

**The Gnomonic Application of  
Sharaf al-Dīn al-Ṭūsī's Linear Astrolabe**

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**Abstract**

The linear astrolabe was invented by Sharaf al-Dīn al-Ṭūsī (fl. c. 1180) and although several Arabic monographs on the subject have reached us, none have received sufficient scholarly attention. As a result, one of the most important features of Ṭūsī's invention, the gnomonic application, remains unknown. Based on an analysis of all known monographs on the linear astrolabe, this paper will discuss the gnomonic application of the linear astrolabe, which is an alternative to the azimuth lines typical of the plane astrolabe. We will also consider the potential role of Ṭūsī's pupil, Kamāl al-Dīn ibn Yūnus (d. 1242), in the improvement of this instrument. Finally, after surveying the available manuscript copies of the above-mentioned monographs, we present a full edition and translation of one such text and an edition and translation of one chapter, on the gnomonic application, from another text.

**Keywords:** Astrolabe, Gnomon, Kamāl al-Dīn Ibn Yūnus, Linear astrolabe, Sharaf al-Dīn al-Ṭūsī, Sundial.

## Introduction

The linear astrolabe is an astronomical instrument invented by Sharaf al-Dīn al-Ṭūsī, a prominent mathematician, astronomer, and philosopher of the 12<sup>th</sup> century, to replace the regular plane astrolabe. The instrument is known in the literature as “al-Ṭūsī’s staff” (‘*aṣā al-Ṭūsī*). Several Arabic monographs on the linear astrolabe have reached us, two of which are attributed to Ṭūsī, one shorter than the other.<sup>1</sup> Apart from their length, the main difference between these texts is that the longer treatise describes a gnomon which is added to the linear astrolabe. This gnomon is an alternative to the azimuth lines of the plane astrolabe. The first modern study of the linear astrolabe was Carra de Vaux’s edition and French translation of the sections devoted to the linear astrolabe in Abū al-Ḥasan al-Marrākushī’s (fl. second half of thirteenth century) *Jāmi‘ al-mabādi’ wa-al-ghāyāt* (Combination of principles and objectives).<sup>2</sup> Since then, Marrākushī’s text has become the standard source for studies on the linear astrolabe. However, while Marrākushī included the gnomonic application in his description of the instrument, no study on the linear astrolabe has analyzed this feature. Henri Michel, who studied this instrument and constructed a model of it, offered only the following comments on the gnomonic application:<sup>3</sup>

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1. As we will discuss later, there is a revision of Ṭūsī’s longer treatise that is attributed to Ṭūsī himself, an abridgment of Ṭūsī’s longer treatise, and an anonymous treatise on the linear astrolabe.

2. Marrākushī’s work was known in Europe through its French translation published by J. J. Sédillot (in 1834) and L. A. Sédillot (in 1841) based on a manuscript in the Bibliothèque nationale de France. The section on the linear astrolabe was among the part translated by the latter who erroneously attributed its invention to Naṣīr al-Dīn al-Ṭūsī (d. 672/1274). H. Suter, in his article on Jacob’s staff (published in 1895), mentioned the linear astrolabe due the similarity of the title of these two instruments (both of them have “staff” in their title). Carra de Vaux, correcting the misattribution, edited and translated Marrākushī’s text (based on the same BnF MS). See J. J. Sédillot, *Traité des instruments astronomiques des Arabes*, 2 vols. (Paris: Imprimerie Royal, 1834-1835); L. A. Sédillot, *Mémoire sur les instruments astronomiques des Arabes* (Paris: Imprimerie Royal, 1841); H. Suter, “Zur Geschichte des Jakobsstabes,” *Bibliotheca Mathematica* (Neue Folge) 9 (1895): 13-18; M. Le Baron Carra de Vaux, “L’Astrolabe linéaire ou Bâton d’Et-Tousi,” *Journal Asiatique* 5(3) (mai-juin 1895): 464-516.

3. Henri Michel, “L’astrolabe linéaire d’al-Tūsī,” *Ciel et Terre* 59 (1943): 101-107, reprinted in *Islamic Mathematics and Astronomy*, ed. Fuat Sezgin (Frankfurt am Main: Institute for the History of Arabic-Islamic Science, 1998), 94: 331-337. The translation of this passage follows: “Finally, a “scale of the shadow” gives the length of the shadow of a gnomon directed parallel to the axis of the Earth, on a plane parallel to the Equator, at various times of the year. This scale is used mainly for the establishment of a sundial.”

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Enfin une «échelle des ombres» donne la longueur de l'ombre d'un style orienté parallèlement à l'axe de la Terre, sur un plan parallèle à l'Equateur, aux divers moments de l'année. Cette échelle sert principalement à l'établissement d'un cadran solaire.

This description is not precise enough since, as we shall show, the gnomonic application, which was apparently inspired by the equatorial sundials, is one of the most innovative, yet not the most practical, features of Ṭūsī's linear astrolabe as an alternative to the plane astrolabe's azimuth lines. Even in the recent publications related to the topic, such as those by James E. Morrison<sup>1</sup> and Massimo Goretti,<sup>2</sup> this application is neglected.

In what follows, we will give an overview of Ṭūsī's life and the events related to the linear astrolabe and we will also discuss the potential role that Kamāl al-Dīn Ibn Yūnus (d. 639/1242) played in the improvement of the instrument. We will then discuss the gnomonic application of the linear astrolabe, and conclude the paper with a complete edition and translation of Ṭūsī's shorter treatise, as well as an edition of a chapter of the longer treatise.

### Ṭūsī and Kamāl al-Dīn Ibn Yūnus

No biography of Ṭūsī is known in the primary sources and our information about his life comes only from short biographical notices.<sup>3</sup> Al-Qiftī (d. 642/1245),<sup>4</sup> Ibn Abī Uṣaybi'a (d. 668/1269-1270),<sup>5</sup> Ibn

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1. James E. Morrison, *The Astrolabe* (London: Janus Publishing, 2006), 303-309.

2. Massimo Goretti, "The linear astrolabe of al-Ṭūsī-Gnomonic Application," *The Compendium, Journal of the North American Sundial Society*, 2010, vol. 17, no. 2. Although Goretti mentions the gnomonic application in the title of his article, he does not actually discuss the gnomon of the linear astrolabe in the article.

3. For the modern account of his life see Adel Anboubā, "Al-Ṭūsī, Sharaf al-Dīn al-Muẓaffar ibn Muḥammad ibn al-Muẓaffar," *Dictionary of Scientific Biography*, ed. Charles Coulston Gillispie (New York: Charles Scribner's Sons, 1981), 13:514-517; Roshdi Rashed, *Sharaf al-Dīn al-Ṭūsī Oeuvres Mathématiques, Algèbre et Géométrie au XIIe Siècle*, 2 vols. (Paris: Société d'édition «Les Belles Lettres», 1986), 1:XXXII-XLI (French), VIII-XIII (Arabic).

4. Ibn al-Qiftī, 'Alī ibn Yūsuf, *Ta' rīkh al-Ḥukamā'*, ed. Julius Lippert (Leipzig: Dieterich'sche Verlagsbuchhandlung, 1903), 426.

5. Ibn Abī Uṣaybi'a, *Uyūn al-anbā' fī ṭabaqāt al-aṭibbā'*, 2 vols, ed. August Mueller (Cairo: al-Maṭba'ah al-wahbiyya, 1882) 2: 182 (Ibn al-Ḥājib), 2:191 (al-Ḥārithī).

Khallikān (d. 681/1282),<sup>1</sup> and al-Subkī (d. 771/1370)<sup>2</sup> mention him in some of their entries, describing him as a tutor. Adel Anbouba's entry in the *Dictionary of Scientific Biography* is one of the best modern accounts of Ṭūsī's life based on these sources.<sup>3</sup> Here we discuss a few important events in Ṭūsī's life related to our main subject.

Ṭūsī's full name was Sharaf al-Dīn al-Muẓaffar ibn Muḥammad ibn Muẓaffar. In different periods of his life he lived in Ṭūs, Damascus, Aleppo, and Mosul. In addition to the two treatises on the linear astrolabe, two other short texts attributed to him are extant: Ṭūsī's response to a question on geometry<sup>4</sup> and a two-page text on the asymptote problem.<sup>5</sup> The latter may be a part of a lost work, known today as *Kitāb fī al-mu'ādilat* (The book on [algebraic] equations), which has been considered his most important work. The content of this work is only known through an abridgment made by an anonymous author.<sup>6</sup> In addition to the mathematical sciences, he is also known for his knowledge of philosophy.

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1. Ibn Khallikān, Shams al-Dīn Aḥmad ibn Muḥammad, *Wafayāt al-a'yān wa anbā' abnā' al-zamān*, ed. Iḥsān 'Abbās (Beirut: Dār Ṣādir, 1977), 5:314, 6:52-53.

2. Al-Subkī, Ṭāj al-Dīn 'Abd al-Wahhāb ibn 'Alī, *Ṭabaqāt al-Shāfi'iya al-kubrā*, ed. Maḥmūd Muḥammad al-Ṭanāḥī and 'Abd al-Fattāḥ Muḥammad al-Ḥilw (Cairo: Dār Iḥyā' al-Kutub al-'Arabiyya, no date), 8:386.

3. Adel Anbouba, "Al-Ṭūsī, Sharaf al-Dīn," 514-517.

4. Two copies of this text are extant: Colombia University, Smith, or. 45, ff. 15b-18b; Leiden or. 14, pp. 322-327. The latter probably is a direct copy of the former. The text has been edited by Roshdi Rashed in *Sharaf al-Dīn al-Ṭūsī Oeuvres Mathématiques*, 2: 137-143. The incipit of the first copy says: "mas'alatun sa'alahā Shamsu al-Dīni Amīru al-Umarā' 'i al-Nizāmiyyati 'an al-Imāmi ... Sharafī al-Dīni Bahā' 'i al-Islāmi Ḥujjati al-Zamāni Muẓaffari ibn Muḥammadi al-Muẓaffari al-Ṭūsī ... bi-baladi Hamadāna sanati <...> wa-ḥamsami 'ati hijriyyati" (a problem asked by Shams al-Dīn Amīr al-Umarā' al-Nizāmiyya from Imām ... Sharaf al-Dīn al-Ṭūsī in the city of Hamadan). The copyist only wrote the century when dating the treatise and omitted the number of the year. The name Shams al-Dīn in this incipit probably is referring to Shams al-Dīn Muḥammad al-Pahlawān ibn Ilduguz (r. 568-582/1172-1186), the ruler of central Persia. Before succeeding his father in 568, Shams al-Dīn was appointed as the commander of the armies. It seems the title Amīr al-Umarā' is referring to this position. Hence, the treatise probably was written before 568. See Ibn Athīr, *al-Kāmil fī al-tārīkh*, ed. Muḥammad Yūsuf al-Diqāqa (Beirut: Dār al-kutub al-'ilmiyya, 2003), 10:8-9, 46-47; Bosworth, C.E., "Ildēnizids or Eldigūzids", *Encyclopaedia of Islam*, Second Edition, Edited by: P. Bearman, Th. Bianquis, C.E. Bosworth, E. van Donzel, W.P. Heinrichs (Leiden and London: Brill, 1986), 3:1110-13.

5. Two copies of this text are extant: Istanbul, Süleymaniye Kütüphanesi, Ayasofya, MS 2646; Baghdad, Muḥaf, MS 30129.

6. This abridgment is edited by Roshdi Rashed in *Sharaf al-Dīn al-Ṭūsī Oeuvres Mathématiques*.

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Ibn Abī Uṣaybi‘a, in his entry for a certain Muḥadhdhab al-Dīn ibn al-Ḥājib (d. c. 619/1222), says that when Sharaf al-Dīn al-Ṭūsī was in Mosul, Ibn al-Ḥājib went there only to find Ṭūsī already taking leave for Ṭūs (a region and a medieval city in east north Persia). Unable to study with him, Ibn al-Ḥājib stayed in Mosul for a while and then went to the neighboring city of Irbil to study with Abū Shujā‘ Fakhr al-Dīn Muḥammad ibn ‘Alī ibn Shu‘ayb, known as Ibn al-Dahhān (d. 590 or 592/1194 or 1196),<sup>1</sup> with whom he studied Ibn al-Dahhān’s *zīj*.<sup>2</sup> If Şeşen is correct about the attribution of the *zīj* in Kütahaya, MS 2841, to Ibn al-Dahhān,<sup>3</sup> we can estimate the date of Ṭūsī’s travel. The *zīj*, probably called *al-Dustūr al-kāfi* (The sufficient canon),<sup>4</sup> was written around 571/1175<sup>5</sup> for the latitude of Baghdad. Hence Ibn al-Ḥājib’s travel to Irbil must have taken place sometime after 571/1175, shortly after Ṭūsī’s travel from Mosul to Ṭūs. Subkī seems to be referring to Ṭūsī’s travel to Ṭūs when he quotes Kamāl al-Dīn ibn Yūnus (d. 639/1242) as saying that:<sup>6</sup>

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1. About Ibn al-Dahhān see ‘Alī Rafī‘ī, “Ibn al-Dahhān Muḥammad ibn ‘Alī ibn Shu‘ayb,” *Dā‘irat al-Ma‘ārif-i Buzūrg-i Islāmī* (Persian), vol. 3 (Tehran: Markaz-i Dā‘irat al-Ma‘ārif-i Buzūrg-i Islāmī, 1369sh/1996), 13:522-523.

2. Here is the translation of Ibn Abī Uṣaybi‘a’s words:

ولمّا كان شرف الدين الطوسي بمدينة الموصل، وكان أوحده زمانه في الحكمة والعلوم الرياضية وغيرها، سافر ابن الحاجب والحكيم موفق الدين عبد العزيز إليه ليجتمعا به، ويشغلا عليه. فوجداه قد توجه إلى مدينة طوس فأقاما هناك مدة، ثم سافر ابن الحاجب إلى إربل، وكان بها فخر الدين بن الدهان المنجم فاجتمع به، ولازمه وحلّ معه الزيج الذي كان قد صنعه ابن الدهان، وأتقن قراءته عليه ونقله بخطه، ورجع إلى دمشق.

“When Sharaf al-Dīn al-Ṭūsī -who was distinguished in his time in philosophy, mathematical sciences, and other [fields]- was in Mosul, Ibn al-Ḥājib and al-Ḥakīm Muwaffaq al-Dīn ‘Abd al-‘Azīz traveled to join Ṭūsī and study with him. But they found him preparing to go to the city of Ṭūs. They stayed in Mosul for a time and then Ibn al-Ḥājib traveled to Irbil. Fakhr al-Dīn ibn al-Dahhān al-Munajjim was in Irbil, so Ibn al-Ḥājib got together with Ibn al-Dahhān and accompanied him. Ibn al-Ḥājib studied with Ibn al-Dahhān the *zīj* authored by him and perfected the study of the *zīj* with him. He copied the *zīj* and returned to Damascus.”

3. Ramazan Şeşen, *Mukhtārāt min al-makhṭūṭāt al-‘Arabiyya al-nādira fī maktabāt Turkiyā* (Istanbul: İslam Tarih Sanat ve Kültür Araştırma Vakfı, 1997), 888. I should thank Dr. Benno van Dalen for sharing his *A New Survey of Islamic Astronomical Handbooks* (unpublished) with me, through which I got to know this MS.

4. This title is coming from the title of the first extant part of the *zīj* on f.3b, where the author says “*al-fannu al-awwalu min al-Dustūri al-Kāfi fī istikhrāji tawārikhi al-umami wa a’yādihim*”.

5. The author of the *zīj* used the beginning of year 138 Jalālī, which is equivalent to 29 Sha‘bān 571 (13 March 1176), in a number of places in the *zīj*, like on f.31a.

6. Al-Subkī, Tāj al-Dīn ‘Abd al-Wahhāb ibn ‘Alī, *Ṭabaqāt al-Shāfi‘iya al-kubrā*, ed. Maḥmūd Muḥammad al-Ṭanāḥī and ‘Abd al-Fattāḥ Muḥammad al-Ḥilw (Cairo: Dār Iḥyā’ al-Kutub al-‘Arabiyya, no date), 8:386:

I saw the handwriting of Kamāl al-Dīn ibn Yūnus on the first part of Euclid's [*Elements*], corrected by Thābit ibn Qurra, that he wrote: "I studied this section with [...] Sharaf al-Dīn [...] Abī al-Muzaffar [...] when he came back from Ṭūs. I analyzed this section [of the *Elements*], *Almagest*, and some parts of [Apolonus's] *Conics* in Ṭūsī's presence. I asked for the fulfillment of what we intended regarding [Ibn Haytham's] *al-Shukūk*, thus he brought it and I copied it. This note was written by Mūsá ibn Yūnus ibn Muḥammad ibn Mun'a on the same date [as the copy date of the book]." This was the copy of his note. The date of the mentioned book was 19 Rabī' I 576 AH.

According to this quotation, Ṭūsī took a trip to Ṭūs and returned from it shortly before Rabī' I 576/August 1180. The above-mentioned Kamāl al-Dīn was Mūsá ibn Yūnus ibn Muḥammad ibn Mun'a, a *Shāfi'ī* jurist, who spent most of his life in Mosul where many scholars attended his teaching circles. Some of his works on mathematics are extant and indicate that he was a knowledgeable mathematician.<sup>1</sup> Ibn Khallikān (d. 681/1282), who met Kamāl al-Dīn several times, confirms his studies with Ṭūsī. Quoting from Ibn al-Mustawfī's (d. 637/1239) *Tārīkh Irbil* (The history of Irbil),<sup>2</sup> Ibn Khallikān says:<sup>3</sup>

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ورأيت بخط الشيخ كمال الدين بن يونس على الجزء الأول من أقليدس إصلاح ثابت بن قرّة ما نصّه: قرأت على الشيخ الإمام العالم الزاهد الورع شرف الدين فخر العلماء تاج الحكماء أبي المظفر، أدام الله أيامه، بعد عوده من طوس هذا الجزء. وكنت حللته عليه نفسي مع كتاب المجسطي وشيء من المخروطات. واستنجزته ما كان وعدنا به من كتاب الشكوك فأحضره واستنسخته وكتبه موسى بن يونس بن محمد بن منعة في تاريخه. هذا صورة خطه. وتاريخ الكتاب المشار إليه تاسع عشر ربيع الأول سنة ست وسبعين وخمسائة هجرية.

1. On Kamāl al-Dīn's life and works see Younes Karamati and Hanif Ghalandari, "Zindigī-nāma-yi wa kārnāma-yi 'ilmī Kamāl al-Dīn ibn Yūnus Mawṣilī (the biography and scientific legacy of Kamāl al-Dīn ibn Yūnus Mawṣilī)," in *Studies on the History of Sciences from Antiquity to the XVII Century* (Tehran: Mirāth Maktūb, 2011), 93-122; Sabine Arndt, "Judah ha-Cohen and the Emperor's Philosopher Dynamics of Transmission at Cultural Crossroads," (Ph.D., Oxford: University of Oxford, 2016), 98-102; Dag Nikolaus Hasse, "Mosul and Frederick II Hohenstaufen: Notes on Aṭīraddīn alAbharī and Sirāḡaddīn al-Urmawī," in *Occident et Proche-Orient: contacts scientifiques au temps des Croisades*, ed. Isabelle Draelants, Anne Tihon, Baudouin van den Abeele, and Charles Burnett (Turnhout: Brepols, 2000), 145-63.

2. Only fragments of *Tārīkh Irbil* has reached us and this section is not extant.

3. Ibn Khallikān, Shams al-Dīn Aḥmad ibn Muḥammad, *Wafayāt al-a'yān wa anbā' abnā' al-zamān*, ed. Iḥsān 'Abbās (Beirut: Dār Ṣādir, 1977), 5:314:

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The above-mentioned Abū al-Barakāt al-Mubārak ibn al-Mustawfī speaks of Kamāl al-Dīn in *Ta'rikh Irbil*. He says: “He was a senior scholar, well versed in every science. He was among those who were distinguished in the science[s] of the Ancients, such as geometry, logic, and other [sciences]. He studied Euclid [’s *Elements*] and the *Almagest* with the master Sharaf al-Dīn al-Muzaffar ibn Muḥammad ibn al-Muzaffar al-Ṭūsī al-Fārābī, *i.e.*, the inventor of the linear astrolabe which is known as the staff.”

However, the connection between Kamāl al-Dīn and Ṭūsī was not just that the former attended the latter’s teaching circle. Kamāl al-Dīn also had a role in the compilation of Ṭūsī’s longer treatise on the linear astrolabe. Ibn Khallikān again provides us with some valuable information in this regard:<sup>1</sup>

It is said that the inventor of the astrolabe was Ptolemy, the author of the *Almagest*. The reason for his inventing it was that he had a celestial globe in his hand as he was riding, it fell, and

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ولقد ذكره أبو البركات المبارك بن المستوفي المقدم ذكره في تاريخ إربل. فقال: هو عالم مقدم، ضرب في كل علم. وهو في علم الأوائل كالهندسة والمنطق وغيرهما ممن يشار إليه. حل أقليدس والمجسطي على الشيخ شرف الدين المظفر بن محمد بن المظفر الطوسي الفارابي، يعني صاحب الاصطرلاب الخطي المعروف بالعصا.

The English translation presented here is a revised version of William McGuckin de Slane’s translation in *Ibn Khallikan’s Biographical Dictionary* (Paris: Oriental translation fund of Great Britain and Ireland, 1845), 3:470.

1 Ibn Khallikān, Shams al-Dīn Aḥmad ibn Muḥammad, *Wafayāt al-a’yān wa anbā’ abnā’ al-zamān*, ed. Iḥsān ‘Abbās (Beirut: Dār Ṣādir, 1977), 6:52-53.

وقيل إن أول من وضعه بطليموس صاحب المجسطي، وكان سبب وضعه له أنه كان معه كرة فلكية وهو راكب، فسقطت منه، فداستها دابته فحسفتها، فبقيت على هيئة الاصطرلاب. وكان أرباب علم الرياضة يعتقدون أن هذه الصورة لا ترسم إلا في جسم كروي على هيئة الأفلاك، فلما رآه بطليموس على تلك الصورة علم أنه يرسم في السطح ويكون نصف دائرة ويحصل منه ما يحصل من الكرة، فوضع الاصطرلاب ولم يسبق إليه. وما اهتدى أحد من المتقدمين إلى أن هذا القدر يتأتى في الخط ولم يزل الأمر مستمراً على استعمال الكرة والاصطرلاب إلى أن استنبط الشيخ شرف الدين الطوسي المذكور في ترجمة الشيخ كمال الدين بن يونس، رحمهما الله تعالى، وهو شيخه في فن الرياضة أن يضع المقصود من الكرة والاصطرلاب في خط، فوضعه وسماه العصا، وعمل له رسالة بديعة، وكان قد أخطأ في بعض هذا الوضع، فأصلحه الشيخ كمال الدين المذكور، وهذبه. والطوسي أول من أظهر هذا في الوجود ولم يكن أحد من القدماء يعرفه. فصارت الهيئة توجد في الكرة التي هي جسم لأنها تشتمل على الطول والعرض والعمق، وتوجد في السطح الذي هو مركب من الطول والعرض بغير عمق، وتوجد في الخط الذي هو عبارة عن الطول فقط بغير عرض ولا عمق، ولم يبق سوى النقطة، ولا يتصور أن يعمل فيها شيء لأنها ليست جسماً ولا سطحاً، ولا خطاً بل هي طرف الخط كما أن الخط طرف السطح، والسطح طرف الجسم، والنقطة لا تتجزأ، فلا يتصور أن يرسم فيها شيء. وهذا وإن كان خروجاً عما نحن بصدده لكنه أيضاً فائدة، والأطلاع عليه أولى من إهماله.

The English translation presented here is our revision of William McGuckin de Slane’s translation in *Ibn Khallikan’s Biographical Dictionary* (Paris: Oriental translation fund of Great Britain and Ireland, 1845), 3:581-582. About this passage see David A. King, *In Synchrony with the Heavens* (Leiden-Boston: Brill, 2005), 2:594-595.

the animal on which he was riding stepped on it and flattened it so that it became like an astrolabe. The practitioners of the mathematical sciences thought, [till then,] that the image [drawn on the globe] could only be drawn on a spherical body that looked like the orbs. When Ptolemy saw that [flattened globe], he recognized that it could be drawn upon a [plane] surface, as a half of circle, and that such an instrument would yield the same result as that given by the globe. He made the astrolabe, and no one ever preceded him [in this invention]. Still, none of the ancients was led to [recognize] that it can be done upon a line. So they continued to use the globe and the astrolabe till the master Sharaf al-Dīn al-Ṭūsī, who was mentioned in the biography of Kamāl al-Dīn ibn Yūnus and who was the teacher of the latter in mathematics -peace be upon both of them- established what was intended in the globe and the astrolabe on a line. He invented it and called it the staff and wrote an elegant treatise on it. Ṭūsī erred in an aspect of this invention, which was rectified by the above-mentioned master Kamāl al-Dīn. Ṭūsī was the first who created this instrument; none of the ancients ever knew it. The shape [of the celestial sphere] can be produced on a globe, that is, a [solid] body having length, breadth and thickness, and on a surface, which has only length and breadth, without thickness, and on a line having length only without breadth and thickness. Nothing remains other than the point, and it is not conceivable to draw something on it since it is neither a solid, nor a surface, nor a line, but only the extremity of a line, in the same manner that the line is the extremity of the surface, and the surface the extremity of the solid. The point being indivisible, it is impossible to conceive how anything can be drawn upon it. These observations are foreign to our subject, but they furnish some information that is better to know than not.

From this passage, it is important to highlight the following sentence: “He invented it and called it the staff and wrote an elegant treatise on it. Ṭūsī erred in an aspect of this invention, which was rectified by the above-mentioned master Kamāl al-Dīn.” Here, Ibn Khallikān tells us that Kamāl al-Dīn corrected the instrument invented by Ṭūsī.<sup>1</sup> In two

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1. Ibn Khallikān’s words can be interpreted differently such that Kamāl al-Dīn’s correction should be taken as a correction to Ṭūsī’s treatise, not the instrument. However, his use of the term *wad’* leads us to believe that he meant the correction was made regarding the instrument itself.



## 15/ The Gnomonic Application of...

copies of Ṭūsī's longer treatise we find an incipit that addresses what Ibn Khallikān said.<sup>1</sup> The incipit reads:<sup>2</sup>

In the name of God, the beneficent, the merciful and blessing upon our master Muḥammad and upon all his family and his companions. *Risāla fī 'amal 'aṣā al-Sharaf al-Ṭūsī*; he [= al-Ṭūsī] dictated it -a dictation corrected by Kamāl al-Dīn ibn Yūnus. He has another treatise which has been written down by himself, but this one is simpler for the learner.

There is no mention of Kamāl al-Dīn's correction of the instrument itself, but the incipit clearly says that Ṭūsī dictated the treatise and that Kamāl al-Dīn corrected it. So now the question is what exactly was Kamāl al-Dīn's role? Did he modify the instrument itself, or did he just correct the treatise? Ibn Khallikān might have received his information on this correction either through the above-mentioned incipit or through his personal communications with Kamāl al-Dīn himself or his students. As noted above, several monographs on the linear astrolabe have reached us. We can divide these monographs in two groups according to a distinctive characteristic: In one group, none of the applications dealing with the azimuth were discussed while in the other, the linear astrolabe features a gnomon designed for applications related to the azimuth. Now the question emerges: was Ibn Khallikān and the author of the incipit referring to this gnomon when they spoke of Kamāl al-Dīn's correction? One possibility is that Ṭūsī himself improved the linear astrolabe after writing his original composition and reported the final version of his invention in his final treatise. The latter would then have been dictated to Kamāl al-Dīn, who was responsible for the quality of the text.

The second possibility is that there was an original version of the treatise which Ṭūsī then dictated to Kamāl al-Dīn who then edited it. In this case, one of the multiple versions of Ṭūsī's longer treatise would have been written by Ṭūsī himself and another by Kamāl al-Dīn. Those copies of Ṭūsī's longer treatise which bear Kamāl al-Dīn's name might attest to the modified version since, as we will see, these two copies

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1. We will discuss these copies in the following sections.

2. MS B, f.104a; MS C, f.27a (see Appendix 1):

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ وَصَلَّى اللَّهُ عَلَى سَيِّدِنَا مُحَمَّدٍ وَعَلَى آلِهِ وَصَحْبِهِ أَجْمَعِينَ. رسالة في عمل عصاء الشرف الطوسي؛ أملاها إملاءً بإصلاح كمال الدين بن يونس وله رسالة أخرى مؤلفة لكن هذه أسهل على المتعلم وأقرب.

contain a slightly different version of the text: The order of the two parts of the treatise is changed in such a way that the construction part precedes the part on working with the astrolabe. However, this rearrangement was not done carefully and evidence of the original arrangement remains in that text. We will discuss these differences later but, for now, we can say that there is not sufficient evidence to take these changes to be the correction announced in the incipit.

We have yet another manuscript witness that contains a text which is ascribed to Ṭūsī (hereafter “Ṭūsī’s longer treatise II”) and which is similar in content to his longer treatise and which discusses the gnomonic application yet exhibits sufficient differences from the other witness of Ṭūsī’s longer treatise such that we cannot identify the two as being the same text. Yet, the two texts are sufficiently similar such that one can claim that one of these two texts is a predecessor of the other. It is certainly conceivable that one of these two texts was written by Ṭūsī and the other by Kamāl al-Dīn, but we do not yet have enough evidence to judge this hypothesis, let alone determine which text is which. The precise temporal and philological relation between these two texts is not yet clear and requires further research.

Another possibility would be that Ṭūsī and Kamāl al-Dīn had discussed the linear astrolabe, and it was during their discussions that they established together the idea of using a gnomon in the instrument. Ṭūsī would then have dictated the final version of the text to Kamāl al-Dīn.

Yet another possibility would be that the gnomonic part of the linear astrolabe was invented by Kamāl al-Dīn himself. While this possibility is in agreement with Ibn Khallikān’s words, it is not supported by the above-mentioned incipit. A final possibility could be that, in contradiction to what Ibn Khallikān and the aforementioned incipit say, Kamāl al-Dīn had no role in the quality of the text or the instrument itself, but simply had the text dictated to him.

Although the available evidence does not allow us to come to a decisive conclusion, the first three possibilities are the most likely scenarios, as we will discuss in greater detail when treating the texts on the linear astrolabe. But first, we turn to a study of the linear astrolabe and its gnomon in the following section.

## 17/ The Gnomonic Application of...

### **The linear astrolabe according to its inventor**

The most significant difference between Ṭūsī's two treatises on the linear astrolabe is that the shorter treatise does not discuss the gnomonic application. Beyond this, the two texts exhibit some important differences. First of all, the treatises differ in the technical language employed to explain the various applications of the instrument. Moreover, other than the gnomon, the instrument explained in the longer treatise is more elaborate. For example, the second treatise specifies colors and additional signs to be used in the construction of the instrument and offers more details on its operation. It is not trivial to understand the gnomonic application of the instrument as explained in the longer treatise, especially since the theory behind the gnomon is not given in the treatise.<sup>1</sup> In order to show the difference in the language of these two treatises we included the edition and translation of Ṭūsī's shorter treatise and the edition and translation of the section on the gnomonic application in the longer treatise at the end of this paper. Since the basic applications of the linear astrolabe have been discussed by Henri Michel,<sup>2</sup> James E. Morrison,<sup>3</sup> and Massimo Goretti,<sup>4</sup> it would be sufficient here to include a reconstructed diagram of the scales on the linear astrolabe based on the description of these scales in the shorter treatise.<sup>5</sup> The way that these scales should be drawn and the way the instrument can be used is described in the treatise.

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1. As we will discuss in Appendix I, there is an anonymous treatise whose language is closer to the longer treatise, although it does not include the gnomonic application. It is possible that this treatise might also have been written by Ṭūsī.

2. Henri Michel, "L'astrolabe linéaire d'al-Ṭūsī".

3. James E. Morrison, *The Astrolabe*, 303-309.

4. Massimo Goretti, "The linear astrolabe of al-Ṭūsī".

5. The base latitude for al-Ṭūsī seems to be 36°, which is reported in some sources as the latitude of Mosul (see the description of al-Ṭūsī's treatises below). Hence, the reconstruction of the linear astrolabe is here given for this latitude. A comparison between this diagram and what Michel has drawn in his paper shows that his linear astrolabe differs from Ṭūsī's in external appearance. Moreover, Ṭūsī used just two threads, instead of the three threads used by Marrākushī and his modern followers. However, no technical difference follows from the use of two threads over three.

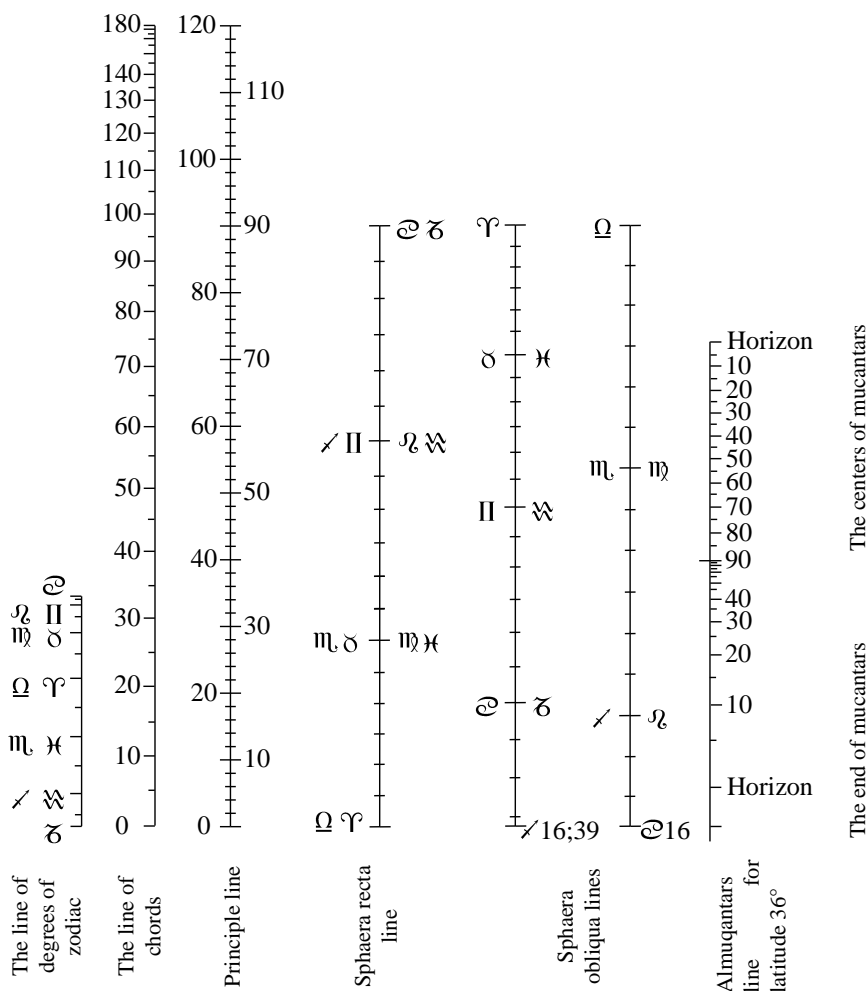


Figure 1. The scales on the linear astrolabe based on Ṭūsī's shorter treatise.

In what follows, we focus on the application of the linear astrolabe for finding the azimuth. In order to reconstruct Ṭūsī's procedure, we reverse the procedure he put forth in configuring the linear astrolabe and thus consider first the final configuration of the linear astrolabe and then explain the theory behind this configuration.<sup>1</sup> The following figure depicts such a final configuration. Let  $MKEqE$  be the equator,  $P$  its

1. We should emphasize that this is not the way that Ṭūsī introduces the gnomon of the linear astrolabe.

### 19/ The Gnomonic Application of...

northern pole,  $Z$  the zenith,  $MZP$  the meridian,  $PLK$  the circle of declination,  $LEq$  the ecliptic,  $L$  the Sun's position on ecliptic,  $Eq$  the autumnal equinox,  $UW$  the horizon and  $ZL$  the altitude circle that passes through Sun's position.

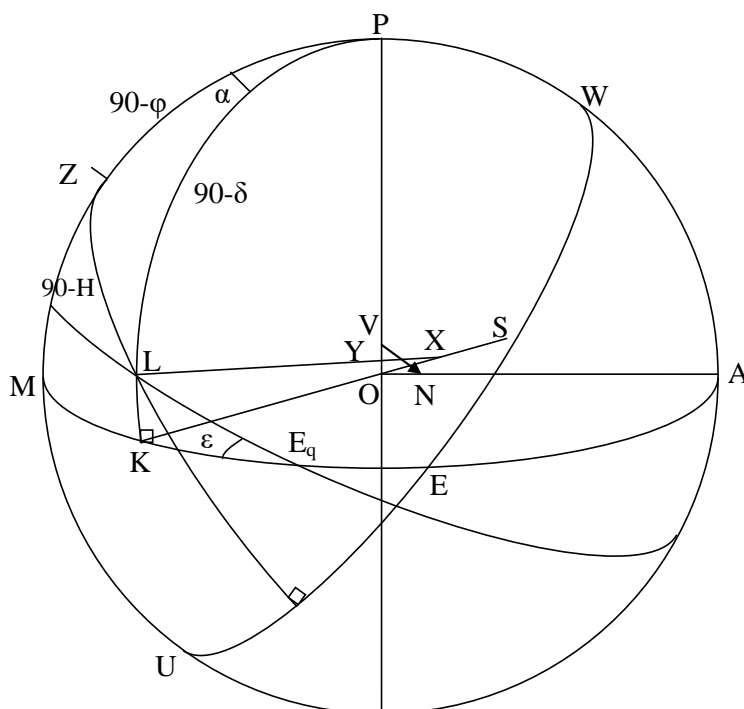


Figure 2. The linear astrolabe configured in the celestial sphere.

Here, the Sun has risen above the horizon but has not yet reached the meridian. Arc  $MK$ , the hour angle, is equal to angle  $\alpha$ , and arc  $LK$  is the declination of the Sun.

Let a gnomon be inserted perpendicular to the body of the linear astrolabe at its pole (*quṭb*) at the distance of 60 parts from the reference point (called *mumsik* by Ṭūsī). Let the linear astrolabe be placed at the middle of the celestial sphere as the gnomon lays on the celestial sphere's axis  $OP$ , and let the body of astrolabe be directed to the point  $K$  on the Equator, right below the Sun. If we draw the line  $MOA$

perpendicular to the axis of celestial sphere, in the plane of meridian, the angle between  $MO$  and the body of the astrolabe will be equal to arc  $MK$ , the hour angle. So, the angle  $SON$  is the hour angle.

There are some holes in the gnomon that allow the rays of the Sun to pass through. Let  $LYX$  be one of the rays which pass through the hole  $Y$  and falls on the body of the linear astrolabe at  $X$ . In this configuration, the angle between the ray and the body of the astrolabe will be equal to the declination of the Sun, arc  $LK$ . Now, if we calculate the declination of the Sun,  $\delta$ , by following equation:

$$\sin \delta = \sin \lambda \times \sin \varepsilon$$

in which  $\lambda$  is the longitude of the Sun, and  $\varepsilon$  is the maximum declination of the ecliptic, we will be able to calculate the proper height of the hole and the distance from the gnomon in which the ray reaches the body of the astrolabe.

Based on the declination, the zodiac signs can be divided in two groups: 6 northern signs and 6 southern. These two groups are symmetrical around the equinoxes. Moreover, each of these two groups is symmetrical around the solstices and so, having the declinations of 3 signs, we will have the declination of all the signs. Ṭūsī uses this symmetry to calibrate the gnomon and the astrolabe. He applies the declination of three signs: Aries, Taurus, and Gemini.

When the Sun is close to the equinoxes, its declination is less than 12 degrees, and when the declination comes close to zero, the ray touches the body of the astrolabe far from the gnomon. Hence, Ṭūsī separates the method of calibration for Aries from that of the other two signs. For Aries, he fixes the place where the ray touches the body and makes different holes in the gnomon for different declinations. But for the other signs, the rays always come from the highest hole of the gnomon<sup>1</sup> and fall on different places on the body of astrolabe that have been already calibrated. The following diagram shows the gnomon and calibration.

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1. The gnomon has 12 parts height.



Now, having the three necessary components we can put the linear astrolabe in the configuration described above. When this is produced, the gnomon shows the direction of the pole of the equator, and the triangle *VON* will be in the plane of the meridian. Thus, by hanging a second plummet to the point *O*, and approaching the astrolabe to the ground, the line between the two plummets on the ground will show the meridian line. If we know the azimuth of other localities, such as that of the *qibla*, we are able to trace the direction of those localities on ground using the chord mechanism.

The two additional scales that Ṭūsī introduced for the gnomon application are shown in the following diagram:

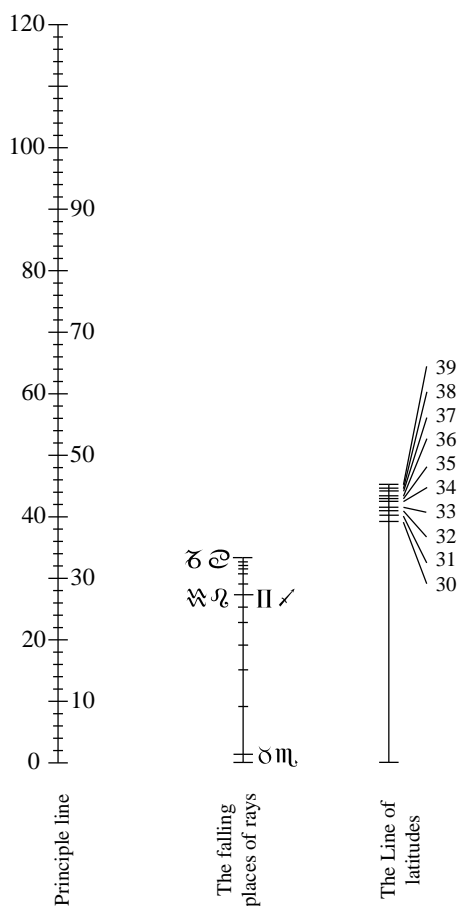


Figure 5. The scales for the gnomonic application.



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As Ṭūsī himself explains, the person who is working with the astrolabe, in the mechanism above, needs to control at the same time three angles with three movements:<sup>1</sup>

For this [configuration] three motions are needed: one of them is that you rotate the astrolabe around itself such that it does not move from its position, and the gnomon rotates around [the body of the astrolabe], so the tip of gnomon goes up and down; the second [motion] is that by which the end of the equalized line inclines upward and downward; and the third [motion] is that by which the end of the diameter pulled by two fingers moves upward and downward.

But this procedure is so complicated that one may question to what extent this gnomonic mechanism is practical. If we compare this mechanism with what is required for the same applications in a regular plane astrolabe, there will be no chance for the linear astrolabe to compete in terms of simplicity. Ṭūsī, in the longest of the three extant introductions to his longer treatise,<sup>2</sup> wherein he compares the linear astrolabe to the plane astrolabe, asserts that not only does the linear astrolabe have no defect, but that it is better than the plane astrolabe:<sup>3</sup>

When we consider how the proofs for this matter [*i.e.*, the different kinds of astrolabes] could be derived, a linear astrolabe became demonstrated to us, constructed on a [piece of] wood, in any possible length, in which points have been determined, and

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1. For the Arabic text see Appendix 2.2, paragraph 8:

وتحتاج في ذلك إلى ثلاث حركات: أحدها أن تدير الاصطرلاب على نفسه بحيث لا ينتقل عن موضعه ويدور الشخص حوله فيصير رأس الشخص أميل إلى جهة فوق أو إلى تحت؛ والثانية أن يصير بها رأس خط الاستواء أميل إلى فوق أو إلى تحت؛ والثالثة أن يصير بها طرف القطر الممدود بالإصبعين أميل إلى فوق أو إلى تحت.

2. As we will discuss in the following section, Ṭūsī's longer treatise reached us with three different introductions/incipits.

3. The Arabic text is:

فلما تفكرنا في استنباط وجوه البراهين في هذا المعنى تبرهن عندنا اصطرلاب خطي تعمل على خشبة بأي طول يتفق ويستخرج فيه نقط ويعلق منه خيط واحد. فينوب عن الاصطرلاب التام الكامل في جميع الأعمال، مع زيادة السهولة والصحة، ولا تقصر عنه في شيء ألبتة بل تزيد عليه في أشياء كما يتبين مع خفة المؤنة والصناعة إذ تقدر كل أحد أن يعمل اصطرلاباً في قريب مدة ساعة. ثم بعد الوفاء بجميع الأعمال الموجودة في الاصطرلاب المعهود على هذه الجهة تأدي الفكر عنها إلى أمور عجيبة جداً في هذا الفن فمن جعلتها أقامة ثلاث نقط في كل واحد واحد من العروض مقام تسعين دائرة من دوائر المقنطرات ومائة وثمانين دائرة من دوائر المقنطرات ومائة وثمانين دائرة سمته مع أن خروج المطلوب من هذا أحسن وأسهل مما عن المائتين والسبعين دائرة. ففي جميع عروض التسعين درجة، التي من خط الاستواء إلى سمت القطب، يقوم مائتان وسبعون نقطة مقام أربعة وعشرين ألفاً وثلاثمائة دائرة.

a single thread is suspended from it. The linear astrolabe displaces the complete [plane] astrolabe in all applications with more ease and accuracy. It not only lacks any deficiency in comparison with the plane astrolabe but, in some cases, it also surpasses it with less complexity, as will be explained. [The linear astrolabe displaces the plane astrolabe considering] the construction, since everyone is able to construct an astrolabe in around one hour. After fulfilling all the applications of the mentioned [plane] astrolabe by this method [of constructing the astrolabe], [our] thinking led to some really wondrous things in this art such as replacing 90 circles of almucantars with three points for each latitude [of localities], and 180 circles of almucantars, and 180 circles of azimuth, considering that deriving the outcomes with this [replacement] is better and easier than that of 270 circles. [This way], in all ninety-degree latitudes which [start] from the equator [and end] at the pole, 270 points stand for 24300 circles.

Having dismissed the complication of the gnomonic application, Ṭūsī's claims about the advantage of the linear astrolabe over the plane astrolabe are correct. Since the construction of the linear astrolabe needs the least equipment, anyone, even an amateur astronomer, using a few tables available in works like al-Farghānī's *al-Kāmil fī ṣanā'at al-uṣṭurlāb* (A complete [work] on the construction of the astrolabe),<sup>1</sup> would be able to make a sample for himself. Moreover, by increasing the length of the astrolabe, we are able to increase the accuracy of the divisions on the scales of the linear astrolabe. Despite all this, some astronomers like Marrākushī were not convinced and were critical of Ṭūsī's invention. Marrākushī, in the beginning of his discussion on the linear astrolabe in the fifth chapter of the third section of his *Jāmi' al-mabādi' wa-al-ghāyāt*, says:<sup>2</sup>

Some people considered this astrolabe [*i.e.*, the linear astrolabe] in the same rank as the southern plane astrolabe and the northern one. But this is wrong since it is seriously defective. We will

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1. This work is cited in the copies K and L of Ṭūsī's longer treatise.

2. The Arabic text:

من الناس من جعل هذا الاصطرلاب في رتبة الاصطرلاب السطحي الجنوبي والشمالى وهو غلط بل هو ناقص عنهما نقصاناً كثيراً وسنين ذلك في كيفية العمل به.

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explain why [in the section] on knowing how to operate [the linear astrolabe].

And later in the 14th chapter of fourth section, he says:<sup>1</sup>

Chapter. Deriving the azimuths by this instrument is seriously defective, since it is not possible at all to find the azimuths of the stars and also the azimuth of the Sun when the Sun is close to the equinoxes. [Even] when the Sun is far from the equinoxes it is possible [to derive], except that it would be inaccurate. The inventor of this instrument did not mention this deficiency. However, finding the meridian, which is one of the advantages of finding the azimuth, has been discussed. The situation in finding the meridian with this instrument is similar to what we said about the azimuth. Finding the meridian using the stars or anything else at night is not possible, and it is not possible to derive the meridian using the linear astrolabe when the Sun is close to the equinoxes. When the Sun is far from the equinoxes, it is possible but with seriously increasing inaccuracy.

As we have discussed above, the gnomonic mechanism works with the Sun's rays, so it is not possible to find the meridian at night. Moreover, as we have shown in the configuration previously explained, the body of the astrolabe lies in the plane of celestial equator. So, when the Sun is on this plane, there will be no intersection between the Sun's rays and the body of the astrolabe. And when the Sun is close to the equinoxes, the point of intersection falls in physical infinity. The equatorial sundials have this same problem. This makes the measurement of the inclination of the Sun in these positions impossible. Hence, Marrākushī's critiques are also valid.<sup>2</sup>

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### 1. The Arabic text:

فصل وأما استخراج السموت بهذه الآلة فناقص جداً من قبل أنه لا يمكن أن يستخرج بها سموت الكواكب أصلاً ولا سمت الشمس إذا كانت قريبة من الاعتدالين. وأما إذا كانت بعيدة عن الاعتدالين فيمكن إلا أنه يتعسف ولم يذكر ذلك المصنف. وإنما ذكر استخراج خط نصف النهار وهو أحد فوايد السموت وأمر هذه الآلة في استخراج خط نصف النهار مثل ما ذكرنا في السموت فإنه لا يمكن أن يستخرج بها خط نصف النهار بالليل من قبل الكواكب ولا غيرها ولا يمكن أن يستخرج بها خط نصف النهار بالنهار إذا كانت الشمس قريبة من الاعتدال. وأما إذا كانت بعيدة من الاعتدال فيمكن إلا أن فيه تعسف زايد جداً.

2. Among other astronomers who criticized the linear astrolabe is Najm al-Dīn al-Miṣrī (d. c. 750/1350). However, his criticism is based on a confusion. His discussion on the linear astrolabe is based neither on Ṭūsī's treatise nor on Marrākushī's work. For this, see François

### The transmission of texts on the linear astrolabe<sup>1</sup>

Although no exemplar of the linear astrolabe itself has been discovered, it seems it was a fairly well-known instrument in the east and west of the Islamic world, as evidenced by a number of extant works on the topic. As noted above, Abū al-Ḥasan al-Marrākushī discussed the linear astrolabe in his *Jāmi' al-mabādi' wa-al-ghāyāt*. Marrākushī's material on the linear astrolabe was later quoted in a treatise entitled *Ṣinā'a ālāt al-raṣād*, attributed to 'Abd al-'Alī al-Bīrjandī (d. 934/1527).<sup>2</sup> The Rasulid ruler of Yemen, 'Umar ibn Yūsuf ibn 'Umar (d. 1297), in his work on the astrolabe entitled *Manhaj al-ṭullāb fī 'amal al-uṣṭurlāb* (A student's course on constructing the astrolabe) also mentioned Ṭūsī and his linear astrolabe.<sup>3</sup> According to Lisān al-Dīn Ibn Khatīb (d. 776/1374), an Andalusian scholar by the name of Ibn al-Arḡam al-Numayrī (d. 1259) also wrote a treatise on the linear astrolabe.<sup>4</sup> Najm al-Dīn al-Miṣrī (d. c. 750/1350) discussed the linear astrolabe, although his discussion was not based on any of Ṭūsī's treatise and he believed that the linear astrolabe was not an astrolabe.<sup>5</sup> In addition to these texts, five other treatises on the linear astrolabe have reached us, three of which have been attributed to Ṭūsī:

- Ṭūsī's shorter treatise (available in two copies);
- Ṭūsī's longer treatise I (available in three MS groups, 5 copies);
- Ṭūsī's longer treatise II (available in one copy);
- The abridgment of Ṭūsī's longer treatise (available in one copy);

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Charette, *Mathematical Instrumentation in Fourteenth-Century Egypt and Syria: The Illustrated Treatise of Najm al-Dīn al-Miṣrī* (Leiden and Boston: Brill, 2003), 62-63.

1. Among the texts discussed in this section Ṭūsī's shorter treatise, Ṭūsī's longer treatise (I), and the sections in Marrākushī's *Jāmi' al-mabādi'* related to the linear astrolabe were edited in my MA thesis "Linear astrolabe: description, structure and usage" at the Institute for the History of Science, University of Tehran, submitted in December 2009.

2. Four copies of this work are known: Germany, Berlin, Staatsbibliothek zu Berlin, Sprenger, 1877 (1b-126b); Iran, Qom, Kitābkhāna-yi Āya'ī Allāh Mar'ashī, 10612 (1b-61a); Iran, Mashhad, Kho'ī, 122; India, Delhi, Hamdard, MS 275.

3. One copy of this is in Iran, Tehran, Kitābkhāna-yi Millī, 1510. The linear astrolabe is mentioned on p. 7.

4. See: Ibn Khaṭīb, *Al-Iḥāṭa fī akhbār gharnāṭa*, ed. Muḥammad 'Abd Allāh 'Anān (Cairo: Dār al-Ma'ārif, 1974), 2:143; Roser Puig, "Ibn Arḡam al-Numayri (m. 1259) Y LA Introduccion en al-Andalus del Astrolabio Lineal," *Nuevos Estudios sobre Astronomia Espanola en el Siglo de Alfonso X*, ed. J. Vernet (Barcelona: Consejo Superior de Investigaciones Cientificas, 1983), 101-3.

5. See François Charette, *Mathematical Instrumentation*.

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- An anonymous treatise (available in one copy).

In what follows, we will discuss each of these monographs in turn.

### **Ṭūsī's shorter treatise**

This treatise was probably Ṭūsī's first composition on the linear astrolabe, two copies of which has reached us:

- 1) A (آ): Istanbul, Topkapi Sarayi Museum, Ahmed III, MS 3464, ff. 151b-154a.<sup>1</sup> This codex contains many important works, some of which are related to middle books.<sup>2</sup> Some of these texts are dated and the dates given vary from 625 to 630/1228 to 1233. The treatise on the linear astrolabe is copied in the same seventh/thirteen century by a later owner of the codex, on the blank pages in the codex. A text on magical squares is copied by the same scribe on ff. 188v-189r.
- 2) N (ن): Ankara Üniversitesi, Mustafa Con, MS A 293, ff. 123b-126a, copied in 1267/1851. This codex contains several witnesses, some of them on astronomical instruments.

Our knowledge about the author of this treatise comes from a title provided on the margin of the f. 151b of MS A, where the copyist of the treatise wrote: "A treatise on the linear astrolabe by Imam Sharaf al-

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1. Max Krause, "Stambular Handschriften islamischer Mathematiker," *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*, Abteilung B, Studien 3 (1936): 437–532, at 490.

2. For the description of the MS and its contents, see: Roshdi Rashed and Athanase Papadopoulos, *Menelaus' Spherics: Early Translation and al-Māhānī/al-Harawī's Version* (Berlin; Boston: De Gruyter, 2017), 493-496; Richard Lorch, *Thābit ibn Qurra on the Sector-Figure and Related Texts*, ed. Fuat Sezgin, *Islamic Mathematics and Astronomy* 108 (Frankfurt am Main: Institute for the History of Arabic-Islamic Science at the Johann Wolfgang Goethe University, 2001), 21-23; Elaheh Kheirandish, *The Arabic Version of Euclid's Optics: Edited and Translated with Historical Introduction and Commentary* (New York, NY: Springer, 1999), xxvi; Nathan Sidoli and Yoichi Isahaya, *Thābit Ibn Qurra's Restoration of Euclid's Data: Text, Translation, Commentary* (Cham, Switzerland: Springer, 2018), 27-28; Nathan Sidoli and Takanori Kusuba, "Al-Harawī's Version of Menelaus' Spherics," *Suhayl* 13 (2014): 160-161; Paul Kunitzsch and Richard Lorch, *Theodosius Sphaerica: Arabic and Medieval Latin Translations* (Stuttgart: Steiner, 2010), 3; Paul Kunitzsch and Richard Lorch, "Theodosius, De diebus et noctibus," *Suhayl* 10 (2011): 13. Sajjad Nikfahm-Khubravan and Osama Eshera, "The Five Arabic Revisions of Autolycus' On the Moving Sphere (Proposition VII)," *Tarikh-e Elm* 16(2) (2019): 7-70, on p. 44.

Dīn al-Ṭūsī – may God have mercy upon him.”<sup>1</sup> Ṭūsī uses very simple and concise language to describe the instrument throughout the treatise. The treatise has been arranged in two parts: the first part on constructing the linear astrolabe (in six chapters) and the second part on working with the instrument (in seven chapters). When discussing the construction, the method of drawing seven scales is explained: i) the principle line; ii) the line of arcs; iii) the line of ascension (*maṭāli*)<sup>2</sup> for the terrestrial equator; iv and v) two lines for the ascension of the locality; vi) the line of almucantars; vii) the line of the zodiacal signs. These lines, based on their description in Ṭūsī’s shorter treatise, have been reproduced in section III of this paper. Of these lines, the three lines of ascension are drawn in MS A, but MS N contains only the diagram for the ascension for the locality. As we have mentioned above, this treatise did not treat the gnomon and the relevant scales on the astrolabe. Hence, it deals with a primitive version of Ṭūsī’s invention. The treatise includes two tables: i) a table of distances of the centers of almucantars and their end (*nihāya*)<sup>3</sup> from the origin of the principle line; ii) a table of the degrees of the zodiacal signs. MS A contains both tables, while MS N only has the first. The first table in both copies has columns for the latitudes 34°, 35°, 36°, 37°, 38°, and 39°; however, the tables found in both copies are not filled completely. Only the column for 36°<sup>3</sup> has been filled in MS A while the columns for 36° and 38° are filled in MS N. This table would probably have been calculated using the tables of almucantars in sources like Farghānī’s work on the construction of astrolabe. Using Farghānī’s table, we would just need to multiply the entries of the tables by two in order to arrive at Ṭūsī’s table. In preparing the edition of the shorter treatise provided in Appendix 1.1 of this paper, we use MS A as the base copy and give the variances of MS N in the endnotes. Some of the terms used in this treatise are different from what we find in Ṭūsī’s longer treatise. In the following table, we give some of the important technical terms.

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1. Ṭūsī’s name as the author of the treatise is repeated in the table of contents of the codex on an old added folio at the beginning of the codex, where the original opening folio of the codex was already lost.

2 This is Ṭūsī’s term. The end of the almucantar is the intersection of the almucantar with the meridian in the plane astrolabe.

3 As we will discuss later, this is the latitude of Mosul.

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Term	Translation	Description
خطّ الأصول	the principle line	The main line/scale on the body of the astrolabe that defines the unit of measurement for all scales. It is divided in 120 divisions.
المبدأ	the origin	The origin of the principle line.
القطب	the pole	The pole in the linear astrolabe stands for the pole in the plane astrolabe. Its distance from the origin is taken to be 60 parts. This distance for the plane astrolabe was given as 30 in many treatises.
خطّ القسي	the line of arcs	The scale for the chords of the arcs.
خطّ المطالع بنخطّ الاستواء	the line of ascension for the terrestrial equator	
خطّي مطالع البلد	two lines of ascension for the locality	
خطّ المقنطرات	the line of almucantars	
نهاية مقنطرة	the end of almucantar	This is equal to the point of intersection of each almucantar with the meridian line in the plane astrolabe.
خطّ أجزاء المنطقة	the line of the parts of the zodiacal equator	
خيّط المحور	the thread of the axis	The thread suspended from the pole with a plummet.
خيّط الوتر	the thread of the chord	The thread suspended from the origin, used for measuring the chords.
المري	<i>almuri</i>	A movable knot on the axis thread.

### **Ṭūsī's longer treatise (I)**

This version of Ṭūsī's longer treatise has been transmitted in three manuscript groups, each of which has a different introduction. We will discuss each of these groups separately in the following sections.

#### **a) Ṭūsī's longer treatise (I) with the longer introduction**

To this group belong two MSS copies, in each of which no title is given to the treatise and the text begins with a long introduction.

- 3) L (ل): Leiden, Leiden University, MS Or. 591, pp. 1-25.<sup>1</sup>  
The codex contains several texts, all of them related to astronomical instruments. Although a copy date is not provided at the end of the treatise on the linear astrolabe, some other witnesses in the codex bear different copy dates, ranging from 610 to 637/1213 to 1239.
- 4) K (ك): Istanbul, Topkapi Sarayı Museum, Ahmed III, MS 3505, ff. 63b-79a.<sup>2</sup> It was probably copied directly from MS L.

On f.63a of MS K, the title of the work is erroneously given as *Kitābun fī ma'rifati al-uṣṭurlābi al-musaṭṭaḥi wa-al-'amali bihi* (A book on the knowledge of the plane astrolabe and operating with it).<sup>3</sup> A similar title is given to MS L in some modern bibliographical sources.<sup>4</sup> However, on the title page of the MS L, the title of the treatise is recorded as *Risālatu Sharafī al-Dīni Muẓaffari al-Ṭūsī fī ṣan'ati <?>*<sup>5</sup> *taqūmu maqamu al-uṣṭurlābi fī a'mālihi* (The treatise of Sharaf al-Dīn Muẓaffar al-Ṭūsī on the construction of <?> that takes the astrolabe's position in its applications). This title is a correct description of the linear astrolabe. The missing word in this title was perhaps the source of the mistake in reporting the subject of the treatise by some scholars.

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1. Jan Just Witkam, *Inventory of the Oriental Manuscripts of the Library of the University of Leiden* (Leiden: The Lugt Press, 2007), 1:249-250; P. Voorhoeve, *Codices Manuscripti*, vol. 7, *Handlist of Arabic Manuscripts in the Library of the University of Leiden and other collections in the Netherlands* (The Hague/Boston/London: Leiden University Press, 1956), 194.

2. Max Krause, "Stambular Handschriften," 490.

3. See: Max Krause, "Stambular Handschriften," 490, who recorded the MS K as "Kitāb fī ma'rifa al-uṣṭurlāb al-musaṭṭaḥ wa'l-'amal bihi" (The book on the knowledge of the plane astrolabe and working with it).

4. For example, Jan Just Witkam, *Inventory of the Oriental Manuscripts of the Library of the University of Leiden* (Leiden: The Lugt Press, 2007), 1:249.

5. One word is not readable due to damage.



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Adel Anbouba was the person who first recognized that the subject of this treatise is the linear astrolabe, not the plane astrolabe.<sup>1</sup>

In the long introduction of these two copies, Ṭūsī describes the regular plane astrolabe and then tells the reader how he came to the idea of the linear astrolabe in which a limited number of dots can replace all the circles in the plane astrolabe. At the end of this introduction, Ṭūsī tells us that the treatise has three parts: i) a part on operating the instrument; ii) a part on constructing the linear astrolabe; and iii) a part on testing the correctness of the instrument. The last part, however, is not included in any of the two aforementioned copies. It is probable, but not certain, that this last part of the treatise was never actually written.

The first part of the treatise is arranged in five chapters, some of which are divided into sub-sections. The first chapter introduces the parts and scales of the linear astrolabe. Chapters 2, 3, and 4 are, respectively, on the determination of the altitude of the Sun and stars; knowing the time and the ascendant (*tāli*) using the altitude; and the equalization of the houses (*taswiya al-buyūt*). The fifth chapter is devoted to the gnomonic application of the linear astrolabe, the edition and translation of which follow in Appendix 1.2 and Appendix 2.2. The second part of the treatise, on the construction, is arranged in two chapters. The first chapter is on the scales that do not pertain to any locality: the principle line (called, by Ṭūsī, the equalized line or *khatt al-istiwā*); the ascension for the terrestrial equator; the parts of the zodiacal equator; the falling place of the rays of the Sun; the chords; and shadows. The second is on the scales for specific localities: almucantars; the two scales of ascension for the locality; and the scale of the evening. Each of these chapters has its own sub-divisions. The calibration of the astrolabe for the falling place of the rays of the Sun is explained in the fifth section of the first chapter (see section III of this paper).

In this version of treatise, two tables are included: i) a table of degrees of the zodiacal signs, which is the same table we find in the shorter treatise; and ii) a table of the falling place of the rays of the Sun on the body of the astrolabe.

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1. Adel Anbouba, "Al-Ṭūsī, Sharaf al-Dīn," 518.

As noted above some of the terms used in Ṭūsī's longer treatise are different than the terms in the shorter treatise. In the following table, some of these terms are listed.

Term	Translation	Description
خط الإستواء	the equalized line	The main line/scale on the body of the astrolabe that defines the unit of measurement for all scales. It is divided in 150 divisions. The name <i>istiwā'</i> has been assigned to this line since all of its divisions are equal.
الممسك	the holding point	The reference point of the principle/equalized line, although it is not at beginning of the line, but in 30 parts from the beginning.
القطب	the pole	The distance of the pole from the holding point is 60 parts of the equalized line.
خط القصي	the line of arcs	The scale for the chords of the arcs. It starts from the holding point and ends at 120 parts from it.
الخيطة الممثلة	the folded thread	This is a two-layer thread suspended from the pole. At the length of 60 parts from the pole, there is a knot on this thread which is called the knot of the end of the diameter. There is a movable knot called <i>almuri</i> on one of the layers of the folded thread.

**b) Ṭūsī's longer treatise (I) with Kamāl al-Dīn ibn Yūnus' incipit**

At the beginning of the two following copies of Ṭūsī's longer treatise (I), there is an incipit in which the name of Kamāl al-Dīn ibn Yūnus is mentioned.

- 5) B (ب): Istanbul, Topkapi Sarayı Museum, Ahmed III, MS 3342, ff. 104a-112a.<sup>1</sup> This codex contains unique copies of some very important treatises on astronomy and mathematics. These texts were copied by different scribes.

1. Max Krause, "Stambular Handschriften," 490.

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There are different copy dates at the end of these treatises, such as 564, 632, 635, and 668.

- 6) C (ج): Istanbul, Topkapi Sarayi Museum, Ahmed III, MS 3494, ff. 27a-65a, copied in 877/1472 by Aḥmad al-Qudṣī.<sup>1</sup> This copy was most probably copied from MS B.

The longer introduction of version 'a' does not exist in this version. Considering the incipit, one may expect to see a different version of the treatise in these two copies than version 'a'. However, some minor differences between the text and its arrangement notwithstanding, the main body of the text is identical in both versions. Version 'b' differs from 'a' in that the part on construction comes before the part on operating the astrolabe. But this rearrangement has not been done carefully, since, in version 'b' the text starts with an introduction on introducing the parts and scales of the linear astrolabe (it starts with the phrases "*fa-yajibu awwalan an nudhakkira al-asmā'a al-wāqi'ata fihā*" (it is necessary first to mention the names [of the parts] that exist in the astrolabe)). This introduction is actually the first chapter of the first part of version 'a' but given here without the title of "chapter." After this introduction, the text continues with the part on construction. The construction part has the title "part two" (*al-qism al-thānī*), although there was no "first part" before it. As we saw above, this part was actually the second part in version 'a.' This means that someone simply copied the second part (construction) before the first part (on operation). At the end of this part on the construction, the copyist wrote:<sup>2</sup>

Thus ends the first part (*maqāla*) on the construction of Ṭūsī's staff, and drawing it, and drawing its other lines. It will be followed in the second part with how to operate the astrolabe and derive the operations which can be [performed] in it.

After this, the part on operating the astrolabe begins. However, the opening chapter is titled "the second chapter" (which is in fact the

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1. Max Krause, "Stambular Handschriften," 490.

2. The Arabic text:

تمت المقالة الأولى من عمل عصا الطوسي ورسمها ورسم سائر خطوطها يتلوه في المقالة الثانية كيفية العمل بها واستخراج الأعمال التي فيها.

second chapter in the version ‘a’). This confirms that the rearrangement in this version ‘b’ was based on a version like version ‘a.’<sup>1</sup>

Version ‘b’ also has two short extra passages that do not exist in version ‘a.’ The first is a passage that bears the title *faṣl* (section), and is added at the end of the part on the construction. This passage explains how one can calculate the distance of the degrees of the zodiacal equators from the reference point on the linear astrolabe. In version ‘a’ there was a table for this matter, which is missing in the version ‘b’ (in fact there are no table at all in version ‘b’). The second passage is added at the end of the treatise and has the title “the sixth chapter and it is the conclusion of the treatise.” This chapter has two parts, the first is on how one would calculate the distances of the centers of almucantars from the reference point using a table like what is in Farghānī’s *al-Kāmil*. However, Ṭūsī’s shorter treatise and version ‘c’ of the longer treatise (see below) include a table for almucantars, while version ‘a’ and ‘b’ lack this table. The second part of the passage is on how to calculate the distances of the falling places of the rays of the Sun on the body of the astrolabe. A table exists in versions ‘a’ and ‘c’ for this purpose. Notice that this concluding chapter, which is related to constructing the astrolabe, comes at the end of the part of the treatise on operating the astrolabe. Although, if the above-mentioned tables existed in the version ‘b’, there would be no need for these complementary passages.

At the end of this version there is a nice diagram which traces the eight lines of the linear astrolabe and the corresponding scales.<sup>2</sup> The caption of the diagram reads:<sup>3</sup>

This is the quality of drawing the lines of Ṭūsī’s staff—and assistance is only with God. Let the reliance be upon what was previously mentioned in the treatise regarding the drawing of the staff such as to avoid the occurrence of error. I understood it.

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1. There are some scribal errors in the text of version ‘b’ as well. Almost five lines of the text are omitted in the introductory part of version ‘b’ which, as we mentioned, is the first chapter of the first part of version ‘a.’

2. This diagram is reproduced in David A. King, *In Synchrony with the Heavens* (Leiden-Boston: Brill, 2005), 2:69.

3. The Arabic text:

وهذه صفة رسم خطوط عصا الطوسي وبالله الاستعانة. وليكن الاعتماد على المذكور في الرسالة متقدماً من كيفية رسم العصا احترازاً من وقوع الخلل. فأفهم ذلك.

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Hence, it seems this diagram is a later addition to Ṭūsī's treatise.

#### c) Ṭūsī's longer treatise (I) with a shorter introduction

The last known copy of Ṭūsī's longer treatise (I) exhibits enough differences from 'a' and 'b' such that we consider it a distinct version 'c.' This copy is:

- 7) M (م) London, British Library, British Museum, MS Or. 5479, ff. 86b-104b.<sup>1</sup> This collective volume contains three other treatises on astronomical instruments. There is no copy date. The scribe's name is mentioned at the end of the codex, f.113r: 'Alī ibn Yūsuf ibn Marwān.

The most important difference between this version and the earlier two is that neither the long introduction nor the incipit that contains Kamāl al-Dīn's name are found in this copy. Instead, we find a short introduction that contains no technical information about the linear astrolabe. The order of divisions in MS M is largely consistent with that of version 'a' and, although there are some differences, some of those are clearly due to scribal mistakes. One of these differences is the misplacement of almost two pages from the first chapter of the first part (on introducing the parts and scales). This segment appears at the middle of the fifth division of the first chapter of the second part (f.99v, line10 – f.100v, line 10). This misplacement likely happened due to a disarrangement of the folios of the original from which MS M was copied. The main difference between 'c' and versions 'a' and 'b' is the omission of certain passages in 'c' which are otherwise found in the other versions. The first such passage is the last section of the first chapter of the part on the construction, which is on drawing the scale of tangent. The second missing section is that on drawing the scale of evening. Still, all three tables mentioned above exist in this version: i) a table of distances of the centers of almucantars and their end from the origin of the equalized line; ii) a table of the degrees of the zodiacal signs; iii) a table of distances in which the Sun's rays touch the body of the astrolabe.

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<sup>1</sup> A. G. Ellis and Edward Edwards, *A descriptive List of the Arabic Manuscripts Acquired by the Trustees of the British Museum since 1894* (London: William Clowes and Sons, 1912), 39.

### Ṭūsī's longer treatise (II)

This text known through a single copy and is closely related to Ṭūsī's longer treatise but sufficiently different such that the two cannot be counted as one text.

- 8) F (ف): Istanbul, Millet Kütüphanesi, Feyzullah Efendi, MS 2178, ff. 134b-156b. This codex contains copies of several works on astronomical instruments. The introductory passage of this treatise (II) is found twice in the codex: once with the treatise and again independently on f. 62a. Apparently, the anonymous scribe of this witness started copying the treatise on this folio but changed his mind and, instead of continuing to copy Ṭūsī's treatise, started another text on f. 62b. On f. 156b the scribe tells us that he copied Ṭūsī's text from a defective copy of the text.

On the top margin of f. 134b, the title of the treatise and the name of its author have been written as *Risālatun li-l-shaykhi Muẓaffari al-Dīni al-Ṭūsī fī al-uṣṭurlābi al-khaṭṭī*. The treatise starts with the longer introduction of version 'a.' although the introduction announces three parts to the treatise, only the first part on operating the instrument exists in the MS F. There are several differences between the text in F and Ṭūsī's longer treatise in MSS B, C, L, K, and M. First, the text in F is divided into following 17 chapters:

باب ١ في أسماء أجزائه التي تصطلح عليها ليسهل تقرير ما يتعلّق بها؛ باب ٢ في أخذ الارتفاع؛ باب ٣ في معرفة الأرمته في النهار والليل؛ باب ٤ أمّا معرفة أوقات الليل من ارتفاع الثوابت؛ باب ٥ في معرفة الطالع بالنهار والليل؛ باب ٦ في معرفة نصف قوس النهار بطريق آخر؛ باب ٧ في تسوية البيوت؛ باب ٨ في معرفة غاية ارتفاع الشمس والكواكب؛ باب ٩ في معرفة درجة طلوع الكوكب الثابت ودرجة غروبه؛ باب ١٠ في معرفة وقت طلوع الكوكب من النهار أو الليل وغروبه؛ باب ١١ في معرفة مطالع البروج؛ باب ١٢ في معرفة ميل الشمس؛ باب ١٣ في معرفة ما بين طلوع الفجر وطلوع الشمس وما بين غروب الشفق ومغيب الشمس؛ باب ١٤ في معرفة أوقات العصر؛ باب ١٥ فيما يتعلق بالمساحة؛ باب ١٦ في السموت؛ باب ١٧ في معرفة الأوزان.

Among these, chapters 15 and 17 do not have any counterpart in Ṭūsī's longer treatise. Chapter 16 here corresponds to chapter 5 of the first part of the longer treatise on gnomonic application. Most of the

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other chapters are included in the third chapter in each version of Tūsī's longer treatise (I). Some of the terms used in the longer treatise (II) are different from those in the longer treatise (I). For example, while the principle line in the longer treatise (I) is consistently referred to as *khatt al-istiwā'* (the equalized line), version (II) uses the name *khatt al-aṣl* (the principle line). The arrangement of the threads of the linear astrolabe in the version (II) is different than that in version (I) and is like that of the shorter treatise. More importantly, although the text in F generally follows the same ideas in the longer treatise, it is sometimes completely different.

Marrākushī clearly used a version of Tūsī's longer treatise similar to the text of MS F. Consider, for example, the corresponding passages from version (I), version (II), and Marrākushī's *Jāmi' al-mabādi'* on the sun rays passing through the gnomon and falling onto the body of the astrolabe:

Tūsī's longer treatise (I)	Tūsī's longer treatise (II)	Marrākushī
إعلم بأن مواقع الشعاعات النافذة من ثقب الشخص وهو مقياس الظل، أما البروج الأربعة التي هي الحمل والسنبلة والميزان والحوت، تكون بحذاء النقط الثلاثة من النقط التي تكون في جوار نقط أجزاء المنطقه، وتكون معها على ضلع واحد وتخالقها في اللون وبُعدها عن محاذاة الممسك بقدر جزئين ونصف من أجزاء الاستواء. ولأوائل الحمل يكون خروج الشعاع من الثقب الأسفل ولكل درجتين ونصف منه، يرتفع ثقبه حتى ينتهي لآخر الحمل إلى الثقب الأعلى ولأول الثور أيضاً. ولخمس درجات من	إعلم أن مقياس الظل أحد نصفه الدقيق تحتاج إليه للبروج الجنوبية لأن تعلق منه الشاقول والثقب الذي على النصف العريض والثقب الأسفل الأقرب إلى القطب هو يكون الشمس على بعد أكثر من درجتين عن نقطة الاعتدال ويقع شعاعه فيما بين الجزء الثاني والثالث من أجزاء السواء عن الممسك. وجميع الخلال الثقب لكل واحد من البروج الأربعة التي يلي الاعتدال يقع شعاعاتها على هذا الموضع. ومن كون الشمس في الدرجة الرابعة إلى السابعة يخرج من ثقب الثاني ومنه إلى التاسعة من الثقب الثالث ومنها إلى	إعلم أن مقياس الظل أحد نصفه وهو الدقيق تحتاج إليه للبروج الجنوبية لأن تعلق منه الشاقول والثقب الاثنى عشر التي على النصف العريض الأسفل منها وهو الأقرب إلى القطب منه يدخل شعاع الشمس إذا كان بعدها أكثر من درجتين عن نقطة الاعتدال ويقع فيما بين الجزء الثاني والثالث من أجزاء السواء. وجميع الثقب يشترك فيها الفصول الأربعة وذلك ظاهر. ومن كون الشمس في الدرجة الرابعة إلى السابعة يخرج الشعاع من الخرم الثاني ومنها إلى ط يخرج من الثالث ومنها إلى يب يخرج من ٤ ومنها إلى يد يخرج من ه ومنها إلى يز يخرج من السادس ومنها إلى يط يخرج من ٧ ومنها إلى كب يخرج من الثامن ومنها إلى كد

<p>الثور تتقدّم نقطة منها إلى جهة القطب من محاذاة تلك النقط الثلاث وهكذا لكلّ خمسة خمسة نقطة نقطة. ولأول الجوزاء نقطتان ثمّ لكلّ خمسة خمسة. نقطة نقطة ولأول السرطان ثلاث نقط. ثمّ ترجع درجات السرطان على درجات الجوزاء، كما في أجزاء المنطقة، حتّى تعود إلى أول السنبله، إلى نقطة الحمل وأول الثور. ثمّ لخمس درجات من السنبله ينزل ثقتين. وهكذا لكلّ خمس ينزل ثقبه حتّى ينتهي عند أول الميزان إلى الثقب الأخير. ثمّ يأخذ في الصعود لكلّ خمس خمس من الميزان ثقتين ثقتين إلى أن ينتهي عند أول العقرب إلى الثقب الفوقاني. ولجميع العقرب والقوس والجدي والدلو يخرج الشعاع من الثقب الفوقاني. وتنتقل على الأجزاء المذكورة للثور وما بعده، حتّى تنتهي في آخر القوس إلى آخر نقط الجوزاء. وترجع الجدي على نقط القوس وعند أول الحوت تعود إلى أول العقرب. ثمّ ينزل لكلّ خمس درجات من الحوت ثقتين حتّى ينتهي عند آخر الحوت إلى الثقب الأخير.</p>	<p>الثانية عشر من الرابع ثمّ من الثانية عشر إلى الرابع عشر من الخامس ثمّ من يد إلى يز من السادسة ومن يز إلى يط من السابع ومن يط إلى كب ومن الثامنة ثمّ إلى كد من التاسعة ثمّ إلى كز من العاشرة ثمّ إلى كط من الحادي عشر والأول برج الثور يكون من الثاني عشر. ثمّ بعد ذلك البرجين الباقيين في الربع يخرج شعاع لجميع أجزائها من الثانية عشر ويصير شعاعاتها أقرب إلى القطب.</p>	<p>يخرج من ٩ ومنها إلى كز يخرج من العاشر ومنها إلى كط يخرج من ١١ ولأول برج الثور يكون من الثاني عشر. ثمّ بعد ذلك البرجين الباقيين من الربع يخرج شعاع جميع أجزائهما من الثاني عشر.</p>
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As discussed above, there is a possibility that the claimed revision made by Kamāl al-Dīn Ibn Yūnus was made on Ṭūsī's longer treatise (II) and the result was Ṭūsī's longer treatise (I). If this were the case, then certain changes in technical terminology, in the arrangement of the threads, and certain changes in the text could be attributed to Kamāl al-Dīn. Of course, this requires further investigation.

#### **The abridgment of Ṭūsī's longer treatise**

This abridgment is known only through an incomplete copy:

- 9) D (د) Dublin, Chester Beatty, MS Ar. 3035, ff. 147b-149b.<sup>1</sup>

The codex contains a number of recensions of middle books, all written by a single hand. Only five pages of the abridgment exist in the codex, and some folios from the end are omitted. The copy date is mentioned at the end of the first witness, f. 126r, as 669/1270.

According to a marginal note at the beginning of the treatise, the abridgment was provided for the Ayyubid prince Al-Malik al-Mu'izz Mujīr al-Dīn Yaqūb (d. 654/1256) by certain Turkish figure. It is apparently based on Ṭūsī's longer treatise since it starts with the longer introduction although, since MS D is incomplete, it is not clear on which of the versions (I) and (b) it is based.

#### **An anonymous treatise on operating the linear astrolabe**

One copy of this treatise is known:

- 10) E (ع) London, British Library, India Office, MS Islamic 461, ff. 2b-7b, copied in 1197/1784.

The name of the author is not mentioned in the treatise. On the title page of the codex, f. 1r, however, it is attributed to "Khawājā Naṣīr," *i.e.*, Naṣīr al-Dīn al-Ṭūsī (d. 672/1274). The codex is a collection of works, including an abridgement of Sharaf al-Dīn al-Ṭūsī's treatise on algebra, *i.e.*, *Kitāb fī al-mu'ādilāt*. There is a possibility that the text on the linear astrolabe was by Sharaf al-Dīn al-Ṭūsī as well. The gnomonic application has not been discussed in this treatise. This treatise is similar to the longer treatises of Ṭūsī in style and in certain textual features. If we accept its attribution to Ṭūsī, it is also possible that this was the original work based on which Kamāl al-Dīn prepared his revision.

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1. Arthur J. Arberry, *The Chester Beatty Library, a Handlist of the Arabic Manuscripts* (Dublin: Emery Walker, 1955), 1:13.

## **Conclusion**

In this paper, we have introduced several treatises on the linear astrolabe. We have highlighted some of the differences and similarities of this text. Some of these treatises were dedicated only to operating with the linear astrolabe while other treatises included sections on both construction and operation. The technical language employed in these treatises is, in some cases, distinct and worthy of attention. That some of these texts treat the gnomonic application of the linear astrolabe while others do not bring to mind the possibility that there was an evolution in the invention of this instrument and in the textual tradition on the linear astrolabe. We have here considered some historical and codicological evidence regarding one aspect of that evolution, namely, the roles of the inventor of the instrument, Sharaf al-Dīn al-Ṭūsī, and his pupil, Kamāl al-Dīn Ibn Yūnus. Nevertheless, the evidence we have so far is not sufficient to yield definitive conclusions about their respective roles and contributions. These questions notwithstanding, we have indicated some aspects of the scientific significance of this instrument and shown how the gnomonic mechanism, which works like an equatorial sundial, could be used to compensate for the lack of a counterpart for azimuth lines in the linear astrolabe.

## **Acknowledgements**

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**Appendix 1.1: Translation of Sharaf al-Dīn al-Ṭūsī's shorter treatise on the linear astrolabe**

**A treatise on the linear astrolabe by Imam Sharaf al-Dīn al-Ṭūsī – may God have mercy upon him**

Then, this is a treatise on the linear astrolabe arranged in two parts: the first part on its construction and the second on its operation.

**Part I: On the construction of the linear astrolabe, in six chapters**

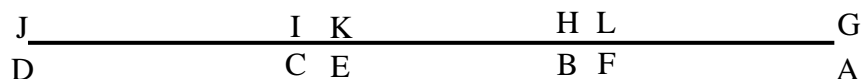
**Chapter I: On drawing the chords and arcs**

We pick a straight ruler made of wood or brass and draw a straight line in the middle of it and divide it into 120 divisions and call it the principle line. Then we make a hole in the beginning of the divisions and call it the origin. [We make] another hole with the distance of 60 parts [from the origin] and call it the pole. We draw another line to the left of the principle line and call it the line of arcs and draw a point on this line opposite the origin [of the principle line]. From the table of arcs and chords, we look for the arc of 5 degrees and see to which chord it corresponds, then [we] take that amount from the principle line and open the compass to that amount. [Using the compass, we] draw a mark on the line of arcs. From the beginning [of the line of arcs] to this mark is 5 degrees of the arc. Likewise is the process for 10 degrees of the arc, 15 degrees, 20 degrees, and the same up to 120 degrees of the arc and more. In practice, the easier [method] is to find the arc of 10 degrees from the table and see to which chord it corresponds. Then, we take that amount from the principle line and subtract the chord of 5 degrees of the arc from it. We open the compass to the remaining amount of the chord and put [one of] its legs on the mark for the arc of 5 degrees and the other leg where the chord of 10 degrees reaches, and we draw there another mark. We operate in the same manner regarding the 15 degrees of the arc and so on to the end of the operation [of drawing the line of arcs].

**Chapter II: On drawing ascension of the zodiacal signs at the [terrestrial] equator**

We draw a line to the right of the principle line and call it the line of ascension at the terrestrial equator. We look at the table of ascension at the equator and find the ascension of 5 degrees of Aries. We take from

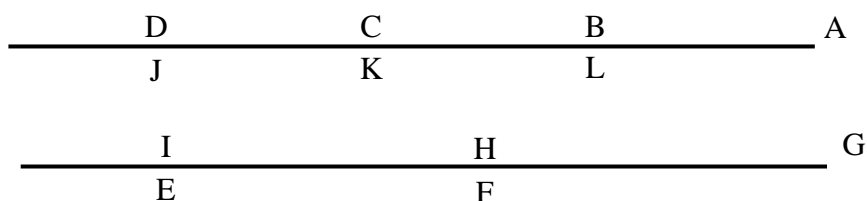
the origin of the principle line a distance [equal to] that amount, and opposite to the endpoint, drawing a mark on the line of ascension at the equator. So, from the beginning of [this] line up to that mark is the ascension of 5 degrees of Aries. So, we proceed for 10 degrees, 15 degrees, 20 degrees of Aries, up to the end of Aries. We do the same for Taurus and Gemini, till it reaches the end of Gemini, at 90 parts of the principle line. We write the mark for Aries at the beginning of the line, then the mark for Taurus at the end of the ascension of 30 degrees of Aries, and the mark for Gemini at the end of the ascension of Taurus. Then we write the mark for the other zodiacal signs like the [following] illustration, to obtain the ascension of signs at the terrestrial equator, some in the sequence and some in counter-sequence:



[Figure 6]

### Chapter III: On drawing the ascension of the zodiacal signs at the locality

We draw two lines to the right of the principle line and call them the two lines of ascension at the locality. We look at the ascension table for the locality and find the ascension of 5 degrees of Aries. We take from the origin of the principle line a distance [equal to] that amount, and opposite to the endpoint, drawing a mark on one of the lines of the ascension at the locality. So, we proceed for 10 degrees of Aries, [till] we finish drawing the ascension on two lines of the ascension at the locality, like the [following] illustration, to obtain the ascension of zodiacal signs, some in the sequence and some in counter-sequence:



[Figure 7]

**Chapter IV: On drawing the horizon and the almucantars**

We draw a line to the right of the principle line and call it the line of almucantars. Opposite the origin of the principle line, we draw a point on this line and write on it the latitude of the horizon [of the locality]. Then, we look at the table of the distances of the centers of almucantars for the latitude of the locality and open the compass to the amount of the distance of the horizon at that latitude from the origin of the principle line, and we draw a mark there [*i.e.*, where the second leg of the compass reaches on the line of almucantars]. This mark is the center of the horizon. We open the compass to the amount of the distance of the center of the almucantar of 5 degrees and draw a mark there. In the same way, we draw the centers of all almucantars. Then we look for the end of the almucantar of the horizon at the latitude of locality and its distance from the origin of the principle line and obtain its end on this line [of almucantars]. Then we obtain the end of almucantar of 5 degrees, 10 degrees, and so on, till we obtain the ends of all almucantars. This is the representation of the table of the distances of the centers of the horizon and the almucantars and their ends from the beginning of the principle line:

The distances of the center of horizon and almucantars and their ends from the beginning of the principle line												
Latitude	34		35		36		37		38		39	
Circles	center	end	center	end	center	end	center	end	center	end	center	end
	d m	dm	d m	dm	d m	dm	d m	dm	d m	dm	d m	dm
Horizon					5 54	7246						
5					12 54	7054						
10					18 14	69 3						
15					22 28	67 16						
20					25 48	65 30						
25					28 24	63 48						
30					30 46	62 4						
35					32 38	60 20						
40					34 10	58 38						
45					35 28	56 54						
50					36 32	55 10						
55					37 24	53 24						

60					38 10	51 40						
65					38 46	49 52						
70					39 12	48 12						
75					39 32	46 40						
80					39 48	44 8						
85					39 56	42 6						
90					39 58	39 58						

**Chapter V: On drawing the divisions of the [zodiacal] equator**

We draw another line to the left of the principle line and call it the line of the divisions of the [zodiacal] equator. Opposite the origin of the principle line, on this line we draw a point. We put [a] leg of the compass and open it to the amount of one part of principle line. We draw a point [on this line] at the endpoint of the other leg. [From] the origin of the principle line to the drawn mark, 15 degrees of Capricornus has passed. We divide it into three parts such that each part is 5 degrees of Capricornus. Then we look for what is opposite to the other degrees of Capricornus in the table of the divisions of the [zodiacal] equator, [we] open the compass to that amount from the principle line and put the leg of the compass [on the beginning of the line]. Wherever the other leg of the compass falls, we draw a point as a marker for those degrees [of Capricornus]. Like so, we continue until we reach 10 degrees of Gemini. Then we open [the compass] to the amount of 34 1/3 parts for the beginning of Cancer, [and draw its mark]. We divide what is between it and 10 degrees of Gemini in four parts. [In this way,] the parts of the [zodiacal] equator from the beginning of Gemini up until Cancer are divided. This is the table:

The divisions of the [zodiacal] equator													
Degrees	Capricorn		Aquarius		Pisces		Aries		Taurus		Gemini		
	d	m	d	m	d	m	d	m	d	m	d	m	
0	0	0	3	36	11	22	20	52	27	57	32	36	30
5	0	7	4	40	13	23	22	22	28	53	33	6	25

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10	0	25	6	3	14	42	23	22	29	49	33	31	20
15	0	58	6	24	16	21	24	36	30	38	33	50	15
20	1	39	8	38	17	50	25	45	31	23	34	6	10
25	2	36	10	20	19	30	26	53	32	2	34	15	5
		Sagittarius	Scorpio	Libra	Virgo	Leo	Cancer	degrees					

**Chapter VI: On the completion of the discussion on [the construction of] the astrolabe**

Having drawn these lines, we pick a long thread and pass it through the hole which is at the beginning of the divisions of the principle line and stretch it on the astrolabe till it enters the hole of the pole. We call that [part of thread] which comes out of the hole of the pole the thread of the axis, and the part that comes out of the origin of the principle line the thread of the chord. We pick from the thread of the axis a part [equal to] what is between the origin of the principle line and the pole, which is 60 parts, and make there a fixed knot. Then we make another knot [on the thread of the axis], this time movable on the part between the pole and the fixed knot, and we call it the *muri*. We suspend a plummet from the thread of the axis which stretches the thread of the chord and makes it straight. We fix the end of the thread of the chord in the first part of the thread of axis so that it moves with the *muri*. Thus, we end [the construction of] the instrument named the linear astrolabe.

**Part II: On the operation with the linear astrolabe, in seven chapters**

**Chapter I: On measuring the altitude of the Sun in order to know [other] operations**

We superpose the first part of the thread of the axis on the principle line so that the fixed knot lies on the origin of principle line. We put the *muri* on the division of the Sun on the [zodiacal] equator. Then we suspend the plummet and the thread of the axis, and we stand in front of the Sun. We take [in hand] the part of the astrolabe that reaches the origin of principle line and go forward and back until [no] shadow of

the astrolabe remains.<sup>1</sup> [Then] we stretch the chord thread up to the fixed knot of the thread of the axis and we superpose that amount of the chord thread on the principle line. [The mark on the chord line opposite to] the divisions of the principle line that are covered by the chord [thread] is the complement of the altitude of the Sun; we subtract it from 90 and the result is the altitude of the Sun.

**Chapter II: On the knowledge of the hours of the distance between the Sun and the meridian [*i.e.*, the hour angle]**

We put the *muri* on the end of the almucantar of the altitude [of the Sun] of the [current] moment and hold it with the left thumb. Then, with a finger of the right [hand], we stretch the chord thread from the *muri* to the center of that almucantar and superpose the right thumb on the chord thread stretched on the principle line. We suspend the plummet and the axis thread such that the thread [showing the] radius of the almucantar becomes a straight line, while the axis thread is straight [as well]. [Then] we stretch the chord thread from the origin of the principle line to the fixed knot on the axis thread. Then we superpose [the chord thread] on the principle line: The divisions of the arc [line] to which it is opposite, starting from the point opposite the origin of the principle line, are the hours of the distance between the Sun and the meridian [*i.e.*, the hour angle].

**Chapter III: On the knowledge of the ascendant**

We look for the part of the Sun on the line of ascension for the [terrestrial] equator and move from it counter-sequentially in the amount of the hours of the distance [of the Sun] from the meridian, if the altitude of the Sun is eastern, and move from it sequentially in the amount of the hours, if the altitude is western. The degree which we reach is the degree of the tenth. The degree opposite to it on [the lines of] the ascension of the locality is the ascendant.

**Chapter IV: On the knowledge of the half of the arc of daylight**

Its method is similar to what we mentioned for the hours of the distance from the meridian, except that we leave the *muri* in its place and we apply the end of the horizon instead of the end of the almucantar, and the center of the horizon instead of the center of the almucantar. Then

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1. In both copies we find ‘*an yabqá li-l-uşurlābi zillun*’, but it should be ‘*an [lā] yabqá li-l-uşurlābi zillun*’.



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we measure the hours of the distance from the meridian, which is the half of the arc of daylight. Doubling it would be the daylight.

##### **Chapter V: On the knowledge of the passed hours of the day**

If the altitude of the Sun is eastern, we subtract the hours of the distance from the half of the arc of daylight; and, if it is western, we add it to it. The result is the arc of revolution, we divide it by 15 and it represents the hours of the day which have passed. For the [number of] hours [until] noon, we divide the half of the arc of daylight by 15, the result would be the hours of noon, and doubling it would be the hours of daylight.

##### **Chapter VI: On the equalization of the houses**

We find the half of the arc of daylight for the degree of the ascendant. Then we take one third of the half of the arc of daylight and pick the degree of the tenth on [the line of] ascension for the [terrestrial] equator and move sequentially the amount of one third of the arc of daylight, the degree which we reach is the degree of the eleventh. Then we move sequentially from the degree of the eleventh by the amount of one third of the arc of daylight, and the degree which we reach is the degree of twelfth. Then we subtract one third of daylight from 60, what remains would be one third of the *nadir*. We move counter-sequentially from the degree of the tenth by the amount of one third of the *nadir*, [the degree] which we reach is the degree of the ninth. We move counter-sequentially from the degree of the ninth by the amount of one third of the *nadir*, [the degree] which we reach is the degree of the eighth. We set the degrees of the houses [which are] opposite to these [above-mentioned] houses [to be] equal to them, thus we obtain the degrees of the other houses.

##### **Chapter VII: On [the knowledge of] the altitude of the star and deriving the actions from it**

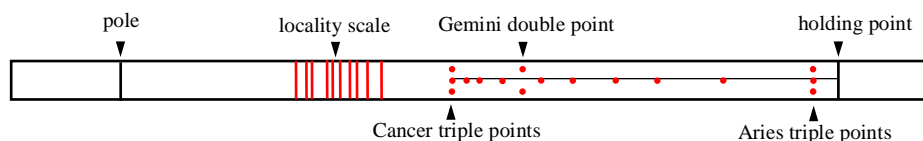
We take the altitude of a star whose distance from the pole is less than 60 parts. Apply that degree [of the star] in place of the degree of the Sun [in order] to derive the actions, except when deriving the passed hours, we move from the degree of the tenth to the *nadir* of the Sun, in this way we obtain the hours of the distance of the Sun from [the hidden half of] the meridian. Then we subtract these hours from half of the arc of the night, if the movement were sequential, and add to it if it were counter-sequential—what remains or stands out is the arc of revolution

from the sunset. We divide it by 15, the result is the hours which have passed of the night. Thus, the treatise is complete, with the assistance of God and best of his help.

## Appendix 1.2: Translation of Sharaf al-Dīn al-Ṭūsī's longer treatise (I) on the linear astrolabe, part 1, chapter 5

### Chapter V: On finding the direction of the *qibla*, the pole and the meridian line, and the north and south points

[1] Be aware that the places of the rays passing through the hole of the gnomon—which is the scale of the shadow— [for] four zodiacal signs, that is, Aries, Virgo, Libra, and Pisces, [on the body of astrolabe] are in front of those three points that are next to the points of the divisions of the [zodiacal] equator, and are on the same side (*zil* ' with [the divisions but] in different color, and their distance from [the point] in front of the holding point (*mumsik*) is 2;30 parts from the divisions of the equalized line (*al-istiwā* '). The rays of the beginning of Aries exit from the lowest hole [on the gnomon], and for every 2;30 degrees [of Aries, the exit place] elevates hole by hole until it reaches the highest hole for the end of Aries. For the beginning of Taurus, as well, [the rays exit from the highest hole].



[Figure 8 (our reconstruction)]

[2] For the 5 degrees of Taurus, [the falling place of the rays] proceeds forward one of those [points on the body of astrolabe] from the alignment of the triple points in the direction of pole. Likewise for every five [degrees], point by point. For the beginning of the Gemini there are two points, then for every five [degrees] point by point and for the beginning of Cancer there are three points. Then degrees of Cancer return along the degrees of Gemini, as was the case for the [zodiacal] equator divisions, until it returns [for] the beginning of Virgo to the point of Aries and the beginning of Taurus.

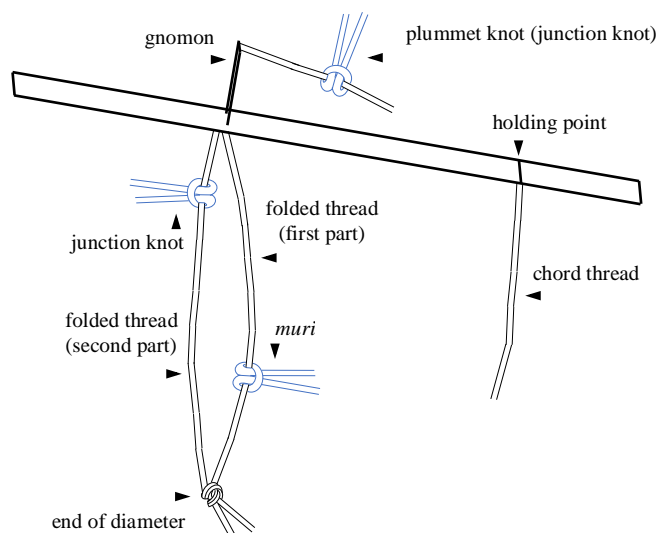
[3] Then, for 5 degrees of Virgo, [the exit place of the rays on the gnomon] descends two holes and, likewise, for every five [degrees] it descends one hole until it reaches the last hole for the beginning of Libra. Then it starts to rise two holes for every five [degrees] of Libra till, for the beginning of Scorpio, it reaches the highest hole. For all [the

degrees of] Scorpio, Sagittarius, Capricorn, and Aquarius, the rays come out from the highest hole and [the falling place] passes through the mentioned parts for Taurus and [through] what is after that, till, [for] the end of Sagittarius, it reaches the end of points of Gemini. [Then] Capricorn returns through the dots of Sagittarius [till], at end of the Pisces, [the falling place] returns to the beginning of Scorpio. Then [the exit place of the rays on the gnomon] descends two holes for every five degrees of Pisces until, for the end of Pisces, it reaches the last hole.

[4] **Section on the quality of operation [with the astrolabe].** Let all the holes of the scale [of the shadow, *i.e.*,] the gnomon, be closed except the one through which the ray should come out, based on the position of the Sun [according to] its [zodiacal] sign and degree. Then, if the Sun is in the northern signs, hang the thread of plummet from the twelfth hole and, for the southern [signs], from the sharpened ending of the gnomon which is not on the surface of equalized line.

[5] Be aware that, in this operation, you need a variety of motions. It is necessary to make a knot —similar to the knot of the end of [the previously] mentioned [thread of] diameter— in the thread of the plummet which is hanged from the tip of the gnomon, such that it moves on this thread. Make also a similar moving knot on one of the layers of the second part of the folded thread (*muthannā*) on which the *murī* is not. Put this part on the base of the gnomon and stretch the folded thread and put the end of the diameter upon the holding point (*mumsik*). This knot is called the knot of junction (*multaqā*), and also the knot of plummet thread [is called the knot of junction]. When the end of the diameter is joined to the holding point, superpose the junction knot upon the point from the line of [the falling] places of the rays (*khatt mawāqi' al-shu'ā'āt*), that is aligned with the letter mark for the latitude of the locality. Also, the thread of this plummet [should] remain always suspended from the highest hole. Also, pull the knot until it reaches and coincides with the point with which the other knots coincide, which is aligned with the mark of the latitude of the locality.

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[Figure 9 (our reconstruction)]

[6] Take the astrolabe by one of your hands from a point close to the falling place of the ray, based on the Sun's position. With the thumb and index finger of the other hand, take the end of the diameter from the knot and, with the [other] three upper fingers, take the chord [thread] close to the holding point (*mumsik*) such that [the chord thread] first passes through [the space between] the thumb and index finger, below the folded thread (*muthannā*), at the end of the diameter. It first passes leaning on a point on the index finger close to its tip, the knot of the end of the diameter being above it. Take the tip of the thumbnail above the end of the diameter to hold it on the chord [thread] such that this knot [of the end of the diameter] and its passing on the chord [thread] under [the thumb] is visible. Then pass the rest of the chord [thread] behind the body of the other three fingers and hold it by them.

[7] Then, when needing to increase [the length of] the chord [thread], which is between the end of the diameter and holding point (*mumsik*), loosen this hold and pull the end of the diameter in the opposite direction of the astrolabe so the end of the diameter moves on the chord [thread] and its distance from the holding point increases. Then hold [it] strongly such that the chord [thread] does not increase in distance [from the holding point]. When needing to decrease [the length], bring the hand holding the end of the diameter and the chord [thread] close to the

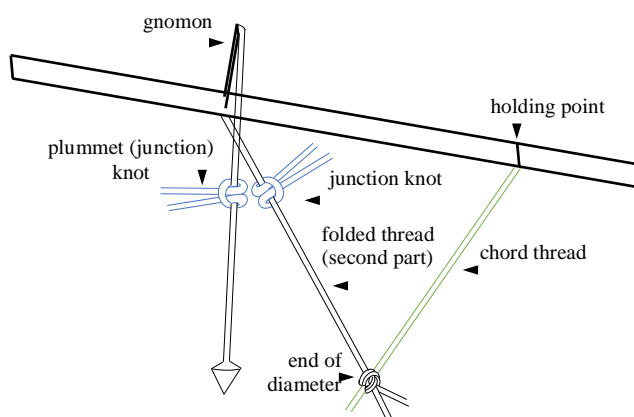
hand holding the astrolabe. Then grab the excess of the chord [thread] —which exceeds the end of diameter— with the tip of the thumb and middle finger which are surrounding the astrolabe. Release the chord [thread] from the three fingers and continue to hold gently [the chord thread] at the end of diameter between the thumb and index finger. Then pull this hand from the astrolabe and holding point (*mumsik*) [until] the part of the chord [thread] between the end of the diameter and holding point (*mumsik*) passes and comes out through the thumb and index finger and becomes [part] of what is between the places taken by the fingers of the two hands.

[8] Then, after the reaching a [desired] length of the chord [thread] between the holding point (*mumsik*) and the [end of] the diameter using this method, you need to superpose the two junction knots (*multaqá*). For this [configuration], three motions are needed: one of them is that you rotate the astrolabe around itself such that it does not move from its position, and the gnomon rotates around [the body of the astrolabe] so the tip of gnomon goes up and down; the second [motion] is that by which the end of the equalized line inclines upward and downward; and the third [motion] is that by which the end of the diameter pulled, by two fingers, moves upward and downward.

[9] Then, to superpose the two [junction] knots, after taking a length of the chord [thread] between the holding point and the end of the diameter, pull the end of the diameter and rotate the astrolabe by the first motion and the end of diameter by the third motion until the plummet knot touches the diameter thread. If, then, [the plummet knot] superposes on the knot of the diameter, that is what was desired. If the knot of the diameter is below the tangency position, rise the head of the astrolabe by the second motion and the end of diameter by the third motion and lower the tip of the gnomon by the [first] motion, then the two knots will be superposed. If the knot of the diameter is above the tangency point, lower the head of the astrolabe and the end of diameter by the second and third motions, and raise the tip of the gnomon by the first motion. Also, if the two knots are superposed upon a length of the mentioned chord [thread] and [then] the chord has been increased a little bit, [to return the superposition of the knots] lower the head of the astrolabe by the second motion and the tip of the gnomon by the first motion and the end of the diameter by the third motion. If the [length

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of] the chord has been decreased, raise these three by the three motions so the superposition returns.



[Figure 10 (our reconstruction)]

[10] After these preliminaries, if the Sun is in the northern [zodiacal] signs, face the *qibla* such that the west is on your right hand and the east is on your left hand. Take the astrolabe by the hand that is not in the direction of the Sun, from a place close to the alignment of the falling place of the Sun's rays. Take the thumb above the equalized line on the [same] place aligning the [falling] place [of the rays of the Sun]. Take the end of the diameter by the hand which is in the direction of the Sun, *i.e.*, if the Sun is eastern, take the astrolabe by the right [hand] and the end of the diameter by the left [hand], and the opposite [in] the opposite [case].

[11] Take a length of the chord [thread] —between the end of the diameter and the holding point— that is thought to be the hours of the Sun's distance from the meridian. The greater the distance, the longer you take the chord [thread]. Then, look at the shadow of the astrolabe cast on your clutched fingers, *i.e.*, if the shadow is toward the east, turn your whole body toward west until the shadow of the gnomon returns to the equalized line. If the ray that passes through the hole [of the gnomon] does not reach the tip of the thumb, decrease the chord [thread] which is between the end of the diameter and the holding point and revert the superposition of the knots. And if [the ray] exceeds it, increase the chord [thread] and revert the superposition of the knots. Then, examine the shadow of the gnomon with regard to the equalized

line and test [the falling ray] similarly, and increase [or] decrease the chord thread until the ray falls on the tip of the thumb.

[12] Upon that [configuration], the tip of the gnomon is directed toward the pole of the world such that if an observer looks on the alignment [of the gnomon] his eyes fall on the northern pole. The triangle formed from the gnomon, the plummet thread, and the half of the diameter will be in the plane of the meridian circle, and the plane shaped from the half of the diameter and the equalized line will be the plane of the equinoctial, such that if someone faces the direction of the half of the diameter, he will be facing toward true south and north.

[13] Then, superpose the chord thread, which is from the holding point, to the end of the diameter on the equalized line. The number of divisions of the arcs [on the astrolabe] which are in front of this chord from the holding point to the end [of the chord thread] is the hours of the distance [of the Sun] from the meridian.

[14] However, if the Sun is in the southern [zodiacal] signs, hang the plummet from the sharpened ending of the gnomon and take the astrolabe such that the narrow half of the gnomon is above and the wider [half] is below and place the east on your right hand and the west on your left and your sight on the equalized line. Look at it and at the gnomon from below. This [configuration] obtains if you put the equalized line perpendicular to the horizon and the end of the line is above and you look at it. Then, incline its upper head toward yourself and raise it up in this configuration. The plummet suspended from the sharpened ending of the gnomon touches the side of the astrolabe and it almost touches the equalized line, if the thickness of the astrolabe does not push it. The astrolabe should be in the hand that is not in the Sun's direction, as we mentioned, and the end of diameter in the hand that is in its direction, as we mentioned in the northern [zodiacal signs].

[15] Then, take a length of the chord [thread] based on the approximate hours of the Sun's distance from the meridian. By the first motion, incline the sharpened tip of the gnomon toward the direction of the Sun and the half of the diameter, and superpose the two knots by those three motions. Then, look at the shadow of the astrolabe cast on your fingers clutching the astrolabe, as we mentioned. Rotate your whole body in the opposite direction [of the shadow] until the shadow of the gnomon falls on the equalized line. If the [falling] ray does not reach the tip of the thumb, increase [the length] of the chord thread and lower the head



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of the astrolabe with the second motion and the sharpened tip of the gnomon with the first motion, and revert the superposition of the two knots and turn until the ray comes back to the equalized line. If [the ray falls] longer, it is necessary to raise all of those and decrease the chord [thread] and test it likewise until the ray falls on the tip of the thumb. In this [configuration], then, the sharpened tip [of the gnomon] is directed toward the north pole and the triangle [formed] by the gnomon, the plummet thread, and the half of the diameter is in the plane of the meridian circle, and the plane formed by the equalized line and the half of the diameter [is] the plane of the equinoctial, and the mentioned chord [thread] is equal to the hours of the [Sun's] distance [from the meridian], and that is what was desired.

[16] **As for the knowledge of the direction of the *qibla***, it is necessary to increase significantly the length of the plummet thread and hang another heavy plummet from the end of another thread. Make this thread pass through the clutched thumb and the index finger at the end of the diameter and the chord [thread]. [Then], execute the mentioned operation until you determine the length of the chord [thread] — between the holding point and the end of the diameter— and the ray falls on the desired place. Then, release this second plummet while preserving the length of the chord thread which is between the end of the diameter and the holding point such that it does not increase or decrease. Then, lower the two hands with the astrolabe while preserving the superposition of the knots and the falling of the ray on the necessary position and [preserving] the length of the chord and the tautness of the end of the diameter, until the plummet of the tip of the gnomon reaches the ground. The tip of the mentioned suspended plummet also has reached the ground such that both [plummets] touch together on the ground. Then the line connecting them is the line of the meridian. If the plummet of the tip of the gnomon reaches the ground, but then the plummet of the end of the diameter does not reach, release the excess of the thread of this plummet which is between the thumb and the index finger. This [can be done] since that the plummet has extra weight and you loosen the hold between the thumb on the index finger which grabs the end of diameter and this thread until a length of the thread is pulled by the weight of the plummet, and then grab it [strongly] and thus the length [of the thread] has been increased. Test it until, by the mentioned operation, both plummets together reach the ground and the meridian

line will obtain. Then, place the astrolabe on [the meridian line] such that the end of the equalized line is toward the south and its beginning is toward you and the equalized line is facing the sky. Take a length of the chord [thread] from the holding place and superpose it on the equalized line up to a place such that the divisions of the arc aligned with it up to its end equal the inclination of the locality [from the *qibla*]. Take with the thumb and index finger the end of the diameter on the end of this length of the chord [thread] and pull them until both of them straighten. If the inclination of the locality is eastward, then pull them toward the west, and if it is westward [pull them] toward the east. Then, [at this configuration], the alignment of the half of the diameter is toward the *qibla*, and that is what was desired.

## Appendix 2.1: Edition of the Arabic Text of Sharaf al-Dīn al-Tūsī's shorter treatise on the linear astrolabe (MSS A and N)

رسالة في الاصطراب الخطي للإمام شرف الدين الطوسي رحمه الله<sup>١</sup>  
وبعد فهذه رسالة في الاصطراب الخطي، مرتبة على قسمين: الأول في صنعة؛ والثاني<sup>٢</sup> في العمل  
به.

### القسم الأول في صنعة الاصطراب الخطي وفيه ستة أبواب

#### الباب الأول في رسم الأوتار والقسي

نأخذ مسطرة معتدلة من خشب أو صفر<sup>٣</sup> ونخط في وسطها خطاً مستقيماً ونقسمه بمائة<sup>٤</sup>  
وعشرين<sup>٥</sup> قسماً ونسميه خط الأصول. ونثقب في أول الأجزاء ثقباً ونسميه بالمبدأ، وعلى بعد  
ستين<sup>٦</sup> جزءاً ثقباً آخر نسميه بالقطب. ونخط على الأيسر من هذا الخط خطاً آخر نسميه خط  
القسي، ونجعل بحذاء المبدأ على هذا الخط مركزاً ونطلب من جدول القسي والأوتار قوس<sup>٧</sup>  
خمس درجات وننظر كم نصيبه من الوتر ثم نطلب ذلك القدر من خط الأصول ونفتح الفرجار<sup>٨</sup>  
بمقداره ونعمل<sup>٩</sup> على خط<sup>١٠</sup> القسي علامة، فمن<sup>١١</sup> الأول إلى هذه العلامة خمس درجات من  
القوس. وهكذا العمل لعشر<sup>١٢</sup> درجات من القوس، ولخمس<sup>١٣</sup> عشرة درجة،<sup>١٤</sup> ولعشرين<sup>١٥</sup> درجة،  
وهكذا إلى مائة وعشرين درجة من القوس أو أكثر. والأسهل في العمل أن نطلب قوس عشر  
درجات من الجدول وننظر كم نصيبه من الوتر، ثم<sup>١٦</sup> نطلب ذلك القدر من خط الأصول وننقص  
منه وتر خمس درجات من القوس ونفتح الفرجار<sup>١٧</sup> بقدر ما يبقى من الوتر ونضع رأسه على  
علامة خمس درجات ونضع الرأس الآخر بإزاء منتهى وتر عشر درجات ونعلم هناك علامة  
أخرى، وبهذا الطريق نعمل لخمس عشرة<sup>١٨</sup> درجة من القوس، وهكذا إلى آخر العمل.

#### الباب الثاني<sup>١٩</sup> في رسم مطالع البروج بخط الاستواء

نخط على أيمن خط الأصول خطاً نسميه خط المطالع بخط الاستواء، وننظر في جدول مطالع  
البروج بخط الاستواء ونطلب مطالع خمس درج من الحمل ونأخذ من مبدأ خط الأصول أجزاء  
بذلك القدر ونعلم بحذاء آخر جزء منه على خط مطالع البروج بخط الاستواء علامة،<sup>٢٠</sup> فمن أول  
الخط إلى تلك العلامة هو مطالع خمس درج من الحمل. وكذا نعمل لعشر<sup>٢١</sup> درج من الحمل  
ولخمس عشرة<sup>٢٢</sup> ولعشرين إلى آخر الحمل. وكذا نعمل للثور والجوزاء حتى ينتهي إلى آخر  
الجوزاء عند تسعين جزءاً<sup>٢٣</sup> من خط الأصول. فنكتب<sup>٢٤</sup> علامة الحمل في مبدأ الخط، ثم علامة  
الثور عند تمام مطالع ثلاثين<sup>٢٥</sup> درجة من الحمل، ونكتب علامة الجوزاء عند تمام مطالع الثور.  
ثم نكتب علامة سائر البروج على هذه الهيئة ليحصل مطالع البروج بخط الاستواء بعضها على  
التوالي وبعضها<sup>٢٦</sup> على خلافه:





ل	لو <sup>٧١</sup>	لب	نز <sup>٧٠</sup>	كز	نب <sup>٦٩</sup>	ك	نب	يا	لو <sup>٦٨</sup>	ج	و	و	و
كه	وه <sup>٧٥</sup>	لج	نج	كح	كب	كب	كج	يج	م <sup>٧٣</sup>	د	ز <sup>٧٢</sup>	و	و
ك	لا	لج	مط	كط	كب	كج	مب	يد	ج	و	كه	و	ي
يه	ن	لج	لح	ل	لو <sup>٧٩</sup>	كد	كا <sup>٧٨</sup>	يو	كد	و <sup>٧٧</sup>	نح	و	يه
ي	و	لد	كج	لا	مه	كه	ن	يز	لح <sup>٨٠</sup>	ح	لط	ا	ك
ه	يه	لد	لب	لب	نج	كو	ل <sup>٨٣</sup>	يط	ك	ي	لو <sup>٨١</sup>	ب	كه
الأجزاء	السرطان	الأسد	السنبلة	الميزان	العقرب	القوس							

#### الباب السادس في تَمَّة القول في الاصطربلاب<sup>٨٦</sup>

إذا رسمنا<sup>٨٧</sup> هذه<sup>٨٨</sup> الخطوط، نأخذ خيطاً<sup>٨٩</sup> طويلاً ونخرجه من الثقب التي في مبدأ أجزاء خطِّ الأصول. ونمده على الاصطربلاب حتى يدخل في الثقب التي<sup>٩٠</sup> هو<sup>٩١</sup> القطب؛ فالذي يخرج<sup>٩٢</sup> من ثقب القطب نسميه خيط المحور، والذي يخرج من ثقب مبدأ خطِّ الأصول نسميه خيط الوتر.<sup>٩٣</sup> ونأخذ من خيط المحور مقدار<sup>٩٤</sup> ما بين مبدأ خطِّ الأصول والقطب- وهو ستون جزءاً- ونعقد هناك عقدة ثابتة. ثم نعقد عليه عقدة أخرى، متحركة في القسم الذي بين<sup>٩٥</sup> القطب والعقدة الثابتة ونسميها بالمرى. ونعلق بخيط المحور شاقولاً، يمد خيط الوتر<sup>٩٦</sup> ويعدله. ونسد طرف<sup>٩٧</sup> خيط الوتر<sup>٩٨</sup> على القسم الأول من خيط المحور بحيث يتحرك مع المرى. وحينئذ<sup>٩٩</sup> تتم الآلة الموسومة بالاصطربلاب الخطي.

#### القسم الثاني في العمل بالاصطربلاب الخطي وفيه<sup>١٠٠</sup> سبعة أبواب

##### الباب الأول في<sup>١٠١</sup> أخذ ارتفاع الشمس لمعرفة<sup>١٠٢</sup> الأعمال

نطبق<sup>١٠٣</sup> القسم الأول من خيط المحور على خطِّ الأصول حتى تنطبق العقدة الثابتة على مبدأ خطِّ الأصول، ونضع المرى على جزء الشمس من أجزاء المنطقة. ثم نعلق الشاقول وخيط المحور ونقف مقابل<sup>١٠٤</sup> الشمس<sup>١٠٥</sup> ونأخذ الجانب الذي يلي<sup>١٠٦</sup> مبدأ خطِّ الأصول من الاصطربلاب ونتقدم ونتأخر إلى أن يبقى<sup>١٠٧</sup> للاصطربلاب ظل. ونمد خيط الوتر إلى العقدة الثابتة من خيط المحور ونطبق ذلك القدر من خيط الوتر على خطِّ الأصول فما ينطبق عليه الوتر من أجزاء<sup>١٠٨</sup> خطِّ الأصول، فهو تمام ارتفاع الشمس، فننقصه من تسعين<sup>١٠٩</sup> فما بقي فهو ارتفاع الشمس<sup>١١٠</sup>.

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### الباب الثاني في معرفة أزمنة بعد الشمس من نصف النهار

نضع المري على نهاية مقنطرة ارتفاع الوقت، ونمسكه<sup>١١١</sup> بالإبهام اليسرى. ثم نمد<sup>١١٢</sup> خيط الوتر من المري بالأصبع اليمنى إلى مركز<sup>١١٣</sup> تلك المقنطرة ونطبّق الإبهام اليمنى على خيط الوتر ممتداً على خط<sup>١١٤</sup> الأصول. ونرسل الشاقول وخيط المحور حتى يصير خيط نصف قطر المقنطرة خطأً مستقيماً مع استواء خيط المحور. ونمد خيط الوتر من مبدأ خط الأصول إلى العقدة الثابتة على خيط المحور. ثم نطبّقه على خط الأصول، فما يحاذيه من أجزاء القوس - مبتداه من محاذاة مبدأ<sup>١١٥</sup> خط الأصول - هي أزمنة بعد الشمس من نصف النهار.<sup>١١٦</sup>

### الباب الثالث في معرفة الطالع

نطلب جزء الشمس في مطالع خط الاستواء، وننتقل منه على<sup>١١٧</sup> خلاف التوالي بقدر أزمنة البعد عن نصف النهار إن كان ارتفاع الشمس شرقياً وننتقل منه على التوالي بقدر الأزمنة إن كان الارتفاع غربياً. فالدرجة التي انتهينا إليها هي درجة العاشر والدرجة المقابلة له من مطالع البلد هو الطالع.

### الباب الرابع في معرفة نصف قوس نهار الشمس

وطريقه مثل ما ذكرناه في أزمنة ساعات البعد عن نصف النهار، إلا أننا نترك المري على حاله ونستعمل نهاية الأفق بدل نهاية المقنطرة ومركز الأفق بدل مركز المقنطرة، ونستخرج أزمنة البعد عن نصف النهار، فما كان فهو نصف قوس النهار وضعفها هو قوس النهار.

### الباب الخامس في معرفة الساعات الماضية من النهار

فإن كان ارتفاع الشمس شرقياً، ننقص أزمنة البعد من نصف قوس نهار الشمس، وإن كان غربياً، نزيدها عليه. فما بلغ أو بقي فهو الدائر، فنقسمه على خمسة عشر،<sup>١١٨</sup> فما كان فهو الساعات الماضية من النهار. وأما ساعات نصف النهار فنقسم<sup>١١٩</sup> نصف قوس النهار على خمسة عشر،<sup>١٢٠</sup> فما كان فهو ساعات نصف النهار<sup>١٢١</sup> وضعفها ساعات النهار.

### الباب السادس في<sup>١٢٢</sup> تسوية البيوت

نستخرج نصف قوس النهار درجة الطالع. ثم نأخذ<sup>١٢٣</sup> ثلث نصف قوس النهار ونطلب درجة العاشر في مطالع خط الاستواء وننتقل منها بمقدار<sup>١٢٤</sup> ثلث نصف قوس<sup>١٢٥</sup> النهار إلى التوالي، فما انتهى إليه فهو درجة الحادي عشر. ثم ننتقل من درجة الحادي عشر<sup>١٢٦</sup> إلى التوالي<sup>١٢٧</sup> بمقدار ثلث نصف قوس النهار، فما انتهى إليه فهو درجة الثاني عشر. ثم ننقص ثلث نصف قوس النهار من ستين، فيبقى ثلث النظر. فننتقل من درجة العاشر بمقدار ثلث النظر على خلاف التوالي، فما انتهى إليه فهو درجة التاسع. وننتقل من درجة التاسع بمقدار ثلث النظر على خلاف التوالي، فما انتهى إليه فهو درجة الثامن. ونجعل درجات البيوت المقابلة بهذه<sup>١٢٨</sup> البيوت مساوية لها، فنحصل درجات سائر البيوت.

الباب السابع في<sup>١٢٩</sup> ارتفاع الكوكب واستخراج الأعمال منه  
نأخذ ارتفاع كوكب يكون بعده عن<sup>١٣٠</sup> القطب أقلّ من ستين<sup>١٣١</sup> جزءاً. ولتعمل ذلك الجزء بدل  
جزء الشمس في استخراج الأعمال، إلا أن في استخراج الساعات الماضية، ننتقل من درجة  
العاشر إلى نظير جزء الشمس، فنحصل أزمنا بعد الشمس عن نصف الليل. ثمّ ننقص هذه الأزمنة  
عن نصف قوس الليل إن كان الانتقال على التوالي ونزيده عليه إن كان على خلاف التوالي، فما  
بلغ أو بقي، فهو الدائر من وقت غروب الشمس. فنقسمه على خمسة عشر،<sup>١٣٢</sup> فما كان فهو  
الساعات الماضية من الليل. تمّت الرسالة بعون الله تعالى وحسن توفيقه.<sup>١٣٣</sup>



**Appendix 2.2: Edition of the Arabic Text of Sharaf al-Dīn al-Ṭūsī's longer treatise on the linear astrolabe, part 1, chapter 5 (MSS B, C, L, K, and M)**

الباب الخامس في استخراج سمت القبلة والقطب وخط نصف النهار ونقطة الشمال والجنوب

[١] إعلم بأن مواقع الشعاعات النافذة<sup>١</sup> من ثقب الشخص<sup>٢</sup> وهو مقياس الظل، أما البروج الأربعة التي هي الحمل والسنبلة والميزان والحوت، تكون بحذاء النقط الثلاثة<sup>٣</sup> من النقط التي تكون في جوار نقط أجزاء المنطقه، وتكون معها على ضلع واحد وتخالفها<sup>٤</sup> في اللون وبعدها عن<sup>٥</sup> محاذاة الممسك بقدر جزئين ونصف من أجزاء الاستواء. ولأوائل الحمل يكون خروج الشعاع من الثقب الأسفل ولكل درجتين ونصف منه، يرتفع ثقبه ثقبه<sup>٦</sup> حتى ينتهي لآخر الحمل إلى الثقب الأعلى ولأول الثور أيضاً.<sup>٧</sup>

[٢] ولخمس درجات من الثور تتقدم نقطة منها إلى جهة القطب من محاذاة تلك النقط<sup>٨</sup> الثلاث وهكذا لكل<sup>٩</sup> خمسة خمسة نقطة نقطة<sup>١٠</sup>. ولأول الجوزاء نقطتان ثم لكل خمسة خمسة<sup>١١</sup> نقطة نقطة ولأول السرطان ثلاث نقط<sup>١٢</sup>. ثم ترجع درجات السرطان على درجات الجوزاء، كما في أجزاء<sup>١٣</sup> المنطقه، حتى تعود إلى أول السنبلة، إلى نقطة<sup>١٤</sup> الحمل وأول الثور.

[٣] ثم لخمس درجات من السنبلة<sup>١٥</sup> ينزل ثقتين<sup>١٦</sup>. وهكذا لكل<sup>١٧</sup> خمس ينزل ثقبه حتى ينتهي عند أول<sup>١٨</sup> الميزان إلى الثقب الأخير<sup>١٩</sup>. ثم يأخذ في الصعود لكل خمس خمس من الميزان ثقتين ثقتين<sup>٢٠</sup> إلى أن ينتهي عند أول العقرب إلى الثقب الفوقاني. ولجميع العقرب والقوس والجددي<sup>٢١</sup> والدلو يخرج الشعاع من الثقب الفوقاني. وتنتقل على الأجزاء المذكورة للثور وما بعده، حتى تنتهي في آخر القوس<sup>٢٢</sup> إلى آخر نقط<sup>٢٣</sup> الجوزاء. وترجع<sup>٢٤</sup> الجدي على نقط<sup>٢٥</sup> القوس وعند أول الحوت تعود إلى أول العقرب. ثم ينزل لكل خمس درجات من الحوت ثقتين<sup>٢٦</sup> حتى ينتهي<sup>٢٧</sup> عند آخر الحوت إلى الثقب الأخير<sup>٢٨</sup>.

[٤] فصل في كيفية العمل. فلتكن جميع ثقب المقياس، وهو الشخص، مسدوداً<sup>٢٩</sup> إلا الواحد الذي يجب أن<sup>٣٠</sup> يخرج منه الشعاع بحسب موضع الشمس في برجه ودرجته. ثم إن كان الشمس في البروج الشماليّة تعلق خيط الشاقول من الثقب الثاني عشر، وللجنوبيّة<sup>٣١</sup> من الطرف الدقيق الذي للمقياس وليس على وجه خط الاستواء.

[٥] واعلم بأنك تحتاج في هذا العمل إلى أصناف من الحركات. فيجب أن تعمل في خيط الشاقول المعلق من رأس الشخص، عقداً شبيهاً<sup>٣٢</sup> بعقد طرف القطر المذكور، بحيث<sup>٣٣</sup> يمكن نقله على<sup>٣٤</sup> هذا الخيط<sup>٣٥</sup>. وتجعل في طبقة واحدة من القسم الثاني من الخيط المثانة<sup>٣٦</sup> الذي ليس عليه المري، عقداً شبيهاً بهذا أيضاً منتقلاً. وتجعل هذا القسم في أصل الشخص وتمد الخيط المثانة<sup>٣٧</sup> وتطبق طرف<sup>٣٨</sup> القطر<sup>٣٩</sup> على الممسك. وهذا العقد نسميه عقد الملتقى وكذا

عقد خيط الشاقول. فتطبق عقد الملتقى أيضاً<sup>٤٠</sup> عند إطباق طرف القطر على الممسك، على الموضع المحاذي لعلامة حرف عرض المدينة من خطّ مواقع الشعاعات. وكذلك يكون خيط هذا الشاقول أبداً معلقاً من الثقب<sup>٤١</sup> الفوقاني. وتمد أيضاً عقدة حتى ينتهي وينطبق على الموضع الذي انطبق عليه العقد الآخر من محاذاة علامة عرض<sup>٤٢</sup> المدينة.

[٦] فتأخذ الاصطرلاب بأحدي<sup>٤٣</sup> اليدين، عند الموضع القريب من موقع الشعاع، بحسب<sup>٤٤</sup> موضع الشمس. وتأخذ طرف القطر، عند العقد، بإبهام اليد الأخرى وسبابته وتأخذ الوتر قرب الممسك، بالأصابع الثلاثة<sup>٤٥</sup> الفوقانية، بحيث يمر<sup>٤٦</sup> أولاً فيما بين الإبهام والسبابة، تحت<sup>٤٧</sup> الخيط المشاة<sup>٤٨</sup> عند طرف القطر. فيمر أولاً معتمداً على الموضع القريب من رأس<sup>٤٩</sup> السبابة ويكون فوقه عقد<sup>٥٠</sup> طرف القطر. وتجعل فوق طرف القطر رأس ظفر الإبهام لتمسكه على الوتر بحيث يكون هذا العقد ووقوعه<sup>٥١</sup> على الوتر تحته ظاهراً. ثم تمر بقية الوتر تحت باطن الأصابع الثلاثة<sup>٥٢</sup> الباقية وتقبض بها عليه.

[٧] ثم عند الحاجة إلى زيادة وتر، ما بين طرف القطر والممسك،<sup>٥٣</sup> تُرخي هذا القبض وتمد<sup>٥٤</sup> طرف القطر إلى خلاف جانب الاصطرلاب فيمر طرف القطر على الوتر ويزيد بعده عن الممسك. ثم تقبض بشدة<sup>٥٥</sup> لئلا يزيد الوتر بعد<sup>٥٦</sup>. وعند الحاجة إلى النقصان تقرب اليد الماسكة<sup>٥٧</sup> لطرف القطر والوتر في اليد الماسكة<sup>٥٨</sup> للاصطرلاب<sup>٥٩</sup>. فتقبض من<sup>٦٠</sup> بقية الوتر<sup>٦١</sup> المجاوزة عن<sup>٦٢</sup> طرف القطر بطرف<sup>٦٣</sup> الإبهام والوسطى<sup>٦٤</sup> المحيطين بالاصطرلاب. وتخلي الوتر من الأصابع الثلاث وتحفظ إمساكه بين الإبهام والسبابة<sup>٦٥</sup> عند طرف القطر برفق. ثم تمد هذا<sup>٦٦</sup> اليد عن<sup>٦٧</sup> الاصطرلاب والممسك فيمر من<sup>٦٨</sup> الوتر الذي بين طرف القطر والممسك فيما بين الإبهام والسبابة ويخرج فيصير فيما بين الموضعين المقبوضين بأصابع<sup>٦٩</sup> اليدين.

[٨] ثم بعد تحصيل<sup>٧٠</sup> قدر من الوتر بين الممسك والقطر على هذه<sup>٧١</sup> الجهة تحتاج إلى تطابق عقدتي<sup>٧٢</sup> الملتقى. وتحتاج في ذلك إلى ثلاث حركات: أحدها<sup>٧٣</sup> أن تدير الاصطرلاب على نفسه بحيث لا ينتقل عن موضعه ويدور الشخص حوله فيصير رأس الشخص<sup>٧٤</sup> أميل إلى جهة فوق أو إلى تحت؛ والثانية أن يصير بها رأس خطّ الإستواء أميل إلى فوق أو إلى تحت؛ والثالثة أن يصير بها طرف القطر الممدود<sup>٧٥</sup> بالإصبعين أميل إلى فوق أو إلى تحت.

[٩] ثم لإطباق العقد بعد أخذ مقدار من الوتر بين الممسك وطرف القطر تمدّ طرف القطر وتدير الاصطرلاب<sup>٧٦</sup> بالحركة الأولى وطرف القطر بالحركة الثالثة حتى يصير عقد الشاقول مماساً لخيط القطر. فإن كان عند ذلك واقعاً على عقد القطر فهو المراد. وإن كان عقد القطر أسفل من موضع المماسّة تشيل<sup>٧٨</sup> رأس الاصطرلاب بالحركة الثانية وطرف القطر بالحركة الثالثة<sup>٧٩</sup> وتحط رأس الشخص بالحركة فيتطابق<sup>٨٠</sup> العقدان. وإن كان عقد القطر أرفع من موضع المماسّة فتحط رأس الاصطرلاب وطرف القطر بالحركتين الثانية والثالثة وتشيل رأس الشخص بالحركة الأولى. وأيضاً إذا كان العقدان متطابقين<sup>٨١</sup> عند أحد مقدار من<sup>٨٢</sup> الوتر المذكور وزدت

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في الوتر قدرأ، فتحط رأس الاصطرلاب بالحركة الثانية ورأس الشخص<sup>٨٣</sup> بالحركة الأولى وطرف القطر بالحركة الثالثة.<sup>٨٤</sup> وإن كنت نقصت<sup>٨٥</sup> من الوتر فتشيل هذه الثلاثة بالحركات الثلاثة فيعود التطابق.

[١٠] وبعد هذه<sup>٨٦</sup> المقدمات إن كان<sup>٨٧</sup> الشمس في البروج الشماليّة تستقبل القبلة بحيث يكون المغرب على يمينك والمشرق على يسارك. وتأخذ الاصطرلاب باليد التي<sup>٨٨</sup> لا تلي جهة الشمس في الموضع القريب من محاذاة<sup>٨٩</sup> موقع شعاع الشمس وتجعل الإبهام<sup>٩٠</sup> فوق خط الإستواء على النقطة المحاذية لتلك النقطة. وتأخذ طرف القطر باليد التي<sup>٩١</sup> تلي جهة الشمس أعني إن كان الشمس شرقياً<sup>٩٢</sup> تأخذ الاصطرلاب باليمين وطرف القطر باليسار وعكسه عكسه.<sup>٩٣</sup>

[١١] وتأخذ بين طرف القطر والممسك قدرأ من الوتر بحسب ما يظنّ أزمنة بعد<sup>٩٤</sup> الشمس عن نصف النهار وكلّما<sup>٩٥</sup> كان أبعد تأخذ الوتر أطول. ثمّ تنظر إلى ظلّ الاصطرلاب الواقع على أصابعك القابضة، أعني إن كان الظلّ إلى جهة الشرق<sup>٩٦</sup> فتدور بجميع شخصك إلى جهة الغرب<sup>٩٧</sup> حتّى يعود ظلّ الشخص إلى خطّ الإستواء. فإن كان الشعاع النافذ في الثقب يقصر<sup>٩٨</sup> عن البلوغ إلى رأس الإبهام فتتقص<sup>٩٩</sup> من خيط الوتر الذي بين<sup>١٠٠</sup> طرف القطر والممسك، وترد تطابق العقدين وإن كان يتجاوزه فتزيد<sup>١٠١</sup> في خيط الوتر وترد تطابق العقدين. ثمّ تعتبر<sup>١٠٢</sup> ظلّ الشخص إلى خطّ الإستواء وتمتحن كذلك فتزيد<sup>١٠٣</sup> في الوتر وتنقص حتّى يقع الشعاع على رأس الإبهام. [١٢] فعند ذلك<sup>١٠٤</sup> يكون رأس الشخص مسامتاً<sup>١٠٥</sup> لقطب العالم حتّى لو نظر ناظر<sup>١٠٦</sup> على استقامة<sup>١٠٧</sup> حينئذ يكون بصره على القطب الشماليّ. ويكون المثلث الحاصل من الشخص وخيط الشاقول ونصف القطر في سطح دائرة نصف النهار والسطح الحاصل من نصف القطر وخطّ الإستواء هو سطح معدّل النهار؛ حتّى لو توجه أحد<sup>١٠٨</sup> حينئذ على استقامة نصف القطر يكون متوجهاً إلى حقيقة الجنوب والشمال.

[١٣] ثمّ تطبق خيط الوتر الذي من الممسك إلى طرف القطر على خطّ الإستواء، فعدد أجزاء القوس المحاذي لهذا الوتر من الممسك إلى طرفه يكون أزمنة البعد عن نصف النهار.

[١٤] أمّا إن كان<sup>١٠٩</sup> الشمس في البروج الجنوبيّة تعلق الشاقول من<sup>١١٠</sup> الطرف الدقيق للمقياس وتأخذ الاصطرلاب على وضع يكون النصف الدقيق من<sup>١١١</sup> الشخص أميل إلى فوق والعريض إلى تحت وتجعل المشرق على يمينك والمغرب على يسارك ونظرك على خطّ الإستواء. تنظر إليه وإلى الشخص من أسفل وذلك<sup>١١٢</sup> يحصل بأن ينصب خطّ الإستواء قائماً على الأفق ويكون آخره إلى جهة فوق ونظرك عليه. ثمّ تميل<sup>١١٣</sup> رأسه الفوقاني إلى جهتك وترفعه على هذا الوضع. فيكون الشاقول المنحدر من الرأس الدقيق للمقياس يماس جانب الاصطرلاب ويكاد يماس خطّ الإستواء لو لم يزااحمه غلظ الاصطرلاب. وينبغي أن يكون الاصطرلاب في اليد التي<sup>١١٤</sup> لا<sup>١١٥</sup> تلي جهة الشمس كما ذكرنا وطرف القطر في اليد التي<sup>١١٦</sup> تلي جهته كما ذكرنا<sup>١١٧</sup> في الشمال.

[١٥] ثم تأخذ قدرًا من الوتر بحسب أزمنة بُعد الشمس عن<sup>١١٨</sup> نصف النهار بالتخمين. وتميل رأس الشخص الدقيق بالحركة الأولى إلى جهة الشمس ونصف القطر وتطبق العقدين بتلك الحركات الثلاثة. ثم تنظر إلى ظل الاصطرلاب الواقع على أصابعك القابضة عليه كما ذكرنا. وتدور بجميع شخصك إلى خلاف جهته حتى يصير ظل الشخص على خط الإستواء. فإن كان الشعاع قاصراً عن طرف الإبهام فتزيد<sup>١١٩</sup> في خيط الوتر وتحط<sup>١٢٠</sup> رأس الاصطرلاب بالحركة الثانية ورأس المقياس الدقيق بالحركة الأولى وترد تطابق العقدين وتدور حتى يعود الشعاع إلى خط الإستواء. وإن كان أطول من الواجب ترفع كل واحد من هولاء وتنقص من الوتر وتمتحنها كذا<sup>١٢١</sup> حتى يحصل الشعاع على طرف الإبهام. فيكون الرأس الدقيق حينئذ متوجهاً إلى القطب الشمالي والمثلث الذي من الشخص وخيط الشاقول ونصف<sup>١٢٢</sup> القطر في سطح دائرة نصف النهار والسطح الحاصل من خط الإستواء ونصف القطر سطح دائرة معدّل النهار والوتر المذكور بقدر أزمنة البعد وهو المراد.

[١٦] أما<sup>١٢٣</sup> معرفة القبلة. فيجب<sup>١٢٤</sup> أن يطول في خيط الشاقول زيادة تطويل، وتعلق شاقولاً آخر من طرف خيط<sup>١٢٥</sup> آخر له ثقل. وتجعل هذا الخيط ماراً بين الإبهام والسبابة القابضة على طرف القطر والوتر. وتعمل العمل المذكور حتى تتعين<sup>١٢٦</sup> مقدار الوتر<sup>١٢٧</sup> الذي بين الممسك وطرف القطر ويقع الشعاع على الموضع المطلوب. فترسل بعد ذلك هذا الشاقول الآخر مع حفظ<sup>١٢٨</sup> مقدار خيط<sup>١٢٩</sup> الوتر الذي بين طرف القطر والممسك، لئلا<sup>١٣٠</sup> يزيد ولا ينقص. ثم تخفض<sup>١٣١</sup> اليدين مع الاصطرلاب ومع حفظ تطابق العقدين ووقع<sup>١٣٢</sup> الشعاع على الموضع الواجب ومقدار<sup>١٣٣</sup> الوتر ومدّ طرف القطر، حتى ينتهي شاقول طرف الشخص إلى<sup>١٣٤</sup> الأرض. فإن كان قد انتهى أيضاً الشاقول المرسل المذكور مع انتهائه إلى الأرض حتى وقعا كلاهما معاً<sup>١٣٥</sup> على الأرض. فالخط<sup>١٣٦</sup> الواصل بينهما خط نصف النهار. وإن كان وصل شاقول طرف الشخص إلى الأرض ولم يصل بعد شاقول طرف القطر فترسل من خيط هذا الشاقول زيادة ما ممّا بين الإبهام والسبابة وذلك بأن يكون في الشاقول زيادة ثقل وتجعل إطباقك الإبهام<sup>١٣٧</sup> على السبابة الماسكة لطرف القطر وهذا الخيط رخواً<sup>١٣٨</sup> حتى ينجذب بثقل<sup>١٣٩</sup> هذا<sup>١٤٠</sup> الشاقول قدر من خيطه خيطه ثم تمسكه وقد زاد في طوله. وتمتحن حتى يتفق وقوع كلي الشاقولين<sup>١٤١</sup> معاً على الأرض بالعمل المذكور فيحصل خط نصف النهار. فتطبق<sup>١٤٢</sup> الاصطرلاب عليه بحيث يكون آخر خط الإستواء إلى جهة الجنوب وأوله إلى جهتك ويكون خط الإستواء مواجهاً للسماء. فتأخذ من الممسك قدرًا من الوتر وتمده على خط الإستواء إلى موضع تكون أجزاء<sup>١٤٣</sup> القوس المحاذية<sup>١٤٤</sup> له إلى طرفه بقدر أجزاء<sup>١٤٥</sup> إنحراف المدينة<sup>١٤٦</sup>. وتأخذ من<sup>١٤٧</sup> طرف هذا القدر من الوتر طرف نصف القطر بطرف الإبهام والسبابة ثم تمدّهما<sup>١٤٨</sup> حتى يمتد<sup>١٤٩</sup> كلاهما<sup>١٥٠</sup>. فإن كان الإنحراف شرقياً فتمدّهما<sup>١٥١</sup> إلى جهة المغرب وإن كان غربياً فإلى جهة المشرق. ويكون<sup>١٥٢</sup> نصف القطر حينئذ متوجهاً إلى القبلة على استقامة وهو المراد.

### Appendix 2.3: Apparatus Tūsi's shorter treatise

(١) رسالة في الاضطراب الخطي للإمام شرف الدين الطوسي رحمه الله [بسم الله الرحمن الرحيم الحمد لله رب العالمين واصلوة على نبيه محمد خير النبيين وعلى آله وصحبه اجمعين: ن. (٢) والثاني] الثاني: ن. (٣) أو صفر] واصفر: ن. (٤) بمائة] ثماسة: آ. (٥) بمائة وعشرين] ١٢٠: ن. (٦) بعد ستين] ٦٠: ن. (٧) جزءاً] حروا: آ. (٨) قوس] قوسا؟: ن. (٩) الفرجار] البركار: ن. (١٠) نعمل] نعلم: ن. (١١) خط] وجه: ن. (١٢) فمّن] وفمّن: آ. (أظنّ «و» مشطوب). (١٣) لعشر] بعشر: آ = بعشر: ن. (١٤) ولخمس] ؟: ن. (١٥) درجة] -ن. (١٦) ولعشرين] وبعشرين: ن. (١٧) نطلب قوس عشر درجات من الجدول وننظر كم نصيبه من الوتر، ثم] -ن. (١٨) الفرجار] البركار: ن. (١٩) لخمس عشرة] بخمسة عشر: ن. (٢٠) في] + معرفة: ن. (٢١) علامة] + خمس: ن. (٢٢) لعشر] بجزء عشر: ن. (٢٣) ولخمس] وإلى خمسة عشر: ن. (٢٤) جزءاً] جزء: ن. (٢٥) فنكتب] فيكتب عليه: ن. (٢٦) ثلاثين] (بياض): ن. (٢٧) على التوالي] وبعضها] -ن. (٢٨) في] + معرفة: ن. (٢٩) ونسميها] ونسميها: آ. (٣٠) نخطّ خطين على أيمن خطّ الأصول ونسميها خطي مطالع البلد] -ن. (٣١) درج] درجات: ن. (٣٢) وكذا] كذى: آ. (٣٣) في] + معرفة: ن. (٣٤) الفرجار] البركار: ن. (٣٥) فتلك] وتلك: ن. (٣٦) الفرجار] البركار: ن. (٣٧) وبعدها من] بعد هاتين: آ = وبعدها عن: ن. (٣٨) ونهايتها] ونهايات: ن. (٣٩) أبعاد مراكز الأفق والمقنطرات ونهاياتها من مبدأ خطّ الأصول] أبعاد الأفق ونهاياتها عن البلدان والمساكن: م. (٤٠) أفق] -م. (٤١) مو] مز: آ. (٤٢) ه] ه: م. (٤٣) سب] سد: آ = سه: م. (٤٤) لح] يح: آ = ح: م. (٤٥) ند] لد: آ. (٤٦) لط] -م. (٤٧) لب] ل: م. (٤٨) لط] -م. (٤٩) نو] نز: م. (٥٠) في] + معرفة: ن. (٥١) من] -ن. (٥٢) الفرجار] البركار: ن. (٥٣) تمر] بخمس: ن. (٥٤) يه] ١٥: ن. (٥٥) درجة] (?): ن. (٥٦) التي للجدي] الى الجدي: ن. (٥٧) نفتح] ونفتح: ن. (٥٨) الفرجار] البركار: ن. (٥٩) ونضع] وضع(?): ن. (٦٠) الفرجار] البركار: ن. (٦١) تلك] لتلك: ن. (٦٢) عشر] عشرة: ن. (٦٣) أربعة وثلاثين جزءاً] ٣٤ جزء: ن. (٦٤) وثلاث] (?): ن. (٦٥) جدول أجزاء المنطقة] -آ = جدول لجيوي على أجزاء المنطقة: م. (٦٦) الأجزاء] -آ = العدد: ل. (٦٧) الدلو] الدالي: م. (٦٨) لو] له: ل. (٦٩) نب] مب: ل. (٧٠) نز] ند: ل. (٧١) لو] لز: آ. (٧٢) ز] (غير مقروء في «ل»). (٧٣) م] مز: ل. (٧٤) كب] ب: ل. (٧٥) و] ه: آ. (٧٦) مب] نو: ل. (٧٧) و] ز: ل. (٧٨) كا] كد: ل. (٧٩) لو] لز: آ. (٨٠) لح] نا: ل. (٨١) لو] يو: آ. (٨٢) يط] ك: م. (٨٣) ل] ه: م. (٨٤) كو] كز: م. (٨٥) الأجزاء] -آ. (٨٦) في] + صنعة: ن. (٨٧) رسمنا] رسمت: ن. (٨٨) هذه] بعد: ن. (٨٩) خيطاً خطأ: آ. (٩٠) التي] الذي: ن. (٩١) هو] + في: ن. (٩٢) يخرج] + من: آ (مشطوب). (٩٣) الوتر] المري: ن. (٩٤) مقدار] بمقدار: آ = بمقدار: ن. (٩٥) بين] من: آ. (٩٦) الوتر] المري: ن. (٩٧) ونسّد طرف] (?): ن. (٩٨) الوتر] المري: ن. (٩٩) وحينئذ]

وح: ن. (١٠٠) وفيه [فيه: ن. (١٠١) في] + امعرفة: ن. (١٠٢) لمعرفة [بمعرفة: ن. (١٠٣) نطبّق] (؟): ن. (١٠٤) مقابل [المقابل: ن. (١٠٥) الشمس] السمتين: ن. (١٠٦) يلي [ن. (١٠٧) يبقى] آ، ن («لا يبقى» صحيح). (١٠٨) أجزاء [آخر: ن. (١٠٩) من تسعين] (؟): ن. (١١٠) الشمس] + والله اعلم: ن. (١١١) ونمسكه] وتمسك: ن. (١١٢) نمدّ [تخط: ن. (١١٣) مركز] + مقنطرة: آ (مشطوب). (١١٤) خطّ [خيظ: ن. (١١٥) مبدأ] - ن. (١١٦) النهار] + والله اعلم: ن. (١١٧) على [آ (فوق السطر). (١١٨) خمسة عشر] ١٥: ن. (١١٩) فنقسم [فنقسمه: آ. (١٢٠) خمسة عشر] ١٥: ن. (١٢١) النهار] + وضعفها: آ (مشطوب) = - ن. (١٢٢) في] + معرفة: ن. (١٢٣) نأخذ [نأخذ: آ. (١٢٤) بمقدار] بقدر: ن. (١٢٥) نصف قوس] - ن. (١٢٦) عشر] عشرة (؟): آ. (١٢٧) من درجة الحادي عشر إلى التوالي] - ن. (١٢٨) بهذه] لهذه: ن. (١٢٩) في] + معرفة: ن. (١٣٠) عن [بين: آ = من: ن. (١٣١) ستين] ٦٠: ن. (١٣٢) خمسة عشر] ١٥: ن. (١٣٣) تمت الرسالة بعون الله تعالى وحسن توفيقه] والله تع اعلم تمت في يوم الأحد ٢٩ من ربيع الأول سنة سبعة وستين وماتين والالف: ن.

### Tūsi's longer treatise

(١) الشعاعات النافذة] الشعاع النافذ: م. (٢) الشخص] + حاشية الشخص يكون يب ثقباً على عدد الشهور لأجل نزول الشمس: ب (في الهامش). (٣) الثلاثة] الثلث: م. (٤) وتخالفها] يخالفها: م. (٥) عن [من: م. (٦) ثقبه] - ج. (٧) أيضاً] وأيضاً: ل. (٨) النقطة] النقطة: ل. ك. (٩) لكل] الكل: م. (١٠) نقطة] - ب، - ج. (١١) خمسة] - ب، - ج. (١٢) نقط] - م. (١٣) أجزاء] آخر: ب، ج، م. (١٤) نقطة] نقط: ل، ك. (١٥) السنبله] + إلى نقطه الحمل واول الثور لخمسة درجات من السنبله: م. (١٦) ثقبتين] + حاشية ما أدري قوله ثقبتين من قوله نقطتين [غير مقروء] أو ثقبتين فتدبر ذلك: ب. (١٧) لكل] الكل: م. (١٨) عند أول] لأول: ب. (١٩) الأخير] الآخر: ب. (٢٠) ثقبين ثقبين] ثقبين ثقبين: ل. (٢١) والجدي] وللجدي: م. (٢٢) القوس] إلى القوس: م. (٢٣) نقط] نقطه: م. (٢٤) ويرجع] ويرج: م. (٢٥) نقط] نقطه: ب، ج، م. (٢٦) ثقبين] ثقبين: ل، ك. (٢٧) ينتهي] - م. (٢٨) الأخير] + فافهمه: ب، ج. (٢٩) فصل في كيفية العمل فلتكن جميع ثقب المقياس وهو الشخص مسدوداً] ويجب ان يكون جميع ثقب الشخص الذي هو المقياس مسدودة: م. (٣٠) يجب أن] - م. (٣١) وللجنوبيه] والجنوبيه: م. (٣٢) شبيهة] شبيهة: ب، ج. (٣٣) بحيث] وبحيث: ب، ج. (٣٤) على] عن: ل. (٣٥) الخيط] الجز: م. (٣٦) المثناة] المثناة: م. (٣٧) المثناة] المثناة: م. (٣٨) طرف] + طرف: ب، ج. (٣٩) القطر] الخيط: م. (٤٠) أيضاً] - ب. (٤١) الثقب] البيت: ب، ج. (٤٢) عرض] ب (في الهامش). (٤٣) بأحدي] بأحد: ب، ج. (٤٤) بحسب] بحيث: م. (٤٥) الثلاثة] الثلث: م. (٤٦) يمر] مر: ل. (٤٧) يمر أولاً فيما بين الإبهام والسبابة تحت] - م. (٤٨) المثناة] المثناة: م. (٤٩) رأس] - م. (٥٠) عقد] عند: م. (٥١) ووقوعه] ووقوعه: م. (٥٢) الثلاثة] - ل، - ك. (٥٣) والممسك] او الممسك: ب، ج. (٥٤) القبض وتمدّ] القبض ثم نمدّ: ب. (٥٥) بشدة]

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يده: م. (٥٦) بُعد - م. (٥٧) الماسكة [الماسك: ل، ك، م. (٥٨) الماسكة] الماسك: ل، ك. ك.  
لطرف القطر والوتر في اليد الماسكة] - ب. (٥٩) للاصطرلاب [الاصطرلاب: ب. (٦٠) من  
منه: ب. (٦١) في اليد الماسكة للاصطرلاب فتقبض من بقية الوتر] - م. (٦٢) عن [على: م.  
(٦٣) بطرف] لطرف: م. (٦٤) والوسطى [الوسطى: ب، ج. (٦٥) الإبهام والسبابة] السبابة: م.  
(٦٦) هذا] بهذا: م. (٦٧) عن [من: م. (٦٨) من [بين: ج. (٦٩) بأصابع] بأصبع: م. (٧٠)  
تحصيل] تحصل: م. (٧١) هذه] هذا: ك، ل، م. (٧٢) عقدتي] عقدي: ك، ل، م. (٧٣) أحدها]  
أحديها: م. (٧٤) حوله فيصير رأس الشخص] - م. (٧٥) الممدود] الممدود: م. (٧٦) أميل إلى  
فوق أو إلى تحت ثم لإطباق العقدتين بعد أخذ مقدار من الوتر بين الممسك وطرف القطر تمد  
طرف القطر وتدير الاصطرلاب] - م. (٧٧) وإن [وإن: م. (٧٨) تشيل] ليسل: م. (٧٩) الثالثة]  
الثالث: م. (٨٠) فيتطابق] فيتطابق: ب. (٨١) متطابقين] متطابقين: ب = متطابقان: م. (٨٢)  
من] - ب. (٨٣) ورأس الشخص] واس الشخصط: م. (٨٤) الثالثة] الثانية: م. (٨٥) نقصت]  
نقصته: ب، ج. (٨٦) هذه] - م. (٨٧) كان] كانت: ج. (٨٨) التي] الذي: ب، ل، ك، م. (٨٩)  
محاذاة] + موضع: ب. (٩٠) الإبهام] + من: ب. (٩١) التي] الذي: ب، ل، ك. (٩٢) شرقياً]  
مشرقياً: ج. (٩٣) عكسه] - م. (٩٤) بُعد] ب (في الهامش). (٩٥) وكلما] وكل ما: م. (٩٦)  
الشرق] المشرق: ب. (٩٧) الغرب] المغرب: ب. (٩٨) يقصر] يقتص: م. (٩٩) فتتقص] -  
م. (١٠٠) بين [من: م. (١٠١) فتزيد] وتزيد: ل، ك. (١٠٢) ثم تعتبر] ثم تعيد: ب، ج = ويعتبر:  
ل، ك = ثم بعد: م. (١٠٣) وتزيد] فتزيد: ب، ج، م. (١٠٤) ذلك] - م. (١٠٥) مسامتاً] مماسا:  
ك، ل. (١٠٦) نظر ناظر] نظرنا: م. (١٠٧) استقامة] الاستقامة: ل = استقامته: م. (١٠٨) أحد]  
واحد: ب، ج. (١٠٩) كان] كانت: ج. (١١٠) من] + من (مطموس): ب. (١١١) من] ك (تحت  
السطر بخط غير خط الكاتب) = - ل. (١١٢) وذلك] + وذلك: م. (١١٣) ثم تميل] يميل: ك  
= ميل: ل. (١١٤) التي] الذي: ك، ل، ب. (١١٥) لا] - م. (١١٦) التي] الذي: ب، ك، ل.  
(١١٧) ذكرنا] - ك، - ل، - م. (١١٨) عن [من: م. (١١٩) فتزيد] وتزيد: ل، ك. (١٢٠) وتحط]  
وخيطة: م. (١٢١) كذا] - م. (١٢٢) ونصف] ونصف: ب. (١٢٣) أمأ] وأمأ: م. (١٢٤) فيجب]  
فأعلم إنه يجب: ب. (١٢٥) خيط] اخيط: ب = خط: م. (١٢٦) تتعين] تعين: ل. (١٢٧)  
الوتر] + على طرف القطر والوتر وتعمل العمل المذكور حتى تتعين مقدار الوتر: م. (١٢٨)  
حفظ] - ك، - ل. (١٢٩) خيط] جيد: م. (١٣٠) لئلا] له لا: ل، ك. (١٣١) تخفض] بحفص:  
ب = يخفض: ج = تخفض: ك = يحفظ: ل = تخصص: م. (١٣٢) ووقع] ورفع: م. (١٣٣)  
ومقدار] مقدار: م. (١٣٤) إلى] + وتد: ب، ج. (١٣٥) معاً] - م. (١٣٦) فالخط] كالخط: م.  
(١٣٧) الإبهام] لإبهام: م. (١٣٨) رخوًا] رخو: ب، ج = + جداً: م. (١٣٩) بثقل] يفعل: م.  
(١٤٠) هذا] - ب. (١٤١) الشاقولين] الشاقول: ج. (١٤٢) فتتطبق] فتطبق: ل، ك. (١٤٣)  
أجزاء] اجز: ب = اخر: م. (١٤٤) المحاذية] المحاذي: ل، ك. (١٤٥) أجزاء] اخر: م. (١٤٦)

المدينة] + عن: ب (مشطوب)، ج. (١٤٧) من] + الممسك قدرأ من الوتر وتمده على خط الاستواء إلى موضع يكون اخر القوس المحاذية له إلى طرفه بقدر اخر انحراف المدينة وتأخذ من: م. (١٤٨) ثمّ تمدّهما] وتمدّهما: ب. (١٤٩) يمتدّ] تمد: ك. (١٥٠) يمتدّ كلاهما] ييند لي بهما: م. (١٥١) فتمدّهما] فتمدّها: ب. (١٥٢) ويكون] فيكون: ك، ل، م.



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