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What makes a Rhythm to be Bad

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ABSTRACT

Deciding whether a musical rhythm is good or not, depends on many factors like geographical conditions of a region, culture, the mood of society, the view of rhythm over years, and so on. In this paper, we want to make a decision from the scientific point of view, using geometric features of rhythms, about bad ones. The researchers who are investigating the relationship between geometry and music, certainly realize that there is a big vacuum in this regard, not using computers to detect a good or bad rhythm. Here, using computer programming and applying geometric features to more than four thousand rhythms, we decide on the bad musical rhythms. Then we present algorithms for deciding about bad rhythms using geometrical features.

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1 Introduction

The connection between two subjects of mathematics and music has been of long interest to many mathematicians as well as musicians. Establishing a connection between these

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two worlds that may seem separate from each other, has been accomplished with geometry. These studies have been started since 1960. For a better understanding of the subject, it should be noted that addressing music from a geometry gate initially begins by imaging musical rhythms on paper that has introduced in [2] for the first time and has improved in [16,18]. Then obtaining geometric properties and continues with the examination of these features happens. The obtained features become a standard for the recognition of the strong or weak rhythms of music from a scientific point of view. In this paper, we will focus only on the geometric factors of musical rhythms and ignore mathematics of sounds [19], tuning methods [20], signal representations [14], construction of music instruments [19,17,3] and geometric symmetry transformations [10]. At the rest of this paper, in Section 2, we first review some basic concepts and geometric image of music rhythms are studied. In Section 3, the geometric properties of bad rhythms are extracted from the following 11 characteristics:

- Maximal evenness
- Rhythmic oddity
- Beatness
- Weighted beatness
- Metric complexity
- Open rhythms
- Distinct durations
- Distinct adjacent durations
- Depth of rhythms
- Asymmetry
- Tallness

Section 4 is devoted to applying these features to all the rhythms in the sixteen-pulse cycle with five onsets and obtained the total amount of badness for each of six rhythms. These results are obtained by rating each function and finally summation the points for each of rhythms. Three worst rhythms are shown in $0, 1$ notation and also by using circular display.

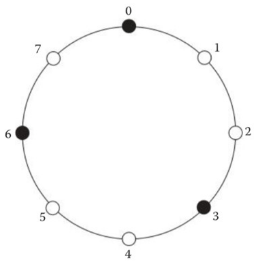

Figure 1: The cuban timeline [18]

2 Geometry and music, basic concepts

Two very important subjects in music are: Rhythm and Melody. Rhythm is time-dependent while melody is in contrast to the rhythm. Melody is not time dependent and is a vertical direction in music. The time-shift model is called the general definition of rhythm and a pattern configuration of notes, is the specific definition of rhythm. For example, a steady heartbeat or tick tak of a clock without end are simple sequences of rhythm, while musicians say that these unique sequences are not rhythms since they do not include any recognizable audible patterns. There are two kinds of pulses in music, onset and off-set. When a note is played, that pulse is called onset [5] and when the note isnt played, the pulse is called off-set [6]. Usually in the classic world there is a special rhythm that is an indicator. Sometimes this basic feature is the synchronic onsets and non-interrupted. But other times music is characterized by unique periodic patterns. These special rhythms are generally called time-lines. An example of a Cuban time-line is shown in Fig. 1. If a rhythm is repeated continuously throughout a piece of music, it is called a repetitive rhythm.

2.1 Geometric Image of Music Rhythms

As seen in Fig. 1, a particular rhythm which includes a set of onsets and off-sets, can be depicted on a circle. There are different types of pulse numbers including 5, 7, 16, and 32. In Fig. 1, the cycle consists of 7 pulses. Imagine a clock instead of 12 numbers has 16 numbers and its numbers start from zero. If we remove the clock and minute counter from it, and leave only the second counter, we can depict a repeating rhythm on such a circle, while the distance between two pulses may not be exactly one second. Black points on the circle represent onsets and white points indicating the off-sets in a particular music rhythm. In this paper all rhythms are depicted on a 16-pulse cycle and also contain 5 onsets and 11 off-sets; [16,18]. The number of rhythms in this series is 4368. Applying rhythms to a circular cycle has done in [16] earlier in 1962. As we can see in Fig. 2, the left part is a box notation of a particular rhythm that has applied on the circle on right. Box notation is also an old method to represent rhythms on paper. It has introduced in [13] and used in Korea for hundred years [11]. There are six rhythms that have not been lost for many years but have been very lasting with migrations from continents to

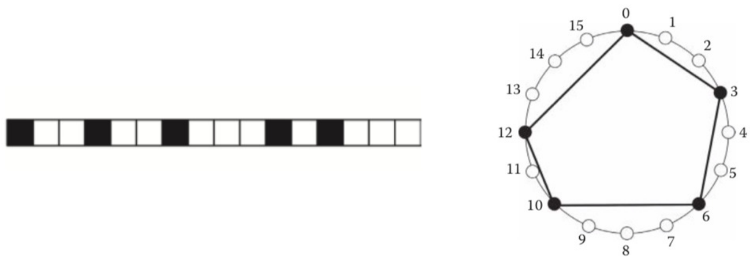

Figure 2: The display of a rhythm on a circle [18]

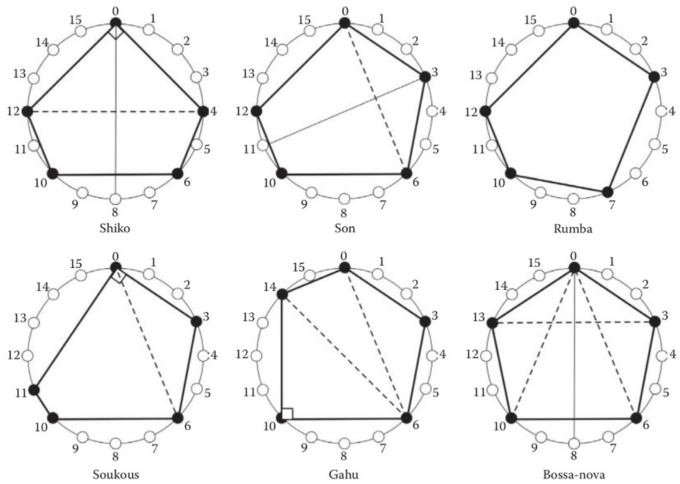

Figure 3: The picture of six famous timelines on sixteen-pulse cycle [18]

continents and gained great popularity among different cultures. These six rhythms are called:

- Son,
- Rumba,
- Gahu,
- Bossa-nova,
- Soukous and
- Shiko.

If these six rhythms of music are pictured on a 16-pulse cycle, the following circles are obtained as in Fig. 3. If we look at Fig. 3, we notice that there are pentagons in all 16-pulse rhythms, including the six famous rhythms. These polygons are obtained

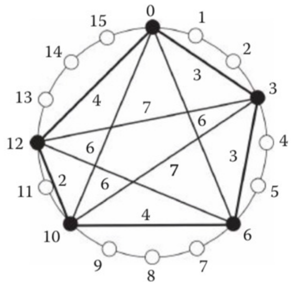

Figure 4: The full interval polygon of Son [18]

by connecting the onsets in the clockwise order for each rhythm. By examining these polygons, we find a set of geometric features that are the basis for deciding about bad rhythms.

2.2 Characteristics of polygons

If there is an onset in the cycle which produces the same rhythm in the forward or backward direction, rhythm has the characteristic which is called two-way orientation. Son has this feature, because it also has an axis; see [18] for more details. Looking at the image of the six rhythms, we notice the Equilateral triangles which means the existence of an onset that has a similar time interval to the third onset. There are such triangles in Shiko, Son, Soukous and Gahu [18]. If the pentagons have an angle of 90 degrees, the following two perspectives are considered. Geometric View: There are two onsets on circle that are definitely contrasted with each other. Musical View: There are two onsets that divide the cycle into two half-cycles. It is an important feature in music, which is called the degree of regularity [18]. In pulse segmentation there are two kinds of pulses: weak and strong. Syncopation happens when there is an onset on a weak pulse and an off-set on a strong pulse. While the Rumba differs only in one onset with Son, Rumba is more complicated because it doesnt have equilateral triangle and symmetry axis [18]. Syncopation also have more definitions that are introduced in [9]. For grouping and comparing these six rhythms, another feature that is useful is converting polygons to a histogram. It can be shown for full interval polygon or not-full interval polygons. Much more details about full interval vectors have been reviewed in [15] for the first time. Full interval polygon of a particular rhythm is shown in Fig. 4.

The histograms of Fig. 5 are formed according to the full interval graph of the pentagons in the sixteen-pulse cycle shown in Fig. 4. Every edge has a weight that is actually the number of pulses between two onsets that have created the edges. As we can see in Fig. 5, the horizontal axis of the full interval histogram is divided into eight parts, since the maximum distance between two onsets is eight. For example, the Son shown in Fig. 4 has an edge with weight 2, two edges of 3, two edges of 4, three edges of 6, and two edges of 7.

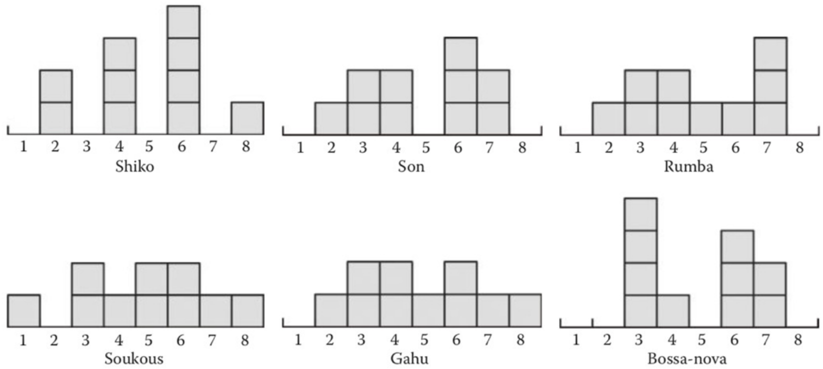

Figure 5: The full interval histogram for six famous rhythms [18]

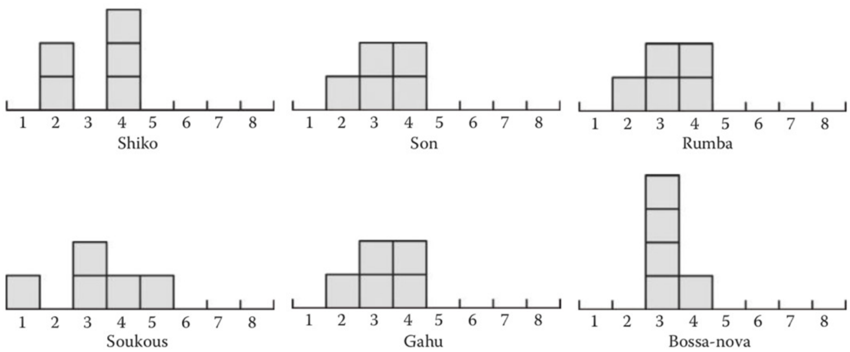

Figure 6: The adjacent interval for six famous rhythms [18]

```
bool f1(int a[]){
    int dur = 0, k;
    for(int i=0; i<16 && k<16; i++){
        while(a[i]!=1) i++;
        k = i+1;
        while(a[k]!=1) k++;
        dur = k-i;
        if(dur%2==0) return false;
        i=k-1;
    }
    return true;
}
```

Figure 7: Maximal evenness

3 Geometric Properties of Bad Rhythms

The following geometrical features of bad rhythms will be studied in this section.

3.1 Maximal evenness

This feature occurs when all the adjacent onsets in a rhythm have the same distance. In other words, it occurs when a rhythm is completely regular, or rhythm onsets have odd distances from each other [11]. For a rhythm to be a bad timeline, the five onsets should be distributed almost as oddly as possible within the 16-pulse span. The excessive order makes the rhythm unattractive. Although many years of such rhythms have been used to lead the soldiers. In this paper we use this definition of evenness but there are other ways to compute evenness; [1,4]. Evenness is a feature that with other geometric properties, has many applications in computational music theory; [6,17]. An example of a regular rhythm is given as $[X.X.X.X.X]$. The function to compute this characteristic is shown in Fig. 7.

3.2 Rhythmic oddity

This property occurs when there are no two opposite onsets in a rhythm. or in other words, do not divide the cycle into two equal parts [18]. Of the six popular rhythms, Gahu, Shiko, and Bossa-nova, have this feature. As shown in Fig. 8, there are not two onsets that divide the cycle into two equal parts by connecting them together. This shape belongs to the Son, one of the popular rhythms in the world.
To function for rhythmic oddity is given in Fig. 9.

3.3 Beatness

This feature occurs when the onsets of a rhythm is in pulses of zero, four, eight, and twelve. These pulses are called main pulses. Off-beat: Apart from the main pulses, the rest of the pulses are considered off-beat. Double off-beat: The onset next to a main

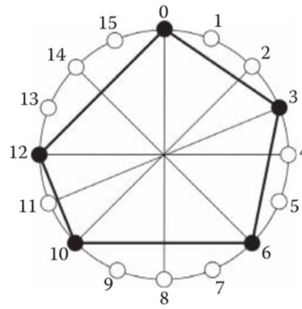

Figure 8: The rhythmic oddity of Son [18]

```
// Rhythmic Oddity
bool f2(int a[]){
    int m=n/2;
    for(int i=0; i<m; i++){
        if(a[i]==1 && a[i]==a[i+m])
            return false;
    }
    return true;
}
```

Figure 9: Rhythmic oddity

pulse is called double off-beat. As shown in Fig. 10, the Son has a off-beat onset in pulse number three.
The beatness feature is implemented in Fig. 11.

3.4 Weighted beatness

Beatness means occurring onsets in non-main pulses. This is one of the characteristics of the goodness of a rhythm. to measure the value of this characteristic, we count off-beat pulses once and double off-beat twice and then we add two numbers together. Now, in order to measure bad rhythm, we reverse the situation. This means that the rhythm with highest score gets the lowest amount of beatness. For example, if we count the Off-beat

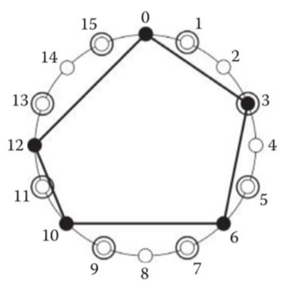

Figure 10: The beatness of Son [18]

```
// Offbeatness
int f3(int a[]){
    int res=0;
    if(a[1]==1) res++;
    if(a[3]==1) res++;
    if(a[5]==1) res++;
    if(a[7]==1) res++;
    if(a[9]==1) res++;
    if(a[11]==1) res++;
    if(a[13]==1) res++;
    if(a[15]==1) res++;
    return res;
    // [0,5]
}
```

Figure 11: Beatness

```
// Weighted Offbeatness
int f4(int a[]){
    int res=0;
    int weight[16] = {0, 2, 1, 2, 0, 2, 1, 2, 0, 2, 1, 2, 0, 2, 1, 2};
    for(int i=0; i<n; i++){
        res += a[i]*weight[i];
    }
    return res;
    // [1,10]
}
```

Figure 12: Weighted beatness

pulses in Fig. 10, the result is four. In this way, the onset at pulse three is counted twice and the onset in the sixth and tenth pulses are counted once. Computing the maximum of weight is a special problem in graph theory [6]. This feature is shown in Fig. 12.

3.5 Metric Complexity

For a sixteen-pulses cycle, we consider a certain weight for the entire set. According to these weights, the maximum complexity for a rhythm is equal to seventeen. To find the complexity of each rhythm, we reduce its simplicity from 17. In this way, the complexity of each rhythm is achieved. The greater value of this number makes the rhythm more spaced than the standards of goodness. The weight sequence (5, 1, 2, 1, 3, 1, 2, 1, 4, 1, 2, 1, 3, 1, 2, 1) is used to measure this property which is known as Lerdahl and Jackendoff algorithm [18]. This sequence is shown in the histogram in Fig. 13. By adapting each of the rhythms with this histogram and counting the squares of each column and obtaining their total, the simplicity of each rhythm is obtained. For example, this number is equal to thirteen for Son, and by diminishing it from the seventeen, we reach four. As a result, the metric complexity for Son is equal to four.
The following function in Fig. 15 is related to the implementation of the metric complexity.

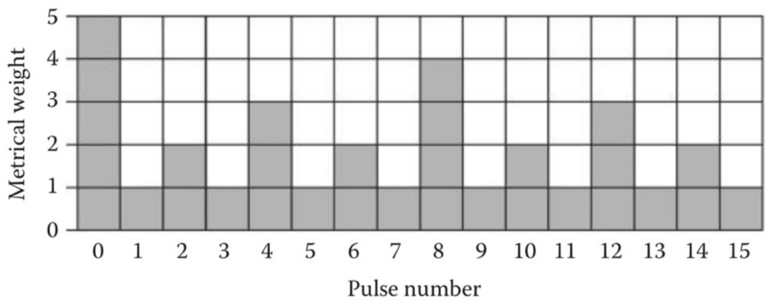

Figure 13: The standard histogram of Lerdahl and Jackendoff for metric complexity

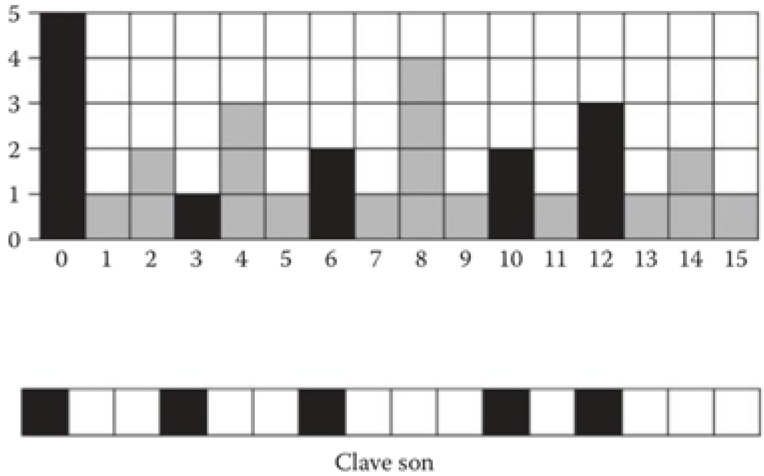

Figure 14: The adjust of Son with standard histogram

```
// Metrical Complexity
int f5(int a[]){
    int res=0;
    int weight[16] = {5, 1, 2, 1, 3, 1, 2, 1, 4, 1, 2, 1, 3, 1, 2, 1};
    for(int i=0; i<n; i++){
        res += a[i]*weight[i];
    }
    return 17-res;
    // [0,12]
}
```

Figure 15: Metric complexity

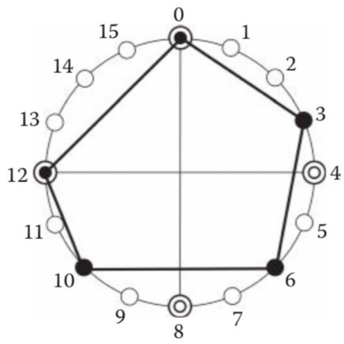

Figure 16: Showing the closure of the Son [1]

```
// Main-Beat and Closure
int f6(int a[]){
    int res=0;
    if(a[0]==1) res++;
    if(a[4]==1) res++;
    if(a[8]==1) res++;
    if(a[12]==1) res++;
    return res;
    // [0,4]
}
```

Figure 17: Open rhythms

3.6 Open rhythms

As explained in the previous features, there are four main pulses per cycle of sixteen pulses. By definition, the rhythm is closed if it has an onset in the pulse number twelve. So, a rhythm without this feature is an open rhythm; see [18]. For example, according to Fig. 16, the Son has an onset in pulse number twelve, so Son is a closed rhythm.
The following function is responsible for the openness of a rhythm; see Fig. 17.

3.7 Distinct durations

In the previous features, methods for measuring the complexity of rhythms were presented. Another method for measuring the rhythms complexity is the entropy characteristic which is obtained by counting the number of distinct durations of the full interval histogram of the rhythm. the higher number makes the rhythm worse in terms of geometric characteristics. For example, this number is five for Son [18].

3.8 Distinct adjacent durations

As we know, the seventh feature sometimes fails. Therefore, for a better result, we use the adjacent interval histogram of a rhythm, to measure distinct duration. This feature is similar to the previous feature. The more number is obtained, we approach our goal to

```
// Distinct Durations
int f7(int a[]){
    int s=0, du=0, res=0;
    int histo[8]= {0,0,0,0,0,0,0,0};
    while(a[s]!=1) s++;
    for(int i=s; i<n; i++){
        while(a[i]!=1 && i<n) i++;
        for(int k=i+1; k<n; k++){
            if(a[k]==1){
                du=k-i;
                if(du>8) du=16-du;
                histo[du-1]++;
            }
        }
    }

    for(int i=0; i<8; i++){
        if(histo[i]!=0) res++;
    }

    return res;
    // [4,8]
}
```

Figure 18: Distinct adjacent durations

Shiko	0	0	0	0	1	2	3	4
Son	0	0	0	1	2	2	2	3
Rumba	0	0	1	1	1	2	2	3
Soukous	0	1	1	1	1	2	2	2
Gahu	0	1	1	1	1	2	2	2
Bossa	0	0	0	0	1	2	3	4

Figure 19: The Table derived from full interval histogram with ascending order [18]

find bad rhythms. The following functions in Fig. 18 have been implemented for features distinct durations and distinct adjacent durations.

3.9 Depth of rhythm

The rhythm that has no two columns in its full interval histogram with same height, is called a deep rhythm. Indeed, except for columns with a height of 0; see [18]. As mentioned above, the value of this characteristic is obtained from the full interval histogram. In order to get the depth of each rhythm, we use the table in Fig. 19. This table is sorted in ascending order for the six distinguished rhythms. By reducing the numbers associated with Shiko (or Bossa-Nova) from each other row in Figue 19, we calculate the depth of each rhythm. For example, the depth of Son is equal to four. This feature has been discussed with more details in [15, 8, 20, 12].
The following function in Fig. 20 is related to the implementation of deepness.

```
// Deepness
int f10(int a[]){
    int s=0, du=0, diff=0;
    int shikoHisto[8] = {0,0,0,0,1,2,3,4};
    int histo[8] = {0,0,0,0,0,0,0,0};
    while(a[s]!=1) s++;
    for(int i=s; i<n; i++){
        while(a[i]!=1 && i<n) i++;
        for(int k=i+1; k<n; k++){
            if(a[k]==1){
                du=k-i;
                if(du>8) du=16-du;
                histo[du-1]++;
            }
        }
    }
    sort(histo,histo+8);
    for(int i=0; i<8; i++){
        if(histo[i]!=shikoHisto[i]){
            diff += abs(histo[i]-shikoHisto[i]);
        }
    }
    return diff;
    // [0,8]
}
```

Figure 20: Depth of rhythm

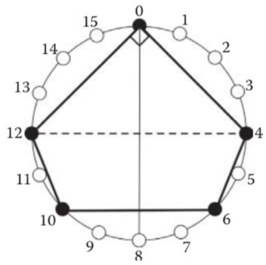

Figure 21: The Symmetry in Shiko [18]

3.10 Asymmetry

The symmetry of polygons in circular cycles is divided to vertically, horizontally and diagonally. This feature is often considered as a good feature for rhythm. Since the polygons are full of symmetry lines, we consider the best of them, the diagonal symmetry line. So, if the rhythm does not have a diagonal line of symmetry, we consider it as a bad rhythm. According to Figure 23, the Shiko rhythm has no diagonal symmetry line [18]. The implementation of the symmetry function is shown in Fig. 22.

3.11 Tallness

This characteristic is obtained by measuring the height of the tallest column of the adjacent interval histogram of a rhythm. The larger value of this attribute, the rhythm has less chance of having distinct durations [18]. This property is inversely related to the

```
// Mirror Symmetry
bool f12(int a[]){
    bool isSym, target = 0;
    for(int i=0; i<8; i++){
        isSym = true;
        for(int k=i+1; k<8+i; k++){
            target = (((2*i)+16)-k)%16;
            if(a[k]^a[target]){
                isSym = false;
                break;
            }
        }
        if(isSym) return true;
    }
    return false;
}
```

Figure 22: Asymmetry

```
// Tallness
int f11(int a[]){
    int s=0, du=0;
    int histo[8] = {0,0,0,0,0,0,0,0};
    while(a[s]!=1) s++;
    for(int i=s; i<n; i++){
        while(a[i]!=1 && i<n) i++;
        for(int k=i+1; k<n; k++){
            if(a[k]==1){
                du=k-i;
                if(du>8) du=16-du;
                histo[du-1]++;
            }
        }
    }
    sort(histo,histo+8);
    return histo[7];
    // [2,4]
}
```

Figure 23: Tallness

complexity of the rhythm. less amount of this attribute means the higher complexity of the rhythm. For example, according to Fig. 6, this number is equal to 3 for the Son. In Fig. 23, the function for computation of tallness of a rhythm is discussed.
As previously stated, the rhythms examined were all sixteen pulse rhythms with five onsets. If we want to examine good rhythms, the advantage of the above method is that at the very first stage, a large number of rhythms will be eliminated from the competition and thus finding good rhythms, according to the geometric properties, becomes much easier. On the other hand if we want to find bad rhythms, this method can identify these rhythms and also draw conclusions about their geometric properties, and even determine their degree of badness.

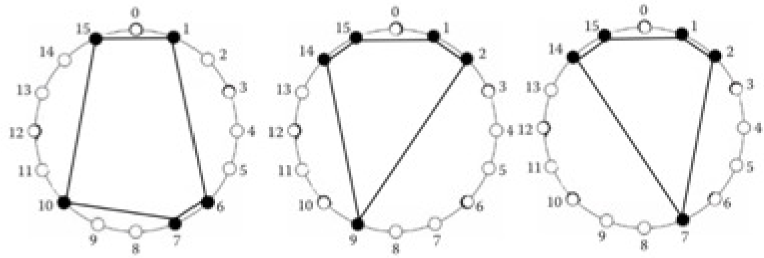

Figure 24: The image of three rhythm with the worst score on sixteen-pulse cycle

4 Results and Implementations

With the help of the material in Section 2, we extracted the features of the bad rhythms. These eleven characteristics are the bad geometric properties of the rhythms, which are applied to all the rhythms in the sixteen-pulse cycle with five onsets. Programming of these features has been done in C++. Thus, by applying these features, results are specified to identify the worst rhythms. These results are obtained by rating each function and finally summation the points for each of rhythms. In this way, for Boolean functions, numbers zero and one are considered, and for functions with numerical outputs, the number is returned to a scale in $[0, 1]$. Based on the largest numbers obtained, the worst rhythms are presented below. These are three worst rhythms in this case and shown by 0,1 notation and also using circular display according to Fig. 3.

$$[0, 1, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 1] >> 8.95556$$
$$[0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 1] >> 8.95556$$
$$[0, 1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 1] >> 8.95556$$

For a closer look, the results of each function for six famous time-lines are specified in Table 1. These results show the numerical values of scores for the named rhythms. Note that larger results assigned to each rhythm proves that rhythm is less ear-catching or in other words, is worse.
The total amount of badness for each of six rhythms is shown in Table 2. These values are the summation of result functions for every single rhythm. According to these values, soukous has the most score.
The results are obtained due to geometrical features of badness which are defined in Section 2. As we know, these results are relative because more factors could be involved to decide about a rhythm; such as geographical conditions of a region, culture and the mood of society, view of rhythm over years, and so on.

Table 1: The results of each function for six famous time-lines

f1		f2		f3		f4		f5		f6	
Son	1	Son	1	Son	1	Son	4	Son	4	Son	2
Shiko	0	Shiko	0	Shiko	0	Shiko	2	Shiko	2	Shiko	3
Rumba	0	Rumba	1	Rumba	2	Rumba	5	Rumba	5	Rumba	2
Gahu	0	Gahu	0	Gahu	1	Gahu	5	Gahu	5	Gahu	1
Bossa	0	Bossa	1	Bossa	2	Bossa	6	Bossa	6	Bossa	1
Soukous	0	Soukous	0	Soukous	2	Soukous	6	Soukous	6	Soukous	1
f7		f8		f9		f10		f11		f12	
Son	5	Son	3	Son	16	Son	4	Son	3	Son	1
Shiko	4	Shiko	2	Shiko	16	Shiko	0	Shiko	4	Shiko	1
Rumba	6	Rumba	3	Rumba	19	Rumba	4	Rumba	3	Rumba	0
Gahu	7	Gahu	3	Gahu	17	Gahu	6	Gahu	2	Gahu	0
Bossa	4	Bossa	2	Bossa	16	Bossa	0	Bossa	4	Bossa	1
Soukous	7	Soukous	4	Soukous	17	Soukous	6	Soukous	2	Soukous	0

Table 2: The total amount of badness for each of six rhythms

Rhythm	Total Amount of Badness
Son	5.8
Shiko	3.55556
Rumba	6.57222
Gahu	7.77222
Bossa	5.54444
Soukous	7.79444

5 Conclusion

In this paper we have decided about rhythms from a geometric point of view. The other named properties could be involved in future works. Obviously considering these features will give us a more accurate result about musical rhythms. Its also an open problem in the subject of finding bad rhythms.

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