

# Optimization of horizontal drain dimensions in homogeneous earth dams using neural network

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## ABSTRACT

Designing and optimizing the dimensions of drainage systems is very important for keeping the downstream shells dry and preventing the increase of pore water pressure in the body of earth dams. By optimizing the drainage dimensions, the minimum factor of safety, and consequently the construction costs, can be reduced. The purpose of this research was to optimize the size of horizontal drainage that is affected by some important parameters of the dam. In this study, a homogeneous earth dam was modeled using the Geostudio software. The minimum factor of safety was obtained by changing drainage dimensions, materials, and the slope of the dam body. A two-layer neural network was used to predict the least factor of safety resulted from different scenarios created in the software. By training the neural network based on the data obtained from homogeneous dams, the minimum factor of safety for drainage optimization was extracted. For optimal, an Mfile was fitted to the trained neural network function, by which the optimal values of the dam parameters were calculated. The results showed that the optimum values of drainage dimensions obtained for homogeneous dams for three heights of 10, 20, and 30 m could be generalized to other heights between 10 and 30 m with a simple interpolation.

**Keywords :** Horizontal drainage, Homogeneous dam, Minimum Factor of safety, Optimization

## 1. Introduction

A drainage system is typically used to guide the flow of water in earth dams. The construction of a horizontal drain for collecting and guiding water drainage formed by the dam is intended to keep the bottom slope of the downstream dam dry and prevent the increase in pore water pressure in the body of the dam. This type of drain is one of the most efficient drains, whose main task is to collect and drain water from the body of the earth dam. Also, this type of drainage, while stabilizing the earth dam body, should also have minimum dimensions. Regarding this, it seems that finding a way to calculate the position of dimensions and the optimal shape of the horizontal drain in the earth dams is of great economic and technical importance. In order to achieve a suitable drainage plan, different design parameters such as drain length and width, number of drains, and the location for these drains should be taken into considerations. If the optimal dimensions are found, the performance of these dams will be improved, and in terms of costs, the construction of the dam will be very short and cost-effective. Tesarik & Kealy (2005) modeled a number of earth dams, providing graphs for the estimation of the horizontal drainage function in lowering the phreatic Line. Based on these graphs, the researchers analyzed two earth dams without drainage and with horizontal drainage (drainage of specified length), using computer and laboratory models [1]. Xu et al. (2002) investigated the optimization of earth dams with two different materials by phreatic Line drop [2]. Chahar (2004) has determined the location of the phreatic Line of the CASA Grande drawing, and in this way, has estimated the minimum and maximum horizontal drain lengths in homogeneous earth dams [3]. Mishra and Singh (2005) studied the flow of water within the embankment, the phreatic Line position, the minimum distance between the free path and the downstream slope,

and considered the position and the effective length of the horizontal drain, taking into account the water capillary effect in the soil [4]. Alonso & Pinyol (2008) presented the problem of water level reduction as a flow deformation problem for saturated/unsaturated conditions. They discussed the role of different soil properties in explaining the phenomena that occur at the time of discharge [5]. Najafpour et al. (1993) implemented a physical model of a homogeneous earth dam in a laboratory flume with the aim of determining the optimal and anomalous angle in designing the toe drain. Then, the pore water pressure in the flume body and the volume of leakage discharge were measured volumetrically using the piezometers they had made. Finally, the pressure and flow rates measured in the laboratory were compared with those of the Plaxis software using the SAS statistical software at 95% confidence level, 45 and 60-degree angles, and  $P/h = 0.35$  as the optimal criteria for designing the toe drain design [6]. Malekpour et al. (2012) studied the effect of horizontal drain length and thickness on steep gradient stability under Drawdown conditions in a homogeneous earth dam [7]. Lowe & Karafiath (1980), Baker et al. (1993), and the American Army Corps of Engineers at (2003) are among those who have studied the stability of slopes in non-drained conditions [8, 9 and 10]. Also, Svano & Nordal (1987), Wright and Duncan (1987), Lane & Griffiths (2000), and Berilgen (2007) to investigate the stability of soil gradients after surface changes Reservoir water examined the effective shear strength parameters of the soil under drainage conditions. The research emphasizes the importance of the role of drainage as one of the sustainability factors of the slopes of the earth [11, 12, 13, and 14]. Some researchers, including Duncan and Wright (2005), have emphasized the role of the T parameter, which is called the time without the dimension of consolidation, in determining the drainage state (drainage behavior) and the earthy slope stability. According to their research, more than 99 percent of the permeate overpressure from the reservoir drop is

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depreciated over  $3 \geq T$ , and for longer periods, the drainage behavior should be taken into account in the analysis of the gradient stability [15]. By measuring and installing drainage upstream of dams, Zomoradiyan & Abdollahzadeh (2012) measured the reduction of pore water pressure and calculated the rate of improvement in the factor of safety and stability [16]. Bahrehbor et al. (2017) conducted some experiments on drainage systems in earth dams to select the most appropriate type of drainage system to prevent the leakage and internal erosion of the dam body. To analyze the leakage conditions, they tested a laboratory model of a homogeneous earth dam in several types of drainage systems with sealing in different conditions. The results showed that the average length and thickness of the toe drainage system was 18.55 m/day in the laboratory flume and 17.55 m/day in the Plaxis software. The results also showed that the toe drainage system had the best slip stability with a factor of safety of 1.656 [17]. Kalantari & Nazeri (2016) investigated the stability of the dam with the specification of materials used in the dam. They showed the effect of different material quality on the stability of earth dams by modeling using the Geostudio software [18]. Yazdaniyan & Afshon (2016) investigated the effect of altitude on the static standing of heterogeneous earth dams. Their models considered heterogeneous earth dams with the same material characteristics and different heights, as well as fast leakage and different permeability values. They modeled two models with different heights of 62 and 133 cm with a crest width of 6 cm in the Geostudio software. After comparing the results of the analysis of both models, it was found that the factor of safety for shorter dams was higher than the taller dams [19]. Fattah et al. (2017) studied the saturation phenomena in earth dams using the Geostudio software. Their results showed that during the rapid discharge of a reservoir, the pore water pressure in the dam body decreases linearly, indicating a steady-state flow [20]. Darabi et al. (2017) investigated the effect of drainage geometry on the dynamic response of a homogeneous dam. Their results showed that the vertical chimney drainage caused by the earthquake produced less pore water pressure than the oblique drainage [21]. In this study, the Marvak earth dam is modeled with real parameters using the Geostudio software, and a minimum coefficient of reliability for the dam is obtained with changing the parameters that affect the factor of safety, including the dimensions of drainage, material specifications, and the slope of the body. Then, horizontal drain dimensions will be optimized for different scenarios of software results using the horizontal neural network. Also, the percentage effect of different parameters of the dam in the factor of safety will be minimized.

## 2. Materials and methods

### 2.1. Drainage system

Drainage systems in the body and foundation of earth dams are designed to collect and direct leakage of water to downstream areas. In terms of shape and position of the drainage system in the dam body, various methods such as horizontal drain, toe drain, and chimney drain have been established so far. The drainage system should be such that the leak line at the bottom of the dam is completely controlled, and the bottom shell is always in a dry condition. Also, by stabilizing the earth dam body, it also has minimum dimensions. In almost all homogeneous dams, drainage should be used to reduce the water-free surface (phreatic line), increase downstream slope stability, control the downstream outlet water flow, and prevent piping. The typical kinds of drains in homogeneous earth dams are shown in Fig. 1.

The dimensions and permeability of the drainage shall be such that it can transfer the calculated water flow rate with a sufficient factor of safety. The permeability coefficient of the drainage material should preferably be 100 times the permeability coefficient of the drainage material. Chimney drainage is the best type of drainage, and it is used in most major dams. Toe drain is used in dams with a height of less than 20 meters. In this study, horizontal drainage was used in the homogeneous dam model. This type of drain is located down the slope and is located at the base of the dam, which directs all the branches inside the body and the main part of the foundation.

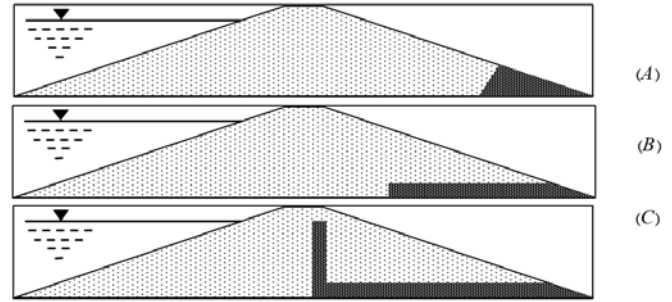


Fig. 1. Drainage used in homogeneous dams: (A) Toe drain, (B) horizontal drain, (C) chimney drains.

Another role that a horizontal drain may have is that the foundation is reinforcing and permeable layer. In this case, the presence of a layer of horizontally drained can facilitate and accelerate the lowering of the bottom layer due to the barrier load. Horizontal drains are used in dams with a height of more than 20 meters. When using these drains, it should be noted that due to the nature of the thickened soils during the construction of the dam, the horizontal permeability of these soils can be much larger than its permeability, and there is a risk that in dense soils, the flow of water inside the dam does not pass through the drain, and this type of drainage will lose its efficiency.

## 3. Modeling Homogeneous Dams in Geostudio

Three homogeneous models were used for measuring 10, 20, and 30 meters. The width of the dam, according to the USBR, was selected for short dams up to 30 meters (1).

$$B = \frac{H}{5} + 3 \quad (1)$$

In this equation, B is the crest width, and H is the height of the dam. These factors are affecting the factor of safety, the slope of the dam, the angle of friction of the dam materials (the permitted range of variations from 27 to 35), and drainage (the allowed range of variations is 33 to 40). The slope of the body is considered based on the type of soil materials, foundation condition, and height of the dam. Usually, the coarse-grained earthy material has a higher slope, and the finer the slope, the lower the slope. The slope of the slopes is usually from 1:2 to 1:3. Obviously, by decreasing the slope of the dam body, the volume of earth operations will be reduced, and the cost of implementation will be reduced accordingly. Table 1 shows the different states for the homogeneous dam.

Table 1. Dimensions for the homogeneous dam.

Slope	Length of foundation (m)	Crest width (m)	Height (m)
1:2	45	5	10
1:2	86	7	20
1:3	126	7	20
1:3	62	5	10
1:2	130	9	30
1:3	189	9	30

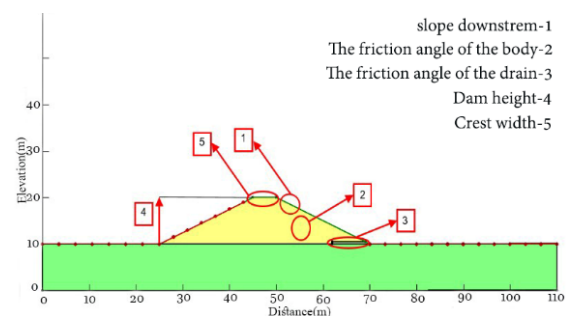


Fig. 2. Homogenous dam modeling in the Geostudio software.

The Mohr-Coulomb behavioral model was used to model the homogeneous dam. Finally, the best possible model with a height of 10 meters, a crest width of 5 meters, a length of 45 meters, and a slope of 1:2 was considered for modeling. As shown in Fig. 3, the factor of safety of the model was 1.818.

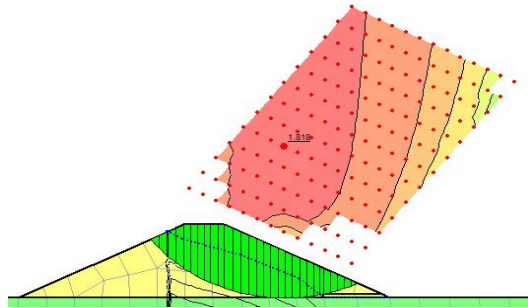


Fig. 3. A minimum factor of safety for the homogeneous dam model.

#### 4. Homogeneous dam factor of safety using neural networks

But the use of neural networks, one can calculate the factor of safety of homogeneous dams for different parameters of the dam without having to use the Geostudio software. In this research, the Toolbox Network was used to teach the neural network. The neural network used for the homogeneous dam had two layers, six inputs, and one output. The neural network's inputs included the internal friction angle of the body of the dam, the internal friction angle of the drainage material, the slope of the dam, the thickness of the drain, the height of the dam, and the length of the drain. Also, the output of the neural network was equal to the reliability of the dam. When training the neural network, only 70% of the data was used, and the remaining 30% of the data was used to test the network. During the training process, the neural network of the homogeneous dam was used from 35 neurons in the first layer and one neuron in the second layer. It should be noted that the number of neurons in the first layer was defined based on trial and error so that the trained neural network had the lowest number of neurons and the highest accuracy in estimating the data. In addition, the presence of a low neuron has the advantage that the neural network function that is obtained after training will be able to analyze the dam factor of safety in the shortest possible time. The training algorithm was used for the Bayesian Regularization Neural Network, which has a longer running time and more accuracy in neural network training than other training algorithms. Neural network training is stopped after 1000 iterations. The R and MSE parameters of the training process indicate that the value of R is very close to one, and the MSE value is very close to zero, indicating the proper training of the neural network. The Toolbox Network charts of the MATLAB neural network are shown in Figs 4 and 5 for dam data.

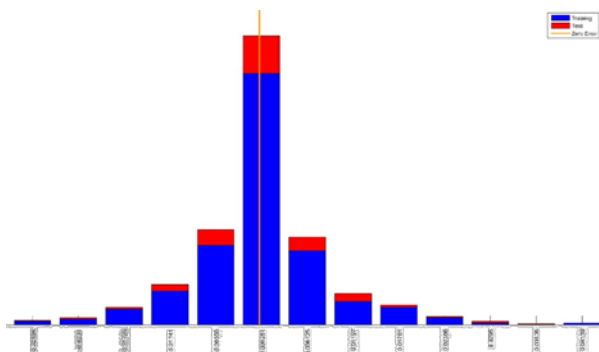


Fig. 4. Shows the error histogram of the homogeneous dam data.

Figs 4 and 5 show the proper training of the dam's neural network. Fig. 4 shows the frequency of learning errors for different error values. As seen, the training error is close to zero for most of the data. Also, in

Fig. 5, the data have a very high linear correlation. Once the training process of the neural network is completed, we extract the function of the neural network of the homogeneous dam. This function takes six inputs for the homogeneous dam and calculates the damping coefficient.

The factor of safety values calculated by the neural network represents only +0.0031 (equivalent to +0.147, i.e., Less than 1%) of the error in calculating the dam factor of safety. After training the dam's neural network, the extracted function is used to optimize the dimensions of the drain. It should be noted that the task of the neural network is to optimize the production of a dam factor of safety.

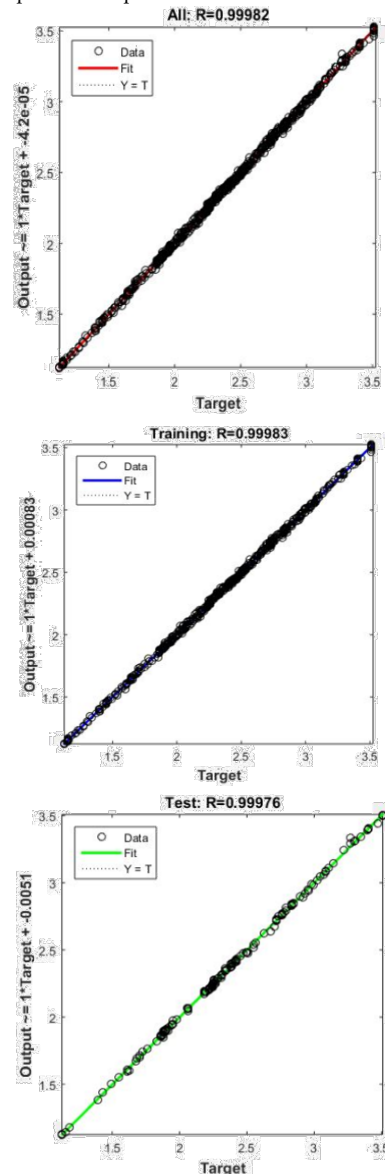


Fig. 5. Regression diagram for the homogeneous dam data.

#### 5. Optimization of homogenous dam parameters

Due to the high cost of constructing the dams, a minimum factor of safety should be considered. Since several parameters affect the dam factor, it is preferable that the minimum factor of safety of the dam is to be met by parameters that cost-effective for dam construction. Dam gradient is one of the important parameters in the dam factor of safety, but providing the minimum dam factor of safety with this parameter dramatically increases the costs of dam construction. The dimensions of dam drainage are one of the factors affecting the dam factor of safety, but less effective than the slope dam. If it is possible to change the

dimensions of the drain, a minimum factor of safety of the dam is provided, and then the cost of constructing the dam decreases dramatically. In this research, a minimum factor of safety was considered for the homogenous dam, and the drain for the dam has attempted the minimum factor of safety. Therefore, an Mfile program was written in MATLAB that fully complied with the neural network function of homogenous dams and was able to provide a minimum factor of safety of the dam so that, in addition to reducing the cost of dam construction, the dam drain dimensions was also optimized. The reason for not using MATLAB optimization towers was the existence of some limitations in the data available for training the neural network. The program was composed of 5 sections, as follows:

- **First Section:** In this section, the allowed range of the neural network inputs for homogenous dams is firstly determined, and then, the neural network produces dam construction factor of safety.
- **Section 2:** Among the confidence factor of safety, the factor of safety is chosen in a way to be within the range of  $F_w + 0.2F_w$ , where  $F_w$  is the minimum required factor of safety of the dam. The reason for considering 20% Tolerance for  $F_w$  is that the input and output data of the neural network are discrete, and the exact amount of  $F_w$  may not be achievable.
- **Section 3:** Among the data in the second section, the data are selected to provide the maximum possible slope of the dam, as the slope of the dam slows down dramatically. If such data are not available, the slope of the dam should be reduced to the point where the coefficient of confidence would be 2.
- **Section 4:** Data from the third section is selected for the data that is the product of the drain length in the drainage thickness minimum. In this case, from the mentioned four stages, a data set is provided that can reduce the volume of ground operations (by maximizing the slope of the dam), and optimizes the drainage dimensions of the dam as well. Also, the minimum factor of safety required for the dam is provided. In some cases, the program writes a zero-length for a dam drain, which means that there is no need for drainage to provide a minimum factor of safety of the dam. The drainage should be used to drain water from the dam. Therefore, in this case, we performed the process according to the fifth section.
- **Section 5:** In this section, the minimum effective drain length is selected as the optimal drain length.

In Table 2, the use of the operator  $0 \rightarrow b$  means that, in this case, no drainage is required to provide the minimum factor of safety. For this reason, the minimum effective drain length of  $b$  meters is selected as the drain length. After calculating the optimum values of the homogeneous barrier parameters using MATLAB neural network and coding, the modeling results of the Geostudio software were compared with those of the neural network. The comparison results show a very small error and generally a very good fit for neural network and software modeling. The error can also be due to a neural network training error, but the error is very negligible. If the height of the homogeneous dam is 10 meters, then the dam parameters are in accordance with Table 3.

**Table 2.** Optimization of homogenous dam parameters using MATLAB neural network and coding.

Factor of safety	Drain thickness (m)	Drain length (m)	Slope dam	Dam height (m)	The friction angle of the drain (degree)	Friction angle of body (degree)
2.0330	0.5	12.5	0.5	10	33	27
2.0020	0.5	16.5	0.334	10	33	27
Always less than 2	-	-	0.5	20	33	27
2.0050	1	33	0.334	20	33	27
2.2733	1	0→33	0.5	30	33	27
2.0140	1	52	0.334	30	33	27
2.3290	0.5	0→8	0.5	10	40	33
2.7610	0.5	0→15	0.334	10	40	33
2.0012	1	25	0.5	20	40	33
2.4378	1	0→27	0.334	20	40	33
2.2450	1.5	0→14	0.5	30	40	33
2.3690	1	0→45	0.334	30	40	33

**Table 3.** Optimal parameters of the homogeneous dam with a height of 10 meters.

Factor of safety	Drain thickness (m)	Drain length (m)	Slope dam	Dam height (m)	The friction angle of the drain (degree)	Friction angle of body (degree)
2.3290	0.5	0→8	0.5	10	40	33

For a homogeneous dam with a height of 10 meters, the minimum required factor of safety is provided, the slope of the dam is selected as high as possible, and the drainage dimensions of the dam are minimal, all of which means lower construction costs.

- If the height of the homogeneous dam is 20 meters, then the dam parameters are in accordance with Table 4.

**Table 4.** Optimal parameters of the homogeneous dam with a height of 20 meters.

Factor of safety	Drain thickness (m)	Drain length (m)	Slope dam	Dam height (m)	The friction angle of the drain (degree)	Friction angle of body (degree)
2.0012	1	25	0.5	20	40	33

For a homogeneous dam with a height of 20 m, the minimum required factor of safety is provided, the slope of the dam is selected as high as possible, and the dimensions of the dam drainage are minimal, all of which means lower construction costs.

- If the height of the homogeneous dam is 30 meters, then the dam parameters have optimal values according to Table 5.

**Table 5.** Optimal parameters of the homogeneous dam with a height of 30 meters.

Factor of safety	Drain thickness (m)	Drain length (m)	Slope dam	Dam height (m)	The friction angle of the drain (degree)	Friction angle of body (degree)
2.2450	1.5	0→14	0.5	30	40	33

In a homogeneous dam with a height of 30 meters, the minimum factor of safety of the dam is provided, the slope of the dam is at the maximum possible size, and the dimensions of the drainage of the dam are also minimal.

### 6. Interpolation of optimal drainage area in the homogeneous dam

In a homogeneous dam with a height of 10 meters, the optimal drainage dimensions are  $0.5 \times 12$  m, and for a height of 30 m, the optimal dimensions of the drain or equal to  $1 \times 30$  m. With a simple interpolation, we found out that the ratio of dam height with optimal drainage dimensions is in the form of relation (2).

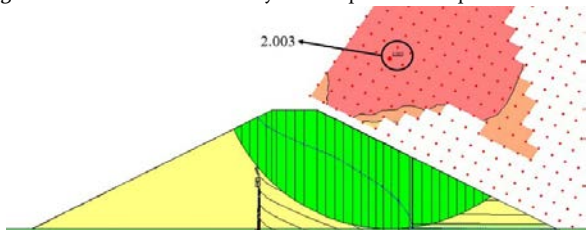
$$A = 1.2H - 6 \tag{2}$$

In equation (2), the coefficient  $H$  is 1.2, where  $h$  is the height of the dam, and  $A$  is the drainage area of the dam. For example, by using the equation (2), the optimal drainage dimensions of the dam for the heights of 17 and 25 meters will be 14.4 and 24 square meters, respectively. Also, the optimal areas should provide at least a factor of safety of 2 for a homogeneous dam. Then, using the Geostudio software, dams with the heights of 17 and 25 meters are modeled. The drainage area can be calculated using the software, and the try and the error should be used to achieve the drain dimensions. Finally, the optimal dimensions were  $0.75 \times 20.25 = 15.25$  m/s for a homogeneous dam with a height of 17 m and  $95 \times 28 = 26.6$  m/s for a homogeneous dam with a height of 25 m. The optimal precision dimensions and optimal dimensions are close to each other, and at the same time, the accurate precision dimensions are capable of providing a minimum factor of safety of 2 for the dam. The results of the Geostudio software are shown in Figs 6 and 7.

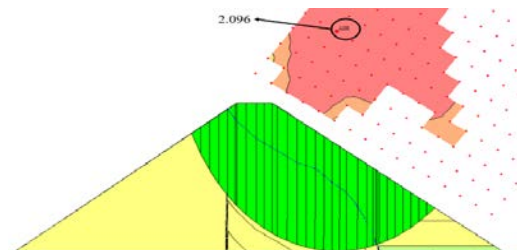
### 7. Conclusion

In this study, the main factors affecting the homogeneous dam factor

of safety were investigated using the Geostudio software and neural networks. For this purpose, a trained neural network function was used to generate a dam factor of safety in the optimization problem.



**Fig. 6.** Homogeneous dam with a height of 17 m with drainage with dimensions of  $0.75 \times 20.25 = 15.2 \text{ m}^2$ .



**Fig. 7.** Homogeneous dam with a height of 25 m with a drain with dimensions of  $0.95 \times 28 = 26.6 \text{ m}^2$ .

The purpose of optimization in this study was to minimize the dimensions of dam drainage by reducing the factor of safety, and consequently, reducing the construction costs. In optimization, reducing the cost of dam construction over-optimizing dam drainage is a priority. To provide the minimum dam factor of safety (which was selected equal to 2 in this study), the slope of the dam should be as high as possible, and the minimum dimensions should be selected for dam drainage under such conditions. This will further reduce the cost of dam construction. In all cases, the maximum possible slope of the dam is permissible to provide the minimum factor of safety of homogeneous dams. The results show that the optimal values of drainage dimensions for homogeneous dams for three heights of 10, 20, and 30 m can be generalized to other heights between 10 and 30 m with a simple interpolation.

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