ldots, \pm(2m{+}3t{+}8), \pm(m{+}t{+}14), \pm(m{+}t{+}18), \ldots, (m{+}3t{+}10), \pm8, \pm10, \ldots, \pm(t{+}6)\}$.
Hence f is an edge pair sum labeling of $SP(1^m, 3^t, 4)$.
Case (ii) m is even.
For $1 \leq i \leq \frac{m}{2}$, $f(e_i) = (2i + 5) = -f\left(e_{\frac{m}{2}+i}\right)$, for $1 \leq i \leq \frac{t}{2}$, $f(e'_i) = (m + 2i + 5) = -f\left(e'_{\frac{t}{2}+i}\right)$, $f(e'_{t+1+i}) = (m + t + 5 + 2i) = -f\left(e'_{\frac{3t+2}{2}+i}\right)$ and $f(e'_{2t+2+i}) = (6 + 2i) = -f\left(e'_{\frac{5t+4}{2}+i}\right)$, $f(e'_{3t+3}) = -1$, $f(e''_1) = -2$, $f(e'_{t+1}) = 2$ and $f(e'_{2t+2}) = 4$. The induced vertex labeling are for $1 \leq i \leq \frac{m}{2}$, $f^*(v_{1+i}) = (2i + 5) = -f^*\left(v_{\frac{m}{2}+i}\right)$, for $1 \leq i \leq \frac{t}{2}$, $f^*(u_i) = (2m{+}t{+}10{+}4i) = -f^*\left(u_{\frac{t}{2}+i}\right)$, $f^*(u_{t+1+i}) = (m{+}t{+}11{+}4i) = -f^*\left(u_{\frac{3t+2}{2}+i}\right)$

and $f^*(u_{2t+2+i}) = (6+2i) = -f^*\left(u_{\frac{5t+4}{2}+i}\right)$, $f^*(u_{3t+3}) = -3 = -f^*(u_{2t+2})$, $f^*(v'_1) = 2 = -f^*(u)$, $f^*(u_{t+1}) = 6$. From the above arguments we get $f^*(V(SP(1^m, 3^t, 4))) = \{\pm 2, \pm 3, \pm 7, \pm 9, \pm 11, \ldots, \pm(m+5), \pm(2m+t+14), \pm(2m+t+18), \ldots, \pm(2m+3t+10), \pm(m+t+15), \pm(m+t+19), \ldots, \pm(m+3t+11), \pm 8, \pm 10, \ldots, \pm(t+6)\} \cup \{6\}$.
Hence f is an edge pair sum labeling of $SP(1^m, 3^t, 4)$. $\square$

References

[1] J. Gross and J. Yellen, *Graph Theory and its applications*, CRC Press,(1999).

[2] J. A. Gallian, A dynamic survey of graph labeling, *Electronic J.Combin.*, **17** (2013), #DS6.

[3] P. Jeyanthi and T. Saratha Devi, Edge pair sum labeling, *Journal of Scientific Research*, **5**(3) (2013), 457–467.

[4] P. Jeyanthi and T. Saratha Devi, On edge pair sum labeling of graphs,*International Journal of Mathematics Trends and Technology*, **7**(2) (2014), 106–113.

[5] P. Jeyanthi and T. Saratha Devi, *Some results on edge pair sum labeling*, (preprint).

[6] P. Jeyanthi and T. Saratha Devi, *Some edge pair sum graph*,Journal of Discrete Mathematical Science and Cryptography, (inpress).

[7] P. Jeyanthi and T. Saratha Devi, *Edge pair sum labeling of some graph*, (preprint).

[8] R. Ponraj and J. V. X. Parthipan, Pair Sum Labeling of Graphs, *The Journal of Indian Academy of Mathematics*, **32**(2) (2010), 587–595.

[9] Ping-Tsai Chung, Sin-Min Lee, On the Super Edge-graceful Spiders of Even orders, *J. Combin. Math. Combin. Comput.*, **64** (2008), 3–17.