

Coordination of R&D Effort, Pricing, and Periodic Review Replenishment Decisions in a Green Supply Chain through a Delay in Payment Contract

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Abstract

This study contributes to the literature on supply chain coordination by coordinating the research and development (R&D) effort, retail price, and inventory decisions. It investigates a real case in the home appliance industry. The main purpose of this study is to examine the optimal values of R&D effort, pricing, and inventory decisions under decentralized and centralized structures. Then, a delay in payment contract is proposed under coordinated structure to simultaneously coordinate all decisions. Moreover, to share the extra profit obtained from the coordinated model between both parties, a profit allocation strategy is proposed. The findings reveal that the coordinated model not only enhances the green supply chain (GSC) profitability in comparison with the decentralized structure but also enhances the profits of all members. Further, the proposed contract improves the GSC performance from both environmental and economic viewpoints.

Keywords

Green supply chain coordination, R&D effort, Delay in payment contract, Periodic review inventory system.

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Introduction

In recent decades, GSC management has attracted the attention of companies around the world. The GSC as a modern management issue aims at reducing environmental impacts and optimizing resource efficiency (Song & Gao, 2017). Thus, many companies are stimulated to adopt measures to improve the green level. For instance, some GSCs invest in the R&D effort to enhance the eco-saving performance of products (Dai, Zhang, & Tang, 2017). Moreover, according to a global research by Accenture, about 80% of consumers consider the greenness of products in purchasing (Hong & Gao, 2019). In such a situation, the market demand of superior eco-saving products is enhanced. Therefore, adopting green measures creates a competitive advantage for the companies to absorb more customers and improve the demand and profitability. Therefore, to maintain the competitive advantage, the manufacturers cannot ignore the environmental subjects (Paydar, Hassanzadeh, & Tajdin, 2016). For instance, Haier, a household appliance company, receives government subsidy because of the compliance of its household appliances with EU A+ energy standard (Dai et al., 2017).

This study is motivated by the issues of a real home appliance company. Due to confidentiality reasons, the dummy name PC is used for the company. PC produces washing machines with energy labels and sells them through the retail channel. In today's business environment, competition is one of the most significant subjects which needs to be considered when investigating market conditions. Accordingly, the PC Company tries to gain competitive advantages through investigation in the R&D effort as a green effort. More precisely, PC makes the R&D effort to decrease the energy consumption of washing machines with the aim of protecting the environment. Therefore, by increasing the public awareness, the demand for the washing machines pulls up and the company can achieve more profit. Thus, in this real case, the main issue of PC is determining the optimal R&D effort. The decision of the manufacturer not only influences the profit of the company but also influences the profit of the retailer.

The green washing machines are sold by the retailer. The retailer applies a periodic review inventory policy to replenish the inventory. Under this inventory model, the retailer reviews the inventory level in

each review period and replenishes the inventory up to the order-up-to level (Nematollahi, Hosseini-Motlagh, & Heydari, 2017). Accordingly, under this inventory policy, the retailer determines the review period and the order-up-to level. Moreover, the retailer determines the retail price of washing machine, which in turn has impacts on the washing machine demand. The stochastic demand of the washing machines depends on the price and the R&D effort with a normal distribution. Because of the stochastic nature of demand, the retailer may face partially backordered shortage. In such a situation, the order-up-to level, review period, and retail price decisions not only influence the profit of the retailer, but also affect the profitability of the company. In the current situation, the company and the retailer determine their decisions to enhance their own profitability; consequently, their decisions may not necessarily be optimal from the viewpoint of whole GSC. Hence, a useful mechanism is needed for obtaining the best performance of the whole GSC and its members.

In such a situation, using an appropriate coordination mechanism can help the members to achieve the optimal profit. To this end, in this paper, a delay in payment mechanism is proposed to coordinate the GSC. Under the coordinated contract, the retailer is encouraged to order larger lots, whereby the manufacturer permits the retailer to settle its account in a determined time (Aljazzar, Jaber, & Moussawi-Haidar, 2016). The main problem of this research is to improve the performance of GSC from both environmental and economic viewpoints. To this end, we propose a delay in payment contract to simultaneously coordinate the R&D effort, retail price, and inventory decisions. Accordingly, this study aims to answer the following questions:

- (1) What are the optimal values of R&D effort, retail price, and inventory decisions under different decision-making structures?
- (2) Can a delay in payment mechanism coordinate the GSC so that the manufacturer and retailer take part in the coordinated structure?
- (3) Is the coordinated structure capable of improving the GSC performance from both environmental and economic viewpoints?

To answer the abovementioned questions, three decision-making structures are examined. First, under the decentralized model, a Nash game is played between the members, and they independently determine their decisions. Then, under the centralized model, all decisions are determined from the whole GSC viewpoint. Although

under the centralized structure, the whole GSC profit increases in comparison with the decentralized structure, the members' profit may decrease compared to the decentralized structure. To solve this issue, under the coordinated structure, a delay in payment mechanism is applied to coordinate the GSC. Eventually, to divide the extra profit, a profit allocation strategy is proposed.

The main contributions of the current study are as follows. First, this study simultaneously analyzes the effects of R&D effort and replenishment inventory decisions on the performance of GSC. Second, the market demand is considered stochastic and depends on the R&D effort and retail price. Third, a delay in payment contract is developed to simultaneously coordinate the R&D effort, pricing, and replenishment inventory decisions. Furthermore, to fairly divide the surplus profit, a profit allocation strategy based on the members' profit in the decentralized structure is applied.

Literature review

The related papers are reviewed in the following subsections: green supply chain, periodic review inventory, and supply chain coordination. Moreover, we address the research gaps and contributions.

Green supply chain

Nowadays researchers pay more attention to the GSC management. Swami and Shah (2013) coordinated the green efforts in a GSC. Dai et al. (2017) considered a GSC in which both the upstream and downstream invest in the R&D effort. Song and Gao (2018) investigated a GSC coordination that the manufacturer invests in the R&D effort. Ebrahimi and Hosseini-Motlagh (2018) examined a GSC coordination, where the manufacturer invests in technology to enhance the green quality of products. Hong and Guo (2019) coordinated a GSC considering environmental responsibilities. Although all the above studies have investigated the coordination of the green effort, they have ignored the replenishment decisions in the GSC. This study simultaneously investigates the effects of R&D effort and replenishment decisions in a GSC. Moreover, in the current study, the R&D effort, retail price, and replenishment decisions are coordinated through a delay in payment contract. In addition, in all the above papers, the market demand is considered to be deterministic. In

contrast to the existing literature, in this study, the market demand is stochastic and depends on the R&D effort and retail price.

Periodic review inventory

Periodic review inventory system is one of the main systems to control the inventory, which is used in many real-world cases (Nematollahi et al., 2017). Nematollahi, Hosseini-Motlagh, Ignatius, Goh, and Nia (2018) coordinated a periodic review inventory policy in a pharmaceutical supply chain through a collaborative model. Johari, Hosseini-Motlagh, Nematollahi, Goh, and Ignatius (2018) studied the coordination of a supply chain with price-credit dependent demand under a periodic review inventory policy. Hosseini-Motlagh, Ebrahimi, Nami, and Ignatius (2018) used a lead time crashing coordination scheme to coordinate a supply chain under a periodic review inventory policy. Hosseini-Motlagh, Nouri-Harzvili, and Zirakpourdehkordi (2019) coordinated the retail price, service level, quality level, and order-up-to level decisions under a periodic review inventory policy. All of the reviewed papers have studied the coordination of periodic review inventory policy. However, none of them have considered the periodic review inventory decisions with the R&D effort as a green effort. Therefore, this paper simultaneously coordinates periodic review inventory decisions and the R&D effort with a delay in payment mechanism.

Supply chain coordination

In the coordination literature, many papers have coordinated the supply chain by applying various contracts such as quantity discount (Johari, Hosseini-Motlagh, & Nematollahi, 2016), revenue sharing (Liu, 2019), cost sharing (Hosseini-Motlagh, Nouri, & Pazari, 2018), and collaborative model (Hosseini-Motlagh, Nematollahi, Johari, & Sarker, 2018). Another incentive mechanism that plays a significant role in the business environments is the delay in payment mechanism. According to this contract, the retailer settles its account to the upstream after an agreed period. Heydari (2015) coordinated the replenishment decisions with a delay in payment contract. Heydari, Rastegar, and Glock (2017) applied a two-level delay in payment contract to coordinate a supply chain. Jazinaninejad, Seyedhosseini, Hosseini-Motlagh, and Nematollahi (2019) investigated the coordination of supply chain by a delay in payment contract. Ebrahimi, Hosseini-Motlagh, and Nematollahi (2019)

coordinated the promotional effort in a supply chain through a delay in payment contract. Although all these studies have used delay in payment contract to coordinate the supply chain, they have ignored the simultaneous coordination of R&D effort and replenishment decisions. In this study, a delay in payment mechanism is developed to coordinate the R&D effort, retail price, and replenishment decisions in a GSC.

Research gaps and contributions

The main contributions of the current study are mentioned as follows. First, in the related literature, most of the papers have investigated the coordination of green effort. However, they have ignored replenishment decisions in the GSC. To fill this research gap and come closer to the real world situations, our paper simultaneously investigates the effects of R&D effort and replenishment decisions in a GSC. Moreover, this study coordinates the R&D effort and replenishment decisions through a delay in payment contract. To the best of our knowledge, scholars have not yet studied the simultaneous coordination of R&D effort, pricing, and replenishment inventory decisions. Second, most of the papers in the field of green supply chain coordination have considered the market demand to be deterministic. However, in the current study, the market demand is stochastic and depends on the R&D effort and retail price. Third, although delay in payment mechanism is applied to coordinate the supply chain, this contract is not used to simultaneously coordinate the R&D effort and replenishment decisions. In the current paper, to coordinate the R&D effort, retail price, and replenishment decisions, a delay in payment contract is developed. Moreover, to fairly divide the surplus profit, a profit allocation strategy based on the members' profit in the decentralized structure is applied.

Problem definition

The current study is motivated by a real home appliance company, namely PC. The home appliance company (i.e., the manufacturer) produces the washing machines and makes the green effort to gain more market share in competition with other home appliance companies. In other words, the company invests in R&D effort to reduce the energy consumption of the washing machine products. Investment in the R&D effort plays a significant role in environmental

protection through the production of green products. Moreover, by increasing the environmental awareness, the consumers tend to buy the green products, and the demand for the company's products increases with the increase in the R&D effort. Thus, the manufacturer decides on the R&D effort level. On the other hand, the retailer uses a periodic review inventory system to replenish the inventory. Under this inventory model, the retailer periodically reviews the inventory and orders a sufficient number of products to boost the inventory to an order-up-to level R . The order quantities are received by the retailer after L units of time. The retailer's decision variables are the length of a review period and the safety factor, and washing machine retail price. The stochastic demand of products depends on the manufacturer's R&D effort and the retail price with a normal distribution. The demand of the washing machine can be calculated as $D = (d + \tau q - \theta p)$, where d is the potential market demand. Furthermore, because of stochastic nature of demand, the retailer faces partially backordered shortage. Thus, determining a low service level through inefficient decision on safety stock level reduces his market share and that of the manufacturer and the whole GSC.

Each member's decisions (i.e., R&D effort, retail price, review period, and safety factor) influence not only its own profit but also the other GSC members' profits. However, in the current situation, the members determine their decisions under the decentralized structure. Therefore, the performance of the whole GSC is not optimal. In this paper, the main problem of the study is to develop an incentive mechanism which is capable of improving the profitability of the whole GSC and both members. In the following sections, three decision-making structures are investigated. Under the coordinated structure, an incentive mechanism is developed to entice members to take part in the coordinated structure. Furthermore, the surplus profit achieved from the coordinated structure is divided between the members according to a profit sharing strategy.

Notations

The variables and parameters are expressed in Table 1.

Table 1. Parameters and variables that are used in the current paper

Decision variables	
p	Retail price per unit
R	Order-up-to level
T	Length of review period
q	Manufacturer's R&D efforts level
Parameters	
D	Demand faced by the retailer
d	Potential market demand
A_r	Unit ordering cost per order for the retailer
h_r	Unit inventory holding cost per unit time of the retailer
π	Shortage cost per unit
L	Lead time
k	The retailer's safety factor
w	The manufacturer's wholesale price
α	Fraction of the lost shortage, $0 < \alpha < 1$
τ	Manufacturer's R&D efforts elasticity coefficient of demand
θ	Retail price elasticity coefficient of demand
η	Cost efficiency coefficient for the R&D effort
x	Protection interval $(T + L)$ demand that has a normal distribution function
c	Purchasing price per unit for the manufacturer
Δ	The extra profit under the coordinated structure
i	Annual rate of return on investment
$t^{sharing}$	The exact value of the length of delay period
t^{min}	The minimum value of the delay period length
t^{max}	The maximum value of the delay period length

Model formulation and computational models

Three different structures are formulated: decentralized, centralized, and coordinated. The profit functions are modeled and the optimal decisions are computed under each structure.

Decentralized structure

In the decentralized model, each member independently optimizes its own profitability (Nouri, Hosseini-Motlagh, Nematollahi, & Sarker, 2018). Under the decentralized structure, when the members have the same decision power, the Nash game should be used to model the problem (Xie & Neyret, 2009). Accordingly, in this paper, the Nash game is played between the members. Under the Nash game, the

members simultaneously decide on their decisions without considering the effects of their actions on the strategies of other members. Considering such a Nash game approach is also used in the related literature such as Xie and Neyret (2009). In the following lines, the member's profit function is modeled and solved.

Manufacturer's problem

In the proposed GSC, the manufacturer invests in the R&D effort to reduce the energy consumption of the washing machines. Therefore, the manufacturer's decision variable is the R&D effort, (q). The R&D cost is a quadratic function and can be calculated as $\frac{1}{2}\eta q^2$. In the

literature, such a quadratic function is applied (e.g., Dai et al., 2017). Since the retailer faces partial shortage, the total demand for the manufacturer is equal to the demand for the retailer minus lost sales. Therefore, the manufacturer's profit function consists of the revenue obtained from selling products and the cost of the R&D effort, and is formulated as follows:

$$\Pi_m(q) = (w - c) \left(d + \tau q - \theta p - \frac{\alpha \sigma (T + L)^{\frac{1}{2}} G(k)}{T} \right) - \frac{1}{2} \eta q^2 \tag{1}$$

Proposition 1. The profit function of manufacturer is concave w.r.t. q under the decentralized Nash game.

By solving Eq. $\frac{\partial \Pi_m(q)}{\partial q} = 0$, the optimal value of q is as follows:

$$q^* = \frac{\tau(w - c)}{\eta} \tag{2}$$

Retailer's problem

In the investigated GSC, a periodic review inventory system is applied by the retailer. The retail price, the order-up-to level (R), and review period (T) are determined to optimize the profit of the retailer. The retailer pays for ordering, holding and shortage costs in each period. The ordering cost can be computed as $\frac{A_r}{T}$. According to Montgomery, Bazaraa, and Keswani (1973), the expected holding cost is calculated

as $h_r \left[R - DL - \frac{DT}{2} + \alpha E(x-R)^+ \right]$, where $E(x-R)^+$ is the expected shortage at the end of each cycle. The expected shortage cost is modeled as $\frac{\pi + \alpha(p-w)}{T} E(x-R)^+$ in which $\pi + \alpha(p-w)$ is the unit cost of lost sale.

Thus, the retailer's profit function consists of the revenue and the costs, and is calculated as follows:

$$\begin{aligned} \Pi_r(T, R, p) = & (p-w)(d + \tau q - \theta p) - \frac{A_r}{T} - \frac{\pi + \alpha(p-w)}{T} E(x-R)^+ \\ & - h_r \left[R - DL - \frac{DT}{2} + \alpha E(x-R)^+ \right] \end{aligned} \quad (3)$$

The market demand (i.e., $D = d + \tau q - \theta p$) follows a normal distribution with mean $D(T+L)$ and standard deviation $\sigma(T+L)^{\frac{1}{2}}$. Therefore, the order-up-to level is $R = D(T+L) + k\sigma(T+L)^{\frac{1}{2}}$, in which k is the safety factor. The expected shortage at the end of each cycle is calculated as:

$$E(x-R)^+ = \int_R^\infty (x-R) f_x(x) dx = \int_k^\infty \sigma \sqrt{T+L} (Z-K) f_z(z) dz = \sigma \sqrt{T+L} G(k) \quad (4)$$

where, $G(k) = \int_k^\infty (z-k) f_z(z) dz = \varphi_z(k) - k[1 - \phi_z(k)]$, $\varphi_z(k)$ denotes the standard normal density function (*p. d. f*) and $\phi_z(k)$ shows the standard normal cumulative distribution function (*c. d. f*). Therefore, by substituting order-up-to level and according to Eq. (4), the retailer's profit function can be transformed to:

$$\begin{aligned} \Pi_r(T, k, p) = & (p-w)(d + \tau q - \theta p) - \frac{A_r}{T} - \\ & h_r \left[\frac{(d + \tau q - \theta p)T}{2} + k\sigma(T+L)^{\frac{1}{2}} + \alpha\sigma(T+L)^{\frac{1}{2}} G(k) \right] \\ & - \frac{\pi + \alpha(p-w)}{T} \sigma(T+L)^{\frac{1}{2}} G(k) \end{aligned} \quad (5)$$

Proposition 2. The profit function of the retailer is concave w.r.t. k and p , for a given T under the decentralized Nash game.

Setting $\frac{\partial \Pi_r(T, k, p)}{\partial k} = 0$ and $\frac{\partial \Pi_r(T, k, p)}{\partial p} = 0$, the optimal values of k and p are calculated as follows:

$$k^* = \phi_z^{-1} \left(1 - \frac{h_r}{\left(h_r \alpha + \left(\frac{\pi + \alpha(p-w)}{T} \right) \right)} \right) \tag{6}$$

$$p^* = \frac{d + \frac{\tau^2(w-c)}{\eta} + \theta \left(w + \frac{h_r T}{2} \right) - \frac{\alpha \sigma (T+L)^{\frac{1}{2}} G(k)}{T}}{2\theta} \tag{7}$$

Theorem1. In the decentralized structure, the upper bound of review period T can be calculated as follows:

$$T^0 = \frac{\pi + \alpha(p-w)}{h_r - h_r \alpha} \tag{8}$$

The retailer’s optimal decisions are determined by developing an optimal algorithm as follow:

Optimal algorithm of the retailer

- Step1.** Set $T = \varepsilon$, where ε is the minimum possible value for T .
- Step2.** Set $p = w$, where w is the minimum possible value for p .
- Step3.** Calculate k^* using Eq.(6).
- Step4.** Calculate p^* using Eq.(7).
- Step5.** If the difference between two successive values of p is negligible, then go to Step 6, otherwise, go to Step 3.
- Step6.** Calculate the retailer profit function using Eq.(5) for the values (T, k^*, p^*) obtained in steps (3) and (4).
- Step7.** If $T > T^0$ stop the algorithm; otherwise, set $T = T + \varepsilon$ and go to step2.
- Step8.** A combination of (T, k^*, p^*) which leads to the maximum retailer’s profit is the optimal solution.

Centralized structure

In this structure, the optimal decisions are determined from the whole GSC viewpoint (Asl-Najafi, Yaghoubi, & Azaron,2018). The profit function of GSC is obtained from the sum of the members’ profit functions. Thus, the GSC profit function is formulated as:

$$\begin{aligned} \Pi_{sc}(T, k, p, q) = & \Pi_m(q) + \Pi_r(T, k, p) = (p-c)(d + \tau q - \theta p) - \frac{A_r}{T} \\ & - h_r \left[\frac{(d + \tau q - \theta p)T}{2} + k\sigma(T+L)^{\frac{1}{2}} + \alpha\sigma(T+L)^{\frac{1}{2}} G(k) \right] \\ & - \frac{\pi + \alpha(p-c)}{T} \sigma(T+L)^{\frac{1}{2}} G(k) - \frac{\eta q^2}{2} \end{aligned} \quad (9)$$

Proposition3. The GSC profit function is concave w.r.t. k , p , and q for a given T under the centralized structure.

Setting $\frac{\partial \Pi_{sc}(T, k, p, q)}{\partial k} = 0$, $\frac{\partial \Pi_{sc}(T, k, p, q)}{\partial p} = 0$ and $\frac{\partial \Pi_{sc}(T, k, p, q)}{\partial q} = 0$, the optimal decisions are equal to:

$$k^{**} = \phi_z^{-1} \left(1 - \frac{h_r}{h_r \alpha + \frac{\pi + \alpha(p-c)}{T}} \right) \quad (10)$$

$$q^{**} = \frac{2(p-c)\tau - h_r \tau T}{2\eta} \quad (11)$$

$$p^{**} = \frac{2Td + 2T\tau q + 2c\theta T + h_r \theta T^2 - 2\alpha\sigma(T+L)^{\frac{1}{2}} G(k)}{4\theta T} \quad (12)$$

Theorem2. In the centralized structure, the upper bound of review period T is equal to:

$$\tilde{T} = \frac{\alpha(p-c) + \pi}{h_r - h_r \alpha} \quad (13)$$

To obtain the optimal decisions in this structure, an algorithm is proposed as follows:

Optimal algorithm of the whole GSC

Step1. Set $T = \varepsilon$, where ε is the minimum possible value for T .

Step2. Set $p = c$, where c is the minimum possible value for p .

Step3. Calculate k^{**} using Eq. (10).

Step4. Calculate q^{**} using Eq. (11).

Step5. Calculate p^{**} using Eq. (12).

Step6. If the difference between two successive values of p is negligible, then go to step7, otherwise, go to step 3.

Step7. Calculate the whole GSC profit function using Eq. (9) for the values $(T, k^{**}, p^{**}, q^{**})$ obtained in steps (3), (4) and (5).

Step8. If $T > \tilde{T}$ stop the algorithm, otherwise, set $T = T + \varepsilon$ and go to step2.

Step9. A combination of $(T, k^{**}, p^{**}, q^{**})$ which leads to the maximum GSC's profit is the optimal solution.

Under the centralized model, the GSC profitability improves compared to the decentralized structure because the decisions are calculated from the whole GSC viewpoint. However, the profit of the members does not necessarily improve in the centralized structure compared to the decentralized structure. Therefore, the member who incurs loss may not participate in the centralized model. In such a situation, an incentive contract is proposed to encourage all members to participate in the coordinated model.

Coordinated structure

Under the coordinated model, by an incentive contract, each GSC member determines its own decisions (i.e., the R&D effort level, retail price, review period, and safety factor) equal to that of the centralized structure. In the coordinated structure, not only the profit of the GSC improves compared to the decentralized structure but also each member's profitability increases in comparison with the decentralized structure. In the current paper, a delay in payment mechanism is used as an incentive mechanism. Under this mechanism, the manufacturer offers the retailer to settle its account within a permissible time after receiving the products. In the proposed contract, the unpaid money can be invested with the interest rate of i by the retailer. Thus, the retailer can gain profit from the delay in payment under the coordinated structure. In the proposed contract, the length of the delay period is a significant factor that must be acceptable to both members. Therefore, under the coordinated structure, the retailer's profit function is formulated as follow:

$$\begin{aligned}
\Pi_r^{co}(t_{T^{**}, k^{**}, p^{**}, q^{**}}) &= (p^{**} - w)(d + \tau q^{**} - \theta p^{**}) - \frac{A_r}{T^{**}} \\
&- h_r \left[\frac{(d + \tau q^{**} - \theta p^{**}) T^{**}}{2} + k^{**} \sigma(T^{**} + L)^{\frac{1}{2}} + \alpha \sigma(T^{**} + L)^{\frac{1}{2}} G(k^{**}) \right] \\
&- \frac{\pi + \alpha(p^{**} - w)}{T^{**}} \sigma(T^{**} + L)^{\frac{1}{2}} G(k^{**}) + iwt \left[(d + \tau q^{**} - \theta p^{**}) - \frac{\alpha \sigma(T^{**} + L)^{\frac{1}{2}} G(k^{**})}{T^{**}} \right]
\end{aligned} \tag{14}$$

where the last term is the obtained profit from investing the unpaid money within the delay period. This term contains four parts, the annual rate of return i , the wholesale price w , the period of the delay t and the total number of products that are purchased by the retailer. The retailer takes part in the coordinated structure if and only if its profitability improves in the coordinated structure in comparison with the decentralized model. Accordingly, the following condition is satisfied under the coordinated structure:

$$\Pi_r^{co}(t_{T^{**}, k^{**}, p^{**}, q^{**}}) \geq \Pi_r^{dec}(T^*, k^*, p^*, q^*) \tag{15}$$

Using Eq. (15) and substituting the profit function of retailer in the coordinated and decentralized models, the lowest value of t that entices the retailer to accept the coordinated model is calculated as follows:

$$\begin{aligned}
t^{min} &= \left[(p^* - w)(d + \tau q^* - \theta p^*) - (p^{**} - w)(d + \tau q^{**} - \theta p^{**}) + A_r \left(\frac{1}{T^{**}} - \frac{1}{T^*} \right) \right. \\
&- h_r \left[\frac{(d + \tau q^* - \theta p^*) T^*}{2} + k^* \sigma(T^* + L)^{\frac{1}{2}} + \alpha \sigma(T^* + L)^{\frac{1}{2}} G(k^*) \right] \\
&+ h_r \left[\frac{(d + \tau q^{**} - \theta p^{**}) T^{**}}{2} + k^{**} \sigma(T^{**} + L)^{\frac{1}{2}} + \alpha \sigma(T^{**} + L)^{\frac{1}{2}} G(k^{**}) \right] \\
&- \frac{\pi + \alpha(p^* - w)}{T^*} \sigma(T^* + L)^{\frac{1}{2}} G(k^*) + \frac{\pi + \alpha(p^{**} - w)}{T^{**}} \sigma(T^{**} + L)^{\frac{1}{2}} G(k^{**}) \left. \right] \\
&/ \left[iw \left((d + \tau q^{**} - \theta p^{**}) - \frac{\alpha \sigma(T^{**} + L)^{\frac{1}{2}} G(k^{**})}{T^{**}} \right) \right]
\end{aligned} \tag{16}$$

Furthermore, under the proposed mechanism, the manufacturer loses the investment opportunities during the delay period. Thus, the manufacturer incurs costs. Under the coordinated structure, the profit function of manufacturer is modeled:

$$\begin{aligned} \Pi_m^{co}(t_{T^{**},k^{**},p^{**},q^{**}}) &= (w-c) \left(d + \tau q^{**} - \theta p^{**} - \frac{\alpha\sigma(T^{**}+L)^{\frac{1}{2}}G(k^{**})}{T^{**}} \right) \\ &- \frac{1}{2}\eta q^{**2} - iwt \left(\left(d + \tau q^{**} - \theta p^{**} \right) - \frac{\alpha\sigma(T^{**}+L)^{\frac{1}{2}}G(k^{**})}{T^{**}} \right) \end{aligned} \tag{17}$$

where the last term is the cost that the manufacturer pays in this contract. The manufacturer takes part in the coordinated structure if and only if its profit increases compared to the decentralized structure. Thus, the following condition is satisfied in the coordinated model:

$$\Pi_m^{co}(t_{T^{**},k^{**},p^{**},q^{**}}) \geq \Pi_m^{dec}(T^*,k^*,p^*,q^*) \tag{18}$$

Using Eq. (18), the upper limit for t can be calculated as:

$$\begin{aligned} t^{max} &= \left[(w-c) \left[\tau(q^{**}-q^*) + \theta(p^*-p^{**}) + \alpha\sigma \left(\frac{(T^*+L)^{\frac{1}{2}}G(k^*)}{T^*} - \frac{(T^{**}+L)^{\frac{1}{2}}G(k^{**})}{T^{**}} \right) \right] + \frac{\eta}{2}(q^{**2}-q^{*2}) \right] \\ &/ \left[iw \left(\left(d + \tau q^{**} - \theta p^{**} \right) - \frac{\alpha\sigma(T^{**}+L)^{\frac{1}{2}}G(k^{**})}{T^{**}} \right) \right] \end{aligned} \tag{19}$$

When the parameter representing the delay period is in the interval $[t^{min}, t^{max}]$, the GSC is coordinated by the delay in payment mechanism. When parameter t is equal to t^{min} , the entire extra profit is achieved by the manufacturer and when t is equal to t^{max} , the retailer gains all the extra profit. Therefore, to gain the exact value for the contract parameter (t) and fairly divide the surplus profit, a profit allocation strategy based on the members' profits in the decentralized model is proposed in the following subsection.

Profit allocation strategy

In this subsection, a profit allocation strategy for allocating the surplus benefit to the members is developed. Under this strategy, the exact value of the contract parameter (t) is calculated based on the members' profits in the decentralized structure. The obtained surplus profit from the coordinated model can be calculated by $\Delta = \Pi_{sc}^{co}(T^{**}, k^{**}, p^{**}, q^{**}) - \Pi_{sc}^{dec}(T^*, k^*, p^*, q^*)$. Note that the manufacturer's profit in the decentralized structure is shown by A and the retailer's profit in the decentralized structure is B , and $A + B$ is the entire GSC profit in the decentralized model. According to the members' profits in the decentralized structure, $\frac{A}{A+B}$ percent from the surplus profit (Δ) is the share of the manufacturer under the coordination model, and so we have:

$$\Pi_m^{co}(t_{T^{**}, k^{**}, p^{**}, q^{**}}) = \Pi_m^{dec}(T^*, k^*, p^*, q^*) + \frac{A}{A+B} \Delta \quad (20)$$

By replacing Eqs. (17) and (1) into Eq. (20), the exact value of $t^{sharing}$ is obtained as follows:

$$t^{sharing} = \left[(w-c) \left[\tau(q^{**} - q^*) + \theta(p^* - p^{**}) + \alpha \sigma \left(\frac{(T^* + L)^{\frac{1}{2}} G(k^*)}{T^*} - \frac{(T^{**} + L)^{\frac{1}{2}} G(k^{**})}{T^{**}} \right) \right] + \frac{\eta}{2} (q^{*2} - q^{**2}) - \frac{A}{A+B} \Delta \right] / \left[iw \left((d + \tau q^{**} - \theta p^{**}) - \frac{\alpha \sigma (T^{**} + L)^{\frac{1}{2}} G(k^{**})}{T^{**}} \right) \right] \quad (21)$$

Note that this value is in the interval $[t^{min}, t^{max}]$, and the retailer's gained profit is exactly equal to $\frac{B}{A+B} \Delta$.

Numerical experiment

To evaluate the performance of the proposed models, the data of a real home appliance company is analyzed. In the investigated company, the manufacturing process brings a cost with coefficient $c = \$350$ /washing machine. In addition, to enhance the market share, the manufacturer (i.e., PC) invests in R&D effort which imposes the R&D cost with coefficient $\eta = 300$. The R&D effort of the company influences the washing machine demand with coefficient $\tau = 9$. The company sells the washing machine through the retailer. The washing

machine is sold to the retailer with the wholesale price of $w = \$400$ /washing machine. The retailer determines the retail price which has impacts on the demand with coefficient $\theta = 12$. The potential demand is $d = 6000$ washing machine. The level of washing machine demand uncertainty is computed based on the historical data of the company as $\sigma = 1500$. Accordingly, the retailer's demand can be shown as $D = 6000 + 9q - 12p$. The retailer applies a periodic review inventory model to replenish its stock. Thus, he reviews the inventory at every T units of time and places orders up to the level R . The retailer's ordering cost is $A_r = \$250$ /order. The order is delivered to the retailer within $L = 3$ days. The retailer's holding cost is $h_r = \$7$ /washing machine. The backordered cost for the retailer is $\pi = \$1$ /washing machine. The data of the real case study is demonstrated in Table 2.

Table 2. The data of the case study

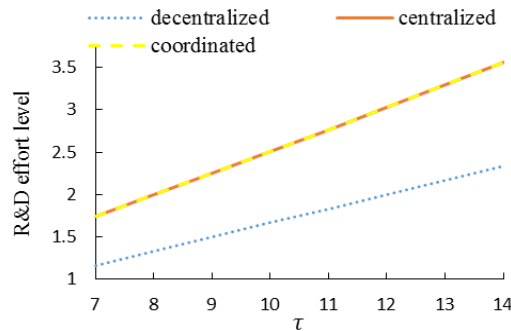
parameters	d, τ, θ	h_r, A_r, π	w, c	L	σ	η	i	α
Data	6000, 9, 12	7, 250, 1	400, 350	3	1500	300	0.13	0.3

Table 3 shows the results of examining the case study under three structures. According to Table 3, under the centralized structure, the profits of whole GSC and manufacturer increase in comparison with the decentralized structure. However, in the centralized structure, the profitability of the retailer decreases compared to the centralized structure. Thus, to convince the retailer to adopt the centralized decisions, a delay in payment mechanism is proposed to coordinate the proposed GSC. Under the coordinated structure, the profits of all members and the whole GSC increase compared to the decentralized structure. Therefore, all members are satisfied to accept the coordination model. Moreover, under the coordinated structure, the manufacturer's R&D effort level is more than its effort under the decentralized structure. It is concluded that the developed mechanism not only enhances the profits of the whole GSC and its members but also enhances the environmental performance of the GSC. Moreover, under the proposed mechanism, the retail price is less than that of the decentralized model, which leads to more market demand and extra profit.

Table3. Results of the real case under the three different models

	Decentralized structure	Centralized structure	Coordinated structure
$T(day)$	38.7	34.46	34.46
k	1.68	1.91	1.91
p	449.61	425.33	425.33
q	1.5	2.25	2.25
Π_r	20445.1	13227.18	23663.73
Π_m	29206.96	44241.49	33804.95
Π_{sc}	49652.06	57468.68	57468.68
$t^{min}(day)$	-	-	56.29
$t^{max}(day)$	-	-	117.25
$t^{sharing}(day)$	-	-	81.39

In the following lines, a set of sensitivity analysis is provided based on analyzing some important parameters. Figure 1 examines the effect of consumer sensitivity to R&D effort on the R&D effort level under all three models. Figure 1 reveals that with the increase in the consumers' environmental awareness, τ , the manufacturer's R&D effort increases under all three models. However, in the coordinated model, the R&D effort is more than that under the decentralized model considering the different values of τ . In addition, under a high value of τ , the difference in the R&D effort level of the manufacturer in the decentralized and coordinated models increases. Therefore, the developed mechanism is able to improve the green effort in comparison with the decentralized structure. It is concluded that the developed contract is efficient from the environmental viewpoint.

**Fig. 1. Effect of consumer sensitivity to R&D effort on the R&D effort level**

The effect of consumer sensitivity to R&D effort on the GSC profit and retailer's profit is shown in Fig. 2. It is revealed in Fig. 2 that under all structures, the profit of GSC and the retailer increase with respect to consumer sensitivity to R&D effort. However, in the coordinated structure, the GSC profit and the retailer's profit are more than their values under the decentralized structure for all values of τ . This finding illustrates the capability of the developed contract. As a result, the proposed contract is efficient from economic viewpoint.

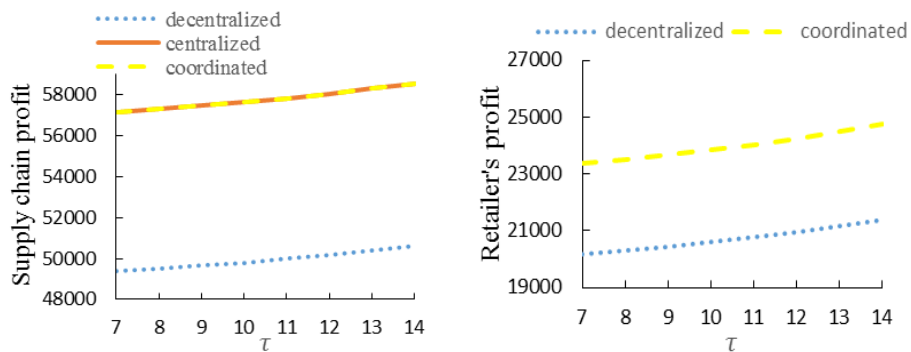


Fig. 2. Effect of consumer sensitivity to R&D effort on the GSC profit and retailer's profit

The impacts of price elasticity of demand on the retail price and the manufacturer's profitability are demonstrated in Figure 3. With the increase in θ , the retail price is reduced under all three models. Moreover, as can be seen, in the coordinated structure, the retail price is less than that of the decentralized structure for all values of θ . It is concluded that the contract is capable of coordinating the GSC even with high values of θ . On the other hand, with the increase in θ , the manufacturer's profitability decreases under all models. In the centralized model, the highest profit is achieved by the manufacturer compared to the other models but the centralized model is not acceptable for the retailer and he would not participate in the centralized structure. As it can be seen, with the application of the contract, the manufacturer's profit is more than its profit under the decentralized structure. It can be concluded that the proposed contract is more profitable for the manufacturer than the decentralized

structure. As a result, although managers have to decrease the price under high price elasticity of demand, they can compensate the decrease in the profitability by participating in the coordinated model.

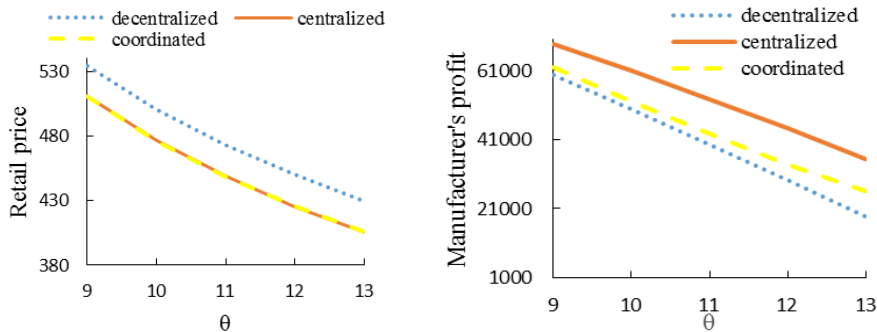


Fig. 3. Effect of price elasticity of the demand on the price and the profit of manufacturer

The effect of demand uncertainty on the GSC profit is illustrated in Figure 4. The GSC profitability decreases under three models with respect to the changes in σ . However, for all values of σ , the profitability of the GSC in the coordinated model is more than that in the decentralized model. It can be concluded that the developed mechanism can coordinate the GSC even under high values of σ .

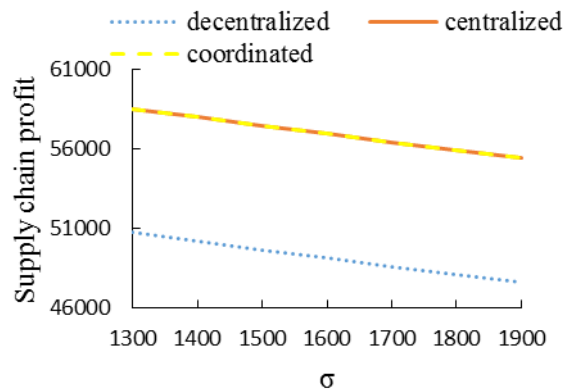


Fig. 4. Effect of demand uncertainty on the GSC profit

Figure 5 examines the impacts of changes in the lost sale rate on the review period length, the safety stock, and the profitability of retailer. As it can be seen, with the increase in α , more volumes of safety stock are hold by the retailer to prevent shortages. According to Figure 5, in the coordinated model, the retailer holds more safety stock compared to the decentralized model. Holding more safety stock leads to higher inventory-holding costs to the retailer. However, the retailer's profitability improves in the coordinated structure compared to the decentralized structure. Accordingly, the proposed contract is useful from social viewpoint. In other words, under the coordination model, the retailer can improve its service level, which in turn is beneficial for customers. That is why the coordination model is of social benefit. Moreover, with the increase in α , the retailer reduces the review period. Figure 5 illustrates that in the coordinated structure, the review period is less than that of the decentralized structure. However, the coordination structure increases the retailer's profitability more than the decentralized structure. Accordingly, the developed contract is beneficial for the retailer even under high values of the lost sale.

The trend of the coordinated parameter by changing the annual rate of return on investment is shown in Figure 6. Figure 6 demonstrates that the interval $[t^{min}, t^{max}]$ is not vacant for each rate of return on investment. Therefore, the GSC can be coordinated with all values of i . Moreover, the interval $[t^{min}, t^{max}]$ is larger in the low values of interest rate and it becomes smaller by increasing i . Thus, with the delay in payment contract, coordinating the proposed GSC will be easier for low values of i . Furthermore, with all values of i , t is in the interval $[t^{min}, t^{max}]$. It can be concluded that the profit allocation strategy is capable of finding the exact value based on the members' profit in the decentralized structure.

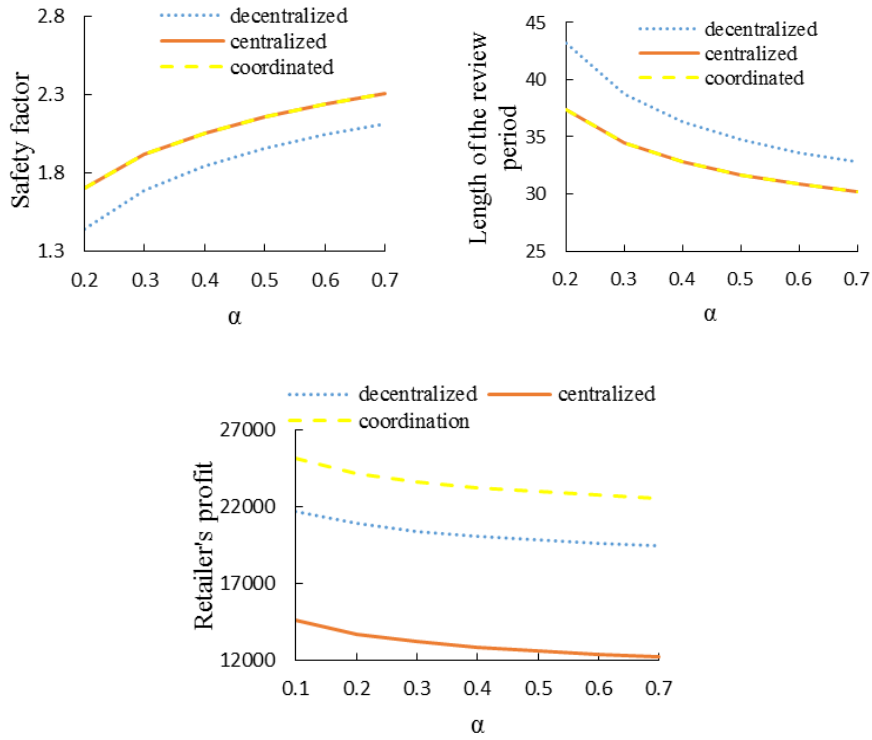


Fig. 5. Effect of increasing lost sale on the review period length, the safety factor, and the retailer's profit

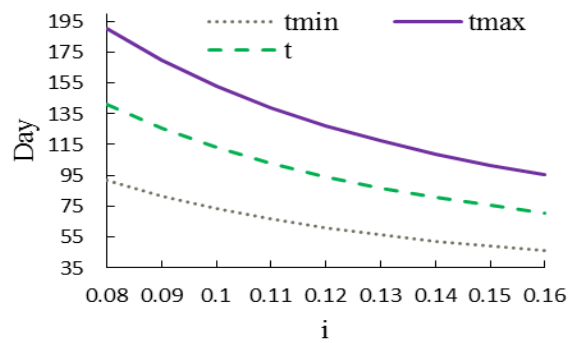


Fig. 6. Trend of coordinated parameter by increasing annual rate of return on investment

Conclusion

We investigated a real case of home appliance industry in this research. The proposed GSC consists of one manufacturer and one retailer. The company produces the washing machines and makes the R&D effort to enhance the degree of green of products. The washing machines are sold by the retailer. The retailer determines the safety factor, review period, and retail price. The stochastic market demand function depends on the R&D effort and the retail price. Three structures were modeled: decentralized, centralized, and coordinated. The Nash game was followed by the members in the decentralized structure. In the coordinated structure, a delay in payment mechanism was proposed which improved the members' profit. The minimum, maximum, and exact values of the delay period were calculated under the coordinated model. The data of the case study were applied and the results showed 15% improvement in the profit of GSC under the coordinated model compared to the decentralized structure. After that, the sensitivity analyses on some parameters were analyzed. The results indicated that under the delay in payment contract, not only the profit of whole GSC increased compared to the decentralized structure but also the profit of its members improved compared to the decentralized model. Moreover, the developed contract was able to coordinate the proposed GSC even under high levels of demand uncertainty. In addition, the delay in payment contract was capable of enhancing the GSC performance from the environmental viewpoint by improving the R&D effort level.

This study has several limitations that can be investigated in the future studies. In this paper, we considered one manufacturer in the GSC while in the real world, more than one manufacturer exist which compete with each other. As a future study, the competition on R&D effort among the manufacturers can be investigated. Furthermore, in this study, the expected value is used to calculate the shortage at the end of each cycle. However, the measures such as VaR, CVaR, EVaR can be used to calculate the expected shortage. Since these measures reflect better behavior in stochastic optimization, these measures can be considered for future studies. Moreover, a Nash game is applied to model the decentralized model in this study. Different game structures can be used to model the GSC for future researches.

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Appendix.

Proof of Proposition1. To prove the concavity of the manufacturer’ profit function, the second derivation of Π_m w.r.t. q must be negative. According to $\partial^2\Pi_m(q)/\partial q^2 = -\eta < 0$, the manufacturer’s profit function is concave.

Proof of Proposition2. To prove the concavity of the retailer’s profit function, we calculate the following Hessian matrix.

$$H(\Pi_r(T, k, p)) = \begin{bmatrix} \frac{\partial^2\Pi_r(T, k, p)}{\partial k^2} & \frac{\partial^2\Pi_r(T, k, p)}{\partial k\partial p} \\ \frac{\partial^2\Pi_r(T, k, p)}{\partial p\partial k} & \frac{\partial^2\Pi_r(T, k, p)}{\partial p^2} \end{bmatrix} \tag{A1}$$

$$H_{11} = -\left[h_r\alpha + \frac{\pi + \alpha(p-w)}{T} \right] \sigma(T+L)^{\frac{1}{2}} \phi_z(k) < 0 \tag{A2}$$

$$H_{22} = \left(2\theta \left[h_r\alpha + \frac{\pi + \alpha(p-w)}{T} \right] \sigma(T+L)^{\frac{1}{2}} \phi_z(k) \right) - \left(\frac{\alpha}{T} \sigma(T+L)^{\frac{1}{2}} (\phi_z(k)-1) \right)^2 > 0 \tag{A3}$$

The first minor (H_{11}) is negative. The second minor (H_{22}) is positive under the following condition:

$$2\theta \left[h_r\alpha + \frac{\pi + \alpha(p-w)}{T} \right] \phi_z(k) > \frac{\alpha^2}{T^2} \sigma(T+L)^{\frac{1}{2}} (\phi_z(k)-1)^2$$

Proof of theorem 1. Under the decentralized structure, an upper bound for the review periodic (T) is calculated by Eq. (6). In Eq. (6), $\phi_z(k)$ is the normal distribution function and is always positive.

Accordingly, $1 - \phi_z(k) \leq 1$ and we have $T^0 = \frac{\pi + \alpha(p-w)}{h_r - h_r\alpha}$.

Proof of Proposition3. To prove the concavity of the whole GSC profit function, we calculate the following Hessian matrix as:

$$H(\Pi_{sc}(T, k, p, q)) = \begin{bmatrix} \frac{\partial^2 \Pi_{sc}(T, k, p, q)}{\partial p^2} & \frac{\partial^2 \Pi_{sc}(T, k, p, q)}{\partial p \partial k} & \frac{\partial^2 \Pi_{sc}(T, k, p, q)}{\partial p \partial q} \\ \frac{\partial^2 \Pi_{sc}(T, k, p, q)}{\partial k \partial p} & \frac{\partial^2 \Pi_{sc}(T, k, p, q)}{\partial k^2} & \frac{\partial^2 \Pi_{sc}(T, k, p, q)}{\partial k \partial q} \\ \frac{\partial^2 \Pi_{sc}(T, k, p, q)}{\partial q \partial p} & \frac{\partial^2 \Pi_{sc}(T, k, p, q)}{\partial q \partial k} & \frac{\partial^2 \Pi_{sc}(T, k, p, q)}{\partial q^2} \end{bmatrix} \quad (A4)$$

$$H_{11} = -2\theta < 0 \quad (A5)$$

$$H_{22} = \left(2\theta \left[h_r \alpha + \frac{\pi + \alpha(p-c)}{T} \right] \sigma(T+L)^{\frac{1}{2}} \varphi_z(k) \right) - \left(\frac{\alpha}{T} \sigma(T+L)^{\frac{1}{2}} (\phi_z(k)-1) \right)^2 > 0 \quad (A6)$$

The first minor (H_{11}) is always negative. The second minor (H_{22}) is positive under the following condition:

$$2\theta \left[h_r \alpha + \frac{\pi + \alpha(p-c)}{T} \right] \varphi_z(k) > \frac{\alpha^2}{T^2} \sigma(T+L)^{\frac{1}{2}} (\phi_z(k)-1)^2.$$

$$H_{33} = \sigma(T+L)^{\frac{1}{2}} \varphi_z(k) \left(h_r \alpha + \frac{\pi + \alpha(p-c)}{T} \right) (\tau^2 - 2\theta\eta) + \frac{\eta \alpha^2 \sigma^2(T+L) (\phi_z(k)-1)^2}{T^2} < 0 \quad (A7)$$

The third minor (H_{33}) is negative under the following condition:

$$\varphi_z(k) \left(h_r \alpha + \frac{\pi + \alpha(p-c)}{T} \right) (\tau^2 - 2\theta\eta) < - \frac{\eta \alpha^2 \sigma(T+L)^{\frac{1}{2}} (\phi_z(k)-1)^2}{T^2}.$$

Proof of theorem 2. Under the centralized structure, an upper bound for the review periodic (T) is determined by Eq. (10). In Eq. (10), $\phi_z(k)$ is the normal distribution function and is always positive.

Accordingly, $1 - \phi_z(k) \leq 1$ and we have $\tilde{T} = \frac{\alpha(p-c) + \pi}{h_r - h_r \alpha}$.