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Numerical investigation of natural convection phenomena in uniformly heated trapezoidal Cylinder inside an elliptical Enclosure

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ABSTRACT

A numerical study of the natural convection of the laminar heat transfers in the stationary state was developed in a horizontal ring and compared between a heated trapezoidal internal cylinder and a cold elliptical outer cylinder. This annular space is traversed by a Newtonian and incompressible fluid. The Prandtl number is set to 0.7 (air case) for different Rayleigh numbers. The system of equations controlling the problem was solved numerically by the calculation of code fluent based on the finite volume method. In these simulations the Boussinesq approximation was considered. The inner and outer surfaces are kept at a constant temperature. The study is performed for Rayleigh numbers ranging from 103 to 105. Indeed, the effects of different numbers of thermal Rayleigh on natural convection were studied. The results are presented in the form of isotherms, isocurrents, local and average numbers of Nusselt. The purpose of this work is to study the influence of the thermal Rayleigh number, and the change of the angle of inclination of the trapezoidal lateral walls on the structure of the flow and the distribution of the temperature.

1. Introduction

The phenomenon of natural convection refers to the heat transfer process that results from the movement of fluid particles between zones with different temperatures. This results in a combination of fluid particles, which subsequently exchange energy and momentum with each other. The structure and intensity of natural convection depend on the external thermal stresses that cause them, the nature of the fluid and the geometry of the space where the process takes place. The importance of the natural convection is confirmed by the exploitation of this phenomenon in many different fields such as industrial water heating, thermal insulation, heat exchangers, solar collector-receiver, vapor condenser for water distillation, cooling of electronic systems, electrical machinery, geophysics, nuclear reactors, etc. The study of the natural convection in closed enclosures has been the subject of numerous theoretical and experimental studies. Many published works have been elaborated concerning natural convection in varied geometries, such as parallelepiped [1-3], cylindrical [4, 5], spherical [6], ellipsoidal [7] the form cylindrical rings [8, 9], spherical [10], and elliptical rings [11, 12]. There are also cone shaped [13], lunette [14] or cylindrical annular enclosures [15]. The natural convection in a square enclosure containing an equilateral triangular cylinderit was studied numerically with CFD techniques, with outer cold walls of square enclosure and warm inner walls of triangular cylinder [16]. The Rayleigh number ranged from 104 to 106 and the angles of orientation from 00 to 1050 for a step of 150 for each case. Rana and Natoosh [17] studied numerically the stable laminar natural convection in a square enclosure, containing four hot triangular cylinders. Costa and Raimundo [18] considered a conductive rotating cylinder inserted in the center of a square enclosure. They concluded that the thermo physical properties of the cylinder object were important for the entire process of heat transfer through the enclosure. Roslan, Saleh, and Hashim [19] studied numerically the natural convection heat transfer in a differentially heated square enclosure containing a conductive polygonal object. The left wall is heated and the right wall is cooled, while the horizontal walls remain adiabatic. Hussain and Hussein [20] analyzed the effects of inserting a conductive

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rotating cylinder at different vertical locations within a differentially heated square cavity. A study of the literature review reveals the importance of the study of the natural convection in an annular space delimited by two horizontal axis cylinders, heated trapezoidal internal cylinder and cold elliptical outer cylinder. In fact, the annular space traversed by a Newtonian fluid and incompressible has not been studied, so that will be the motivation of this study.

In this work, we studied the effect of the angle of inclination of the lateral walls of the trapezium, and the effect of the Rayleigh number on the flow structure, the temperature distributions and the current function as well as the heat transfer rates represented by the local and mean numbers of Nusselt. The detailed numerical simulations of the dynamic and thermal fields of natural convection flows are developed in the proposed configuration at a variable Rayleigh number in the range of 103to105 for different inclination angle equal to 60 °, 70 °, 80 °, 90 °, 100 ° and 110 °.

2. Physical model

Figure 1.a shows the physical model of the current work. The present problem we are dealing with is a trapezoidal cylinder characterized by the length h = 2cm and the angle of inclination of the side walls (α), located inside an elliptical cylinder enclosure with an eccentricity e= 0,7. The outer elliptical fence wall has been constant low temperature TF. However, the trapezoidal inner cylinder is maintained at constant high temperature TC. In this work the number of Prandtl is 0.7. The thermal Rayleigh number varies between103 and 105.The Newtonian fluid properties are also constant, and the Boussinesq approximation is applied to model the buoyancy effect. Indeed the acceleration due to gravity acts in the negative direction y and the viscous dissipation effects are negligible.

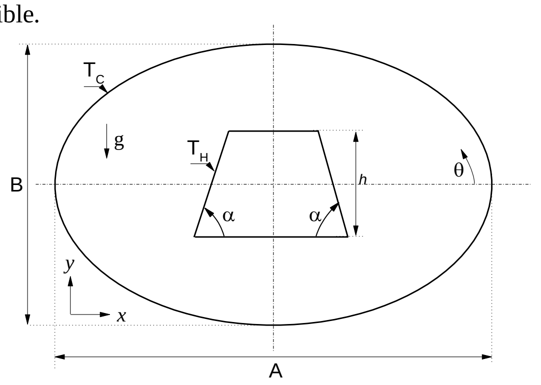


Figure 1.a. Schematic physical model.

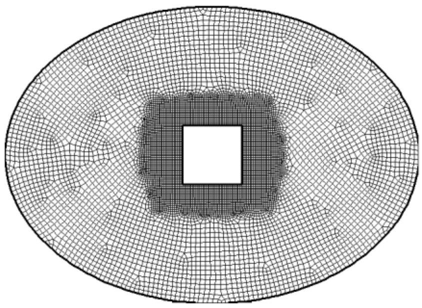


Figure 1.b. Nodes(230×130).

3. Numerical model

3.1 Mathematical formulation

The characteristic quantity used for dimensionless of the problem are the differences in temperature (ΔT=TH-TC) between the walls of the system and thermal diffusivity α of the fluid. The controlling equations are transformed into dimensionless forms under the following non-dimensional variables [21, 22]:

$$\theta = \frac{T - T_c}{T_H - T_c}, X = \frac{x}{B}, Y = \frac{y}{B}, U = \frac{uB}{\alpha}, V = \frac{vB}{\alpha}$$

$$\boldsymbol{P} = \frac{\boldsymbol{p\,B^2}}{\boldsymbol{\rho\alpha^2}}, \boldsymbol{Pr} = \frac{\boldsymbol{v}}{\boldsymbol{\alpha}} \textbf{ and } \boldsymbol{Ra} = \frac{\boldsymbol{g\beta(T_H - T_C)B^3}}{\boldsymbol{\alpha v}} \quad (1)$$

The Equation of Continuity:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (2)$$

The equations of momentums are written as follows:

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \quad (3)$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \Pr\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + RaPr\theta \quad (4)$$

The equation of energy is written as follows:

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \quad (5)$$

The initial condition:

$U = V = 0$

(6)

(6)

$$\theta = \theta_0 \quad (7)$$

(7)

The conditions of the system limits are defined in the inner cylinder as follows:

$$U = V = 0 \quad (8)$$

$$\theta = 1 \quad (9)$$

In the outer cylinder, we can write:

$$U = V = 0 \quad (10)$$

$$\theta = 0 \quad (11)$$

The local Nusselt number:

$$Nu_L = L\frac{\partial \theta}{\partial \vec{n}}\bigg|_{wall} \quad (12)$$

The average Nusselt number for the square and ellipse:

$$\overline{Nu}_s = \frac{1}{P}\int_0^P Nu_L\, dP \quad (13)$$

$$\overline{Nu}_e = \frac{1}{P_e}\int_0^{P_e} Nu_L\, dP_e \quad (14)$$

The average Nusselt number:

$$Nu_{avg} = \frac{\overline{Nu}_e + \overline{Nu}_s}{2} \quad (15)$$

3.2 Simplifying hypotheses

Fluid flows subjected to buoyancy forces are modeled by the Boussinesq approximation:

$$\rho(T) = \rho_0[1 - \beta_t(T - T_0)] \quad (16)$$

The Natural convection is laminar and permanent.

3.3 Meshing choice

In this article, several meshes were arbitrarily used for the configuration presented in the figure 1.b to Rat = 105, we have studies the effect of the meshing on the results. Table 1showtherefore the variation of the average Nusselt number as a function of the nodes number. The observed results allow us to choose the 230x130 mesh in all the simulations, in fact the error relative to the values of the average Nusselt numbers, between the two meshes is minimal.

Table .1 Effect the number of nodes on the average Nusselt number Rat = 105

Nodes	180*80	200*100	210*110	220*120	230*130	240*140
Nu (avg)	5.8826	5.6496	5.5818	5.4955	5.4319	5.4250
Relative error (%)	4.1242	1.2147	1.5704	1.1709	0.1272	

3.3 Numerical approach

The equations are processed sequentially by using the isolated method. The numerical procedure retained by the CFD code Fluent for solving the equations controlling the natural convection with the imposed boundary conditions, is based on the finite volume method. This method has the advantage of satisfying the conservation of the momentum, the mass and the energy in all the considered volumes as well as in all the field of computation. The physical domain is meshed by the Gambit code. In these simulations, a structured quadrilateral mesh was adopted; this mesh was made with cells whose size varied gradually.
To ensure a good resolution in regions with a high temperature gradient, non-uniform structured mesh close to the walls' boundaries was considered. Spatial terms in the equations are discretized using weighted body force. The weighted volume force diagram implied this type of scheme is recommended for flows involving large volume forces.

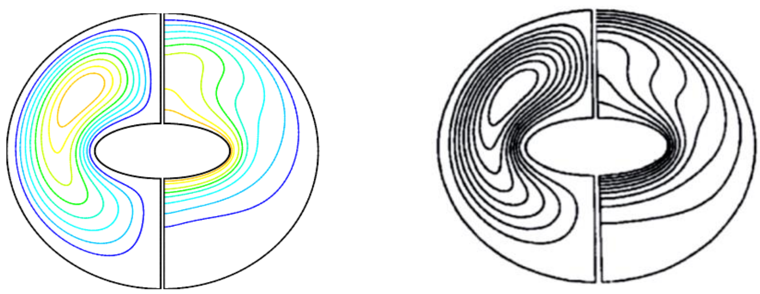
The second order scheme was considered since it allows some stability and minimizes the digital diffusion but it can make the calculation diverge. The simple algorithm of Patankar and Spalding [23] was used for speed-pressure coupling. In addition, the computational residue was used to ensure the convergence and the stability of the solution

4. Results and discussion

4.1 Validation of numerical results

We study the natural convection heat transfer for four values of the thermal Rayleigh number in the case of an annular space delimited by two cylinders, an elliptical outer cylinder and a trapezoidal inner cylinder. The results were presented as isotherms, streamlines, local Nusselt numbers as presented in figures 2 and 3.

Natural convection between confocal horizontal elliptical cylinders by Elshamy [24] was chosen for the validation of the present study. The validations were presented as isotherms and streamlines for two different Rayleigh numbers (Figure 2), and the local Nusselt number was compared to reference [24] for different numbers of Rayleigh ranging from 104 to 105 (Figure.3).For the case of two confocal horizontal elliptical cylinders, the internal and external eccentricities were taken 0.9 and 0.4, respectively, and the Rayleigh number was 104 (Figure.2).The local Nusselt numbers of the inner and outer elliptical cylinders considered in this paper and those of ElShamy are plotted in Figure 3.The results indicate an acceptable agreement with the results that presented by ElShamy et al [24]. In these cases, the results show that two symmetrical recirculation cells are formed on the right and the left. This fact is due to the buoyancy force produced by the temperature gradient.



(a)Present work (b) Elshamy[24]

Figure 2. Streamline (left half) and isotherms (right half) to $Ra_t = 10^4$.

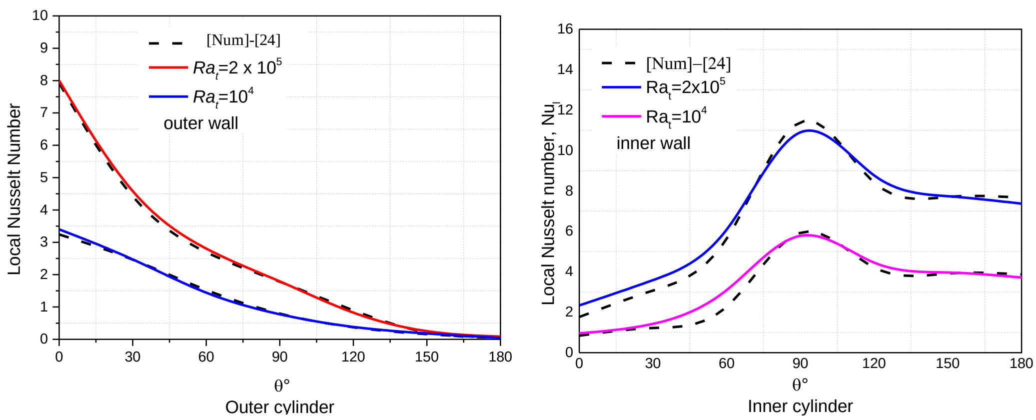


Figure 3. Local Nusselt number along inner and outer ellipses

4.2. Rayleigh effect

Figures 4, 5, and 6 show the isotherms and lines of current at different numbers of Rayleigh and tilt angle of the side wall of the inner cylinder. According to these results, it was observed that at $Ra_t = 10^3$, the isotherms and current lines are symmetrical with respect to the median fictitious vertical plane and change very slowly whatever the angle of inclination. The conduction heat transfer becomes the dominant transfer in all forms of internal cylinders. The current lines of the same figure show that the flow takes place in two main cells which rotate very slowly in opposite directions. The fluid particles move upward under the effect of Archimedes' thrust along the inner warm wall, and then descend under the action of gravity forces along the cold wall of the outer elliptical cylinder. For $Ra_t = 10^4$ and $Ra_t = 10^5$, the isothermal lines change and end up in the shape of a mushroom. The temperature distribution is decreasing from the hot wall to the cold wall. The direction of the deformation of the isotherms is in conformity with the direction of rotation of the current lines. In a laminar regime, it can be said that under the action of the movement of the particles that take off from the hot wall at the axis of symmetry, the isothermal lines move away from the wall at this point.

The values of the current functions increase, which means that the convective heat transfer begins to take place. With the increase of the inclination angle of the side walls of the inner cylinder, the results presented above allow us to notice that whatever the annular space used (that is to say, whatever the value of d angle of inclination of the side wall of the inner cylinder used) when the value of the Rayleigh number is increased, the heat transfer rate and the values of the maximum current function increase, on the one hand, on the other hand, the increase of the inclination angle of the side wall causes an increase in the area of the hot roll, and therefore an increase in the heat transfer coefficient, for the current lines, the decrease of the inclination angle of the side wall causes a widening of the annular space between the two rolls, and an increase in the values of the current function.

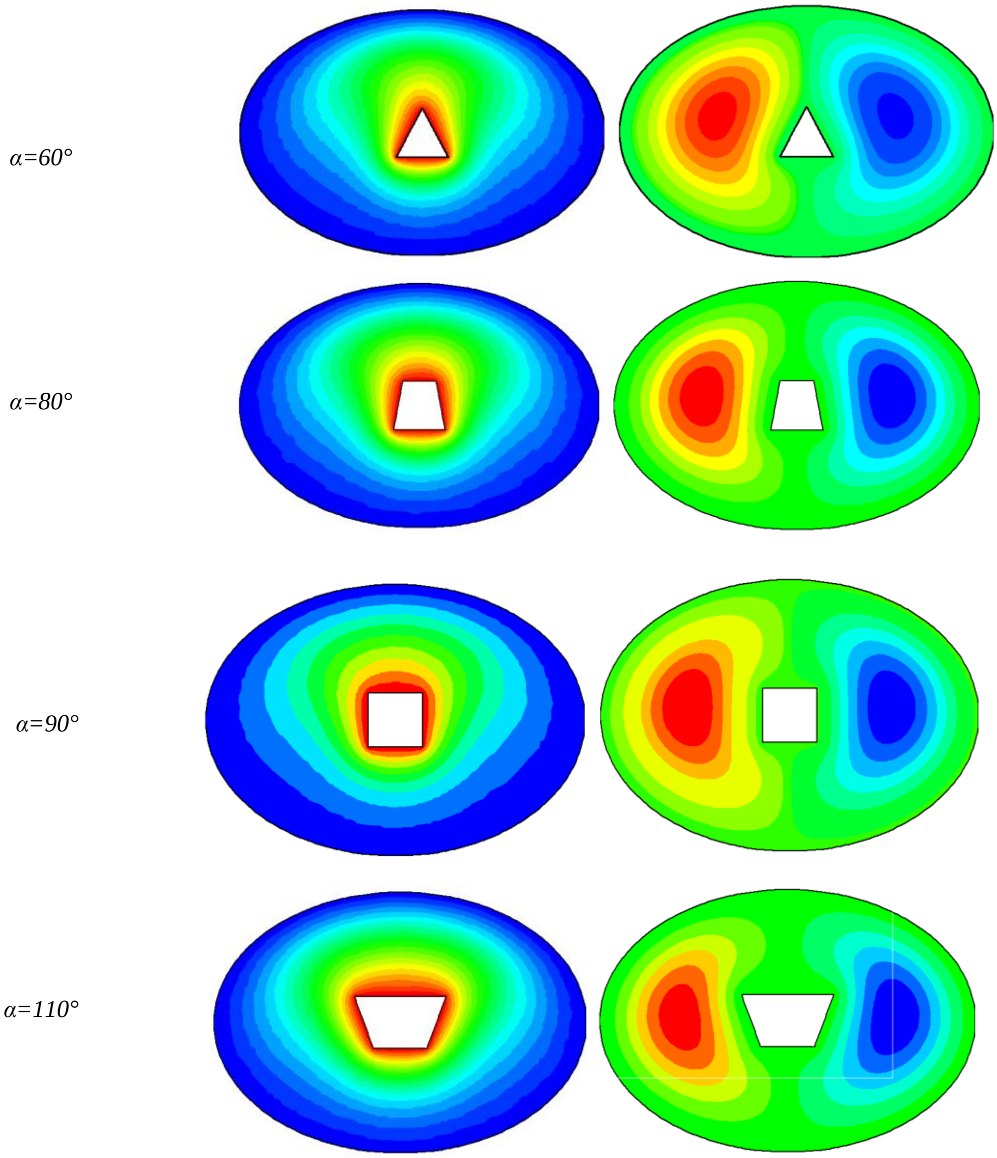


Figure 4. Isotherms and lines of currents for different angles to Ra_t=10^3**.**

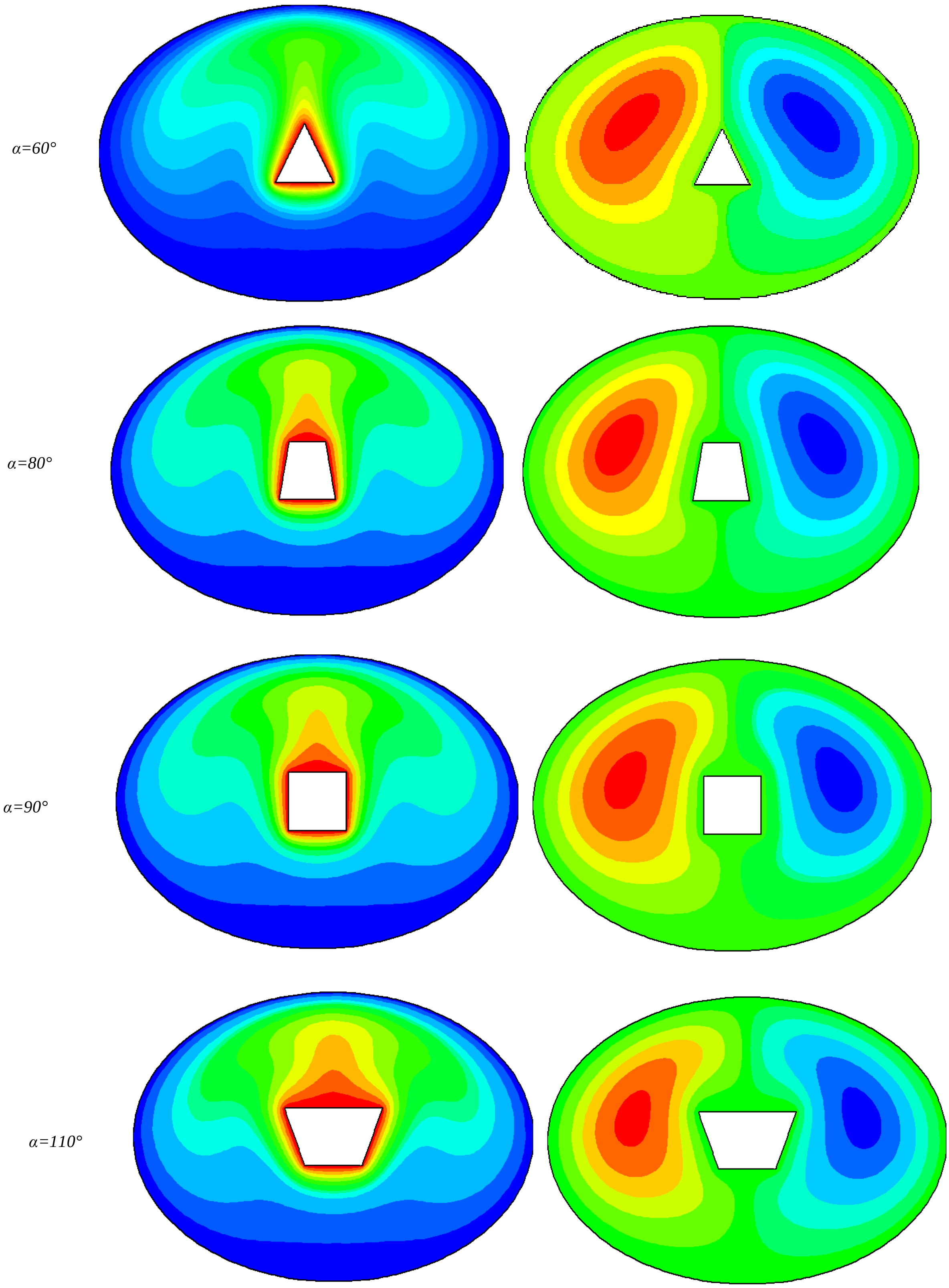


Figure 5. Isotherms and lines of currents for different angles to Ra_t=10^4

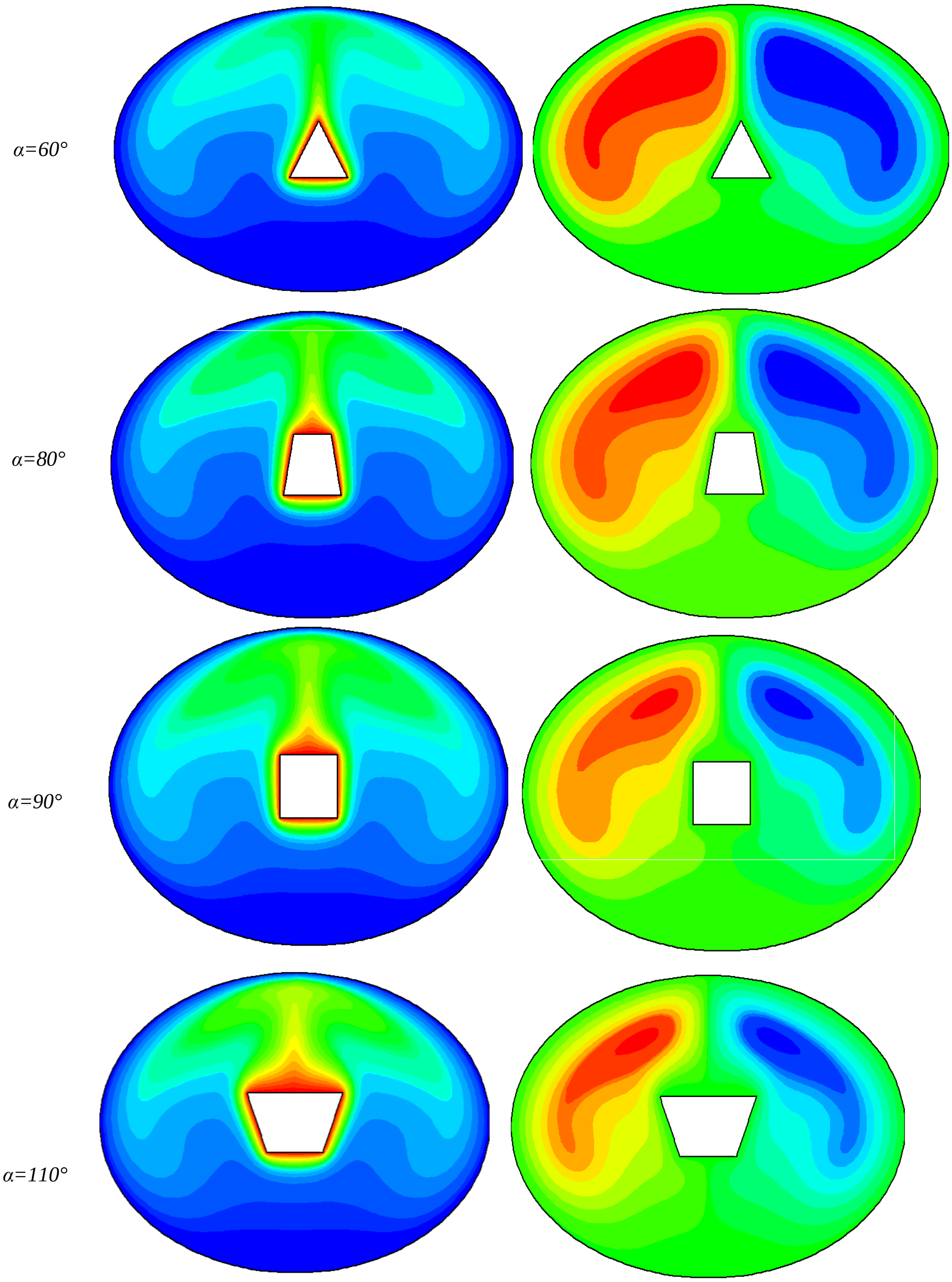
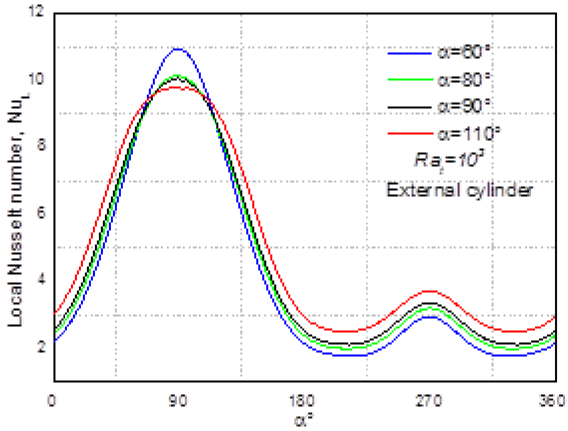


Figure 6. Isotherms and lines of currents for different angles to $Ra_t=10^5$

4.3. Variation of the local Nusselt number along the hot wall:

Figures 7, illustrates the inclination influence of the hot side wall on the variation of the Nusselt numbers of the inner cylinder, these results confirms that with the increase of the Rayleigh number, the values of the local number Nusselt increase, regardless of the shape of the inner cylinder. For the case of an inclination angle $\alpha = 60^o$ corresponding to an inner cylinder with a triangle shape, the variation of the local Nusselt number shows the existence of two maximum values corresponding to the two lower corners of the triangle. The minimum value appears near the top vertex of the triangle. For the case where, the inclination angle is other than 60^o, it has noted in this enclosure that the local Nusselt number is important in all the corners of the trapezium, indeed, it maximum values on the two upper corners presenting the maximum value of the rate of heat by comparison to the two bottom corners. On the rest of the trapeze, the local Nusselt number presents weak value. Also, it has been noted that when the angle of inclination α increases, the maximum value of the local Nusselt number increase too. In these conditions, the range of values corresponding to the minimum local Nusselt number also increases. This fact is due to the increase of the length of the upper side of the hot wall.



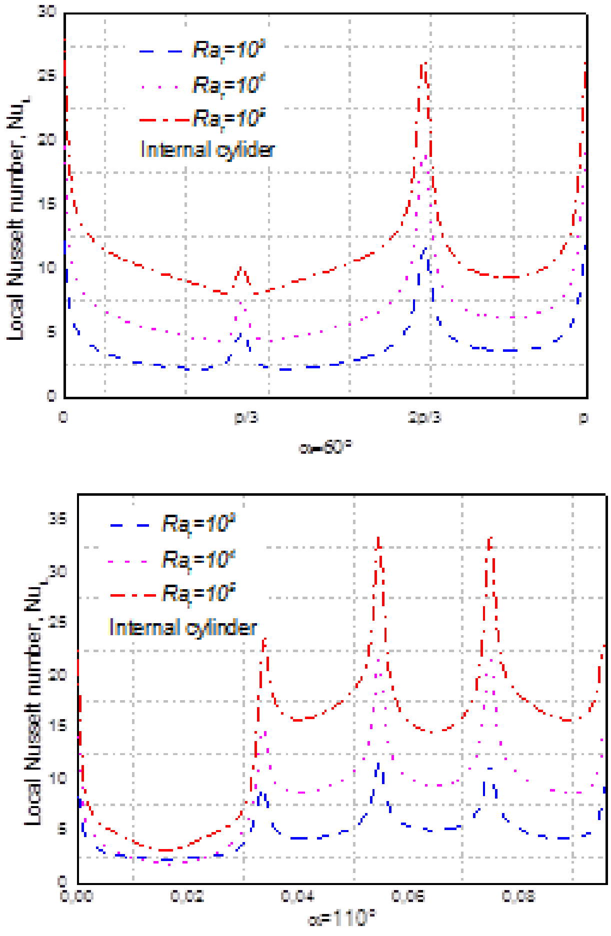


Figure 7. Variation of the number of local Nusselt number along the hot wall for different angles.

4.4. Variation of the local Nusselt number along the cold wall:

Fig 8 shows the variation of the local Nusselt number on the wall of the outer elliptical cylinder. According to these results, it is clear that the increase in the number of Rayleigh, and the value of the local Nusselt number increases. In addition, the variations along the walls are closely related to the isothermal and isocurrent distributions.

For a low Rayleigh number, the heat transfer within the annulus is essentially controlled by simultaneous thermal conduction processes for all inner cylinder shapes. However, for large values of the Rayleigh number, the value of the local Nusselt number is reached at the angular position θ = 90° by a maximum value, and for a minimum value in the lower part of the elliptical cylinder (θ = 270°).In this range of Rayleigh numbers, most of heat transfers are done by convection, which is in accordance with figure 6.

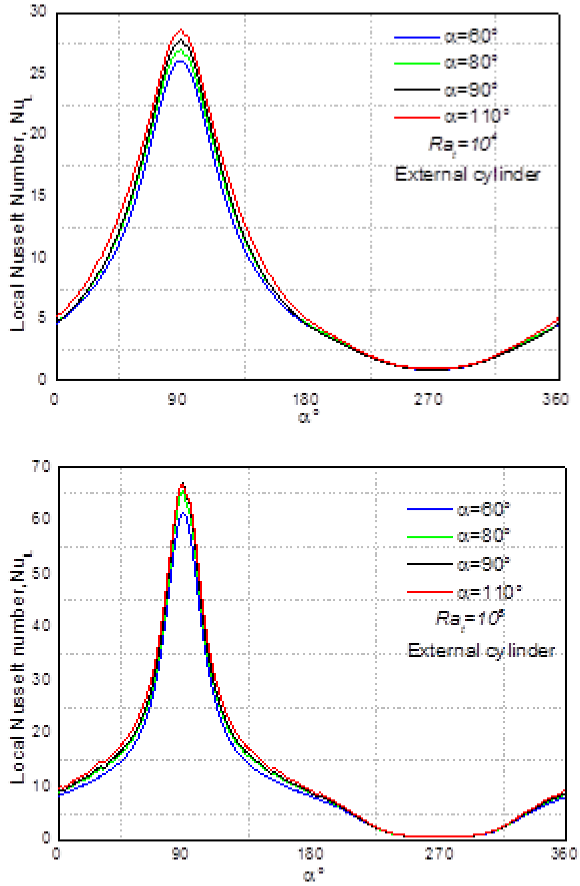


Figure 8. Variation of the local Nusselt number along the cold wallfor different angles.

4.5. Variation of vorticity

Figure 9 illustrates the variation of the vorticity as a function of the inclination angle of the lateral hot walls. The curve can be divided into four different zones. In the first zone, vorticity decreases sharply with an increasing inclination angle. In this area, the upper rib is shorter than the lower rib. In the second zone the value of the vorticity remained constant with the increase of the inclination angle. Therefore, the increase of the angle does not affect the value of the vorticity in this angle interval (70°- 80°) and find an optimal angle α = 75°. However, in the third zone the vorticity also decreases with the increase of the inclination angle. In the fourth zone, the vorticity decreases slowly with the increase of the trapezoid angle since the upper side is shorter than the lower side of the trapezium. Therefore, the increase in the inclination angle of the side wall causes a decrease in the annular space between the two cylinders and a decrease in the values of the vorticity.

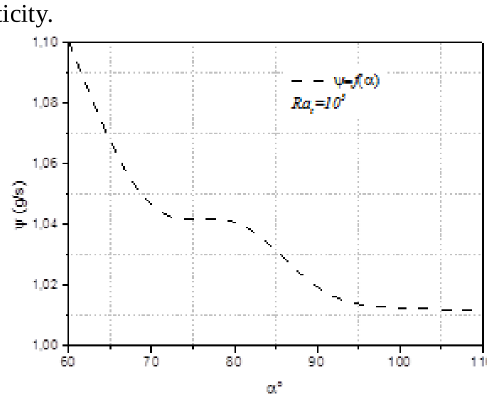


Figure 9. Variation of vorticity according to the angle of inclination of the lateral hot wall.

5. Conclusion

We have been able to highlight the fact of the study in a laminar regime inside a horizontal eccentric ring between a heated trapezoidal internal cylinder and a cold elliptical outer cylinder. Our investigation made it possible to highlight the effects of the inclination of the trapezoidal lateral walls on the intensity of the flow in the annular space and on the heat transfer through the lateral walls.

The results show that the conduction mode is predominant in heat transfer as long as the Rayleigh number is not high. However, with the increase in Rayleigh number, it has been noted a birth of a boundary layer that is becoming thinner. As the thermal Rayleigh number is increased, thermal plumes appear over the hot wall, isotherms writhe increasingly to the top of the enclosure while remaining stratified beneath the hot wall involving that the convection's mode dominates. With the increase of the inclination angle of the side walls of the inner cylinder, the obtained results above mentioned allow us to notice that whatever the annular space used (that is to say, whatever the value of angle of inclination of the side wall of the inner cylinder used) when the value of the Rayleigh number is increased, the heat transfer rate and the values of the maximum current function increase. On the other hand, the increase in the angle of inclination of the side wall causes an increase in the area of the hot cylinder, therefore an increase in the heat transfer coefficient, whereas for the current lines the decrease in the angle of inclination of the side wall, causes a widening of the annular space, and an increase of the values of the current function.

On the hot wall, we notice that when the angle of inclination α increases, the maximum value of the number of Nusselt increases. This fact is due to the increase in the length of the upper side of the hot wall. The local Nusselt number is minimal if the fluid moves away from the wall and, maximum, if the fluid is supplied to the wall. It is found that, large values of the Rayleigh number, the local Nusselt number reached a maximum at $\theta = 90°$ and minima at $\theta = 270°$.

These results will be used for the design of engineering and the process improvement of the heat exchangers, the drying processes, the cooling of electronic circuits and the cooling of nuclear reactors. In the future, we propose to study double-diffusive natural convection.

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