# Determination of Asphalt Binder VECD Parameters Using an Accelerated Testing Procedure

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**ABSTRACT:** Fatigue characteristics of asphalt binder have an important role in asphalt mix resistance against cracking. Viscoelastic Continuum Damage (VECD) analysis of asphalt binders has been successfully used in highway research works in order to predict fatigue behavior of hot mix asphalt (HMA). In this method an intrinsic property of the material, called damage function is obtained which is independent of damage path. However, achieving damage function needs application of various loading paths and a trial and error procedure. In this study, a quick characterization procedure has been proposed to implement VECD analysis that results in fatigue prediction of HMA. The procedure is comprised of a testing setup, along with the analysis required to derive VECD parameters from experimental data. The test consists of a stepwise loading scheme including a few strain levels with relatively large increments in between. Subsequently, an optimization method has been introduced to be performed on the test results, to yield damage function, i.e. modulus as a state function of Internal State Variable (ISV). The analytical framework leading to the optimization problem, along with its solution methods are presented. Consequently, the fatigue life prediction model has been obtained, relating the change in shear modulus to loading conditions such as strain level and frequency. Eventually, the introduced characterization method was validated, comparing the results with those achieved in conventional procedure. The validation showed that the results of optimization and conventional methods agree, with an acceptable precision.

**Keywords**: Asphalt Binder, Fatigue, Fatigue Accelerated Test, Viscoelastic Continuum Damage.

#### INTRODUCTION

Fatigue cracking, caused as a result of repeated traffic loading, is one of the major distress modes in HMA pavements. For many years, significant research efforts were conducted to develop fatigue prediction models, relating fatigue life to loading conditions, and material's undamaged properties (Wen and Li, 2012; Kavussi et al., 2016). Such relationships could be derived by generating regression models on data acquired from testing many samples under different loading conditions (Partl et al.,

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2012). These phenomenological models were simple to use and understand, although require extended time and expenses for the experiments (Cucalon et al., 2016).

With the application of mechanistic approach, performance of HMA could be characterized by testing fundamental properties of binders or mixtures (Kim, 2009; Norouzi and Kim, 2017; Taherkhani and Afroozi, 2017). Continuum Damage Mechanics (CDM) has been widely used to model distresses in asphalt binders and mixes. In this technique the sample is assumed to suffer from a generic "damage" that is not considered as cracks or any disintegration. Instead, it is an internal state variable (ISV) associated with the overall change of internal structure of the substance (Holzapfel, 2000; Darabi et al., 2012).

Among various VECD theories, those of Schapery's works are the most highlighted ones. Schapery developed a series of viscoelastic constitutive equations and damage models that were based on thermodynamics of irreversible processes (TIP) (Schapery, 1991a,b). Schapery used pseudo-strain concept to purge material's response dependency to loading history (Schapery, 1975, 1984). These led to introduction of the well-known damage evolution power-law in viscoelastic materials (Park et al., 1996). Damage was defined as a path-independent internal state variable accountable for any loss of modulus due to disintegrity. Thus the variation of the material stiffness due to changes in microstructure of the material (i.e. damage), namely, damage function, was shown to be an intrinsic property, independent of loading rate. Determination of the damage state function of a material will lead to the prediction of its fatigue life (Holzapfel, 2000; Kelly, 2019).

A variety of tests can be performed to acquire data required in VECD analysis, ranging from monotonic to cyclic and controlled strain to controlled stress tests

(Wang et al., 2017). Many researchers have worked in developing test procedures performed by ordinary DSR machines that decrease testing duration, while the data could still be adequate for VECD modeling. These efforts led to development of Linear Amplitude Sweep (LAS) Test (Johnson, 2010), which was then standardized in two revisions of AASHTO Standard (AASHTO. 2018). It was shown that this standard test is able to collect all the data required to develop VECD analysis, while the test duration is rather short (Johnson, 2010; Hintz et al., 2011; Hintz and Bahia, 2013). In this testing method, the exponent of the damage evolution equation (represented by  $\alpha$  in Eq. (1)) is estimated based on rheological properties of the sample (Park et al., 1996; Lee and Kim, 1998; Underwood, 2016).

However, some researchers recommended that the exact value the exponent should be determined with the condition that damage function would be identical under different loading patterns. Thus, this approach acquires multiple replications of test at different loading rates, in order to perform a trial and error procedure to find the exact value of the exponent (Little et al., 1998; Little and Lytton, 2002). The rheological-based value of the exponent in the former approach was suggested to be used as the initial estimate in this method (Lytton et al., 2001). Since the uniqueness of the damage state function is a key fundamental in TIP (Schapery, 1991a; Kelly, 2019), the latter approach was employed to determine the exponent value in this study.

Fatigue properties of SBS-modified binders have been evaluated in many studies. It is believed that asphalt modification, in most cases, convert simple binders to complex ones (Bahia et al., 2001; Behnood and Olek, 2017; Taherkhani, 2016). Unlike fatigue behavior of the simple binders that is characterized by measuring linear undamaged responses, characterization of fatigue in complex binders requires further testing that measure damage tolerance of the material (Kim, 2009). On that account, in different studies conducted on the use of VECD theory, the experimental investigations is performed on complex binders (Rooholamini et al., 2017). In this research SBS-modified binders are selected to perform experimental evaluation of the new characterization procedure.

Prior studies performed to evaluate SBS engagement in asphalt binders stated that this polymer can improve the strength and elasticity by linking the two-dimensional asphalt molecules to form three-dimensional grids (Isacsson and Lu, 1995; Ding et al., 2013). SBS, if used in effective amounts (3% to 7% by weight of bitumen), can swell to 9 times its initial volume by absorbing asphalt oil, resulting in significantly improved asphalt characteristics, at a temperature above the glass transition (Read and Whiteoak, 2003; Liang et al., 2015).

### VISCOELASTIC CONTINUUM DAMAGE MECHANICS

Extensive application of VECD follows Schapery's works on damage evolution theories using work potential theory and thermodynamics of irreversible processes (TIP). Using pseudo-strain concept, Schapery eliminated the dependency on loading history. This led to the introduction of the following damage evolution law (Park et al., 1996; Lee and Kim, 1998):

$$\dot{S} = \left(-\frac{\partial W^R}{\partial S}\right)^{\alpha} \tag{1}$$

where  $\dot{S} = \frac{dS}{dt}$  (internal state variation rate), S: is the internal state variable (damage),  $W^{R}$ : is pseudo-strain energy density,  $\alpha$ : is the exponent, determining energy dissipation rate during loading; and *t*: is time. In thermodynamics, damage (S) is usually defined as an independent property which represents the structural failures of the material (Holzapfel, 2000). The damage is usually chosen as a (internal) state variable, which means that the structural state of the system can completely be described by that, regardless of the path (i.e. loading condition) that the system has gone through (Kelly, 2019).

It is also important to note that the term "damage" in continuum damage mechanics is defined as any deleterious structural change in a system. Its definition and formulation are based on TIP which is general enough for continuum damage mechanics principles to be applicable not only to fatigue cracking, but also to any breakage of the bonds between material particles which leads to modulus loss. Such generality lets the accelerated testing procedures (e.g. LAS) to be analyzed in VECD to yield fatigue life prediction, even though the testing procedure does not precisely simulate fatigue phenomena (Park et al., 1996; Lytton et al., 2001).

Since the dependency of  $W^{R}$  on time (or loading cycle) is not clear before testing, an exact solution for *S* cannot be acquired. Hence, in a cyclic test, an approximate recursive form of Eq. (1) is proposed to calculate *S* in every cycle:

$$\Delta S \cong (-\Delta W^R)^{\left(\frac{\alpha}{1+\alpha}\right)} \times (\Delta t)^{\left(\frac{1}{1+\alpha}\right)}$$
(2)

The pseudo-strain energy density  $(W^{\mathbb{R}})$  can be determined based on loading conditions and sample geometry. This parameter, in a repetitive test, is the area of a cycle loop in stress-pseudo-strain curve. It can be shown that if the response data could be acquired at stress peaks in each cycle, pseudo-parameters can be replaced with real ones, submitting acceptable approximation (Schapery, 1991a; Lytton et al., 2001). Hence, for a DSR sample in a cyclic constant-strain test, Eq. (2) can be rewritten as:

$$S_{i} \cong S_{i-1} + \left(\pi . G_{0} . \gamma_{0}^{2} . (C_{i-1} - C_{i})\right)^{\left(\frac{\alpha}{1+\alpha}\right)} (t_{i} - t_{i-1})^{\left(\frac{1}{1+\alpha}\right)}$$
(3)

where is cycle *i*: the number,  $G_0$ : is the initial shear modulus (dynamic modulus norm at the first cycle),  $\gamma_0$ : is applied constant shear strain amplitude, C: is the relative modulus  $(G/G_0$  while G is the dynamic modulus norm).

Relative modulus (*C*) is a state function that depends on the chosen state variables. Consequently, relationship between the internal state variable (*S*) and relative modulus (*C*), namely "damage function C(S)", is unique for an asphalt binder and is independent of the loading pattern (Park et al., 1996; Kelly, 2019; Holzapfel, 2000). However, since the original damage function is governed by Eq. (1), it is dependent on the quantity of  $\alpha$ .

Quantifying  $\alpha$  has been the subject of some research works; some of which suggest correlations with rheological properties of the binders (Park et al., 1996: Underwood et al., 2012; Lee and Kim, 1998). However, a rigorous method to find  $\alpha$  can be perceived, considering the fact that damage function, being a thermodynamic state function, is independent of loading rate. Based on this fact, it is proposed to repeat the test procedure (e.g. Time Sweep test), applying different loading patterns, such as different strain levels.  $\alpha$  value that provides the identical trend can be determined as the accurate exponent (Lytton et al., 2001). This method, however, needs more testing replications.

As internal state parameter is calculated at each loading cycle, the damage function trend will be known. Using this trend, Eq. (1) can lead to a fatigue life prediction model. Any function that could provide the trend of C(S)can be used for substitution in Eq. (1). The two-term power model and the exponential model of Eqs. (4) and (5) were proposed in previous research works (Underwood et al., 2012; Foroutan Mirhosseini et al., 2017), while the elliptical model of Eq. (6) is suggested and evaluated in this research.

$$C(S) = c_0 - c_1 \cdot S^{c_2} \tag{4}$$

where  $c_0$ ,  $c_1$  and  $c_2$ : are regression parameters ( $c_0 = 1$ ,  $c_1 > 0$ , and  $0 < c_2 < 1$ ).

$$C(S) = e^{c_1 S^{c_2}}$$
(5)

where  $c_1$  and  $c_2$ : are regression parameters  $(c_1 < 0, \text{ and } c_2 > 0)$ .

$$C(S) = 1 - \frac{\sqrt{p^2 - (p - S)^2}}{p}$$
(6)

where *p*: is the shape parameter and the semi-major axis of the ellipse.

# **OBJECTIVES AND OUTLINE OF THE STUDY**

The goal of this research was to introduce an accelerated characterization method to predict fatigue behavior of asphalt binders. In this method, firstly, a testing procedure is performed that provides adequate data to run VECD analysis. Secondly, VECD analysis is implemented using an optimization method to yield parameters required to constitute a fatigue life prediction model. One of these parameters is the exponent of the damage evolution law ( $\alpha$ ) which is recommended to be determined based on the data acquired from different loading conditions and a trial and error procedure. The testing and the optimization procedure introduced within research can provide the data and implement the analysis required to achieve fatigue prediction model, as a substitute of multi replications of tests and trial and error procedure.

Post-SHRP research works postulated the need of damage tolerance characterization testing on complex asphalt binders (Bahia et al., 2001). Hence, the introduced method was evaluated using samples of neat and modified asphalt binders. Asphalt binders were modified using SBS polymer to convert a simple binder to a complex one. It should be noted that the aim of this research was not to characterize asphalt binders at different conditions. In fact, validation data provided here may be too little for such tasks. Instead, it is to develop a characterization method. Therefore, the validation process was performed on a limited variety of complex binders, and at two temperatures only.

The performance grades of asphalt binders, used to validate the method, are reported in Table 1. Twelve samples were prepared from each specified binder. Three different tests, namely two Time Sweep tests and the new testing method with incremental strain pattern were performed at two temperatures of 15 °C and 25 °C. The first Time Sweep testing consisted of applying 4% strain amplitude at 10 Hz frequency, while the second consisted of applying 2% strain at 5 Hz. All the tests were replicated to verify the repeatability of the results.

Table 1. Performance grades and notations of the							
asphalt binders tested							

Notation	Modification	Performance grade			
Neat1	-	PG 64-16			
S14	Neat1 + 4% SBS	PG 70-16			
S16	Neat1 + 6% SBS	PG 76-16			
Neat2	-	PG 58-16			
S24	Neat2 + 4% SBS	PG 64-22			
S26	Neat2 + 6% SBS	PG 70-22			

### ANALYTICAL DEVELOPMENT

# METHOD

In this chapter the new fatigue characterization method is developed, along with the required analytical framework. The following sections are presented as a background for the main procedure. At first the impact of loading amplitude variation on the shape of damage function is evaluated. Second, a study is conducted to find the best model to fit damage data. The main contribution of the research is then presented next, in which the chosen model is used in an optimization procedure to yield VECD parameters. Subsequently, based on the optimization result the fatigue prediction model is developed.

## Damage Function under Stepwise Loading Pattern

The modulus state function C(S) (also known as damage function) is determined obtaining the values of relative modulus (C) and ISV (S) in each cycle during the test. The former can be measured directly while the latter is calculated using Eq. (3). However, the parameter  $\alpha$  is required to be determined for the calculation of ISV. The precise method of quantifying  $\alpha$  is to repeat the test at different loading patterns and find the value for  $\alpha$  that can generate identical C(S). However. a procedure is time such consuming and requires more testing replicates and; which in turn, contradicts the main goal of establishing an accelerated fatigue characterization test.

In the first version of LAS standard, the strain amplitude was incremented using a stepwise pattern; in which at a constant strain level, the sample is loaded for 10 seconds at 10 Hz frequency (AASHTO, 2018). The proposed testing method of this research is similar, only to have fewer but greater strain increments to provide higher precision. Such a procedure provides several Time Sweep instances with different strain amplitudes which can lead to precise determination of  $\alpha$ . This can be done based on the fact that: "applying a genuine value of  $\alpha$  would develop a smooth curve of damage function, while, using an improper value to calculate ISV will result in a rippling curve, due to the sudden slope changes, caused by the increments of strain".

In order to prove the hypothesis stated

above, the damage function C(S) is assumed to follow any form of Eqs. (4-6). As the fundamental rule of state variables in thermodynamics, the state function of C(S) is independent of loading pattern (Schapery, 1991; Kelly, 2019). Therefore, if the value of exponent  $\alpha$  is correctly chosen, the model parameters ( $c_1$ ,  $c_2$  and p) will be the same at any strain level. Contrariwise, if ISV is calculated applying an improper  $\alpha$ , C(S) trend varies according to the strain amplitude. Now, if the strain follows a stepwise pattern during the test, the C(S) curve stays continuous anyhow, because ISV is obtained through a recursive calculation (Eq. (3)).

The case is different for C(S) slope; it can be expressed in an explicit form (nonrecursive) and may be discontinuous. The differential ratios of *C* with regard to *S* (slope of C(S)) are as presented in Eqs. (7-9). As it can be seen for all the three equations, slope of C(S) is dependent to model parameters ( $c_1$ ,  $c_2$  and p). Thus, if an improper value of  $\alpha$  is used to calculate ISV during a stepwise strain test, the model parameters will vary at different strain amplitudes and the slope will be discontinuous. Mathematically stated, C(S) curve will be continuous of order 0, but not of order 1 and, accordingly, it lacks smoothness.

$$\frac{\partial C}{\partial S} = -c_1 c_2 . S^{c_2 - 1} \tag{7}$$

$$\frac{\partial C}{\partial S} = c_1 c_2 \cdot S^{c_2 - 1} e^{c_1 S^{c_2}} \tag{8}$$

$$\frac{\partial C}{\partial S} = \frac{S - p}{p \cdot \sqrt{p^2 - (p - S)^2}} \tag{9}$$

Figure 1 shows a schematic of damage function C(S), applying different  $\alpha$  values under the first version of LAS test. This illustration is concentrated on a portion of three strain levels with the least random errors, in order to demonstrate the effect of  $\alpha$  variation on the trend of damage function. Two Time Sweep tests were performed on the sample at two different strains in advance,

and after several tries and errors, the best  $\alpha$  value which resulted in similar damage functions for both tests was obtained. This  $\alpha$  value was used to generate the damage data of the left-hand curve in Figure 1, which can be seen to have a smooth form, while the data of the right-hand curve has been obtained using rheological correlations. This figure is an example of how an improper value of  $\alpha$  leads to a non-uniform trend such as that of the right side curve in Figure 1.

Considering the fact that a genuine value of  $\alpha$  would develop a smooth curve of damage function in a test with stepwise strain pattern, the new procedure can be introduced: For each value of  $\alpha$ , a regression model can be fitted to the corresponding C(S) data. If the value of  $\alpha$  is chosen correctly, the curve will be smooth and the model conforms to data after being fitted. Otherwise, an improper  $\alpha$ will result in a rippling curve and a poor regression fit. Based on this, an optimization procedure can be performed in order to determine the best value of  $\alpha$ . Before that, a regression model that provides enough proximity to data must be chosen.

#### The Best Model to Fit Damage Function

If the existence of ripplings in the case of improper  $\alpha$  is supposed to affect the goodness of fit, the chosen model is required to completely conform the trend of data points when the proper  $\alpha$  is applied. In other words, the estimator must produce the least residuals compared to the observations. Therefore, the two-term power model, the exponential model and the elliptical model of Eq. (4) to Eq. (6) were considered, and a study of the best model to fit the damage function was conducted. It should be noted that the high accuracy of the curve fitting is required during the optimization procedure only. Besides, for different materials, the damage function may follow different trends, and thus the fittest model might not be always the same. Regarding the simplicity of the power model (Eq. (4)) in terms of differentiation and integration, it can be used to develop fatigue prediction equations after the best value of  $\alpha$  is found.

In order to compare the mentioned models, a proper value for  $\alpha$  should be first determined for all binder samples. This is achieved using two Time Sweep tests with strain amplitudes of 2% and 4%. Then, the value of  $\alpha$  was adjusted (resulting in the change in ISV values), through a trial and error procedure, until all the data from four data sets (two Time Sweeps with replications) fall on the same curve. The obtained values of  $\alpha$  will be presented in method validation process (Table 3). The regression analysis was then performed on damage function data calculated using the proper  $\alpha$ . All the data from four data sets (Time Sweeps and replications) were used for each regression analysis. A summary of the regressions' "goodness of fit", for six binder types at two temperatures, is reported in Table 2.

Comparing the goodness of fit criteria in Table 2 demonstrates that the elliptical model, presented in Eq. (6), had the least discrepancy between the data points and the model. Values of *SSE* and *RMSE* are the criteria for the difference between the observed values and those predicted by estimator. Figure 2 illustrates an example of the general shape of the above mentioned three models fitted to damage data for a binder sample.



Fig. 1. The schematic effect of parameter  $\alpha$  on the trend of damage function

Binder	Temperature	Power model			Exponential model			Elliptical model					
sample	(°C)	<b>R</b> <sup>2</sup>	$R^2$	SSE	RMSE	$R^2$	$R^2$	SSE	RMSE	$R^2$	$\overline{R^2}$	SSE	RMSE
Neat1	15	0.9829	0.9829	0.5335	0.0416	0.9969	0.9969	0.0971	0.0178	0.9987	0.9987	0.0396	0.0113
	25	0.9887	0.9876	0.3932	0.0357	0.9974	0.9973	0.0843	0.0165	0.9988	0.9988	0.0392	0.0113
S14	15	0.9911	0.9910	0.2783	0.0301	0.9976	0.9976	0.0758	0.0157	0.9990	0.9990	0.0322	0.102
	25	0.9842	0.9842	0.4991	0.0403	0.9971	0.9971	0.914	0.0172	0.9987	0.9987	0.0413	0.0116
S16	15	0.9912	0.9912	0.2752	0.0299	0.9975	0.9975	0.0781	0.0159	0.9990	0.9990	0.0313	0.0101
	25	0.9875	0.9875	0.3965	0.359	0.9971	0.9971	0.0928	0.0174	0.9989	0.9989	0.0354	0.0107
Neat2	15	0.9835	0.9834	0.5161	0.0409	0.9962	0.9962	0.1189	0.0196	0.9992	0.9992	0.0255	0.0091
	25	0.9847	0.9847	0.4829	0.0396	0.9964	0.9964	0.1124	0.0191	0.9991	0.9991	0.0274	0.0094
S24	15	0.9881	0.9880	0.3787	0.0351	0.9965	0.9965	0.1103	0.0189	0.9992	0.9992	0.0243	0.0089
	25	0.9882	0.9881	0.3772	0.0350	0.9968	0.9968	0.1017	0.0182	0.9991	0.9991	0.0272	0.0094
S26	15	0.9918	0.9917	0.2568	0.0289	0.9970	0.9970	0.0935	0.0174	0.9993	0.9993	0.0206	0.0082
	25	0.9914	0.9914	0.2681	0.0295	0.9972	0.9972	0.0877	0.0169	0.9992	0.9992	0.0234	0.0087

 $R^2$  = coefficient of determination,  $\overline{R^2}$  = adjusted  $R^2$ , SSE = sum of squared residuals, and RMSE = root mean squared error.

During fitting analysis of the elliptical model, the maximum damage, endured by the sample, can be used as an initial value. However, complexity of this model, hinders its substitution in Eq. (1) for further calculations, and as a result, an explicit fatigue prediction model cannot be achieved. Hence, after  $\alpha$  is determined, the power model will be used for further calculations.

# New Loading Scheme and Optimization Procedure

Results showed that in order to have the best precision and more clear angularity between the curves of different strain amplitude (Figure 1), a strain pattern, having fewer but larger increments, will be more effective. Figure 3 illustrates the proposed strain pattern, which includes three levels of 1%, 5% and 10%, each lasting 30 seconds (after a 10 second pre-load at 0.1% strain level).



S24 at 15 °C)



Fig. 3. Strain pattern of loadings at frequency of 10 (Hz)

Finding the optimum  $\alpha$ , which results in the best fit of Eq. (6) to damage data, is a single variable optimization problem. The single variable is parameter  $\alpha$ , and the goal (objective) function can be any "goodness of fit" parameter (e.g.  $R^2$  or SSE). The objective function should either be maximized (for  $R^2$ ) or minimized (for SSE). This problem can be solved using heuristic approaches, while due to lack of an achievable closed form of objective function, cannot be solved using classic methods. The estimation of  $\alpha$ , based on rheological correlations (inverse of the slope of master curve for strain-controlled loading) can be used as an initial guess here.

To be more specific, initially a function can be defined which takes  $\alpha$  as the input and after fitting the elliptical model to damage data, gives *SSE* as the output. This function can be given to a heuristic optimization method (e.g. Genetic Algorithm) to find the best  $\alpha$  which results in the least *SSE* value (best fit).

#### **Fatigue Prediction Model**

Knowing the relationship between material modulus (C) and internal state variable (S), the crack evolution law (Eq. (1)) can be solved to obtain an equation relating binder modulus to the loading conditions (e.g. stress or strain amplitude, cycle number, loading frequency and rest duration). Eq. (4),

relating C to S, is a proper candidate (due to its simplicity and ease of derivation and integration) to be substituted in crack evolution law.

In order to present a prediction model, modulus is formulated as a function of loading conditions. For a constant strain cyclic loading, the prediction model can be stated as a function of number of cycles, strain, and frequency. Considering the geometry of the samples and the adopted loading mode, the following can be derived from Eq. (1):

$$\left(-\frac{\partial S}{\partial C}\right)^{\alpha+1} \times (-dC) = (\pi G_0 \gamma^2)^{\alpha} \times dt \quad ^{(10)}$$

Differentiating Eq. (4) and substituting it in Eq. (10), prediction model of material modulus, based on constant strain level and other loading conditions, are presented in Eq. (11-a).

$$G = G_0 - G_0 \times c_1 \\ \times \left( \frac{(1 + \alpha (1 - c_2)) \times (\pi G_0 c_1 c_2)^{\alpha}}{f} \times \gamma_0^{2\alpha} \right)^{\frac{c_2}{1 + \alpha (1 - c_2)}}$$
(11-a)

$$N_{f} = \frac{f \times (\frac{1-C_{f}}{c_{1}})^{\frac{1+\alpha(1-c_{2})}{c_{2}}}}{(1+\alpha(1-c_{2})) \times (\pi G_{0}c_{1}c_{2})^{\alpha}} \times \gamma_{0}^{-2\alpha}$$
(11-b)

c

where *G*: is material modulus at the end of loading,  $G_0$ : is the material initial modulus,  $\gamma_0$ : is the constant strain,  $C_f$ : is the relative modulus at failure (failure criteria), *N*: is the number of cycles of loading,  $N_f$ : is the number of cycles to failure (fatigue life), and *f*: is the loading frequency.

Eq. (11-b) estimates the number of loading cycles required to reduce the sample modulus to failure criteria. Eqs. (11-a) and (11-b) predict fatigue of a sample, loaded under a constant applied strain amplitude. Applying other loading patterns, Eq. (1) can be reintegrated to develop the respective prediction model.

A summary of the main procedure, developed in this study to achieve fatigue prediction model is illustrated in Figure 4.

#### VALIDATION OF THE METHOD

this inventive In research some characterization methods of VECD parameters were introduced that required to validated. be Validation process was performed in two steps. First the value of  $\alpha$ obtained from optimization method was evaluated, and then general validity of the new method was tested based on the fatigue lives prediction.

Initially, the optimization was performed for all binder samples, using Genetic Algorithm, applying the strain pattern of Figure 3 and Elliptical model (Eq. (6)). Table 3 represents a comparison of  $\alpha$  values, obtained from the optimization method (values are average of the two replications) and the manual (trial and error) procedure carried out on Time Sweep tests (performed simultaneously on two Time Sweeps and their replications).



Fig. 4. Flowchart diagram of the new procedure to obtain fatigue prediction model

Binder sample	Temperature (°C)	α Obtained from optimization	$\alpha$ Obtained manually	Deviation (%)
Na at 1	15	1.3302	1.31	1.54
Ineat1	25	1.3782	1.42	2.94
C14	15	1.3614	1.40	2.76
514	25	1.4273	1.43	0.19
Q16	15	1.3910	1.35	3.04
510	25	1.4485	1.45	0.10
Noot?	15	1.3827	1.41	1.94
Neat2	25	1.4119	1.41	0.13
524	15	1.4315	1.45	1.28
524	25	1.4705	1.47	0.03
526	15	1.5509	1.545	0.38
520	25	1.5745	1.56	0.93

**Table 3.**  $\alpha$  values, obtained from the optimization and trial and error method

The differently obtained  $\alpha$  values showed good conformity, which means that the optimization method can simulate the trial and error procedure to a large extent. The small discrepancies are mostly due to the manual trials and errors. The damage function curves of four samples (two Time Sweeps with replications) were tried to be adapted manually by trial and error of different  $\alpha$ values, which cannot be as precise as the optimization procedure.

The correlation between optimized and manual  $\alpha$  values are also determined, and the values along with their trend line are illustrated in Figure 5. The correlation can be seen to have a high coefficient of

determination ( $R^2 = 89\%$ ).

In order to evaluate the overall validity of the method, fatigue lives of pilot sweep tests, with two applied loading amplitude were calculated at two temperatures. Failure criterion was considered as 60% loss of modulus, and the number of cycles to the failure was determined. Results of the validation tests are presented in Figure 6. The values included are the fatigue lives ( $N_f$ ) observed in validation tests (average of two replications), along with the predicted values. These latter ones were estimated by the prediction model of Eq. (11-b), where the values of  $\alpha$  were determined from performing the optimization method.



Fig. 5. Correlation between differently obtained  $\alpha$  values



Fig. 6. Nf acquired from validation tests (observed) and the developed method (predicted)

A statistical analysis is performed to evaluate the correlation between the predicted and observed fatigue lives. Results show that in most cases the deviation of prediction is less than 30%. Coefficient of determination ( $R^2 = 95\%$ ) also indicates that the proposed characterization method was able to simulate material deterioration properties to a great extent. It should be noted that the characterization method, developed in this study was founded on VECD theory assumptions (specifically Eq. (1)) and a portion of the 30% error in fatigue prediction is certainly due to the limitations of VECD.

## CONCLUSIONS

In this research, an accelerated procedure was proposed to characterize fatigue properties of asphalt binders. Performing this testing and analytical procedure, Viscoelastic Continuum Damage (VECD) analysis could be implemented, applying considerably less efforts. The main research achievements are listed below:

1. A quick procedure was presented to determine VECD parameters, including a new testing setup, along with the analysis required to be performed on experimental data.

2. The new testing method consisted of applying a sequence of different loading amplitudes (strains of 1%, 5% and 10%) in a staircase scheme which results in a rippled damage curve, if the ISV is calculated with an improper  $\alpha$  value.

3. A regression analysis was conducted to find the model that best fitted the damage function data. The elliptical model was selected as the model with the highest conformity.

4. The elliptical model was applied in an optimization analysis which determines the exponent of the damage evolution law (i.e.  $\alpha$ ) and other VECD characterizing factors.

5. The optimized VECD parameters were used to develop prediction models that estimate modulus variations of a sample subjected to different loading patterns.

6. Eventually, the efficiency of the method was evaluated for  $\alpha$  values and overall fatigue lives, performing validation Time Sweep fatigue tests on samples which were already characterized applying the new procedure. The optimized values of  $\alpha$  complied with the manually obtained values ( $R^2 = 89\%$ ). The predicted values of fatigue lives ( $N_f$ ), also showed promising conformity with the

results of the validation tests. The logarithmic correlation between estimated and observed values resulted in a high coefficient of determination ( $R^2 = 95\%$ ).

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