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Investigation of the effect of angle beam transducer parameters on the lamb wave field in the three–layer plate by normal mode expansion method

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ABSTRACT

The effect of angle beam transducer parameters such as wedge angle and width transducer on the Lamb wave field generated in the elastic-viscoelastic three-layer plate has been investigated using normal mode expansion method. At first, the propagation of Lamb wave in the three-layer plate has been investigated using global matrix method, and all the modes that are propagated in the three-layer plate have been specified. Then, the optimum parameters of angle beam transducer have been obtained to generate a mode with minimum attenuation at a specific frequency. In addition to this mode, other modes are also generated in the three-layer plate, but this mode has maximum energy in the three-layer plate. The results indicate that the energy contribution of the mode with minimum attenuation at a specific frequency is 99.9% of the total energy and this mode has the highest energy contribution.

1. Introduction

Ultrasonic testing is used to inspect and evaluate the health of multilayer structures. Waves damping in multilayer structures is due to the presence of layers with viscoelastic properties and reduces the inspection range, by producing some wave modes at specific frequencies, the attenuation is reduced and the range of wave inspection can be increased. In order to achieve to this objective, some of characteristics of ultrasound waves, such as phase velocity and attenuation at multilayers, must be evaluated using appropriate approaches. There exist various methods to study wave propagation at multilayers, disregarding and/or regarding influence of wave source. The approaches including transfer matrix, global matrix, analytical, transient finite elements, spectral finite elements and Superposition of partial bulk waves are used for the first class, and normal modes expansion is used for the second class. The approaches of global matrix and transfer matrix are applicable for capturing ultrasound waves characteristics in elastic and viscoelastic environment [1,2]. However, they suffer numerical instability for viscoelastic environment in higher frequencies [3,4]. Analytical and transient finite element methods are used to study propagation of guided waves at multiple adhesive layers [5], and Superposition of partial bulk waves method has already been used to study wave propagation in a viscoelastic layer bound to an elastic layer [6]. Propagation characteristics of guided waves in an elastic hollow cylinder coated with a viscoelastic material were extracted by analytical and global matrix methods, and were compared with experimental results [7]. In addition, spectral finite element method was also used for modelling ultrasonic wave propagation at multiple linear viscoelastic layers [8] and in viscoelastic waveguides with arbitrary cross section, and changes in wave characteristic were displayed as curves [9,10].

In practical problems for sending ultrasonic waves inside multilayers, some sensor is located on its surface, which usually has certain dimensions and applies specific pressure on the object surface. Changes in boundary conditions, which is because of, wave source loading, results in propagating some other modes in the structure. Wave propagation have been investigated under these conditions in infinite elastic cylinder using normal modes expansion [11] and in anisotropic single layer when using strip source [12]. Selection of the modes with highest in-plane and out-of-plane displacements and determining suitable parameters for sensor for these modes, disregarding damping effects of adhesive layer in a titanium-aluminium adhesive joints, has been conducted [13].

In this article, at first, Lamb wave propagation characteristics in the three-layer plate without affecting the wave generation source are investigated using global matrix method and the low-attenuation modes are determined. Then, the influence of source on these characteristics are examined using normal modes expansion, and optimum parameters of wave generation source, such as sensor length and beam angle to generate a wave with low attenuation in a specific frequency are determined, which is one of the innovations of this article.

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2. Lamb wave propagation in three-layer plate

Wave propagation into isotropic elastic multilayers using global matrix method has been investigated. By writing Navier's motion equation, Eq. 1, for material particles in each layer and using Helmholtz's decomposition method, displacement and then with the help of displacement-strain and stress-strain relations in every point of layer, normal and shear stresses can be calculated.

$$\mu\nabla^2\vec{u}+(\lambda+\mu)\nabla(\nabla.\vec{u})=\rho\frac{\partial^2\vec{u}}{\partial t^2}. \qquad (1)$$

In this relation, μ, λ and ρ are Lamé constants and layer density, respectively and $\vec{u}$ is displacement vector. For an adhesive layer which is viscoelastic, according to Alfery's Correspondence principle, we can do the same for elastic layers, provided that Lamé constants are considered to be as complex numbers which are a function of frequency [14].

In each layer, longitudinal and shear waves with various amplitudes and obliquely become closer to and/or farther from layer's surfaces (Figure 1). L+ and L- are longitudinal waves, and S+ and S- are shear waves towards bottom and top of plate, and h_1, h_2 and h_3 are thicknesses of layer, respectively.

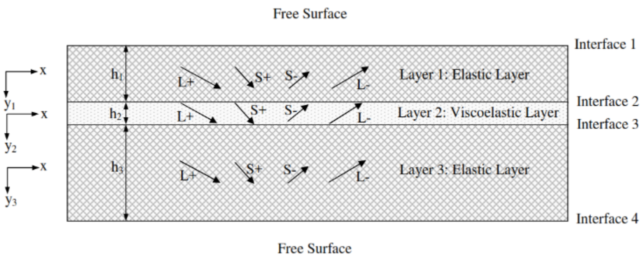

Figure 1. The propagation of Lamb wave in an elastic-viscoelastic three-layer plate.

If the solution of motion equation be assumed as harmonic, for finding displacement and stress in every point from the layer, four constant values of $A_{(L+)}$, $A_{(L-)}$, $A_{(S+)}$ and $A_{(S-)}$ should be calculated in the following relation:

$$\begin{Bmatrix} u_x \\ u_y \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = M \begin{Bmatrix} A_{(L+)} \\ A_{(L-)} \\ A_{(S+)} \\ A_{(S-)} \end{Bmatrix} e^{i(\omega t-kx)}. \qquad (2)$$

where M is layer's matrix with dimensions of 4×4.

By applying boundary conditions for stresses and displacements over the surfaces of every layer, homogeneous equations system with twelve equations and twelve unknowns which are domains or forms of propagating waves modes in the three-layer plates are obtained which its 12×12 matrix determinant coefficients must be zero so that there be nontrivial solutions. By calculating the characteristic equation roots, phase velocity and attenuation curves can be plotted. The computer code for calculating all the roots have been written here.

3. Source influence

If there be compressive stress due to wave generating source on some part of plate surface, by applying new boundary conditions of resulting equations system in global matrix method, will no longer be homogenous, and for this reason, identification of propagating frequencies and wavelengths must be done with another approach. Normal modes expansion method is used for this purpose. With the help of this method, propagating modes have been studies in detail under influence of size and dimensions of source with constant- and variable-pressure distribution and with various beam angles inside an anisotropic elastic plane [15]. By extending and generalizing this method for a three-layer plate, amplitude or shape of propagating waves have been calculated. For doing this, a source with width D which is assembled on a wedge with angle θ_i, has been taken into account (Figure 2).

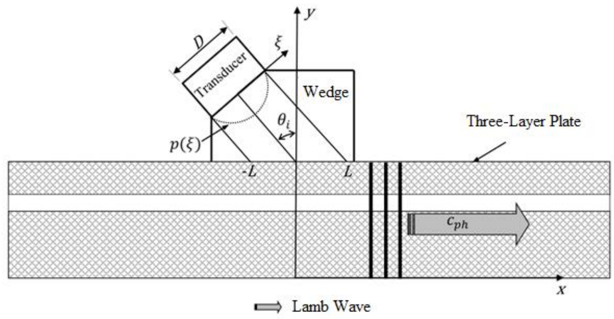

Figure 2. Lamb wave generation by angle beam transducer.

It is assumed that the source generates a harmonic stress wave, which after passing through the wedge, impinges to top surface of plate. In addition, the transducer generates an arbitrary pressure distribution *p(ξ)* in the wedge. This wedge is coupled by a liquid non-viscose film to the top surface of plate. Therefore, just normal reaction is transferred to plate surface and shear reaction is ignorable.

After carrying out required calculations, propagating waves' amplitude factor in *x* direction for constant-pressure distribution of the source are obtained as follows:

$$A_{+n}(x)=\frac{-v_{-ny}(h_1+h_2+h_3)}{4P_{n(-n)}}2\sigma_0\cos\theta_i e^{ik_w(h_1+h_2+h_3)\cos\theta_i} \times\frac{\sin(\frac{(k_n-k_w\sin\theta_i)D}{2\cos\theta_i})}{k_n-k_w\sin\theta_i}e^{-ik_nx},\ x\geq L. \qquad (3)$$

where $A_{+n}(x)$ and v_{-ny} are normal mode amplitude factor for the waves propagated along positive *x* (right) and y component of velocity for the waves propagated along negative *x* (left), respectively. $P_{n(-n)}$ is obtained from Eq. 4 for two modes with wave numbers of k_n and $-k_n$. In this relationship, k_w is wave number in-going to the wedge, which is equal to ω/c_w, c_w is longitudinal wave velocity in the wedge, and L is half length of loaded area, which is obtained from Eq. 5. Other parameters are shown in Figure 2.

$$P_{n(-n)}=-\frac{1}{4}\int_{\text{Thickness}}(\vec{\boldsymbol{v}}_n(y).\boldsymbol{T}_{-n}(y)-\vec{\boldsymbol{v}}_{-n}(y).\boldsymbol{T}_n(y)).\hat{\boldsymbol{e}}_x\mathrm{d}y \qquad (4)$$

$$L=\frac{D}{2\cos(\theta_i)}. \qquad (5)$$

4. Determining source parameters

One the characteristics which is used in determining optimum source parameters in multilayers is average power flow which is transferred using each one of modes along the layer. Once determination of amplitude factors for every mode, one can obtain displacement, velocity and stress, and therefore average power flow for that mode and in each point of multilayer. Also, transferred energy percentage with every mode in relation to total energy of every section of multilayer, are calculated. General shape of average power flow carried by Lamb wave along positive x axis per width unit of three-layer plate used in this study are written as follows:

$$P_{\text{ave}} = \text{Re}(-\frac{1}{2}\int_0^{h_1+h_2+h_3} \vec{v}^*.T.\hat{e}_x \, dy). \quad (6)$$

where, P_{ave} is Lamb wave's average power flow which is stated in terms of W/m. Re is real part of the quantity or parameter inside parentheses and * is conjugate of complex velocity vector and ***T*** is stress tensor on a plane which its normal is $\hat{e}_x$ (coordinate system is shown in Figure 2). Also, average power flow carried out by nth mode along *x* on width unit of plate is obtained from Eq. 7:

$$P_n = \text{Re}(-\frac{1}{2}\int_0^{h_1+h_2+h_3} \vec{v}_n^{\,*}.\overline{T}_n.\hat{e}_x \, dy). \quad (7)$$

where P_n is average power flow carried out by nth mode along *x* axis on width unit of three-layer plate. In above relation $\vec{v}_n^{\,*}$ is conjugate complex vector of n^th mode velocity and $\overline{T}_n$ is n^th stress tensor. Also, using Eqs. 6 and 7, the percentage of carried energy by each mode in proportion to total energy in various distances, can be obtained.

Propagating modes amplitude factor and therefore average power flow and transferred energy percentage by means of them, are dependent to geometrical parameters, namely sensor's width D and beam angle (wedge angle) θ_i. With the help of phase velocity and attenuation dispersion curves by specifying a specific mode which has low attenuation in a certain frequency, given phase velocity of this mode c_{ph}, beam angle is obtained using Snell Law according Eq. 8:

$$\theta_i^s = \sin^{-1}(\frac{c_w}{c_{ph}}). \quad (8)$$

where c_w is longitudinal wave velocity in the wedge.

Here we must specify suitable variables of D and θ_i so that percentage of transferred energy by means of this mode be highest. Using energy percentage graph of this mode, D^{opt} and θ_i^{opt} can be determined for a sensor which its excitation area length is 2L.

5. Results

With the help of a computer code, characteristic equation for a plate which its layers' acoustic properties are summarized in Table 1 [7], were solved numerically and phase velocity and attenuation dispersion curves have been plotted (Figures 3 and 4). With the help of Figure 4 curves, the mode and frequency which have certain attenuation in the plate are identified and by using Figure 3, respective phase velocity and as a result, beam angle θ_i for a point source can be calculated using Snell Law.

Further, by examining attenuation dispersion curve it is observed that for under consideration plate, for some of modes, there are some frequency modes that attenuation amount for them is low, and hence, they are suitable for inspection.

In the case of excitation of three-layer plate by one source with limited dimensions, all the existing modes at three-layer plate with various amplitude factors and energies are produces. So as to find optimum parameters (D^{opt} and θ_i^{opt}) limited source for producing Mode M3 with low attenuation in a three-layer plate aluminium-epoxy-aluminium in terms of angle beam transducer parameters (θ_i and L) at frequency 0.25MHz. Figure 5 shows percentage of energy of mode M3 in a three-layer plate aluminium-epoxy-aluminium in terms of beam angle transducer's parameters at frequency MHz 0.25. After reviewing the graph, it is found that when the wedge beam angle is 16° and half-length of excitation area is 10 mm, the contribution of mode M3 from total energy will be 99.9% and it is the highest energy contribution. Therefore, with regard to Eq. 4, when the optimum wedge angle is 16° and half-length of excitation area is 10 mm, the transducer optimum width will be 20 mm.

Table 1 Geometric and acoustic properties of an elastic-viscoelastic three-layer plate [7].

L	Material	c_1 $km s^{-1}$	α_1/ω $s km^{-1}$	c_2 $km s^{-1}$	α_2/ω $s km^{-1}$	ρ $g cm^{-3}$	h mm
1	Aluminum	6.35	-	3.13	-	2.7	1.6
2	Mereco 303 Epoxy	2.39	0.0070	0.99	0.0201	1.08	0.66
3	Aluminum	6.35	-	3.13	-	2.7	3.175

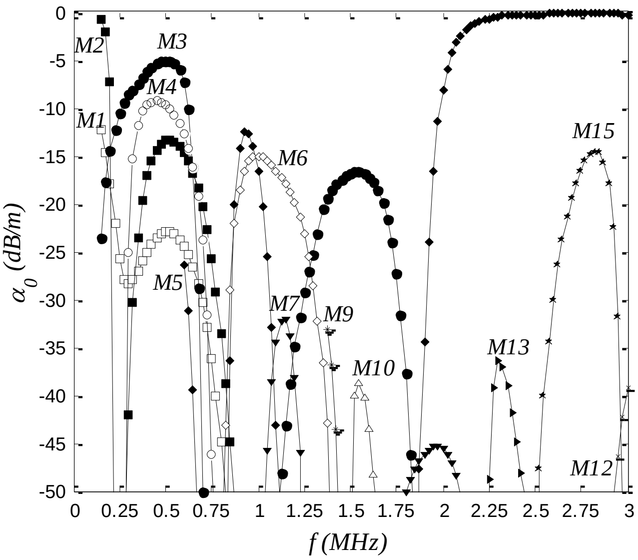

Figure 4. Attenuation dispersion curves in terms of frequency for different modes in three-layer plate: aluminium-epoxy-aluminium..

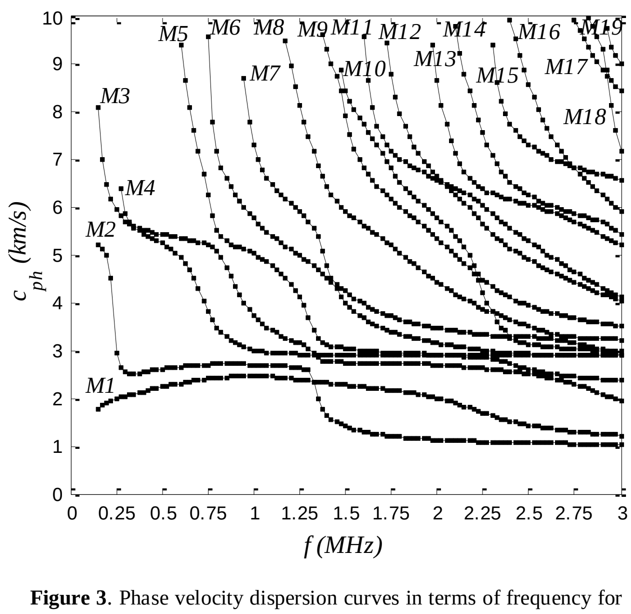

Figure 3. Phase velocity dispersion curves in terms of frequency for different modes in three-layer plate: aluminium-epoxy-aluminium.

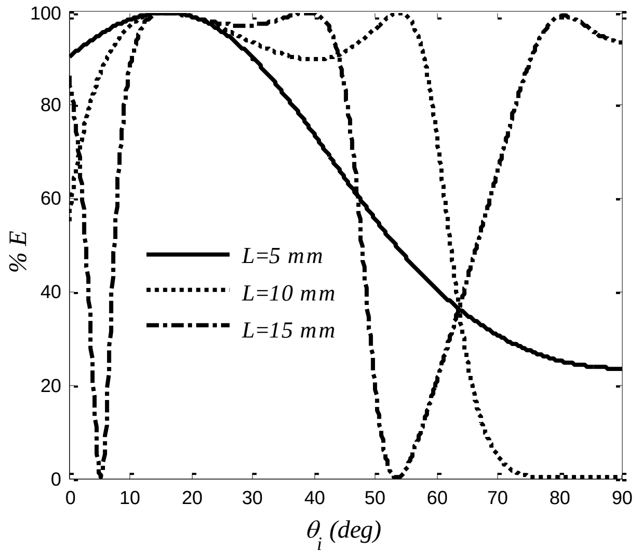

Figure 5. Percentage of energy of mode M3 in a three-layer plate aluminium-epoxy-aluminium in terms of beam angle transducer's parameters at frequency MHz 0.25.

6. Conclusion

By generating Lamb wave mode with low attenuation levels, we can inspect a wide range of structures and elastic-viscoelastic multilayer plate to find defects. In this article, Lamb wave propagation characteristics in a three-layer plate, which consists of phase velocity and attenuation is obtained using global matrix method. By examining Attenuation curve it can be determined that which modes at different frequencies have low attenuation. By producing these modes by angle beam transducer we can inspect a wide range of structures, but the parameters of the wave generation source, which are wedge transducer width and beam angle, have influence on the wave field and produced modes, so the influence of the source parameters on modes generated using normal modes expansion is investigated. Energy and Lamb wave modes energy percentages with low attenuation in terms of source parameters have been investigated, and optimal parameters for producing them have been identified. The Lamb wave that is generated by a beam angle transducer with optimal parameters, has the largest share of energy produced by the source.

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