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4-total mean cordial labeling in subdivision graphs

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ABSTRACT

Let G be a graph. Let $f : V(G) \to \{0, 1, 2, \dots, k-1\}$ be a function where $k \in \mathbb{N}$ and $k > 1$. For each edge uv, assign the label $f(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. f is called k-total mean cordial labeling of G if $|t_{mf}(i) - t_{mf}(j)| \leq 1$, for all $i, j \in \{0, 1, 2, \dots, k-1\}$, where $t_{mf}(x)$ denotes the total number of vertices and edges labelled with x, $x \in \{0, 1, 2, \dots, k-1\}$. A graph with admit a k-total mean cordial labeling is called k-total mean cordial graph.

Keyword: corona, subdivision of star, subdivision of bistar, subdivision of comb, subdivision of crown, subdivision of double comb, subdivision of ladder.

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1 Introduction

Graphs in this paper are finite, simple and undirected. In [3] the concept of k-total mean cordial labeling have been introduced. Also 4-total mean cordial behaviour of several graphs like path, cycle, complete graph, star, bistar, comb, crown have been investigated

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[3]. In this paper, we investigate the 4-total mean cordial labeling of subdivision of star, bistar, comb, crown, double comb, jelly fish, ladder, triangular snake. Let x be any real number. Then $\lceil x \rceil$ stands for the smallest integer greater than or equal to x. Terms are not defined here follow from Harary[2] and Gallian[1]. .

2 k-total mean cordial graph

Definition 2.1. Let G be a graph. Let $f : V(G) \to \{0, 1, 2, \ldots, k-1\}$ be a function where $k \in \mathbb{N}$ and $k > 1$. For each edge uv, assign the label $f(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. f is called k-total mean cordial labeling of G if $|t_{mf}(i) - t_{mf}(j)| \leq 1$, for all $i, j \in \{0, 1, 2, \ldots, k-1\}$, where $t_{mf}(x)$ denotes the total number of vertices and edges labelled with x, $x \in \{0, 1, 2, \ldots, k-1\}$. A graph with admit a k-total mean cordial labeling is called k-total mean cordial graph.

3 preliminary results

Definition 3.1. Let G_1, G_2 respectively be (p_1, q_1),(p_2, q_2) graphs. The corona of G_1 with G_2, $G_1 \odot G_2$ is the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2.

Definition 3.2. If $e = uv$ is an edge of G then e is said to be *subdivided* when it is replaced by the edges uw and wv. The graph obtained by subdividing each edge of a graph G is called the *subdivision graph* of G and is denoted by $S(G)$.

4 Main results

Theorem 4.1. The subdivision of star $K_{1,n}$, $S(K_{1,n})$ is 4-total mean cordial for all n.

Proof. Let u be the vertex of degree n and u_1, u_2, …, u_n be the pendent vertices. Let v_i be the vertex which subdivided the edge uu_i $(1 \leq i \leq n)$.
Clearly $|V(S(K_{1,n}))| + |E(S(K_{1,n}))| = 4n + 1$.
Assign the label 3 to the vertex u.
Now we consider the pendent vertices u_1, u_2, …, u_n. Assign the label 0 to the n vertices u_1, u_2, …, u_n. We now move to the vertices v_1, v_2, …, v_n. Assign the label 2 to the n vertices v_1, v_2, …, v_n.
Clearly $t_{mf}(0) = t_{mf}(1) = t_{mf}(2) = n$, $t_{mf}(3) = n + 1$. □

Theorem 4.2. $S(B_{n,n})$ is 4-total mean cordial for all values of n.

Proof. Let u, v be the vertices of degree $n+1$ and w be the vertex of degree 2 adjacent to both u and v. Let x_i be the vertex of degree 2 adjacent to u and y_i be the vertex of degree 2 adjacent to v. Let u_i and v_i $(1 \leq i \leq n)$ be the pendent vertex adjacent to x_i and y_i respectively.
Obviously $|V(S(B_{n,n}))| + |E(S(B_{n,n}))| = 8n + 5$.
Assign the labels 0, 3, 2 respectively to the vertices u, v, w.

Case 1. n is odd.
Assign the label 1 to the n vertices u_1, u_2, $\ldots$, u_n. We now assign the label 0 to the n vertices $x_1, x_2, \ldots, x_n$. Next assign the label 2 to the $\frac{n+1}{2}$ vertices $v_1, v_2, \ldots, v_{\frac{n+1}{2}}$ and assign the label 3 to the $\frac{n-1}{2}$ vertices $v_{\frac{n+3}{2}}, v_{\frac{n+5}{2}}, \ldots, v_n$. Assign the label 2 to the $n-1$ vertices $y_1, y_2, \ldots, y_{n-1}$ and finally assign the label 3 to the vertex y_n.

Case 1. n is even.
Assign the label 1 to the n vertices $u_1, u_2, \ldots, u_n$. We now assign the label 0 to the n vertices $x_1, x_2, \ldots, x_n$. Next assign the label 2 to the $\frac{n}{2}$ vertices $v_1, v_2, \ldots, v_{\frac{n}{2}}$ and assign the label 3 to the $\frac{n}{2}$ vertices $v_{\frac{n+2}{2}}, v_{\frac{n+4}{2}}, \ldots, v_n$. Assign the label 2 to the n vertices $y_1, y_2, \ldots, y_n$.

Thus this vertex labeling f is 4-total mean cordial labeling follows from the Tabel 1 □

Nature of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
n is odd	$2n+1$	$2n+1$	$2n+1$	$2n+2$
n is even	$2n+1$	$2n+1$	$2n+1$	$2n+2$

Table 1:

Theorem 4.3. $S(P_n \odot K_1)$ is 4-total mean cordial for all values of n.

Proof. Let P_n be the path $u_1u_2\ldots u_n$ and v_i be the pendent vertices adjacent to u_i $(1 \leq i \leq n)$. Let x_i be the vertex which subdivided the edge u_iu_{i+1} $(1 \leq i \leq n-1)$ and y_i be the vertex which subdivide the edge u_iv_i $(1 \leq i \leq n)$.
It is easy to show that $|V(S(P_n \odot K_1))| + |E(S(P_n \odot K_1))| = 8n - 3$.

Case 1. n is odd.
Assign the label 0 to the n vertices $u_1, u_2, \ldots, u_n$. Next assign the label 2 to the $n-1$ vertices $x_1, x_2, \ldots, x_{n-1}$. We now assign the label 3 to the n vertices $v_1, v_2, \ldots, v_n$. Assign the label 0 to the $\frac{n-1}{2}$ vertices $y_1, y_2, \ldots, y_{\frac{n-1}{2}}$. Next assign the label 3 to the $\frac{n-1}{2}$ vertices $y_{\frac{n+1}{2}}, v_{\frac{n+3}{2}}, \ldots, y_{n-1}$ and finally assign the label 2 to the vertex y_n.

Case 2. n is even and $n \geq 4$.
Assign the label 0 to the $\frac{n-2}{2}$ vertices $u_1, u_2, \ldots, u_{\frac{n-2}{2}}$ and assign the label 3 to the $\frac{n+2}{2}$

vertices $u_{\frac{n}{2}}, u_{\frac{n+2}{2}}, \ldots, u_n$. Next assign the label 3 to the $\frac{n-2}{2}$ vertices $x_1, x_2, \ldots, x_{\frac{n-2}{2}}$ and assign the label 2 to the $\frac{n-2}{2}$ vertices $x_{\frac{n}{2}}, x_{\frac{n+2}{2}}, \ldots, x_{n-2}$. Now assign the label 0 to the vertex x_{n-1}. We now assign the label 1 to the n vertices $v_1, v_2, \ldots, v_n$. Finally assign the label 0 to the n vertices $y_1, y_2, \ldots, y_n$.

From Tabel 2, this vertex labeling f is 4-total mean cordial labeling

Nature of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
n is odd	$2n-1$	$2n-1$	$2n-1$	$2n$
n is even	$2n-1$	$2n$	$2n-1$	$2n-1$

Table 2:

Case 3. $n = 2$.
A 4-total mean cordial labeling is given in table 3

□

Vertex	u_1	u_2	v_1	v_2	x_1	y_1	y_2
Label	0	1	3	3	0	2	2

Table 3:

Theorem 4.4. The subdivision of crown $C_n \odot K_1$, $S(C_n \odot K_1)$ is 4-total mean cordial for all n.

Proof. Let C_n be the cycle $u_1u_2 \ldots u_nu_1$ and v_i be the pendent vertices adjacent to u_i $(1 \le i \le n)$. Let x_i be the vertex which subdivided the edge u_iu_{i+1} $(1 \le i \le n-1)$ and x_n be the vertex which subdivided the edge u_nu_1. Let y_i be the vertex which subdivide the edge u_iv_i $(1 \le i \le n)$. Note that $|V(S(C_n \odot K_1))| + |E(S(C_n \odot K_1))| = 8n$.

Case 1. n is odd.
Assign the label 2 to the n vertices $u_1, u_2, \ldots, u_n$. Next we assign the label 0 to the n vertices $x_1, x_2, \ldots, x_n$. We now assign the label 0 to the n vertices $v_1, v_2, \ldots, v_n$. Finally assign the label 3 to the n vertices $y_1, y_2, \ldots, y_n$.

Case 2. n is even.
Now assign the label 0 to the n vertices $u_1, u_2, \ldots, u_n$. Then we assign the label 2 to the n vertices $x_1, x_2, \ldots, x_n$. Now we assign the label 3 to the n vertices $v_1, v_2, \ldots, v_n$. Assign the label 0 to the $\frac{n}{2}$ vertices $y_1, y_2, \ldots, y_{\frac{n}{2}}$ and finally assign the label 3 to the $\frac{n}{2}$ vertices $y_{\frac{n+2}{2}}, y_{\frac{n+4}{2}}, \ldots, y_n$.

From Tabel 4, this vertex labeling f is 4-total mean cordial labeling

□

Nature of n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
n is odd	$2n$	$2n$	$2n$	$2n$
n is even	$2n$	$2n$	$2n$	$2n$

Table 4:

Theorem 4.5. $S(P_n \odot 2K_1)$ is 4-total mean cordial for all values of n.

Proof. Let P_n be the path $v_1v_2\dots v_n$ and u_i,w_i be the pendent vertices adjacent to v_i $(1 \leq i \leq n)$. Let z_i be the vertex which subdivided the edge u_iu_{i+1} $(1 \leq i \leq n-1)$. Let x_i,y_i be the vertices which subdivided the edge u_iv_i,v_iw_i $(1 \leq i \leq n)$.
It is easy to verify that $|V(S(P_n \odot 2K_1))| + |E(S(P_n \odot 2K_1))| = 12n - 3$.

Assign the label 2 to the n vertices u_1, u_2, $\dots$, u_n. Next we assign the label 0 to the n vertices x_1, x_2, $\dots$, x_n. Then we assign the label 0 to the n vertices v_1, v_2, $\dots$, v_n. We now assign the label 2 to the vertex y_1 and assign the label 3 to the $n-2$ vertices y_2, y_3, $\dots$, y_{n-1}. Next assign the label 3 to the n vertices w_1, w_2, $\dots$, w_n. Finally we assign the label 2 to the $n-1$ vertices z_1, z_2, $\dots$, z_{n-1}.
Obviously $t_{mf}(0) = 3n$, $t_{mf}(1) = t_{mf}(2) = t_{mf}(3) = 3n - 1$. □

Theorem 4.6. The subdivision of Book with triangular pages $K_2 + mK_1$, $S(K_2 + mK_1)$ is 4-total mean cordial for all m.

Proof. Let u,v be the vertices of degree $m+1$ and u_i be the vertex adjacent to both u and v. Let w be the vertex of degree 2 which subdivided the edge uv. Let x_i,y_i be the vertices which subdivided the edges uu_i,vu_i $(1 \leq i \leq m)$. Clearly $|V(S(K_2 + mK_1))| + |E(S(K_2 + mK_1))| = 7m + 5$.
Assign the labels 0, 3, 2 respectively to the vertices u, v, w.

Case 1. $m \equiv 0 \pmod 4$.
Let $m = 4r$, $r \in N$.
Consider the vertices u_1, u_2, $\dots$, u_{4r}. Assign the label 0 to the $2r$ vertices u_1, u_2, $\dots$, u_{2r}. Next we assign the label 2 to the r vertices u_{2r+1}, u_{2r+2}, $\dots$, u_{3r}. We now assign the label 1 to the r vertices u_{3r+1}, u_{3r+2}, $\dots$, u_{4r}. Now we consider the vertices x_1, x_2, $\dots$, x_{4r}. Assign the label 0 to the r vertices x_1, x_2, $\dots$, x_r. Then we assign the label 2 to the $2r$ vertices x_{r+1}, x_{r+2}, $\dots$, x_{3r}. We now assign the label 1 to the r vertices x_{3r+1}, x_{3r+2}, $\dots$, x_{4r}. Next we consider the vertices y_1, y_2, $\dots$, y_{4r}. Assign the label 3 to the r vertices y_1, y_2, $\dots$, y_r. Now we assign the label 0 to the r vertices y_{r+1}, y_{r+2}, $\dots$, y_{2r}. Finally we assign the label 3 to the $2r$ vertices y_{2r+1}, y_{2r+2}, $\dots$, y_{4r}.

Case 2. $m \equiv 1 \pmod 4$.
Let $m = 4r + 1$, $r \geq 0$.
As in Case 1, assign the label to the vertices u_i, x_i,y_i $(1 \leq i \leq 4r)$. Finally we assign the

label 1, 0, 2 to the vertices u_{4r+1}, x_{4r+1}, y_{4r+1}.

Case 3. $m \equiv 2 \pmod 4$.
Let $m = 4r + 2$, $r \geq 0$.
Label the vertices u_i, x_i, y_i $(1 \leq i \leq 4r + 1)$ as in Case 2. Next assign the label 0, 2, 3 to the vertices u_{4r+2}, x_{4r+2}, y_{4r+2}.

Case 4. $m \equiv 3 \pmod 4$.
Let $m = 4r + 3$, $r \geq 0$.
In this case, assign the label for the vertices u_i, x_i, y_i $(1 \leq i \leq 4r + 2)$ as in Case 3. We now assign the labels 1, 0, 2 to the vertices u_{4r+3}, x_{4r+3}, y_{4r+3}.

The table 5, given below establish that this vertex labeling f is 4-total mean cordial labeling

□

Nature of m	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$m = 4r$	$7r + 1$	$7r + 1$	$7r + 1$	$7r + 2$
$m = 4r + 1$	$7r + 3$	$7r + 3$	$7r + 3$	$7r + 3$
$m = 4r + 2$	$7r + 4$	$7r + 5$	$7r + 5$	$7r + 5$
$m = 4r + 3$	$7r + 6$	$7r + 7$	$7r + 7$	$7r + 6$

Table 5:

Theorem 4.7. $S(J(n,n))$ is 4-total mean cordial for all values of n where $J(n,n)$ is a jelly fish.

Proof. Let $V(J(n,n)) = \{u, v, x, y, u_i, v_i : 1 \leq i \leq n\}$ and
$E(J(n,n)) = \{ux, uy, xy, vx, vy, uu_i, vv_i : 1 \leq i \leq n\}$.
Let o, p, q, r, s be the vertices which subdivided the edges xy, ux, uy, vx, vy. Let x_i, y_i be the vertices which subdivided uu_i, vv_i $(1 \leq i \leq n)$.
It is easy to verify that $|V(S(J(n,n)))| + |E(S(J(n,n)))| = 8n + 19$.

Assign the labels 0, 2, 0, 1, 1, 0, 1, 3, 3 respectively to the vertices u, v, x, y, o, p, q, r, s. Now we consider the pendent vertices $u_1, u_2, \ldots, u_n$. Assign the label 1 to the n vertices $u_1, u_2, \ldots, u_n$. Next assign the label 0 to the n vertices $x_1, x_2, \ldots, x_n$. We now move to the pendent vertices $v_1, v_2, \ldots, v_n$. Assign the label 3 to the n vertices $v_1, v_2, \ldots, v_n$. We now assign the label 2 to the n vertices $y_1, y_2, \ldots, y_n$.
Note that $t_{mf}(0) = t_{mf}(1) = t_{mf}(2) = 2n + 5$, $t_{mf}(3) = 2n + 4$. □

Theorem 4.8. The subdivision of ladder L_n, $S(L_n)$ is 4-total mean cordial for all n.

Proof. Let $V(L_n) = \{u_i, v_i : 1 \le i \le n\}$ and
$E(L_n) = \{u_iu_{i+1}, v_iv_{i+1} : 1 \le i \le n-1\} \cup \{u_iv_i : 1 \le i \le n\}$.
Let x_i, z_i and y_i be the vertices which subdivide the edges u_iu_{i+1}, v_iv_{i+1} $(1 \le i \le n-1)$ and u_iv_i $(1 \le i \le n)$ respectively. It is easy to verify that, $|V(S(L_n))| + |E(S(L_n))| = 11n - 6$.

Case 1. $n \equiv 0 \pmod 4$.
Let $n = 4r$, $r \ge 2$.
Assign the label 0 to the r vertices u_1, u_2, $\ldots$, u_r, v_1, v_2, $\ldots$, v_r and y_1, y_2, $\ldots$, y_r. Next we assign the label 1 to the r vertices u_{r+1}, u_{r+2}, $\ldots$, u_{2r}, v_{r+1}, v_{r+2}, $\ldots$, v_{2r} and y_{r+1}, y_{r+2}, $\ldots$, y_{2r}. We now assign the label 2 to the r vertices u_{2r+1}, u_{2r+2}, $\ldots$, u_{3r}, v_{2r+1}, v_{2r+2}, $\ldots$, v_{3r} and y_{2r+1}, y_{2r+2}, $\ldots$, y_{3r} and assign the label 3 to the r vertices u_{3r+1}, u_{3r+2}, $\ldots$, u_{4r}, v_{3r+1}, v_{3r+2}, $\ldots$, v_{4r} and y_{3r+1}, y_{3r+2}, $\ldots$, y_{4r}. Consider the vertices z_1, z_2, $\ldots$, z_n. Assign the label 0 to the $r+1$ vertices z_1, z_2, $\ldots$, z_{r+1}. Then we assign the label 1 to the $r-1$ vertices z_{r+2}, z_{r+3}, $\ldots$, z_{2r}. We now assign the label 2 to the $r-1$ vertices $z_{2r+1}, z_{2r+2}, \ldots, z_{3r-1}$ and finally we assign the label 3 to the r vertices $z_{3r}, z_{3r+1}, \ldots, z_{4r-1}$.

Case 2. $n \equiv 1 \pmod 4$.
Let $n = 4r + 1$, $r \ge 2$.
As in Case 1, assign the label to the vertices u_i, v_i, y_i $(1 \le i \le 4r)$ and x_i, z_i $(1 \le i \le 4r-1)$. Finally we assign the labels 2, 3, 0, 0, 3 respectively to the vertices u_{4r+1}, v_{4r+1}, x_{4r}, y_{4r+1}, z_{4r}.

Case 3. $n \equiv 2 \pmod 4$.
Let $n = 4r + 2$, $r \ge 2$.
Label the vertices u_i, v_i, y_i $(1 \le i \le 4r+1)$ and x_i, z_i $(1 \le i \le 4r)$ as in Case 2. Next we assign the labels 0, 1, 0, 2, 3 to the vertices u_{4r+2}, v_{4r+2}, x_{4r+1}, y_{4r+2}, z_{4r+1}.

Case 4. $n \equiv 3 \pmod 4$.
Let $n = 4r + 3$, $r \ge 2$.
In this case, assign the label for the vertices u_i, v_i, y_i $(1 \le i \le 4r+2)$ and x_i, z_i $(1 \le i \le 4r+1)$ as in Case 3. Finally we assign the labels 0, 3, 1, 0, 3 respectively to the vertices u_{4r+3}, v_{4r+3}, x_{4r+2}, y_{4r+3}, z_{4r+2}.

From the Table 6, this vertex labeling f is a 4-total mean cordial labeling of $S(L_n)$

Case 5. $2 \le n \le 7$.
A 4-total mean cordial labeling of $S(L_n)$ is given in Table 7

□

Order of $S(L_n)$	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n = 4r$	$11r - 1$	$11r - 1$	$11r - 2$	$11r - 2$
$n = 4r + 1$	$11r + 1$	$11r + 1$	$11r + 1$	$11r + 2$
$n = 4r + 2$	$11r + 4$	$11r + 4$	$11r + 4$	$11r + 4$
$n = 4r + 3$	$11r + 7$	$11r + 7$	$11r + 6$	$11r + 7$

Table 6:

Theorem 4.9. The subdivision of triangular snake T_n, $S(T_n)$ is 4-total mean cordial.

Proof. Let P_n be the path $u_1u_2 \dots u_n$ and w_i be the vertex adjacent to u_i and u_{i+1}. Let v_i be the vertex which subdivide the edge u_iu_{i+1} $(1 \le i \le n-1)$. Let x_i, y_i be the vertices which subdivided u_iw_i, $u_{i+1}w_i$ $(1 \le i \le n-1)$ respectively.
In this graph, $|V(S(T_n))| + |E(S(T_n))| = 11n - 10$.

Case 1. $n \equiv 0 \pmod 4$.
Let $n = 4r$, $r \ge 1$.
Consider the vertices $u_1, u_2, \dots, u_n$. Assign the label 0 to the r vertices $u_1, u_2, \dots, u_r$. Next we assign the label 1 to the r vertices $u_{r+1}, u_{r+2}, \dots, u_{2r}$. We now assign the label 2 to the r vertices $u_{2r+1}, u_{2r+2}, \dots, u_{3r}$ and assign the label 3 to the r vertices $u_{3r+1}, u_{3r+2}, \dots, u_{4r}$. Consider the vertices $w_1, w_2, \dots, w_n$. Assign the label 0 to the r vertices $w_1, w_2, \dots, w_r$. Then we assign the label 1 to the $r-1$ vertices $w_{r+1}, w_{r+2}, \dots, {}_{2r-1}$. We now assign the label 2 to the r vertices $w_{2r}, w_{2r+1}, \dots, w_{3r-1}$ and assign the label 3 to the r vertices $w_{3r}, w_{3r+1}, \dots, w_{4r-1}$. Consider the vertices $v_1, v_2, \dots, v_n$. Assign the label 0 to the r vertices $v_1, v_2, \dots, v_r$. Next we assign the label 1 to the r vertices $v_{r+1}, v_{r+2}, \dots, v_{2r}$. Now we assign the label 2 to the $r-1$ vertices $v_{2r+1}, v_{2r+2}, \dots, v_{3r-1}$. Now we assign the label 3 to the $r-1$ vertices $v_{3r}, v_{3r+1}, \dots, v_{4r-1}$. Assign the label 0 to the r vertices $x_1, x_2, \dots, x_r$ and $y_1, y_2, \dots, y_r$. Next we assign the label 1 to the r vertices $x_{r+1}, x_{r+2}, \dots, x_{2r}$ and $y_{r+1}, y_{r+2}, \dots, y_{2r}$. We now assign the label 2 to the r vertices $x_{2r+1}, x_{2r+2}, \dots, x_{3r}$ and $y_{2r+1}, y_{2r+2}, \dots, y_{3r}$. Finally we assign the label 3 to the $r-1$ vertices $x_{3r+1}, x_{3r+2}, \dots, x_{4r-1}$ and $y_{3r+1}, y_{3r+2}, \dots, y_{4r-1}$.

Case 2. $n \equiv 1 \pmod 4$.
Let $n = 4r + 1$, $r \ge 1$.
Label the vertices u_i $(1 \le i \le 4r)$ and v_i, x_i, y_i, w_i $(1 \le i \le 4r-1)$ as in Case 1. Finally assign the labels 2, 0, 3, 1, 0 respectively to the vertices u_{4r+1}, w_{4r}, v_{4r}, x_{4r}, y_{4r}.

Case 3. $n \equiv 2 \pmod 4$.
Let $n = 4r + 2$, $r \ge 1$.
As in Case 2, assign the label to the vertices u_i $(1 \le i \le 4r+1)$ and v_i, x_i, y_i, w_i $(1 \le i \le 4r)$. Next we assign the labels 3, 0, 0, 1, 3 respectively to the vertices u_{4r+2}, w_{4r+1}, v_{4r+1}, x_{4r+1}, y_{4r+1}.

Case 4. $n \equiv 3 \pmod 4$.
Let $n = 4r + 3$, $r \geq 1$.
In this case, assign the label for the vertices u_i $(1 \leq i \leq 4r + 2)$ and v_i, x_i, y_i, w_i $(1 \leq i \leq 4r + 1)$ as in Case 3. Finally we assign the labels 2, 1, 3, 2, 0 respectively to the vertices u_{4r+3}, w_{4r+2}, v_{4r+2}, x_{4r+2}, y_{4r+2}.

Thus this vertex labeling f is a 4-total mean cordial labeling of $S(T_n)$ follows from the Tabel 8

Case 5. $n = 2, 3$.
A 4-total mean cordial labeling of $S(T_n)$ is given in Tabel 9

□

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n	2	3	4	5	6	7
u_1	0	0	0	0	0	0
u_2	0	1	1	1	1	1
u_3		2	2	2	2	2
u_4			3	3	3	3
u_5				1	1	1
u_6					0	0
u_7						0
v_1	3	0	0	0	0	0
v_2	3	1	1	1	1	1
v_3		2	2	2	2	2
v_4			3	3	3	3
v_5				3	3	3
v_6					1	1
v_7						3
y_1	0	0	0	0	0	0
y_2	2	1	1	1	1	1
y_3		3	2	2	2	2
y_4			3	3	3	3
y_5				0	0	0
y_6					2	2
y_7						0
x_1	1	0	0	0	0	0
x_2		3	2	2	2	2
x_3			2	2	2	2
x_4				0	0	0
x_5					0	0
x_6						1
z_1	3	2	0	0	0	0
z_2		3	0	0	0	0
z_3			3	3	3	3
z_4				3	3	3
z_5					3	3
z_6						3

Table 7:

Order of $S(T_n)$	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n = 4r$	$11r - 2$	$11r - 3$	$11r - 2$	$11r - 3$
$n = 4r + 1$	$11r + 1$	$11r$	$11r$	$11r$
$n = 4r + 2$	$11r + 3$	$11r + 3$	$11r + 3$	$11r + 3$
$n = 4r + 3$	$11r + 6$	$11r + 5$	$11r + 6$	$11r + 6$

Table 8:

Value of n	u_1	u_2	u_3	w_1	w_2	v_1	v_2	x_1	x_2	y_1	y_2
2	0	1		3		0		2		2	
3	0	0	0	3	3	0	2	2	2	2	2

Table 9: