

## Comparison study between layered and functionally graded composite beams for static deflection and stress analyses

Okan Kirlangiç<sup>a</sup> and Şeref Doğuşcan AKBAŞ<sup>b\*</sup>

<sup>a</sup> The General Directorate of Highways, Ankara, Turkey

<sup>b</sup> Department of Civil Engineering, Bursa Technical University, Bursa, Turkey

---

### ARTICLE INFO

#### Article history:

Received: 20 January 2020

Accepted: 02 February 2020

---

#### Keywords:

Functionally Graded Material

Layered Composites

Beam

Ritz Method

Timoshenko Beam

### ABSTRACT

The aim of this paper is to compare the static deflections and stress results of layered and functionally graded composite beams under static load. In the comparison study, the results obtained for a cantilever beam under point load. The Timoshenko beam and the Euler-Bernoulli beam theories are used in the beam model. The energy based Ritz method is used for the solution of the problem and algebraic polynomials are used with the trivial functions for the Ritz method. Two different materials are considered as layered and functionally graded distribution in a cantilever beam and their static deflections, stress distributions are compared under a point load at free end of the beam. For two different distributions, the formulations of Ritz method are obtained and solved numerically. In the numerical results, the effects of material distribution parameter, aspect ratio on the static deflections and stress distribution of functionally graded beams are obtained and compared with the results of the layered composite beam. Difference among of beam theories are compared for functionally graded and layered beams. Also, some comparison studies are performed in order to validate the using formulations.

### 1. Introduction

Cracking and delamination problems are frequently occurred in layered composite structures because of the stress discontinuities at interfaces. In order to minimize these problems in composite structures, functionally graded materials were used for get rid of discontinuities at interfaces surfaces and the minimize the stress concentrations. Functionally graded materials are special composites whose properties change gradually though direction. In generally, functionally graded materials consist of a mixture of ceramic and metal materials. In the last years, the functionally graded materials have been found in many engineering applications, such as aircrafts, space vehicles and biomedical sectors.

By increasing functionally graded structures, many researchers investigated the static, dynamic and stability analyses of functionally graded structures in last decades. In the literature, some investigations of static behavior of functionally graded and

layered structures are as follows; Lewandowski [1] presented an analytical examination for geometrically nonlinear free vibrations of beams by using Hamilton's principle with the Ritz method. Reddy [2] analyzed functionally graded plates by using finite element models and Navier's solution based on third order shear deformation plate theory. Sankar [3] studied an elastic solution of a functionally graded beam exposed to transverse loads. Deschilder et al. [4] investigated nonlinear structural analysis of a functionally graded beam based on the large deflection theory. Chi and Chung [5] investigated the mechanical behavior of a simply supported functionally graded material plate under the transverse load by using finite element method. Palanivel [6] performed the free vibration analysis of laminated composite beams by using two high-order shear deformation theory and finite elements method. Zhong and Yu [7] investigated the stress analysis of the cantilever functionally graded beams under different loads. Aydogdu and Taskin [8] studied the free vibration analysis of a simply supported functionally graded beams by using the Navier method. Kadoli et al. [9] performed the static analysis of the functionally graded

---

\* Corresponding author. Tel.: +090-224-300-3498; e-mail: [srefda@yahoo.com](mailto:srefda@yahoo.com)

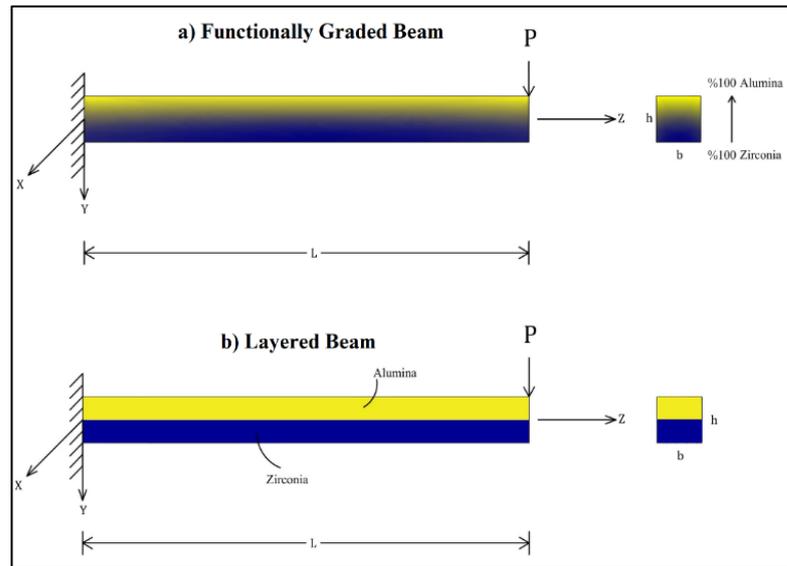
beam with high order shear deformation theory. Li [10] studied a unified approach to analyze the static and dynamic behavior of functionally graded Timoshenko and Euler Bernoulli beams with the effect of rotary inertia and shear deformation. Benatta et al. [11] presented high order beam theory for analysis of short functionally graded symmetric beams under three-point bending. Şimşek [12] performed static analysis of a functionally graded beam by using the Ritz method with the effect of uniformly distributed load. Saidi et al. [13] studied axisymmetric bending and buckling analysis of functionally graded circular plate by using third-order shear deformation theory. Amirani et al. [14] investigated the free vibration analysis of sandwich beams with a functionally graded material as core material by using Galerkin method for the two-dimensional elasticity problems. Kang and Li [15] investigated nonlinear static analysis of functionally graded cantilever beams by considering power-law non-linearity. Mohammadi et al. [16] performed the buckling analysis of thin functionally graded rectangular plates by using the classical plate theory. Akbaş [17-22] performed static, post-buckling, dynamic, geometrically nonlinear analysis of cracked functionally graded and fiber reinforced beams by using finite element method. Kocatürk and Akbaş [23] studied post-buckling analysis of Timoshenko beams exposed to non-uniform thermal loads with the various boundary conditions. Danesh et al. [24] investigated the axial vibration analysis of tapered nanorod based on the non-local elasticity theory by using the differential quadrature method. Mohammadi et al. [25] examined the vibration analysis under the temperature effect of annular graphene sheet embedded in the Visco – Pasternak foundation. Pradhan and Chakraverty [26] performed the free vibration analysis of functionally graded material beams under the different boundary conditions by using Rayleigh-Ritz method within first order shear deformation theory. Su et al. [27] performed a dynamic stiffness method to analyze the free vibration of functionally graded beams. Li and Batra [28] examined the analytical relationships between the functionally graded Timoshenko and Euler Bernoulli beams subjected to axial compressive load in terms of critical buckling load. Akgöz and Civalek [29] examined the free vibration of functionally graded tapered microbeams by using the modified couple stress theory. Asemi et al. [30] performed nonlinear stability analysis of orthotropic single-layered graphene sheet by using the nonlocal theory of Eringen. Safarabadi et al. [31] analyzed the free vibration of the rotating nanobeam and examined the effects of surface energy on vibration frequency. Vo et al. [32] analyzed the free vibration and buckling of the functionally graded sandwich beams with a finite element model using a quasi-3D theory. Avcar [33] investigated free vibration of functionally graded Beams resting on elastic foundation under Axial Force. Akbaş [34,35,36,37] studied the effects of temperature rising on the static and dynamic responses of functionally graded and laminated composite beams. Wattanasakulpong and Mao [38] studied the dynamic response of functionally graded beams with different boundary conditions by using the Chebyshev collocation method within Timoshenko beam theory. Nguyen et al. [39] analyzed static, buckling and vibration analysis of a laminated composite beams by using the high order beam theory. Özütok and Madenci [40] performed static analysis of laminated composite beams by using mixed type finite element method based on high order shear deformation theory. Civalek [41] studied vibration analysis of laminated composite panels with various functionally graded constituents. Mohammadi and Rastgoo [42] studied the vibration behavior of new generation nanostructures under the effect of harmonic loads. Ghasemi and Mohandes [43] compared between the vibrations of fiber-metal laminate and composite cylindrical shells by using beam modal function model. Karamanlı [44] investigated the bending analysis

of laminated composite and sandwich beams by using Timoshenko beam theory and the Ritz method. Mahmoudi et al. [45] investigated effects of micromechanical models on free vibration of functionally graded plates resting on elastic foundation. Moradi et al. [46] investigated vibration of functionally graded nanoplate resting on Visco Pasternak foundation with Magneto-Thermo effects. Zargaripoor et al. [47] presented vibration of functionally graded nanoplates by using nonlocal elasticity theory and finite element method. Sayyad and Ghugal [48] performed bending, buckling and free vibration analysis of functionally graded simple supported beams by using Navier method. Chen et al. [49] performed the buckling and the bending analysis of the functionally graded porous plate by using the Cherbyshev – Ritz method. Hadji et al. [50] and Zouatnia et al. [51] investigated effects of micromechanical models on the static and free vibration responses of functionally graded porous beam by using the Navier method. Yaylı [52] examined Free vibration analysis of a rotationally restrained nanotube. Akbaş [53-64] studied dynamics of functionally graded and layered beam with different conditions. Seyyed Nosrati et al. [65] solved elasto-plastic behavior of thick-walled spherical vessels with functionally graded layers. Yüksel and Akbaş [66] investigated critical buckling loads of fiber reinforced laminated plates with porosity by using the Navier method. Barati et al. [67] presented static torsion of bi-directional functionally graded microtube based on the couple stress theory under magnetic field. Khoram et al. [68] investigated Bending analysis of bidirectional fgm timoshenko nanobeam subjected to mechanical and magnetic forces and resting on winkler–pasternak foundation. Mohammadi et al. [69] investigated the free and forced vibration behavior of a porous functionally graded nanobeam resting on a nonlinear foundation with the effect of electrical and mechanical loads. Mohammadi and Rastgoo [70] examined the nonlinear free and forced vibration analysis of the viscoelastic composite nanoplate.

In this study, static deflection and stress distribution of functionally graded and layered composite cantilever beams are obtained and compared each other. The considered problem is solved by using the Ritz method based on the both Timoshenko and Euler-Bernoulli beam theories. In the comparison study, the static deflections and stresses are compared with different values of material distribution parameters and aspect ratios. In addition, difference among of beam theories are compared for functionally graded and layered beams. In order to validate the using formulations and program, some comparison studies are presented.

## 2. Equations

Cantilever composite beams made of a mixture of ceramic and metal materials are shown in layered and functionally graded distributions in figure 1. The composite beam subjected to a point load ( $P$ ) at free with end with the  $L$  length, the  $b$  height and  $b$  width according to  $X, Y, Z$  the Cartesian coordinate system. In the functionally graded beam, the material properties vary though the height direction as shown figure 1a. In layered beam, two layers are used as shown figure 1b. The geometry properties of layers are equal to each other. In layered beam, the bottom and top layers of the beam is considered as a ceramic material: Zirconia and a metal material: Alumina, respectively. In case these materials grade continuously in the thickness direction, the bottom surfaces of beam are considered as a ceramic material (Zirconia) and the top surfaces is considered as a metal material (Alumina) as shown figure 1.



**Figure 1.** A cantilever composite beam under a point load (P) at free end in case of a) functionally graded distribution  
b) Layered distribution.

Based on the Timoshenko beam theory, the axial strain ( $\epsilon_z$ ) and shear strain ( $\gamma_{zy}$ ) are given as follows;

$$\epsilon_z = \frac{\partial u_0}{\partial z} - Y \frac{\partial \phi}{\partial z} \quad (1a)$$

$$\gamma_{zy} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial z} \quad (1b)$$

where,  $u_0$ ,  $v_0$  and  $\phi$  are axial displacement, vertical displacement and rotation, respectively. The relation between stress and strain is given as follows;

$$\sigma_z = E(Y) \left[ \frac{\partial u_0}{\partial z} - Y \frac{\partial \phi}{\partial z} \right] \quad (2a)$$

$$\sigma_{zy} = G(Y) K_s \left[ \frac{\partial v_0}{\partial z} - \phi \right] \quad (2b)$$

where, E is the Young's Modulus, G shear Modulus,  $\sigma_z$  is the normal stress,  $\sigma_{zy}$  is the shear stresses and  $K_s$  is the shear correction factor. In case the rotation is  $\phi = \partial v_0 / \partial z$ , the beam model reduces to the Euler-Bernoulli beam theory. The material properties (P) of the beam in case of functionally graded material change through height axis based on following power-law function distribution;

$$P(Y) = (P_B - P_T) \left( \frac{Y}{h} + \frac{1}{2} \right)^n + P_T \quad (3)$$

where  $P_B$  and  $P_T$  are material properties of bottom and top surfaces, n is the power-law coefficient (material distribution parameter). According to equation (3), when  $Y = -h/2$ ,  $P = P_T$ , and when  $Y = h/2$ ,  $P = P_B$  when  $n=0$  material of beam gets homogenous full bottom material, and when  $n=\infty$  material of beam gets homogenous full top material. In the layered beam with two layers, the material properties (P) of the beam is defined as follows;

$$P(Y) = \begin{cases} P_B, & 0 \leq h \leq 0.5h \\ P_T, & -0.5h \leq h \leq 0 \end{cases} \quad (4)$$

The elastic strain energy ( $U_i$ ) and potential energy of the external loads ( $U_e$ ) of the beam are expressed as follows;

$$U_i = \frac{1}{2} \int_V \sigma_z \epsilon_z dV + \frac{1}{2} \int_V \sigma_{zy} \gamma_{zy} dV \quad (5a)$$

$$U_i = \frac{1}{2} \int_0^L \left[ A_0 \left( \frac{\partial u_0}{\partial z} \right)^2 - 2A_1 \frac{\partial u_0}{\partial z} \frac{\partial \phi}{\partial z} + A_2 \left( \frac{\partial \phi}{\partial z} \right)^2 \right] dZ + \frac{1}{2} \int_0^L K_s B_0 \left[ \left( \frac{\partial v_0}{\partial z} \right)^2 - 2 \frac{\partial v_0}{\partial z} \phi + \phi^2 \right] dZ \quad (5b)$$

$$U_e = -\frac{1}{2} \int_0^L P v_0 dZ \quad (6)$$

where,  $A_0$ ,  $A_1$ ,  $A_2$  and  $B_0$  are expressed as follows;

$$A_0 = \int_A E(Y) dA, \quad A_1 = \int_A E(Y) Y dA, \quad A_2 = \int_A E(Y) Y^2 dA, \quad B_0 = \int_A G(Y) dA \quad (7)$$

The total potential energy ( $\Pi$ ) of the problem is expressed as follows:

$$\Pi = U_i + U_e \quad (8)$$

In the solution of the problem in Ritz method, approximate solution is given as a series of n terms of the following form:

$$u_0 = \sum_{i=1}^n a_i \alpha_i(z) \quad (9a)$$

$$v_0 = \sum_{i=1}^n b_i \beta_i(z) \quad (9b)$$

$$\phi = \sum_{i=1}^n c_i \gamma_i(z) \quad (9c)$$

where,  $a_i$ ,  $b_i$  and  $c_i$  are unknown coefficients,  $\alpha_i$ ,  $\beta_i$  and  $\gamma_i$  are a set of functions of Z, namely coordinate function defined over the interval [0, L]. The coordinate functions for the cantilever beam are given as algebraic polynomials:

$$\alpha_i(z) = z^i \quad (10a)$$

$$\beta_i(z) = z^{(i+1)} \quad (10b)$$

$$\gamma_i(z) = z^i \quad (10c)$$

where  $i$  indicates the number of polynomials involved in the admissible functions. According to the minimum total potential energy principle, unknown coefficients  $a_i$ ,  $b_i$  and  $c_i$  which correspond to the minimum of the total potential energy are determined by the conditions:

$$\frac{\partial \Pi}{\partial a_i} = 0, \quad \frac{\partial \Pi}{\partial b_i} = 0, \quad \frac{\partial \Pi}{\partial c_i} = 0, \quad i = 1, 2, 3 \dots \dots n \quad (11)$$

Differentiation of  $\Pi$  in respect to unknown coefficients  $a_i$ ,  $b_i$  and  $c_i$  produces the following equilibrium equations:

$$\begin{bmatrix} K_{ij}^{11} & K_{ij}^{12} & K_{ij}^{13} \\ K_{ij}^{21} & K_{ij}^{22} & K_{ij}^{23} \\ K_{ij}^{31} & K_{ij}^{32} & K_{ij}^{33} \end{bmatrix} \begin{Bmatrix} a_j \\ b_j \\ c_j \end{Bmatrix} = \begin{Bmatrix} F_i^1 \\ F_i^2 \\ F_i^3 \end{Bmatrix} \quad (12)$$

where,  $K_{ij}$  and  $F_i$  are the stiffness and load matrixes for the Timoshenko beam theory, respectively. The detail of these expressions are given as follows;

$$\begin{aligned} K_{ij}^{11} &= \sum_{i=1}^n \sum_{j=1}^n \int_0^L A_0 \frac{\partial \alpha_i}{\partial z} \frac{\partial \alpha_j}{\partial z} dz, \quad K_{ij}^{12} = 0, \\ K_{ij}^{13} &= - \sum_{i=1}^n \sum_{j=1}^n \int_0^L A_1 \frac{\partial \alpha_i}{\partial z} \frac{\partial \gamma_j}{\partial z} dz, \quad K_{ij}^{21} = 0, \\ K_{ij}^{22} &= \sum_{i=1}^n \sum_{j=1}^n \int_0^L K_s B_0 \frac{\partial \beta_i}{\partial z} \frac{\partial \beta_j}{\partial z} dz, \\ K_{ij}^{23} &= - \sum_{i=1}^n \sum_{j=1}^n \int_0^L K_s B_0 \frac{\partial \beta_i}{\partial z} \gamma_j dz, \\ K_{ij}^{31} &= - \sum_{i=1}^n \sum_{j=1}^n \int_0^L A_1 \frac{\partial \gamma_i}{\partial z} \frac{\partial \alpha_j}{\partial z} dz, \\ K_{ij}^{32} &= - \sum_{i=1}^n \sum_{j=1}^n \int_0^L K_s B_0 \gamma_i \frac{\partial \beta_j}{\partial z} dz, \\ K_{ij}^{33} &= \sum_{i=1}^n \sum_{j=1}^n \int_0^L A_2 \frac{\partial \gamma_i}{\partial z} \frac{\partial \gamma_j}{\partial z} + \sum_{i=1}^n \sum_{j=1}^n \int_0^L K_s B_0 \gamma_i \gamma_j dz, \\ F_i^1 &= 0, \quad F_i^3 = 0 \\ F_i^2 &= \sum_{i=1}^n \int_0^L P \beta_i dz \end{aligned} \quad (13)$$

By implementing of the rotation  $\phi = \partial v_0 / \partial z$  into the relevant equations, the Ritz formulations for Euler-Bernoulli beam theory are presented as follows;

$$\begin{bmatrix} K_{ij}^{11} & K_{ij}^{12} \\ K_{ij}^{21} & K_{ij}^{22} \end{bmatrix} \begin{Bmatrix} a_j \\ b_j \end{Bmatrix} = \begin{Bmatrix} F_i^1 \\ F_i^2 \end{Bmatrix} \quad (14)$$

where,

$$\begin{aligned} K_{ij}^{11} &= \sum_{i=1}^n \sum_{j=1}^n \int_0^L A_0 \frac{\partial \alpha_i}{\partial z} \frac{\partial \alpha_j}{\partial z} dz, \\ K_{ij}^{12} &= - \sum_{i=1}^n \sum_{j=1}^n \int_0^L A_1 \frac{\partial \alpha_i}{\partial z} \frac{\partial \beta_j}{\partial z} dz, \\ K_{ij}^{21} &= - \sum_{i=1}^n \sum_{j=1}^n \int_0^L A_1 \frac{\partial \alpha_j}{\partial z} \frac{\partial \beta_i}{\partial z} dz \end{aligned}$$

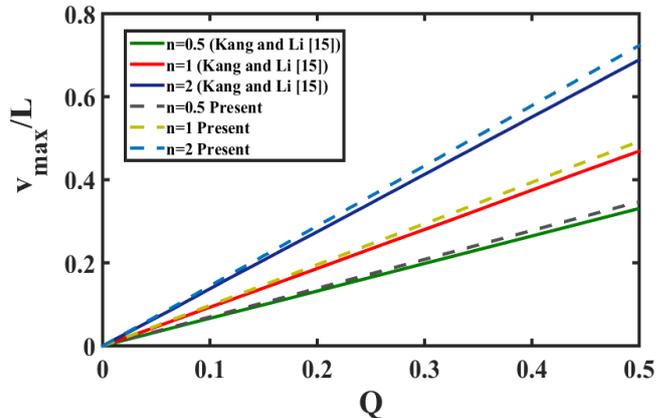
$$K_{ij}^{22} = \sum_{i=1}^n \sum_{j=1}^n \int_0^L A_2 \frac{\partial^2 \beta_i}{\partial z^2} \frac{\partial^2 \beta_j}{\partial z^2} dz,$$

$$F_i^1 = 0 \text{ and } F_i^2 = \sum_{i=1}^n \int_0^L P \beta_i dz \quad (15)$$

### 3. Findings and Discussion

In this section, static deflections and stress distribution of a cantilever beam are obtained for both functionally graded and layered composite materials, and the results of two different composites are compared each other. In addition, the results are compared with different beam theories such as Timoshenko and Euler-Bernoulli beam theories for different aspect ratios. In the numerical examples, the materials of the beams are selected as Zirconia and Alumina, and their distributions are shown in figure 1. The material parameters of these materials are given as follows. Zirconia:  $E=151$  GPa,  $\nu=0.2882$ , Alumina:  $E=70$  GPa,  $\nu=0.31$ .  $\nu$  indicates the Poisson's ratio. The geometry properties of the beam are selected as  $b = 0.1$  m,  $h=0.1$  m and the Length is varied according to the aspect ratios ( $L/h$ ).

In order to validate the formulations and using program for functionally graded beams, the maximum vertical displacements (at free end) are obtained and compared with special results of Kang and Li [15]. In the study of Kang and Li [15], the nonlinear static displacements of a cantilever functionally graded beam are obtained for different material graduation parameters ( $n$ ) under point load like this study. The linear results of Kang and Li [15] in figure 7a are compared with this study in figure 2 according to the parameters of [15]. In this comparison study, the dimensionless results are presented and  $Q$  indicates the dimensionless load which defines as follows;  $Q=PL^2/bh^3Eb$ . It is seen from figure 2 that the displacements of [15] are very close to this study's.



**Figure 2.** Comparison study; dimensionless maximum vertical displacements and load relation for functionally graded cantilever beam for different material graduation parameters ( $n$ ).

To further validate the formulations and using program for layered composite beam, the maximum vertical displacements (at free end) are obtained and compared with ANSYS Workbench 14 program for a point load  $P=100$  kN according to different aspect ratios ( $L/h$ ) in table 1. As seen from table 1, the results of this study are very close to the ANSYS Workbench 14's.

**Table 1.** Comparison study; Maximum vertical displacements ( $v_{max}$ ) of the two layered composite beam for different aspect ratios.

	$v_{max}$ (m)			
	$L/h=7$	$L/h=10$	$L/h=15$	$L/h=30$
Present	0.014	0.04053	0.13628	1.0877
ANSYS Workbench 14	0.017	0.04215	0.13923	1.0886

In figure 3, the vertical maximum displacements (at the free-end of the cantilever beam) versus the increasing of the load are presented in functionally graded and layered beams for different aspect ratios. In addition, the results are presented in both Euler-Bernoulli beam theory (EBT) and Timoshenko beam theory (TBT) in these figures. In the results of the functionally graded materials, different values of the power-law coefficient (material distribution parameter)  $n$  are used in order to compare between the results of functionally graded and layered materials.

It is seen from figure 3 that, the displacements of functionally graded beam increase by increasing the material gradation parameter ( $n$ ). Because Young's Modulus of the beams decrease according to equation (3) and position of the materials, the stiffness of the beam decreases and the displacements increase naturally. With the same materials and their positions in the beam, the displacements of the layered distribution are shown in black color in these figures. The displacements of the layered beams are bigger than the results of functionally graded parameter  $n=1$ . The values of displacements of the layered beams are between the results of  $n=1$  and  $n=3$ . The displacements of  $n<1$  are smaller than the layered beam's. If the value of functionally graded gradation parameter selects less than  $n=1$ , the displacements of functionally graded material is smaller than the layered beam's. So, using the functionally graded material has a superiority over layered beams for  $n<1$ . The results show that the material gradation parameter plays important role on the mechanical behavior of the functionally graded materials and with suitable choosing of  $n$  parameter, functionally graded materials have more advantages than layered materials in regards to mechanical responses.

Another results of figure 3 that the difference between results Euler-Bernoulli beam theory and Timoshenko beam theory decreases with increasing of aspect ratio for two composites significantly. In smaller values of aspect ratio, there is a considerable difference between two theories. In order to get realistic results in smaller values of aspect ratios, the Timoshenko beam theory must be used instead of the Euler-Bernoulli beam theory for two composites.

In figures 4 and 5, normal stress and shear stress distributions along the height at the fixed support of the functionally graded and layered composite beams are presented, respectively, for different material distribution parameters for  $P=10^5$  N,  $L/h=8$  for the Timoshenko beam theory. Figures 4 and 5 shows that the stress distributions in the layered composite are very different from the functionally graded composites. In the layered composite beam, the normal and shear stresses change suddenly at the interface. The stresses are discontinuities in these points. Whereas, the stresses don't change suddenly in the functionally graded beams and distribute as gradually. The results of the stress discontinuities at interfaces in the layered composite, cracking and delamination are

often occurred. So, with the using the functionally graded composites instead of layered composites, fracture problems can be minimizing. Also it is seen from figures 4 and 5, the material gradation parameter has very effects on the stress distribution of functionally graded composites. With suitable choosing of  $n$  parameter, the stresses can be decreased significantly

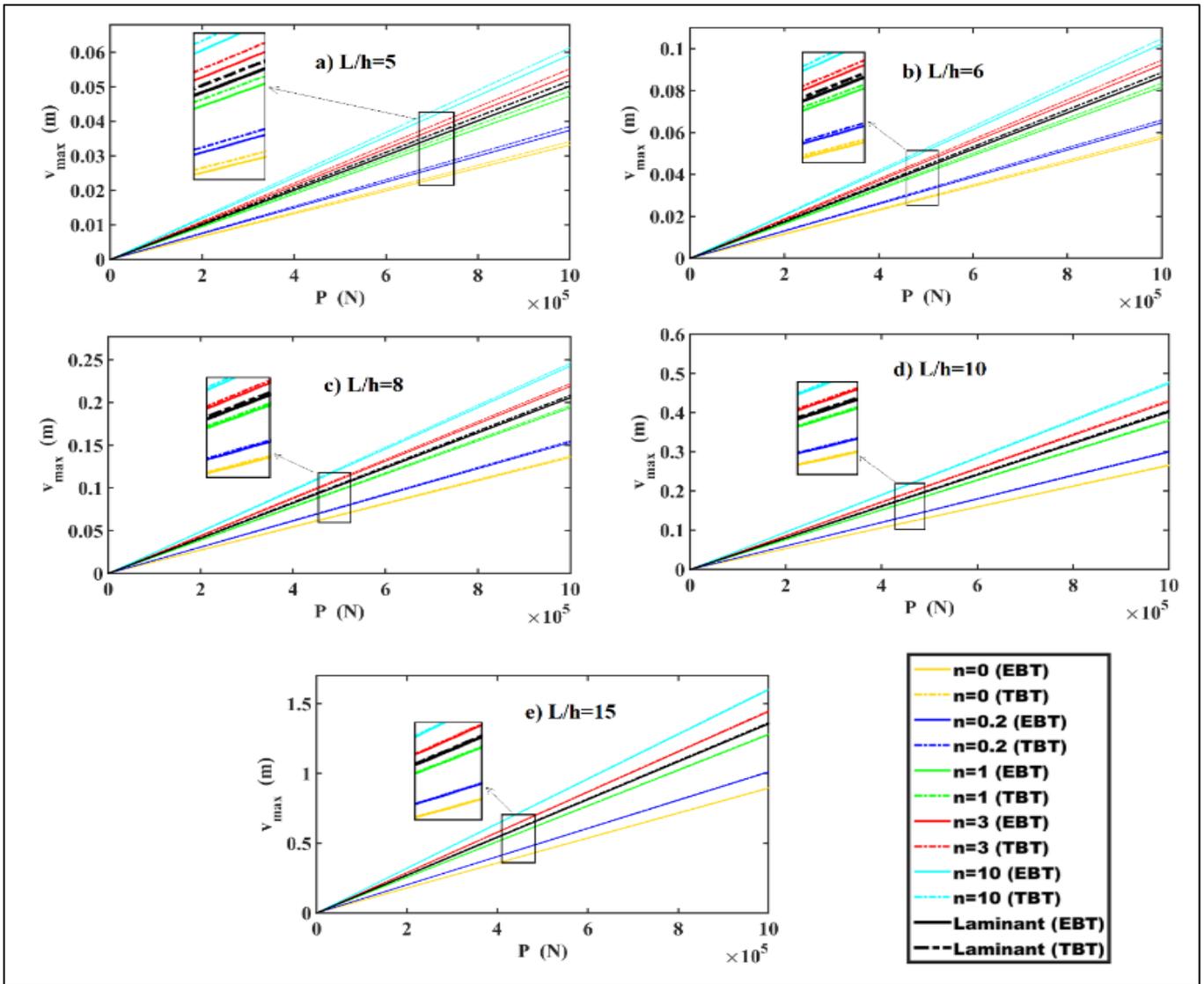
In order to more see the difference between layered and functionally graded beams, the static deflection shapes are presented in figure 6 for layered and different values of material gradation parameters ( $n$ ) for functionally graded material for  $L/h=10$  and  $P=10^5$  N in the Timoshenko beam theory. As seen from figure 6 that the there is a big difference between layered and functionally graded materials in the static deflection shapes. Like the previous results, the  $n$  parameter has important role on the mechanical behavior of functionally graded material. With decreasing of  $n$  parameter, the displacements decrease in contrast with layered composite. This shows that functionally graded materials are more superior than layered composites in the mechanical behavior of the structures.

#### 4. Conclusions

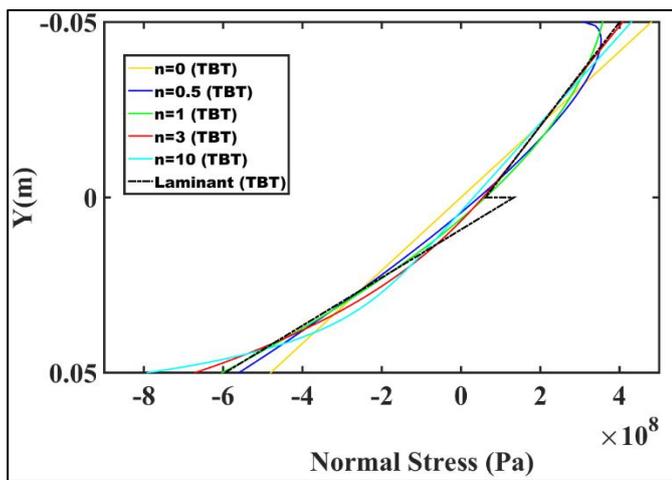
A comparison study between functionally graded and layered composites is investigated for static deflections and stresses of a cantilever beam subjected to a point load. In the solution of the study, the Ritz method is used based on both Timoshenko and Euler-Bernoulli beam theories. For two different distributions, the formulations of Ritz method are obtained and solved numerically. The influences of material distribution parameter, aspect ratio on the static deflections and stress distribution of functionally graded beams are investigated and compared with the results of the layered composite beam in both Timoshenko and Euler-Bernoulli beam theories. Some comparison studies are performed in order to validate the using formulations.

It is seen from results, there is a big difference between static behaviors of layered and functionally graded materials. With changing in the material distribution, the mechanical behaviour of functionally graded beam vary significantly and the difference among two material change considerably. With suitable choosing of  $n$  parameter, functionally graded materials have more advantages than layered materials in regards to mechanical responses. Another result of this study is that the Timoshenko beam theory must be used instead of the Euler-Bernoulli beam theory in smaller values of aspect ratios for both functionally graded and layered beams. In case of suitable choosing of the material distribution parameter, functionally graded materials are more superior than layered composites in the mechanical behavior of the structures.

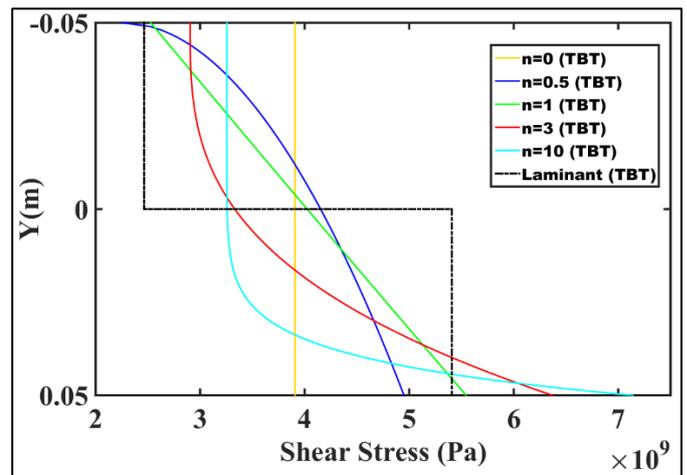




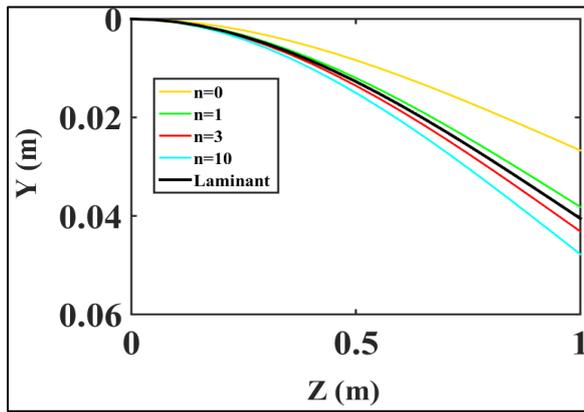
**Figure 3.** The load- displacement relation for functionally graded and layered beams according to different beam theories and material distribution parameters ( $n$ ) for aspect ratios a)  $L/h=5$ , b)  $L/h=6$ , c)  $L/h=8$ , d)  $L/h=10$  and e)  $L/h=15$ .



**Figure 4.** Normal stress distributions along the height at the fixed support of the functionally graded and layered composite beams for different material distribution parameters in Timoshenko beam theory.



**Figure 5.** Shear stress distributions along the height at the fixed support of the functionally graded and layered composite beams for different material distribution parameters in the Timoshenko beam theory.



**Figure 6.** Static deflection shape of functionally graded and layered composite beams for different material distribution parameters in the Timoshenko beam theory.

## References

- [1] Lewandowski R., 1987, Application of the Ritz method to the analysis of non-linear free vibrations of beams, *Journal of Sound and Vibration* 114(1): 91-101.
- [2] Reddy J., 2000, Analysis of functionally graded plates, *International Journal for Numerical Methods in Engineering* 47(1-3): 663-684.
- [3] Sankar B. V., 2001, An elasticity solution for functionally graded beams, *Composites Science and Technology* 61(5): 689-696.
- [4] Deschilder M., Eslami H., Zhao Y., 2006, Nonlinear Static Analysis of a Beam Made of Functionally Graded Material, in: *Proceedings of the 47th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, Newport, Rhode Island.
- [5] Chi S.H., Chung Y.L., 2006, Mechanical behavior of functionally graded material plates under transverse load – Part I: Analysis , *International Journal of Solids and Structures* 43(13): 3657-3674.
- [6] Palanivel S., 2006, Dynamic analysis of laminated composite beams using higher order theories and finite elements, *Composite Structures* 73(3): 342-353.
- [7] Zhong Z., Yu T., 2007, Analytical solution of a cantilever functionally graded beam, *Composites Science and Technology* 67(3-4): 481-488.
- [8] Aydogdu M., Taskin V., 2007, Free vibration analysis of functionally graded beams with simply-supported edges, *Materials & Design* 36(5): 1651-1656.
- [9] Kadoli R., Akhtar K., Ganesan N., 2008, Static analysis of functionally graded beams using higher order shear deformation theory, *Applied Mathematical Modelling* 32(12): 2509-2525.
- [10] Li X.F., 2008, A unified approach for analyzing static and dynamic behaviors of functionally graded Timoshenko and Euler-Bernoulli beams, *Journal of Sound and Vibration* 318(4-5): 1210-1229.
- [11] Benatta M.A., Mechab I., Tounsi A., Bedia E.A.A., 2008, Static analysis of a functionally graded short beams including warping and shear deformation effects, *Computational Materials Science* 44(2): 765-773.
- [12] Şimşek M., 2009, Static analysis of a functionally graded beam under a uniformly distributed load by Ritz method, *International Journal of Engineering and Applied Sciences* 1(3): 1-11.
- [13] Saidi A. R., Rasouli A., Sahraee S., 2009, Axisymmetric bending and buckling analysis of thick functionally graded circular plates using unconstrained third-order shear deformation plate theory, *Composite Structures* 89(1): 110-119.
- [14] Amirani M.C., Khalili S.M.R., Nemati N., 2009, Free Vibration Analysis of Sandwich beam with FG Core Using the Element Free Galerkin Method, *Composite Structures* 90: 373-379.
- [15] Kang YA., Li, X F (2009), Bending of functionally graded cantilever beam with power-law non-linearity subjected to an end force, *International Journal of Non-Linear Mechanics*, 44(6): 696-703.
- [16] Mohammadi M., Saidi A. R., Jomehzadeh E., 2010, Levy Solution for Buckling Analysis of Functionally Graded Rectangular Plates, *Applied Composite Materials* 17(2): 81-93.
- [17] Akbaş ŞD., 2011, Static analysis of a functionally graded beam with edge cracks on elastic foundation, in: *Proceedings of the 9th International Fracture Conference*, Istanbul, Turkey, 70-80.
- [18] Akbaş ŞD., 2013, Geometrically Nonlinear Static Analysis of Edge Cracked Timoshenko Beams Composed of Functionally Graded Material, *Mathematical Problems in Engineering*, doi: 10.1155/2013/871815.
- [19] Akbaş ŞD., 2018, Forced vibration analysis of cracked functionally graded microbeams, *Advances in Nano Research* 6(1): 39-55.
- [20] Akbaş ŞD., 2018, Bending of a cracked functionally graded nanobeam, *Advances in Nano Research* 6(3): 219-242.
- [21] Akbaş ŞD., 2019, Post-buckling analysis of a fiber reinforced composite beam with crack, *Engineering Fracture Mechanics* 212: 70-80.
- [22] Akbaş ŞD., 2019, Nonlinear behavior of fiber reinforced cracked composite beams, *Steel and Composite Structures* 30(4): 327-336.
- [23] Kocatürk T., Akbaş Ş. D., 2011, Post-buckling analysis of Timoshenko beams with various boundary conditions under non-uniform thermal loading, *Structural Engineering and Mechanics* 40(3): 347-371.
- [24] Danesh M., Farajpour A., Mohammadi M., 2012, Axial vibration analysis of a tapered nanorod based on nonlocal elasticity theory and differential quadrature method, *Mechanics Research Communications* 39(1): 23-27.
- [25] Mohammadi M., Farajpour A., Goodarzi M., Mohammadi H., 2013, Temperature effect on vibration analysis of annular graphene sheet embedded on Visco-Pasternak foundation, *Journal of Solid Mechanics*: 5 (3), 305-323.
- [26] Pradhan K.K., Chakraverty S., 2013, Free vibration of Euler and Timoshenko functionally graded beams by Rayleigh – Ritz method, *Composites Part B: Engineering* 51: 175-184.
- [27] Su H., Banerjee J.R., Cheung C.W., 2013, Dynamic stiffness formulation and free vibration analysis of functionally graded beams , *Composite Structures* 106: 854-862.
- [28] Li S., Batra R., 2013 Relations between buckling loads of functionally graded Timoshenko and homogeneous Euler – Bernoulli beams, *Composite Structures* 95: 5-9.
- [29] Akgöz B., Civalek Ö. 2013 Free vibration analysis of axially functionally graded tapered Bernoulli–Euler microbeams based on the modified couple stress theory, *Composite Structures* 98: 314-322.
- [30] Asemi S.R., Mohammadi M., Farajpour A., 2014, A study on the nonlinear stability of orthotropic single-layered graphene sheet based on nonlocal elasticity theory, *Latin American Journal of Solids and Structures*: 11 (9), 1515-1540.
- [31] Safarabadi M., Mohammadi M., Farajpour A., Goodarzi M., 2015, Effect of surface energy on the vibration analysis of rotating nanobeam, *Journal of Solid Mechanics*: 7 (3), 299-311.
- [32] Vo T., Thai T., Nguyen T. K., Inam F., Lee J., 2015, A quasi-3D theory for vibration and buckling of functionally graded sandwich beams, *Composite Structures* 119: 1–12.
- [33] Avcar M., 2016, Free Vibration of Non-Homogeneous Beam Subjected to Axial Force Resting on Pasternak Foundation, *Journal of Polytechnic* 19(4): 507-512.
- [34] Akbaş ŞD., 2019, Nonlinear static analysis of laminated composite beams under hygro-thermal effect, *Structural Engineering and Mechanics* 72(4): 433-441.
- [35] Akbaş ŞD., 2017, Nonlinear static analysis of functionally graded porous beams under thermal effect, *Coupled Systems Mechanics* 6(4): 399-415.

- [36] Akbaş ŞD., 2015, Wave propagation of a functionally graded beam in thermal environments, *Steel and Composite Structures* 19(6): 1421-1447.
- [37] Akbaş ŞD., 2018, Nonlinear thermal displacements of laminated composite beams, *Coupled systems mechanics* 7(6): 691-705.
- [38] Wattanasakulpong N., Mao Q., 2015, Dynamic response of Timoshenko functionally graded beams with classical and non-classical boundary conditions using Chebyshev collocation method, *Composite Structures* 119: 346-354.
- [39] Nguyen T. K., Nguyen N. D., Vo T., Thai T., 2016, Trigonometric-series solution for analysis of laminated composite beams, *Composite Structures* 160: 142-151.
- [40] Özütok A., Madenci E., 2017, Static analysis of laminated composite beams based on higher-order shear deformation theory by using mixed-type finite element method, *International Journal of Mechanical Sciences* 130: 234-243.
- [41] Civalek Ö. 2017, Vibration of laminated composite panels and curved plates with different types of FGM composite constituent, *Composites Part B: Engineering* 122: 89-108.
- [42] Mohammadi M., Rastgoo A., 2018, Primary and secondary resonance analysis of FG/lipid nanoplate with considering porosity distribution based on a nonlinear elastic medium, *Mechanics of Advanced Materials and Structures*: 1-22, doi: 10.1080/15376494.2018.1525453.
- [43] Ghasemi AR., Mohandes M., 2018, Comparison between the frequencies of FML and composite cylindrical shells using beam modal function model, *Journal of Computational Applied Mechanics* 50(2): 239-245.
- [44] Karamanlı A., 2018, Bending analysis of composite and sandwich beams using Ritz method, *Anadolu University Journal of Science and Technology A - Applied Sciences and Engineering* 19(1): 10-23.
- [45] Mahmoudi A., Benyoucef S., Tounsi A., Benachour A., Bedia EAA. 2018, On the effect of the micromechanical models on the free vibration of rectangular FGM plate resting on elastic foundation, *Earthquakes and Structures* 14(2): 117-128.
- [46] Moradi A., Yaghootian A., Jalalvand M., Ghanbarzadeh A. 2018, Magneto-Thermo mechanical vibration analysis of FG nanoplate embedded on Visco Pasternak foundation, *Journal of Computational Applied Mechanics*, 49(2): 395-407.
- [47] Zargaripoor A., Daneshmehr A., Isaac Hosseini I., Rajabpoor A. (2018), Free vibration analysis of nanoplates made of functionally graded materials based on nonlocal elasticity theory using finite element method, *Journal of Computational Applied Mechanics*, 49(1); 86-101.
- [48] Sayyad A.S., Ghugal Y.M, 2018, Bending, buckling and free vibration responses of hyperbolic shear deformable FGM beams, *Mechanics of Advanced Composite Structures* 5: 13-24.
- [49] Chen D., Yang J., Kitipornchai S., 2018, Buckling and bending analyses of a novel functionally graded porous plate using Chebyshev-Ritz method, *Archives of Civil and Mechanical Engineering* 19 (1): 157-170.
- [50] Hadji L., Zouatnia N., Bernard F. 2019, An analytical solution for bending and free vibration responses of functionally graded beams with porosities: Effect of the micromechanical models, *Structural Engineering and Mechanics* 69(2): 231-241.
- [51] Zouatnia N., Hadji L., Kassoul A. 2017, An analytical solution for bending and vibration responses of functionally graded beams with porosities, *Wind and Structures* 25(4): 329-342.
- [52] Yayli MÖ., 2019, Free vibration analysis of a rotationally restrained (FG) nanotube, *Microsystem Technologies*, 25(10); 3723-3734.
- [53] Akbaş ŞD., 2013, Free vibration characteristics of edge cracked functionally graded beams by using finite element method, *International Journal of Engineering Trends and Technology*, 4(10): 4590-4597.
- [54] Akbaş ŞD., 2014, Free vibration of axially functionally graded beams in thermal environment, *International Journal of Engineering and Applied Sciences*, 6(3): 37-51.
- [55] Akbaş ŞD., 2015, Free vibration and bending of functionally graded beams resting on elastic foundation, *Research on Engineering Structures and Materials*, 1(1): 25-37.
- [56] Akbaş ŞD., 2018, Investigation of static and vibration behaviors of a functionally graded orthotropic beam, *Journal of Balikesir University Institute of Science and Technology*, 20(1): 69-82.
- [57] Akbaş ŞD., 2018, Investigation on free and forced vibration of a bi-material composite beam, *Journal of Polytechnic*, 21(1): 65-73.
- [58] Akbaş ŞD., 2019, Forced vibration analysis of functionally graded sandwich deep beams. *Coupled Systems Mechanics*, 8(3): 259-271.
- [59] Akbaş ŞD., 2017, Stability of a non-homogenous porous plate by using generalized differential quadrature method, *International Journal of Engineering and Applied Sciences*, 9(2): 147-155.
- [60] Akbaş ŞD., 2018, Post-buckling responses of a laminated composite beam, *Steel and Composite Structures*, 26(6): 733-743.
- [61] Akbaş ŞD., 2018, Geometrically nonlinear analysis of a laminated composite beam, *Structural Engineering and Mechanics*, 66(1): 27-36.
- [62] Akbaş ŞD., 2018, Geometrically nonlinear analysis of functionally graded porous beams, *Wind and Structures*, 27(1): 59-70.
- [63] Akbaş ŞD., 2019, Hygro-thermal nonlinear analysis of a functionally graded beam, *Journal of Applied and Computational Mechanics*, 5(2): 477-485.
- [64] Akbaş ŞD., 2019, Hygro-thermal post-buckling analysis of a functionally graded beam, *Coupled Systems Mechanics*, 8(5): 459-471.
- [65] Seyyed Nosrati A., Parvizi A., Afzal SA., Alimirzaloo V. 2019, Elasto-plastic solution for thick-walled spherical vessels with an inner FGM layer, *Journal of Computational Applied Mechanics*, 50(1): 1-13.
- [66] Yüksel YZ., Akbaş ŞD., 2019, Buckling Analysis of a Fiber Reinforced Laminated Composite Plate with Porosity, *Journal of Computational Applied Mechanics*, 50(2): 375-380.
- [67] Barati A., Adeli MM., Hadi A., 2019, Static torsion of bi-directional functionally graded microtube based on the couple stress theory under magnetic field, *International Journal of Applied Mechanics*, 12(2): 2050021.
- [68] Khoram MM., Hosseini M., Hadi A., Shishesaz, M., 2019, subjected to mechanical and magnetic forces and resting on Winkler-Pasternak foundation, *International Journal of Applied Mechanics*, <https://doi.org/10.1142/S1758825120500933>.
- [69] Mohammadi M., Hosseini M., Shishesaz M., Hadi A., Rastgoo A., 2019, Primary and secondary resonance analysis of porous functionally graded nanobeam resting on a nonlinear foundation subjected to mechanical and electrical loads, *European Journal of Mechanics – A/Solids*, doi: 10.1016/j.euromechsol.2019.05.008
- [70] Mohammadi M., Rastgoo A., 2019, Nonlinear vibration analysis of the viscoelastic composite nanoplate with three directionally imperfect porous FG core, *Structural Engineering and Mechanics* 69(2): 131-143.