Iranian Journal of Management Studies (IJMS) *http://ijms.ut.ac.ir/*
Vol. 13, No. 2, Spring 2020 *Print ISSN: 2008-7055*
pp. 221-244 *Online ISSN: 2345-3745*
Document Type: Research Paper *DOI: 10.22059/ijms.2019.275192.673500*

An Option-Revenue Sharing Coordination Contract with Price and Sales Effort Dependent Demand

Masoud Rabbani[1*], Neda Manavizadeh[2], Hamed Vafa-Arani[1], Soroush Aghamohamadi-Bosjin[1]
1. Faculty of Industrial Engineering,University of Tehran, Tehran, Iran
2. Faculty of Industrial Engineering, Khatam University, Tehran, Iran

(Received: May 7, 2019; Revised: November 26, 2019; Accepted: December 11, 2019)

Abstract
This study proposes a novel option-revenue sharing coordination contract framework. In the proposed model, the retailer determines the number of order sales effort. The manufacturer sets the price of products for the wholesale strategy. The investigated supply chain problem analyzes the results of different strategies. In the proposed coordination contract problem, two types of games including retailer-based game and manufacturer-based Stackelberg game are considered. In both cases the retailer adopts the value of order and the sales effort and the manufacturer determines the wholesale price. To assess the performance of the proposed contract, a wholesale and a basic selection contract are considered in the model. To obtain the Nash equilibrium in the retailer-based state of the proposed option-revenue sharing coordination contract problem, a hybrid algorithm consisting of a heuristic and a genetic algorithm is proposed by considering the computational complexities of the proposed model. A numerical comparison between the proposed contract and other cases demonstrates that the option-revenue sharing contract significantly dominates the basic option and the wholesale price contract. Finally, we implemented some numerical experiments on the critical parameters of the contract. Based on the results, increasing the price-dependency of demand results in less number of products ordered by the retailer.

Keywords
Supply chain management; Coordination; Option-revenue sharing contract; Price-sales effort dependent demand.

---

* **Corresponding Author, Email:** mrabani@ut.ac.ir

Introduction

In recent years, the application of option contracts has gained the attention of researchers for the supply chain problem. Option contract as an agreement among the parts of supply chain provides a transaction for them. The unit option price and unit allowance price are the basic parameters of the option contract. Unit option price is a type of grant given by the retailer to the manufacturer for keeping one unit of the manufacture capacity. The unit exercise price is the payment by the retailer to the manufacturer for exercising one unit of the option (Basu, Liu, & Stallaert, 2019; Wan & Chen, 2019). Most of companies in retailer-manufacturer supply chains (SCs) interact through a wholesale contract. In a wholesale contract, partners make their decisions independently. The wholesale contract policy sometimes has resulted in conflict among goals of manufacturers and retailers (Zhao, Wang, Cheng, Yang, & Huang, 2010). The wholesale mechanism reduces the benefit of the SC compared to an ideal cooperation situation. In addition, both sides have their own specific goals leading to higher prices and lower demands compared to integrated SCs which is called double marginalization effect Corbett, Zhou, & Tang, 2004). The main goal of supply contract is to provide coordination among the parts of SC system. Because all the basic contracts have some drawbacks, a mixed mechanism can reduce the drawbacks efficiently. Selling the products to the retailer with lower price and getting a portion of profit from retailer are the main structures of revenue sharing contract. This mechanism has a wide application in some industries such as video rental industry (see, e.g. Van der Veen & Venugopal, 2005). Motivated by these evidences from the real world, we have proposed an integrated option-revenue sharing contract in this paper.

In this paper, the hybrid option-revenue sharing contract is formulated in a Stackelberg game framework to address strategies of both members of the SC. The retailer adopts the order quantity of options and the related sales effort. In the hybrid option-revenue sharing contract, the retailer orders a number of options. In the next step, the manufacturer produces according to the number of order. During the marketing period, the retailer analyzes the bought options based on the demand of customers. The manufacturer charges a lower exercise price by taking into consideration the income allocation mechanism. At the termination of the marketing period, the manufacturer sells the unexercised options in a salvage value. The uncertainty of the market demand raises the importance of the coordination contracts. However, the market demand depends on the market price (Chen & Bell, 2011) . The current study investigates the basic questions including:

- The effect of option price on outputs of the option-revenue sharing contract.
- The effect of revenue sharing fraction on outputs of the option-revenue sharing contract.
- The effect of price-dependency of demand on outputs of the option-revenue sharing contract.

The main contributions of this study are stated as follows:

- Proposing a novel option-revenue sharing coordination contract framework
- Considering various decision variables in the structure of the model including number of ordered items, number of produced items and price of items under different scenarios
- Considering retailer-based and manufacturer-based Stackelberg game for the coordination contract problem
- Applying a heuristic-based method to solve the problem

The rest of the paper is organized as follows: section 2 reviews the related studies; sections 3 describes the proposed model of the problem; section 4 reports the results of the study and, section 5 describes the main conclusions of the study.

Literature Review

1. Option Coordination Contract in Supply Chain

A number of previous studies have investigated the option contract as a coordination mechanism in a two-echelon SC, online pricing systems and competitive SCs (Zhang, Zhao, Cheng, & Hua, 2019; Johari & Hosseini-Motlagh, 2019b). Jiao, Du, and Jiao (2007) considered some uncertain variables in a flexible production system with option contract. Gomez_Padilla and Mishina (2009) addressed the option contract in order to integrate the SC system for the single provider-retailer and multiple providers-retailer. Outcomes showed an enhancement in income of the both provider and retailer. Rabbani, Arani, and Rafiei (2015) and Wang, Li, Liang, Huang, and Ashley (2015) studied the application of the option contract in the relief SC. Rabbani et al. (2015) studied a relief SC problem in which an option mechanism was designed for the supply relief products. They incorporated the intensity of the disaster into the number of the options that were exercised to supply the sufficient products for the victims of the disaster. According to Wang et al. (2015), the option contract is able to reach better results than the pre-buying along with buyback and prompt buying with return in a SC. Hua, Liu, Cheng, and Zhai (2019) studied a two-echelon SC considering option contract and applied Stackelberg game to analyze financial problems. Zhao, Choi, Cheng, and Wang (2018) developed a SC with option contract and designed a mechanism for SC to coordinate with information learning. They examined SC competition under different contracts. Song and Gao (2018) proposed a green SC regarding revenue sharing contract and demonstrated that this type of contract improves the performance of a green SC.

Cachon and Lariviere (2005) proposed a study in the filed of revenue sharing and enumerated the profits of the total income reached by the contract. Xu, Dan, Zhang, and Liu (2014) introduced a novel study for the revenue sharing contract of a SC system. In a recent study, Zhang, Liu, Zhang, and Bai (2015) addressed a revenue sharing contract that was used to synchronize the deteriorating items of a SC. Xu (2010) developed an option contract problem considering wholesale policy and assumed the income of supplier and demand as random variables. Zhao et al. (2010) proposed a manufacturer-retailer SC model in which the wholesale price strategy was conducted to set the number of orders. In this regard, Chen and Shen (2012) proposed another study in which the main variables of model included determining the number of ordered and produced items. In addition, some studies have considered other types of contract in the SC. Babich (2006) considered the vulnerability in option contract and assumed uncertain parameters in the model. Li, Ritchken, and Wang (2009) developed an integrated forward option contract for an uncertain SC system. Liu, Chen, Li, and Zhai (2014) compared the effect of option contract and discount policy in SC. Ye, Li, and Yang (2018) analyzed three types of contract for biofuel SC to select the perfect coordination. Nemati, Madhoushi, and Safaei Ghadikolaei (2017) assessed the importance of sales in an integrated SC and applied fuzzy mathematical programming to deal with the uncertainty. Fahimi, Seyedhosseini, and Makui (2017) developed a decentralized competitive SC and set the wholesale policy in a non-cooperative way. Fahimi, Seyedhosseini, and Makui (2018) introduced a competitive SC problem. They determined the price of items according to the dynamic games at the first step and determined the locations of their retailers in simultaneous games. Noh, Kim, and Sarkar (2019) proposed a two-layer SC problem with three scenarios. In the first scenario the retailor was the leader, in the second one manufacturer was the leader and the last scenario regarded a central SC. They determined the value of the lot sizing number, shortage level and price of items under each scenario. Hosseini-Motlagh, Nematollahi, Johari, and Choi (2019) considered a reverse SC and aimed to achieve channel coordination when the collectors participate in the purchase prices presented to the customers and the remanufacturer making decision about the price and environmental decisions. Johari and Hosseini-Motlagh (2019) developed a coordination model for a forward/reverse SC covering the sustainability aspects if SC. Hosseini-Motlagh, Nouri-Harzvili, Choi, and Ebrahimi (2019) assessed the effect of demand disruption in a reverse SC by including a combined two-part-tariff contract in the system. Heydari and Asl-Najafi (2018) developed a SC coordination contract model in which the demand was dependent on the sales effort.

2. Research Gaps

Generally, there are some drawbacks in the literature of option contracts. Firstly, the basic option mechanism cannot reduce the double marginalization effect, which causes inefficiency in attracting customers. The revenue sharing is a complementary mechanism and the current study uses this mechanism, intelligently, to reduce this effect. Consequently, the resulted mixed contract could create a larger market (Arani, Rabbani, & Rafiei, 2016). Secondly, the previous studies often considered the demand independent of the final price, which is an unrealistic assumption, from microeconomics point of view. This paper uses a demand function that depends not only on the market price but also on the sales effort of the retailer. In other words, the demand function is decreasing in the selling price and increasing in the sales effort. It should be noted that some articles have considered demand as price-dependent parameter (Hu, Qu, & Meng, 2018; Shen, Bao, & Yu, 2018; Johari, Hosseini-Motlagh, & Nematollahi 2017). However, the current study considers demand as a price and sales effort dependent demand in a more complex system. Thirdly, the previous studies in the option mechanism limited the decision variables to the order and the production quantities. Fourthly, most of the previous studies modeled a retailer-led SC, while the current paper investigates the manufacturer-led SC as well. In this regard, analyzing the effect of manufacture-led system in Stackelberg games is essential to set the value of order, the sales effort and whole sale price. Fifthly, because of the high computational complexity of the proposed model in case of the retailer-based SC, this paper proposes a heuristic algorithm, for the first time in this context. Finally, the current research examines the performance of the mixed contract through several comparisons.

Problem description

This study introduces a mixed contract which is comprised of a European option as a type of options contract limiting execution data. In other words, the calling action will only take place on the date of option (Arani et al., 2016) and income allocation process as components to integrate the retailer-manufacturer SC. In this regard, the model is developed based on the Stackelberg game. In addition, other situations including basic option contract, wholesale mechanism, centralized SC and option revenue sharing contract are embedded in the model to evaluate the performance of proposed model. In this regard, the model presents different contract strategies in the structure of SC. The related models are in the next sub-sections, assessing the output and behavior of the model. In this model, the retailer, firstly, orders a number of options. Secondly, the manufacturer produces according to the number of order.

1. Notations

Parameters and decision variables are presented as follows:

Parameters

o Unit option price
f Revenue sharing fraction
m The markup that the retailer charges to the final price
v Unit salvage value of the extra products
ρ Unit shortage cost
c Unit production cost of the products
a Slope of the demand curve (Price-dependency of demand)
b Amount of demand which is added per unit of additional sales effort (Marketing efficiency)
l Coefficient of the function of cost of additional effort
x The demand of the market for the option-income sharing contract
ε The random term of the market demand function with $\varphi(x)$ and $\phi(x)$ as the probability density function and the cumulative probability function, respectively
I The constant term of the demand curve
t The market demand in the wholesale mechanism, $\theta(t)$ and $\Theta(t)$ are the distribution and the cumulative functions.
z The market demand in the option-revenue sharing contract, $\psi(z)$ and $\Psi(z)$ are the distribution and the cumulative functions.
ω' The price of wholesale
ep The exercise worth in the basic option contract

Variables

e Sales effort of the retailer
ω The price of items for wholesale policy, which is equal to the price of options
p Unit market price
Q_c Order quantity in the centralized SC
Q Number of retailer's order in the decentralized SC, which is equal to the production quantity of the manufacturer
Q_{WR} Number of retailer's order in the wholesale contract
Q_{WM} Number of produced items in the wholesale contract

The main assumptions are presented here:

- The manufacturer follows a make-to-order manufacturing policy. This means that production process starts only if the retailer orders.
- The manufacturer knows the distribution of the market demand just like the retailer.
- $o + \upsilon < c$, preventing the manufacturer to arbitrage with the option,
- $o + \upsilon < \omega'$, preventing the manufacturer to trade the items in salvage worth instead of content options bought by the retailer,
- The market demand is an independent random variable in all of states.
- The sales effort causes an additional cost for the retailer which is modeled via the concave function $-le^2$.
- The demand is assumed to be the function of the price, sales effort and a random term. It is formulated in Eq. (1).

$$x = I - ap + be + \varepsilon \tag{1}$$

where p is the price of the basic product, as formulated in Eq. (2):

$$p = \omega + m + o \tag{2}$$

Using the price as the main effective factor on the market demand is a common assumption in the microeconomics and also has been used by some previous studies (Chen et al., 2011).

2. Basic option contract

In this section, the results of proposed mixed contract mechanism are compared with a basic contract. The equations stated below show the profit functions:

$$\Pi_R = p.min\{Q.x\} - o.Q - \omega.min\{Q.x\} - \rho.max\{x - Q.0\} \tag{3}$$

$$\Pi_M = \omega.min\{Q.x\} + o.Q - c.Q + \upsilon.\max\{Q - x.0\} \tag{4}$$

The first section of Eq (3) states the revenue earned by selling the items. The prices of option and exercise are shown in the second and the third parts of Eq (3). The last part of Eq (3) shows the cost of shortage. The prices of option and exercise are denoted in the first and second parts of Eq (4). The manufacturing cost and recovering price of the unexercised options are denoted in the last two terms of Eq (4). The best order value of the retailer is $Q^{*}=\Psi^{-1}\left(\frac{p-e+\rho-o}{p-e+\rho}\right)$, (see Zhao et al., 2010).

3. Wholesale mechanism

In wholesale contract, the manufacturer and the retailer set their plans according to their personal information about the market demand. The market price is assumed $(\omega'+m)$, which is a summation of wholesale price and a markup. Therefore, the market demand in the wholesale mechanism is $x=I-a'(\omega'+m)+\varepsilon$. The profit functions of the retailer and the manufacturer are presented as follows:

$$\Pi_{WR}=p.min\{Q_{WR}.x\}-\omega.Q_{WR}-\rho.max\{x-Q_{WR}.0\}+\upsilon.max\{Q_{WR}-x.0\} \quad (5)$$

$$\Pi_{WM}=\omega.min\{Q_{WM}.x\}-c.Q_{WM}-b\rho.max\{x-Q_{WM}.0\}+\upsilon.max\{Q_{WM}-x.0\} \quad (6)$$

The first part of Eq (5) is related to the total income of system. The next item denotes the price of products for wholesale state. The last two items show the deficiency cost and the salvage cost of the additional items. The first part of Eq (6) is the income of the system and the second part is the manufacturing expense. The third and the fourth sections show the deficiency cost and the salvage cost of the additional items. The best values of ordering and manufacturing are stated as follows:

Proposition 1. In a wholesale mechanism that is used in a two-level SC with a price-dependent demand, the best values of ordering and manufacturing are stated as follows:

$$Q^{*}_{WR}=\Theta^{-1}\left(\frac{\bar{\rho}+b-\theta}{\bar{\rho}+b-\upsilon}\right) \quad (7)$$

$$Q^{*}_{WM}=\Theta^{-1}\left(\frac{\theta+b-c}{\theta+b-\upsilon}\right) \quad (8)$$

Appendix A shows the related proof.

4. Centralized SC

Among all the mechanisms, which are presented in the area of contracts, centralized SC has the most similar structure to the main contract. Similar to the main contract, the demand of market (x) is regarded as a function of the expense, sales effort and a random term;

in other words, $x = I - a.p + b.e + \varepsilon$. According to these explanations, the centralized SC's profit function is as follows:

$$\Pi_C(e.Q) = p\,min\{Q_c.x\} - c.Q_c - \rho\,min\{x - Q_c.0\} + \upsilon \max\{Q_c - x.0\} - le^2 \quad (9)$$

where the first term indicates the income of the centralized SC. Next section denotes the manufacturing cost. The last two items show the deficiency cost and the salvage cost of the additional items. Finally, the best number and the sales effort of the centralized SC can be obtained from the following proposition.

Proposition 2. In a cooperative SC that consists of a hypothetical centralized organization, the demand of market is related to the expense of market and the sales effort, and $y = I - a.p + b.e$. The best values of order and sales effort are expressed in the following equations:

$$Q_c^* = \Phi^{-1}\left(\frac{\rho + p - c}{\rho + p - \upsilon}\right) + y \quad (10)$$

$$e_c^* = \frac{b(p - \upsilon)(\rho + p + c) + \rho.b(\upsilon + c)}{2l(\rho + p - \upsilon)} \quad (11)$$

Appendix A shows the related proof.

5. The option-revenue sharing contract

To reach harmony in a SC system with different parts, a contract is proposed in which the option is the basic mechanism and the revenue sharing mechanism plays the role of a supplement. According to the main assumptions of the model, the manufacturer follows a make-to-order manufacturing strategy. To investigate the retailer's behavior in terms of the marketing and sales effort, the retailer sets the value of sales effort besides the order quantity. In addition, the manufacturer determines the price of items for wholesale strategy and the retailer sets the value of order and sales effort as a Stackelberg game in which the aforementioned decision variables are strategies of players; in other words, $x = I - a(\omega + m + o) + be + \varepsilon$. To obtain the optimal state in the Nash equilibrium as a strategy in which no player is interested to be the only one to change, firstly, the income of the retailer is presented in the following equation:

$$\Pi_R(e.Q) = (1 - f)\,p.min\{Q.x\} - o.Q - \omega.min\{Q.x\} - \rho.max\{x - Q.0\} - l.e^2 \quad (12)$$

In equation (12), the income of retailer, option and the exercise expense, deficiency cost and cost of the sales effort are denoted respectively. Besides, the profit of manufacturer is stated in Eq. (13):

$$\Pi_M(\omega) = \omega.min\{Q.x\} + o.Q - c.Q + \upsilon.\max\{Q - x.0\} + f.p.min\{Q.x\} \tag{13}$$

In equation (13), the price of items in wholesale strategy, the price of option, manufacturing expense and salvage worth related to unexercised options are demonstrated respectively.

5.1. Manufacturer-led supply chain

In many real cases, the market is not customer-oriented. In this situation, the manufacturer sets the price of items for wholesale strategy. Next, the retailer sets the value of orders. To obtain the tactics in the Nash equilibrium, the best retailer's decisions are determined. Finally, the ideal plan of the manufacturer is calculated as shown in proposition 3.

Proposition 3. In an option-revenue sharing agreement in which the manufacturer is the head of the two-level SC, the market demand is obtained from Eq. (1), and $y = I - a(\omega + m + o)$. The number of order and sales effort are calculated as follows:

$$Q^* = \Phi^{-1}\left(\frac{\rho + (1-f)p - \omega - o}{\rho + (1-f)p - \omega}\right) + y \tag{14}$$

$$e^* = \frac{((1-f)p - \omega - o)b}{2l} \tag{15}$$

Appendix A shows the related proof.

We used nonlinear optimization algorithms in MATLAB software to achieve the optimal wholesale price.

5.2. Retailer-led supply chain

Most of the modern markets are customer-oriented. In other words, all the members in a SC should match themselves with the customers' demand. In this regard, the key factor of success in competition is demand information. Accordingly, the retailer sets the value of order and the sales effort. Then, the manufacturer decides the wholesale price. Following stages are exactly the same as the stages of the manufacturer-led SC.

5.3. Heuristic algorithm

In the current study a heuristic algorithm is adopted to solve the addressed complex problem. The proposed, basic heuristic algorithm is designed based on solving sequential problems. Because the retailer's problem contains a high level of complexity and applying game theory-based approaches increases the complexity of the problem (Wang & liu, 2018; Sinha, Schlenker, Dmello, & Tambe, 2018), a genetic algorithm into the basic heuristic algorithm is incorporated to facilitate the solution of the problem. In the first step of the heuristic method, the leader's decisions are determined. Next, the follower's response (retailer's decisions) are determined and the main objective function of the model is calculated. By applying crossover and mutation, as the main operators in the Genetic algorithm, the other possible solutions are calculated and compared. Figure 1 shows the summary of research steps.

Analyzing the related studies and identifying the research gaps
Designing the structure of the novel option revenue sharing coordination contract model
Applying the proposed method for solving the model according to following sub steps:

- Generate the initial solution by GA
- Consider the leaders strategy as the current generation
- Calculate the best follower strategy
- Evaluate the fitness of the results
- Apply the mutation and cross over to diversify the answers

Compiling the related data of research according to the opinion of related experts
Reporting and analyzing the results

Fig. 1. The summary of research steps.

Numerical experiments

Here, some numerical experiments are implemented based on random generated data presented in Table 1. The related data are determined through the opinion of related expertise.

Fig. shows the steps of the proposed algorithm.

Table 1. Value of parameters used in the proposed contract

Parameter	Value
Unit option price (o)	4
Revenue sharing fraction (f)	0.2
Unit salvage value (υ)	1
Unit production cost (c)	6
Unit shortage cost (ρ)	10
Coefficient of the cost function of sales effort (l)	4
Added demand per unit of additional sales effort (b)	5
Retailer's markup (m)	20
Slope of the demand curve (a)	-2
Constant term of the demand curve (I)	100
Random term of the demand function in the option-revenue sharing contract $(\varphi(\varepsilon))$	$N(0.400)$
The wholesale price in the wholesale contract (ω')	19
The expense of exercise for the option contract (ep)	14
Market demand in the wholesale contract $(\theta(t))$	$N(22.400)$
Market demand in the basic option contract $(\psi(z))$	$N(24.400)$

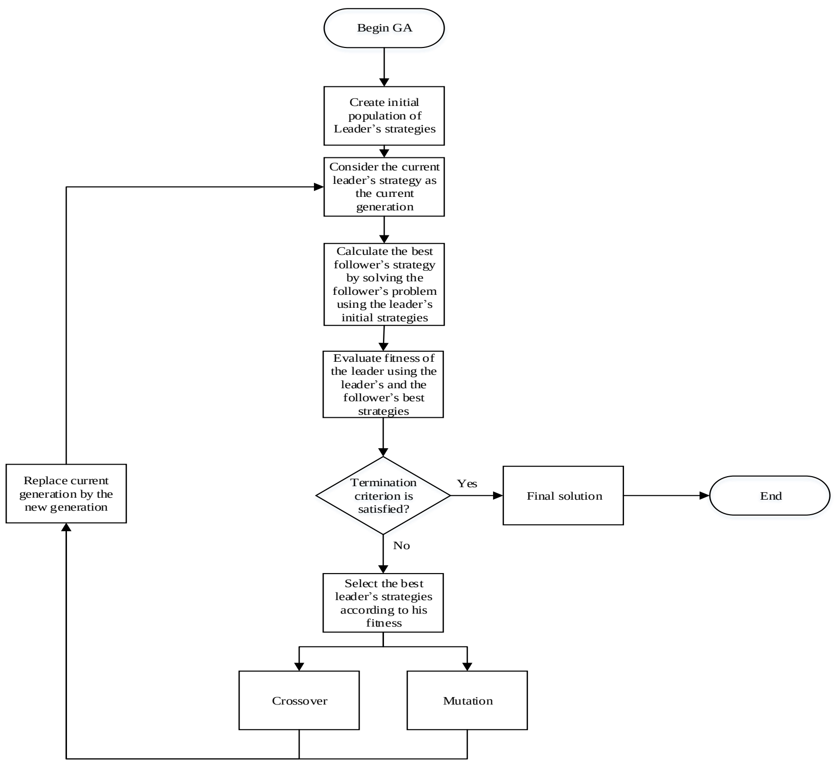

Fig. 2. The proposed hybrid algorithm to solve the retailer-led problem

1. Results

According to the data in Table 1, the outcomes of various contracts are calculated and reported. Also, the outcomes of the comparison criteria are reported.

1.1. Results of the hybrid algorithm

The computational complexity of the retailer-led SC is the main reason to propose a hybrid algorithm. The concept of the heuristic algorithm is used to reach the Nash equilibrium and the genetic algorithm is applied to deal with the retailer's problem. It should be noted that the heuristic algorithm and the genetic algorithm are designed for continuous variables. Therefore, the solution chromosome is a 1×2 border, in which the first array is the order quantity and the second one is the sales effort.

The selection is an operator choosing the best solutions according to the retailer's profit value. The selection operator causes convergence of the algorithm. In the proposed algorithm, a Roulette Wheel method is adopted to select the best solution in the genetic algorithm. The readers are referred to Goldenberg (1989) to find out about the Roulette Wheel method. A crossover operator reproduces offspring from the best solutions that are selected by the selection operator. To do so, two solutions are randomly selected as parents and crossover is implemented on them to reproduce two children. The crossover rate is assumed 0.7 in the proposed algorithm. Finally, genetic algorithm uses a mutation operator to search all the solution space and diversify the algorithm. So, a uniform mutation operator is used in which some solutions are selected randomly and replaced by a random number that is generated according to the uniform distribution among the specified range of the variable. The rate of the mutation for genetic algorithm is assumed 0.1. Fig. demonstrates the outcomes of the proposed hybrid method, which is coded in MATLAB R2011b and ran on an Intel (R) Core 2 Duo CPU 2.53 GHz-based computer with 3 GB of RAM memory. These results will be compared with other situations in the following section.

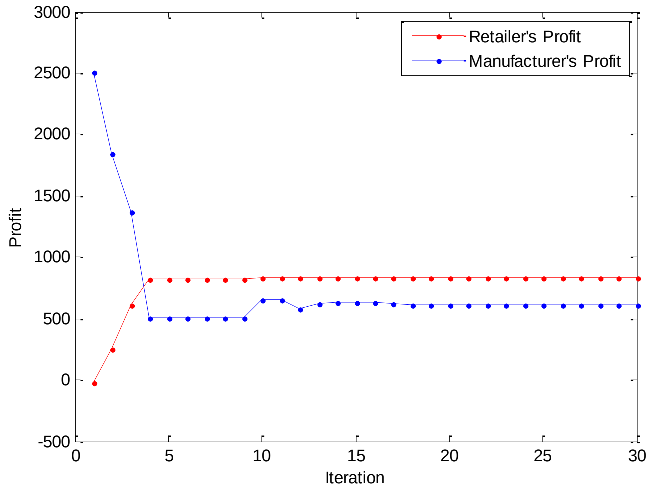

Fig. 3. The income of the retailer and the manufacturer for the hybrid algorithm

1.2. Performance evaluation

Using the results of the hybrid algorithm for the retailer-led SC, we can compare all the situations and contracts. The manufacturer's problem is solved via nonlinear optimization algorithms that exist in MATLAB software. Table 2 presents the results of all the contracts, which are calculated according to the data presented in Table 1. As expected, the basic option contract results in more profit. The order value of retailer is greater in the option-revenue sharing contract. The highest order quantity is resulted in the cooperative SC.

Table 2. A comparison of results of the option-revenue sharing contract with other contracts

	Wholesale	Basic Option	Centralized (Cooperative)	Non-cooperative Decentralized	
				Retailer-led	Manufacturer-led
Order Quantity	27.95	35.31	189.15	43.54	74
Sales Effort	-	-	28.7	1.15	7.32
Wholesale Price	-	-	-	14.1	17.43
Retailer's Profit	38.69	349.97	-	832.77	880.44
Manufacturer's Profit	129.29	135.36	-	614.3	1214.2
Total Profit	167.98	485.33	2270	1447.07	2094.6

2. Experiments

The option-revenue sharing contract is a combination of two mechanisms, each of which having several parameters. These parameters are varied in different markets and industries. To show the results of the contract in different situations, some experiments are implemented in different values of critical parameters of the contract.

2.1. Option price

The first rate to be paid by retailer in order to produce items is the price of option. Table 3 reports the outcomes of the model by changing the value of option price. By enhancing the price of option, the order value of retailer decreases. The main reason is that the retailer pays the related price at the commencement of the process. The higher the option expense, the higher the members' profit. In addition, increasing the option expense leads to less price of exercise for the wholesale strategy.

Table 3. Effect of the option price on outputs of the option-revenue sharing contract

o	*Q*	Retailer's profit	Manufacturer's profit	Total profit	ω	*p*	μ^*
1	86.98	646.83	1128.5	1775.33	20.75	44.75	47.1
2	80.68	743.43	1171.6	1915.03	19.64	43.64	49.32
3	76.8	819.72	1197.2	2016.92	18.53	42.53	51.54
4	74	880.44	1214.2	2094.64	17.43	41.43	53.74
5	71.8	927.75	1226	2153.75	16.32	40.32	55.96

* μ is the deterministic part of the demand function Eq. (1)

2.2. Revenue sharing fraction

Revenue sharing fraction is the proportion of income that belongs to the manufacturer. Table 4 shows the effects of the income distribution fraction variation on the model. These results show that paying a higher proportion of the revenue makes the retailer less interested in participation in the contract. A higher level of revenue sharing fraction decreases the members' profit.

Table 4. Effect of the revenue sharing fraction on outputs of the option-revenue sharing contract for *b*=5 and *b*=4

b	f	Q	Retailer's income	Manufacturer's income	Total income	*ω*	*e*	*p*	*μ*
4	0	68.31	1164.2	917.91	2082.11	23.71	10	47.71	44.58
	0.1	66.79	1025.8	914.83	1940.63	19.3	7.83	43.3	44.72
	0.2	65.33	871.7	911.12	1782.82	15.62	6.03	39.62	44.88
	0.4	62.58	556.76	901.55	1458.31	9.78	3.24	33.78	45.4
5	0	79.56	1164.2	1469.7	2633.9	29.33	12.5	53.33	55.84
	0.1	76.59	1033	1324.5	2357.5	22.63	9.58	46.63	54.64
	0.2	74	880.44	1214.2	2094.64	17.43	7.32	41.43	53.74
	0.4	69.64	577.2	1057	1634.2	9.84	4.03	33.84	52.47

\

2.3. Price-dependency of demand (a)

According to the principles of microeconomics, the market demand is negatively correlated with the market price. Table shows the results of this experiment.

Table 5. Effect of price-dependency of demand on outputs of the option-revenue sharing contract

a	*Q*	Retailer's income	Manufacturer's income	Total income	*ω*	*e*	*p*	*μ*
1	83.19	774.13	2878.9	3653.03	35.98	5	59.98	5.04
1.5	78.81	868.7	1836.5	2705.2	24.56	6.42	48.56	34.98
2	74	880.44	1214.2	2094.64	17.43	7.32	41.43	53.74
2.5	68.98	852.99	812.28	1665.27	12.56	7.92	36.56	66.48
3	63.86	804.19	540.09	1344.28	9.02	8.37	33.02	75.81

According to the results, enhancing the price-dependency of demand leads to less number of items ordered by the retailer. This result proves that, if the demand is dependent on the price, the retailer will not be motivated to use the contract. A higher level of price-dependency of demand makes the manufacturer reduce the wholesale price. This is an action to avoid a remarkable decrease in the market demand.

Conclusion

This study addresses an option-revenue sharing mechanism which decreases double marginalization effect and considers demand as a dependent parameter. The current study is applicable in competitive supply chains in which the pricing strategy plays a vital role in gaining the outputs of the system. The main contributions of this study include: proposing a novel option-revenue sharing coordination contract framework; considering various decision variables in the structure of the model to cover different aspects and make the model more practical; considering retailer-based and manufacturer-based Stackelberg game for the coordination contract problem and, applying a heuristic based method to solve the problem. In this regard, the retailer sets the number of order sales effort, while the manufacturer adopts the price of items for the wholesale strategy. The retailer selects the ideal purchasing option by taking into consideration the demand of market. In terms of the income sharing process, the retailer should pay the predetermined amount of income to the manufacturer. We have formulated the problem in retailer-based and manufacturer-based Stackelberg game. For the retailer-based SC, computational complexity of the problem leads us to propose a hybrid algorithm to obtain the Nash equilibrium. To do so, a genetic algorithm is incorporated into a heuristic algorithm. In order to assess the performance of the option-revenue sharing contract, all the cases are numerically experimented. Finally, some numerical experiments are conducted on the critical parameters of the contract. According to the results, increasing the reliance of demand leads to less number of products ordered by the retailer. In addition, the results show that paying more proportion of the revenue makes the retailer less inclined to contribution in the contract. Although the current study has several novelties, there are some suggestions that the researchers can use to improve it. Firstly, some bargaining models can be used to divide the profit of cooperative situation among the members. The results of bargaining models can be compared with the results of the proposed contract. Considering the effects of disruption on the results of the model and using other novel heuristic algorithms can be used to facilitate the calculation of the Nash equilibrium.

Appendix A.

Proposition 1. The expected value of the members' income function is presented in the Eq. (A.1) and Eq. (A.2):

$$E(\Pi_{WR}) = [(p+\rho).Q_{WR}]\big(1-\phi(Q_{WR}-y)\big) + \upsilon.Q_{WR}\phi(Q_{WR}-y) + [p+\rho-\upsilon]\int_s^{Q_{WR}-y}\varepsilon.\varphi(\varepsilon).d\varepsilon - \rho.\mu - \omega.Q_{WR} \quad \text{(A.1)}$$

$$E(\Pi_{WM}) = [(\omega'+\rho).Q_{WM}]\big(1-\phi(Q_{WM}-y)\big) + \upsilon.Q_{WM}\phi(Q_{WM}-y) + [\omega'+\rho-\upsilon]\int_s^{Q_{WM}-y}\varepsilon.\varphi(\varepsilon).d\varepsilon - \rho.\mu - c.Q_{WM} \quad \text{(A.2)}$$

If we add the dependence term of the demand to its average, the market demand is an independent random variable. Therefore, the average of the market demand in the wholesale contract is $I - a'(\omega' + m)$.

$$E(\Pi_{WR}) = [(p+\rho).Q_{WR}]\big(1-\Theta(Q_{WR})\big) + \upsilon.Q_{WR}\Theta(Q_{WR}) + [p+\rho-\upsilon]\int_s^{Q_{WR}}\varepsilon.\theta(\varepsilon).d\varepsilon - \rho.\mu - \omega.Q_{WR} \quad \text{(A.1)}$$

$$E(\Pi_{WM}) = [(\omega'+\rho).Q_{WM}]\big(1-\Theta(Q_{WM})\big) + \upsilon.Q_{WM}\Theta(Q_{WM}) + [\omega'+\rho-\upsilon]\int_s^{Q_{WM}}\varepsilon.\theta(\varepsilon).d\varepsilon - \rho.\mu - c.Q_{WM} \quad \text{(A.2)}$$

θ denotes the distribution function of the demand for the wholesale mechanism.

Proposition 2. In order to reach the best strategies, firstly, the probable value of the profit function is expressed in Eq. (A.3):

$$E(\Pi_C) = [p.y + (Q_c - y)\upsilon]\phi(Q_c - y) + [p.Q_c + \rho(y - Q_c)]\big(1 - \phi(Q_c - y)\big) + [p - \upsilon + \rho]\int_s^{Q_c - y}\varepsilon.\varphi(\varepsilon).d\varepsilon - \rho.\mu - c.Q_c - le^2 \quad \text{(A.3)}$$

To calculate the optimal values, the sufficient optimality conditions should be checked.

$$\frac{\partial \Pi_C}{\partial Q_c} = \upsilon.\phi(Q_c - y) + (\rho + P)[1 - \phi(Q_c - y)] - c \quad \text{(A.4)}$$

$$\frac{\partial \Pi_C}{\partial e} = b(P - \upsilon)\phi(Q_c - y) - \rho.b[1 - \phi(Q_c - y)] - 2l.e \quad \text{(A.5)}$$

According to these equations, the Hessian matrix of the centralized SC problem is presented in Eq. (A.6):

$$H = \begin{bmatrix} -(P+\rho-\upsilon)\varphi(Q_c - y) & (P+\rho-\upsilon).b.\varphi(Q_c - y) \\ (P+\rho-\upsilon).b.\varphi(Q_c - y) & -(P+\rho-\upsilon).b^2.\varphi(Q_c - y) - 2l \end{bmatrix} \quad \text{(A.6)}$$

To prove that results of Eq. (A.4) and Eq. (A.5) maximize the profit function, we should prove that the Hessian matrix is negative definite. The first minor is equal to the array $-(P + \rho - \upsilon)\varphi(Q_c - y)$ in the matrix. Because $P > \upsilon$ and $\varphi(Q_c - y)$ is a probability value and positive, the term $-(P + \rho - \upsilon)\varphi(Q_c - y)$, which is the odd minor, is negative. The second minor could be expressed as $2l(P + \rho -$

$\upsilon)\varphi(Q_c - y)$ with some algebra. Similarly, since $P > \upsilon$ and $\varphi(Q_c - y)$ is a probability value and positive, this term is positive. Thus, the assumption is proved and the optimal values of variables, which are calculated as $Q_c^* = \Phi^{-1}\left(\frac{\rho+p-c}{\rho+p-\upsilon}\right) + y$ and $e_c^* = \frac{b(p-\upsilon)(\rho+p+c)+\rho.b(\upsilon+c)}{2l(\rho+p-\upsilon)}$, maximize the profit function.

Proposition 3. In this proposition, equilibrium strategies of the retailer are calculated and their optimality is proved:

$$E(\Pi_R) = \left[\left((1-f)p-\omega\right)y\right]\phi(Q-y) + \left[\left((1-f)p-\omega\right)Q - \rho(y-Q)\right]\left(1-\phi(Q-y)\right) + [(1-f)p-\omega+\rho]\int_s^Q \varepsilon.\varphi(\varepsilon).d\varepsilon - l.e^2 \quad \text{(A.7)}$$

Regarding the Q and e, Eqs. (A.8) and (A.9) show the first order derived of income function:

$$\frac{\partial \Pi_R}{\partial Q} = \left((1-f)p-\omega+\rho\right)[1-\phi(Q-y)] - o \quad \text{(A.8)}$$

$$\frac{\partial \Pi_R}{\partial e} = \left((1-f)p-\omega\right)b.\phi(Q-y) - \rho.b[1-\phi(Q-y)] - 2l.e \quad \text{(A.9)}$$

To check the sufficient optimality condition, Hessian matrix of the income is expressed in Eq. (A.10):

$$H = \begin{bmatrix} -\left((1-f)p-\omega+\rho\right)\varphi(Q-y) & \left((1-f)p-\omega+\rho\right).b.\varphi(Q-y) \\ \left((1-f)p-\omega+\rho\right).b.\varphi(Q_c-y) & -\left((1-f)p-\omega+\rho\right).b^2.\varphi(Q-y)-2l \end{bmatrix} \quad \text{(A.10)}$$

Similarly to the previous proposition, the first minor is equal to $-\left((1-f)p-\omega+\rho\right)\varphi(Q-y)$. Obviously, price of market has a higher value compared to the wholesale price. Therefore, we have $(1-f)p>\omega$. Also, $\varphi(Q_c-y)$ is a probability value and positive. Therefore, the term $-\left((1-f)p-\omega+\rho\right)\varphi(Q-y)$ is negative. With some algebra, the second minor is expressed as $2l\left((1-f)p-\omega+\rho\right)\varphi(Q-y)$. Similarly, because $(1-f)p>\omega$ and $\varphi(Q_c-y)$ is a probability value and positive, the second minor is positive. Accordingly, the odd minor is negative, the even minor is positive, and, consequently, the Hessian matrix is negative definite. In this regard, the optimal values are $Q^* = \Phi^{-1}\left(\frac{\rho+(1-f)p-\omega-o}{\rho+(1-f)p-\omega}\right)+y$ and $e^* = \frac{\left((1-f)p-\omega-o\right)b}{2l}$.

References

Arani, H. V., Rabbani, M., & Rafiei, H. (2016). A revenue-sharing option contract toward coordination of supply chains. *International Journal of Production Economics*, *178*, 42-56.

Babich, V. (2006). Vulnerable options in supply chains: Effects of supplier competition. *Naval Research Logistics (NRL)*, *53*(7), 656-673.

Basu, P., Liu, Q., & Stallaert, J. (2019). Supply chain management using put option contracts with information asymmetry. *International Journal of Production Research*, *57*(6), 1772-1796.

Cachon, G. P., & Lariviere, M. A. (2005). Supply chain coordination with revenue-sharing contracts: Strengths and limitations. *Management Science*, *51*(1), 30-44.

Chen, J., & Bell, P. C. (2011). Coordinating a decentralized supply chain with customer returns and price-dependent stochastic demand using a buyback policy. *European Journal of Operational Research*, *212*(2), 293-300.

Chen, X., & Shen, Z.-J. (2012). An analysis of a supply chain with options contracts and service requirements. *IIE Transactions, 44*(10), 805-819.

Corbett, C. J., Zhou, D., & Tang, C. S. (2004). Designing supply contracts: Contract type and information asymmetry. *Management science*, 50(4), 550–559.

Fahimi, K., Seyedhosseini, S. M., & Makui, A. (2017). Simultaneous decentralized competitive supply chain network design under oligopoly competition. *Iranian Journal of Management Studies*, *10*(2), 409-434.

Fahimi, K., Seyedhosseini, S. M., & Makui, A. (2018). Dynamic competitive supply chain network design with price dependent demand and Huff utility function. *Iranian Journal of Management Studies*, *11(2), 271-305*.

Goldenberg, D. E. (1989). *Genetic algorithms: Search, optimization & machine learning*. MA: Addison-Wesley, Inc.

Gomez_Padilla, A., & Mishina, T. (2009). Supply contract with options. *International Journal of Production Economics*, *122*(1), 312-318.

Heydari, J., & Asl-Najafi, J. (2018). A revised sales rebate contract with effort-dependent demand: a channel coordination approach. *International Transactions in Operational Research*, *https://doi.org/10.1111/itor.12556* .Article in Press.

Hosseini-Motlagh, S. M., Nematollahi, M., Johari, M., & Choi, T. M. (2019). Reverse supply chain systems coordination across multiple links with duopolistic third party collectors. *IEEE Transactions on Systems, Man, and Cybernetics: Systems, 1 - 12*.Article in Press.

Hosseini-Motlagh, S. M., Nouri-Harzvili, M., Choi, T. M., & Ebrahimi, S. (2019). Reverse supply chain systems optimization with dual channel and demand disruptions: Sustainability, CSR investment and pricing coordination. *Information Sciences*, *503,* 606-634.

Hu, B., Qu, J., & Meng, C. (2018). Supply chain coordination under option contracts with joint pricing under price-dependent demand. *International Journal of Production Economics*, *205,* 74-86.

Hua, S., Liu, J., Cheng, T. C. E., & Zhai, X. (2019). Financing and ordering strategies for a supply chain under the option contract. *International Journal of Production Economics*, *208,* 100-121.

Jiao, Y.-Y., Du, J., & Jiao, J. (2007). A financial model of flexible manufacturing systems planning under uncertainty: Identification, valuation and applications of real options. *International Journal of Production Research, 45*(6), 1389-1404.

Johari, M., & Hosseini-Motlagh, S. M. (2019). Coordination of social welfare, collecting, recycling and pricing decisions in a competitive sustainable closed-loop supply chain: a case for lead-acid battery. *Annals of Operations Research*, 1-36.Article in Press

Johari, M., Hosseini-Motlagh, S. M., & Nematollahi, M. (2017). Supply chain coordination using different modes of transportation considering stochastic price-dependent demand and periodic review replenishment policy. *International Journal of Transportation Engineering*, *5*(2), 137-165.

Li, H., Ritchken, P., & Wang, Y. (2009). Option and forward contracting with asymmetric information: Valuation issues in supply chains. *European Journal of Operational Research, 197*(1), 134-148.

Liu, Z., Chen, L., Li, L., & Zhai, X. (2014). Risk hedging in a supply chain: Option vs. price discount. *International Journal of Production Economics*, 151, 112–120.

Nemati, Y., Madhoushi, M., & Safaei Ghadikolaei, A. (2017). Towards supply chain planning integration: uncertainty analysis using fuzzy mathematical programming approach in a plastic forming company. *Iranian Journal of Management Studies*, *10*(2), 335-364.

Noh, J., Kim, J. S., & Sarkar, B. (2019). Two-echelon supply chain coordination with advertising-driven demand under Stackelberg game policy. *European Journal of Industrial Engineering*, *13*(2), 213-244.

Shen, X., Bao, L., & Yu, Y. (2018). Coordinating inventory and pricing decisions with general price-dependent demands. *Production and Operations Management*, *27*(7), 1355-1367.

Sinha, A., Schlenker, A., Dmello, D., & Tambe, M. (2018). Scaling-up Stackelberg security games applications using approximations. In *International Conference on Decision and Game Theory for Security* (pp. 432-452). Springer, Cham, Washington, USA.

Song, H., & Gao, X. (2018). Green supply chain game model and analysis under revenue-sharing contract. *Journal of Cleaner Production*, *170*, 183-192.

Rabbani, M., Arani, H. V., & Rafiei, H. (2015). Option contract application in emergency supply chains. *International Journal of Services and Operations Management*, *20*(4), 385-397.

Van der Veen, J. A., & Venugopal, V. (2005). Using revenue sharing to create win–win in the video rental supply chain. *Journal of the Operational Research Society*, *56*(7), 757-762.

Wan, N., & Chen, X. (2019). The role of put option contracts in supply chain management under inflation. *International Transactions in Operational Research*, *26*(4), 1451-1474.

Wang, X., Li, F., Liang, L., Huang, Z., & Ashley, A. (2015). Pre-purchasing with option contract and coordination in a relief supply chain. *International Journal of Production Economics*, *167*, 170-176.

Wang, G., & Liu, T. (2018). Resource allocation for M2M-enabled cellular network using Nash bargaining game theory. *Peer-to-Peer Networking and Applications*, *11*(1), 110-123.

Xu, H. (2010). Managing production and procurement through option contracts in supply chains with random yield. *International Journal of Production Economics*, *126*(2), 306-313.

Xu, G., Dan, B., Zhang, X., & Liu, C. (2014). Coordinating a dual-channel supply chain with risk-averse under a two-way revenue sharing contract. *International Journal of Production Economics*, *147*, 171-179.

Ye, F., Li, Y., & Yang, Q. (2018). Designing coordination contract for biofuel supply chain in China. *Resources, Conservation and Recycling*, *128*, 306-314.

Zhang, J., Liu, G., Zhang, Q., & Bai, Z. (2015). Coordinating a supply chain for deteriorating items with a revenue sharing and cooperative investment contract. *Omega*, *56*, 37-49.

Zhang, J., Zhao, S., Cheng, T. E., & Hua, G. (2019). Optimisation of online retailer pricing and carrier capacity expansion during low-price promotions with coordination of a decentralised supply chain. *International Journal of Production Research*, *57*(9), 2809-2827.

Zhao, Y., Choi, T. M., Cheng, T. C. E., & Wang, S. (2018). Supply option contracts with spot market and demand information updating. *European Journal of Operational Research*, *266*(3), 1062-1071.

Zhao, Y., Wang, S., Cheng, T. E., Yang, X., & Huang, Z. (2010). Coordination of supply chains by option contracts: A cooperative game theory approach. *European Journal of Operational Research*, *207*(2), 668-675.