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Modeling and Analyzing Incremental Quantity Discounts in Transportation Costs for a Joint Economic Lot Sizing Problem

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Abstract
Joint economic lot sizing (JELS) addresses integrated inventory models in a supply chain. Most of the studies in this field either do not consider the role of the transportation cost in their analysis or consider transportation cost as a fixed part of the ordering costs. In this article, a model is developed to analyze an incremental quantity discount in transportation cost. Appropriate equations are derived to compute the costs related to the inventory systems in the buyer and vendor sites. Then, a procedure including five steps is proposed to optimize the model and determine the values of the decision variables. To analyze the performance of the incremental discount, the JELS problem is studied in two other states of transportation costs. These states include fixed transportation cost and all-unit quantity discount. Moreover, some numerical analyses are carried out to show the impact of transportation costs and inventory-related parameters on the system performance. According to the results of the sensitivity analyses, it is observed that all-unit quantity discount leads to a better performance of the system in comparison with the incremental quantity discount.

Keywords
Joint economic lot sizing, All-unit quantity discount, Incremental quantity discount, Integrated inventory model, Transportation.

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Introduction

Traditionally, inventory management of a supply chain (SC) is considered from the viewpoint of individual entities such as producers, distributors, or sellers. In this approach, each entity considers only the parameters directly related to his company. Although this approach optimizes the inventory decisions for an individual company, there is no guarantee that it is optimal for the whole SC. With the seminal work of Goyal (1976), a stream of research has emerged aiming at the coordination of the decisions related to the inventory management in the whole SC rather than individual entities in it. Integrated inventory models in SC literature are usually referred to as joint economic lot sizing (JELS) problem. The underlying problem of a JELS problem can be stated as follows. A supply chain consisting of one vendor and one buyer is considered. The vendor manufactures an item at a rate P and incurs holding cost for each item in inventory and also sets up cost for each produced lot. Each lot produced in the vendor site is delivered to the buyer according to a specific shipment policy, e.g. equal-sized-shipments. The demand at the buyer site is deterministic which is satisfied from the items received from the vendor site. The objective is to determine the size of the lot produced by the vendor and the shipment policy so that the system cost can be optimized.

Considering an infinite production rate and a lot-for-lot shipment policy, the literature of JELS started with the basic model of Goyal (1976). After that, studies of JELS have been developed according to the different shipment policies. We refer the interested readers to Ben-Daya, Darwish and Ertogral (2008) for more information on these policies. Finally, Hill (1999) presented a general model for this problem, placing no restrictive assumptions on the shipment policies. Besides extending the JELS models based on the shipment policy, this topic has recently been developed in various directions, including 1) SC structure; 2) stochastic demand and lead time; 3) price sensitive demand; 4) product quality; 5) product deterioration; 6) set up/order cost or lead time reduction; and 7) learning effect. Considering transportation policies and their costs is another approach for developing JELS models. In comparison with the other directions mentioned, there are a few works in this regard. In fact, most studies in the domain of JELS do not explicitly take into consideration the transportation costs between the actors of different layers of SC. Transportation costs have been usually analyzed as a fixed part of the ordering cost.

In this paper, transportation costs are analyzed in a JELS problem. More specifically, a supply chain is considered to be consisting of one vendor and one buyer in which the vendor is a manufacturer that produces one type of product. For each lot, the vendor incurs a setup cost and transfers the lot in the equal-sized shipment to the buyer. Also, for each shipment, the buyer incurs an ordering cost. The vendor and the buyer incur holding cost in their corresponding warehouses. The problem is to determine the number of equal-sized shipments to the buyer and the size of each shipment, or consequently, the size of the produced lot in the vendor’s site and the number of equal-sized shipments, so that the total cost is minimized. Three scenarios for the transportation costs are considered, including 1) fixed transportation cost; 2) all-unit quantity discount, and 3) incremental quantity discount. First, an equation related to the total inventory costs of the SC is derived. Second, for each scenario, another equation is derived to compute the transportation costs of the system. Finally, corresponding to each scenario, a procedure is proposed to optimize the total cost of the SC, including transportation and inventory costs. The main novelty of this study is the development of a model that takes into account the incremental discount in transportation cost in a JELS problem. Also, the comparative studies of the three scenarios and sensitivity analyses are conducted. It should be noted that discount in transportation costs, which is investigated in this paper, is different from the quantity discount which is related to the purchasing cost. In the literature of inventory models, some studies consider either transportation discount or quantity discount while in some others both of them have been simultaneously analyzed (Darwish, 2008).

The rest of the paper is organized as follows. Section 2 presents a literature review of the JELS. In Section 3, the structure of the problem and the related assumptions are presented. Section 4 addresses the inventory costs for the considered supply chain. Also, in this section, the system is formulated under different transportation policies. Section 5 conducts some numerical analyses and comparative studies about the system. Finally, Section 6 concludes the paper.

Literature review

The economic order quantity (EOQ) model is one of the well-known models in the inventory and operation management. In February 1913, Ford Whitman Harris proposed EOQ model under the assumption of infinite production rate. The second major contribution in the field of inventory management is the development of economic production quantity (EPQ) proposed by Taft (1918). Instead of infinite production rate, EPQ model assumes a finite production rate. Although EOQ and EPQ models have been widely applied by practitioners in industry, these two basic models have some weaknesses. The obvious problem arises from the different restrictive assumptions of the models. For example, in the classic EOQ/EPQ model, the sole objective function is to minimize the total-inventory costs including holding cost and ordering costs. To overcome the weaknesses of EOQ/EPQ models, the inventory management models have been developed in several directions. As stated by (Andriolo *et al.*, 2014), the most notable directions of inventory models can be presented as follows: time varying demand, goods deterioration, quantity discount, inflation, variable lead time, trade credit, process deterioration, shortage and backlogs, imperfect quality items, and environment sustainability. Regarding these topics and extensions, a large body of papers exists and consequently some literature reviews are conducted corresponding to each direction (Engebrethsen & StéphaneDauzère-Pérès, 2018; Seifert, Seifert & Protopappa-sieke, 2013; Khan *et al.*, 2011; Horenbeek *et al.*, 2013).

With the growing interest in the concept of supply chain (SC), different members of a SC notice that inventories across the chain can be more efficiently managed by coordination of the decisions. Thus, integrated inventory models that are also known as JELS emerged by the seminal work of Goyal (1977). The first model of JELS developed by Goyal considers a single-vendor-single-buyer SC while the production rate is assumed infinite and a lot-for-lot shipment policy is applied. Hill (1999) proposed an improved version of Goyal’s model in which no restriction is placed on the shipment policy and the production rate is considered finite. Since then, JELS models have been developed in several directions, including 1) SC structure; 2) stochastic demand and lead time; 3) price and inventory dependent demand; 4) product quality; 5) product deterioration; 6) set up/order cost or lead time reduction; and 7) learning effect. Also, some literature review is done on the JELS and its extensions (Soysal *et al.*, 2019; Glock, 2012; Ben-Daya et al., 2008).

While the basic models of JELS were about the single-vendor-single-buyer SC, the researchers of SC and operation managers have tried to develop the JELS models by considering more realistic SC structures. Thus, the following structures of SC are also investigated: single-vendor-multi-buyer (Chen & Sarker, 2017; Rasay & Mehrjerdi, 2017), multi-vendor-single-buyer (Glock & Kim, 2014), and multi-vendor-multi-buyer (Sajadieh, Saber & Khosravi, 2013)

The primary models of JELS consider deterministic demand and lead times. To conform with the uncertainty of demand and lead times, the assumptions of deterministic demand and lead time are removed by many researchers in the future works (Kilic & Tunc, 2019; Abdelsalam & Elassal, 2014; Fernandes, Gouveia, & Pinho, 2013). Price and inventory dependent demand is another direction of the development of JELS models that are studied by Sajadieh, Thorstenson, and Akbari (2010). A production system may deteriorate with the increase in age and usage, making the production of inferior items inevitable. Thus, the extension of the JELS models to include product quality and process deterioration has been emerged as another direction (Wangsa & Wee, 2019; Kurdhi *et al.*, 2018; Al-Salamah, 2016). Some products such as medical items and food decay with the passing of time. Thus, the duration of time when these items are stored in the warehouse is a key factor in developing their inventory models. Some researchers have studied the JELS problem from this aspect (Lin *et al.*, 2019; Chang, 2014; Chung, Cárdenas-barrón & Ting, 2014).

Another important topic in the development of integrated inventory models regards the transportation costs. Generally, inventory and transportation are two important aspects of a SC. Besides the models of JELS that incorporate transportation costs, inventory routing problem (IRP) and production routing problem (PRP) are two major areas of integrating transportation and inventory decisions. Wangsa and Wee (2017, 2019) developed an integrated inventory model for a single-vendor-single-buyer (SVSB) supply chain while truckload and less-than-truckload shipment policies were considered for transportation costs. Lee and Fu (2014) studied the joint production and delivery policy with transportation cost for a make-to-order production facility in a SC. Using actual shipping rate data, transportation cost was considered as a fitted power function of the delivery quantity. Chen and Sarker (2014) studied a multiple-vendor-single-buyer (MVSB) system consisting of multiple suppliers and one assembler where a JIT was applied to the system. Assembler picked up parts from the suppliers based on a milk run mode, and transportation cost was considered through the vehicle routing problem. Sajadieh et al. (2013) studied a SC that consisted of three layers. In each layer, there were multiple actors. Demand rate was considered deterministic and lead time was stochastic. Transportation cost between each layer was displayed based on the all-unit quantity discount. Since the model derived was NP-hard, to optimize the model, an ant colony algorithm was applied. Rieksts and Ventura (2010) considered two modes of transportation in a SC: truckloads and less than truckload. They analyzed two different structures for the SC: one-warehouse-one-retailer and one-warehouse-multiple-retailer under constant demand rate and an infinite planning horizon. Lee and Wang (2010) studied a JELS in a three levels SC consisting of one supplier, one manufacturer, and one retailer. They considered the scenario of less than track load for transportation cost in which the carrier offered all-unit freight discount. Ertogral and Darwish (2007) studied a SVSB supply chain under two different scenarios for the transportation cost: (1) all-unit quantity discount and (ii2) over declaration. Table 1 presents some recent studies on JELS problem which are closer to our work.

Although in a JELS, the all-unit quantity discount of transportation cost is considered by Ertogral and Darwish (2007) and some other works (Sajadieh et al., 2013; Lee & Wang, 2010), there is no work - to the best of authors' knowledge - that deals with incremental quaintly discount in transportation cost. Therefore, in this paper, we analyze SVSB system under three different states: (1) system with fixed transportation cost; (2) all-unit quantity discount in transportation cost and (3) incremental discount in transportation cost. To reach the optimal solutions in each state, three procedures are suggested. Also, comparative studies are conducted regarding these scenarios.

Table 1. Classification of recent JELS models that incorporate transportation cost

Integrated inventory model	Transportation costs	Supply chain structure	Demand structure
(Ertogral & Darwish, 2007)	All-unit quantity discount	Single-vendor-single-buyer	deterministic
(Rieksts & Ventura, 2010)	Truck-load and less that truck-load-shipment	Single-vendor-single-buyer	deterministic
(Sajadieh et al., 2013)	All-unit quantity discount	Multi-vendor-multi-buyer	deterministic
(Lee & Fu, 2014)	Fitted power function	Single-vendor-single-buyer	deterministic
(Wangsa & Wee, 2017)	Freight cost	Single-vendor-single-buyer	Stochastic
(Wangsa & Wee, 2019)	Truck-load and less that truck-load-shipment	Single-vendor-single-buyer	Stochastic
Current study	Incremental quantity discount and all-unit quantity discount	Single-vendor-single-buyer	Deterministic

Problem statement

Consider a supply chain consisting of one vendor and one buyer in which the vendor is a manufacturer who produces one type of product. For each lot, the vendor incurs a setup cost and transfers the lot in the equal-sized shipment to the buyer. Also, for each shipment, the buyer incurs an ordering cost. The vendor and the buyer incur holding cost in their corresponding warehouses. The problem is to determine the number of equal-sized shipments to the buyer and the size of each shipment, and consequently, the size of the produced lot in the vendor's site and the number of equal-sized shipments, so that the total cost is minimized. Figure 1 displays the system.

Total costs of the system can be classified into (1) the costs of the inventory system, which include the set-up/ordering costs and holding costs of items in the vendor' and buyer's sites, and (2) the cost associated with the transportation of the lots between the vendor and buyer. Three different states are considered for the transportation costs between the buyer and the vendor: 1) transportation cost as a fixed part of the total system cost that is independent from shipment quantity, 2) all-unit quantity discount in the transportation cost, and 3) incremental quantity discount for the transportation cost.

To investigate the system, first an equation is derived which computes the total costs of the inventory system per time unit. Then, transportation costs are analyzed. Three scenarios are assumed for the transportation costs, and in each state, a procedure is presented which optimally determines the decision variables so that the system total cost can be minimized. Decision variables include the number of the shipments between the vendor and buyer in each manufactured lot, which is denoted by n, and the size of each shipment, which is denoted by q. It is obvious that n is an integer value while q is a real positive variable. For example, in Figure 1, the value of n is 5.

Assumptions of the model

1. Demand rate at the retailer site is constant.
2. Shortages are not allowed.
3. Planning horizon is considered infinite.
4. In order to complete the buyer order, production rate is considered greater than demand rate: $P>D$.
5. The produced lot, Q, is transferred from the vendor to the buyer in n equal-sized shipments.
6. The shipment policy is none-delayed, which means that transferring a lot could take place during production phase.
7. It is assumed that the buyer holding cost is greater than the vendor's.

The following notations are used:

i: index representing the state of the system ($i=1,2,3$)

(1 for fixed transportation cost, 2 for all-unit discount in transportation cost, and 3 for incremental discount in transportation cost);

P: the production rate of the vendor;

D: the demand rate of the buyer;

A_v : the production set up cost per cycle;

A_b: the buyer ordering cost per shipment;

h_v: the inventory holding cost in the vendor site;

h_b: the inventory holding cost in the buyer site;

$I_{b,i}$: the average inventory of the buyer in state i;

$I_{v,i}$: the average inventory of the vendor in state i;

$I_{s,i}$: the average inventory of the system in state i.

Decision variables:

n_i: the number of equal-sized shipments under state i;

q_i: the shipment size under state I;

Q_i : the production lot size (Q_i=n_iq_i) under state i.

Equations:

TIC_i (q_i,n_i): the total inventory cost in state i and in unit time that is a function of n_i and q_i;

TSC_i(q_i,n_i) : the total system cost in state i and in unit time that is a function of n_i and q_i;

TC_i (q_i) : the transportation cost under state i that is a function of q_i in state 2,3.

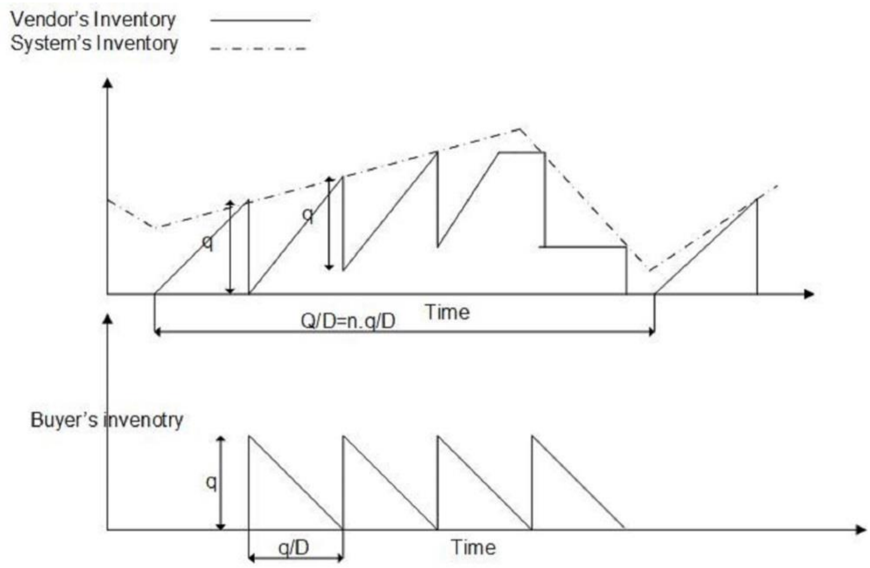

Fig. 1. Inventory level for the vendor, the buyer and the system (Ben-Daya et al., 2008)

Procedures for the optimization of the system total costs in each scenario

In this section, first, the functions related to the costs of the inventory system are derived. Then, the system is formulated under three different states of transportation cost. Finally, an example is presented.

1. Related costs of the inventory system

In this subsection, to calculate the inventory costs of the system, an equation is presented. In state *i*, the average inventory of the system can be expressed as follows:

$$I_{s,i} = \frac{q_i D}{P} + \frac{(p-D)nq_i}{2P} \tag{1}$$

Average inventory of the buyer is: $I_{b,i} = \frac{q_i}{2}$. Also, average inventory of the vendor can be expressed as the average inventory of the system minuses the average inventory of the buyer. Thus, the total inventory cost of the system can be expressed as follows:

$$\begin{aligned} TIC_i(q_i, n_i) &= \frac{(A_v + n_i A_b)D}{Q_i} + h_v(I_s - I_b) + h_b I_b \\ &= \frac{(A_v + n_i A_b)D}{Q_i} + h_v I_s + (h_b - h_v)I_b \qquad \forall i = 1,2,3 \end{aligned} \tag{2}$$

With respect to the decision variables, Eq.(2) is convex. We refer to Hill (1997) for the proof of convexity.

2. System formulation under different states of transportation costs

In the following lines, JELS problem is studied under three different states of transportation costs. These states include 1) fixed transportation cost, 2) all-unit quantity discount, and 3) incremental quantity discount.

Fixed transportation cost

In this state, transportation cost is considered as a fixed part of the total cost of the system. Thus, transportation cost is independent from shipment quantity between the vendor and buyer. Thus, the following equation displays the total costs of the system.

$$TSC_1(q_1,n_1)=TIC_1(q_1,n_1) + TC_1 \tag{3}$$

With respect to the convexity property of Eq. (2), Eq. (3) is also convex, and the optimal values of the variables can be calculated based on these two formulas:

$$n_1{*}=\sqrt{\frac{A_v\,(p\Delta h+2h_v D)}{h_v A_b\,(p-D)}} \tag{4}$$

$$q_1{*}=\left[\frac{D(A_v+nA_b)}{n_1\left(h_v[\frac{D}{P}+\frac{(P-D)n_1}{2P}]+\frac{\Delta h}{2}\right)}\right]^{1/2} \tag{5}$$

Based on the fact that n_1 is integer and considering the above two formulas, i.e., Eq. (2) and Eq. (3), procedure 1 is proposed to calculate n_1 and q_1 and to determine the optimal costs of the system.

Procedure 1.

1. Find the optimal continuous value of n_1 according to Eq. (4)
2. Let $n_1=\lceil n_1^* \rceil$, substitute n_1 in Eq. (5) and find the optimal value of q_1 and the corresponding value of the total cost from Eq. (3)
3. Let $n_1=\lfloor n_1^* \rfloor$, substitute n_1 in Eq. (5) and find the optimal value of q_1 and the corresponding value of the total cost from Eq. (3)
4. The optimal solution is the one that corresponds to the minimum of the solutions found in Steps 2 and 3.

System formulation under all-unit quantity discount in the transportation cost

In this subsection, instead of the consideration of transportation cost as a fixed part of the system total cost, it is explicitly considered in the analyses. The transportation cost is a function of the shipment lot size. Figure 2 displays the transportation cost in this state. As the amount of the shipment increases, the transportation cost for all units decreases based on the following structure:

$$
\begin{array}{ll}
Range & Unit\ transportation\ \text{cost} \\
0 \le q_2 < M_1 & u_0 \\
M_1 \le q_2 < M_2 & u_1 \\
M_2 \le q_2 < M_3 & u_2 \\
. & \\
. & \\
M_{m-1} \le q_2 < M_m & u_{m-1} \\
M_m \le q_2 & u_m
\end{array}
\tag{6}
$$

where $u_0 > u_1 > u_2 \ldots u_{m-1} > u_m$. For a given shipment lot size as q_2, transportation cost per unit time is:

$$
TC_2(q_2) = \begin{cases}
u_0 D & 0 \le q_2 < M_1 \\
u_1 D & M_1 \le q_2 < M_2 \\
. & \\
. & \\
u_{m-1} D & M_{m-1} \le q_2 < M_m \\
u_m D & M_m \le q_2
\end{cases}
\tag{7}
$$

Thus, total system cost in State 2 per time unit is:

$$TSC_2(n_2,q_2)=TIC_2(q_2,n_2)+TC_2(q_2) \tag{8}$$

For each range of q_2 in Eq. (6), Eq. (8) is also convex with respect to the value of n_2 and q_2 because TIC_2 is convex and TC_2 becomes a fixed part of the formula.

The procedure for finding optimal shipment lot size, q_2, and the number of transferred shipments in each produced lot, n_2, is similar to the classical inventory model. To find the optimal values of n_2 and q_2, the following propositions are applied in this subsection. We refer to Ertogral et al. (2007) for proof.

Proposition 1. By increasing the value of q_1, the value of n_1 decreases in Eq. (5)

Proposition 2. If $q_1^* \ge M_m$ then $q_2^* = q_1^*$ and $n_2^* = n_1^*$

Proposition 3. If $q_1^* < M_m$ then $q_2^* \ge q_1^*$ and $n_{2,lower} \le n_2^* < n_{2,upper}$

$$n_{2,lower} = \max\left\{1, \left\lfloor \frac{\sqrt{2A_1 h_v (P-D)DP}}{h_v (P-D) M_m} \right\rfloor \right\} \quad and \quad n_{2,upper} = n_1^*$$

Based on these propositions, to determine the optimal values of n_2 and q_2, the following procedure can be used in State 2.

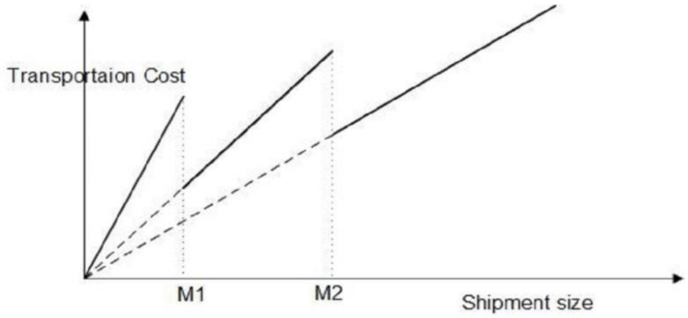

Fig. 2. All-unit quantity discount in transportation cost

Procedure 2.

1. Apply Procedure 1 to find n_1^*, q_1^*.
2. If $q_1^* \geq M_m$ then $q_2^* = q_1^*, n_2^* = n_1^*$, otherwise go to the next step.
3. Find the value of $n_{2,lower}$ and $n_{2,upper}$ using Proposition 3.
4. For each value of $n_2 = n_{2,lower}$, $n_{2,lower+1}, n_{2,lower+2}, \ldots, n_{2,up}$ find the optimal solution as follows:
   a. Find the optimal value of q_2 using Eq. (5) and let r be the largest range index in Eq. (7) such that $M_r \leq q_2$.
   b. Calculate the following total system costs per time unit using Eq. (8): $TSC_2(q_2,n_2)$, $TSC_2(M_{r+1},n_2), TSC_2(M_{r+2},n_2), \ldots, TSC_2(M_m,n_2)$. Among these solutions, a solution with a minimum cost is the optimal solution for the fixed value of n_2.
5. Among the optimal solutions computed for each value of n_2 in Step 4, find a minimum solution. It is the final optimal solution, and the corresponding values of n_2 and q_2 are optimal values for n_2 and q_2.

System formulation under incremental quantity discount in transportation cost

In this subsection, a model is presented for the system under incremental discount in transportation cost. Figure 3 displays transportation cost in this state. Consider the following incremental discount for the system:

$$\begin{array}{ll} 0 \le q_3 \le M_1, & u_0 \\ M_1 + 1 \le q_3 \le M_2, & u_1 \\ M_2 + 1 \le q_3 \le M_3, & u_2 \\ . & \\ . & \\ M_{m-1} + 1 \le q_3 \le M_m, & u_{m-1} \\ M_m + 1 \le q_3, & u_m \end{array} \tag{9}$$

If the value of shipment size, q_3, lies in the range of $M_{j-1} \le q_3 < M_j$, then the transportation cost for each shipment can be calculated from the following recursive formula:

$$TC_3(q_3) = TC_3(M_{j-1}) + u_j(q_3 - M_{j-1}) \tag{10}$$

Thus, the transportation cost for each unit is:

$$\frac{TC_3(q_3)}{q_3} = \frac{TC_3(M_{j-1}) - u_j M_{j-1}}{q_3} + u_j \tag{11}$$

Hence, the total transportation cost per time unit is:

$$\frac{D}{q_3} TC_3(q_3) = \frac{D}{q_3}\left[TC_3(M_{j-1}) - u_j M_{j-1}\right] + u_j D \tag{12}$$

Therefore, under the incremental discount, the total transportation cost is concluded as follows:

$$\begin{aligned} TSC_3(q_3, n_3) = (h_b - h_v)\frac{q_3}{2} + h_v\left[\frac{Dq_3}{P} + \frac{(P-D)n_3 q_3}{2P}\right] + \\ \frac{D}{q_3}\left[\frac{A_v + n_3 A_b}{n_3} + TC(M_{j-1}) - u_j M_{j-1}\right] + u_j D \end{aligned} \tag{13}$$

In Eq. (13), the coefficient of D/q_3 can be stated as follows:

$$\frac{A_v + n_3[A_b + TC_3(M_{j-1}) - u_j M_{j-1}]}{n_3} \tag{14}$$

Let's define A'_b as follwos:

$$A_b^{'} = A_b + TC_3(M_{j-1}) - u_j M_{j-1} \tag{15}$$

Substituting Eq. (15) in Eq. (13) leads to the following equation:

$$TSC_3(q_3, n_3) = (h_b - h_v)\frac{q_3}{2} + h_v\left[\frac{Dq_3}{P} + \frac{(P-D)nq_3}{2P}\right] + \frac{D}{q_3}\left[\frac{A_v + n_3 A_b^{'}}{n_3}\right] + Du_j \tag{16}$$

For each range of q_3 in Eq. (9), the last term of Eq. (16), Du_j, is a fixed part. Also, the first three terms of Eq. (16) are similar to Eq. (2). Thus, for each range of q_3, Eq. (16) can be optimized using Procedure 1. Therefore, to optimize the system performance under incremental quantity discount in transportation cost, the following procedure is proposed.

Procedure 3.

1. For each range of q_3 in Eq. (9), calculate $A_b^{'}$ based on Eq. (15).

2. Replace each obtained value of $A_b^{'}$ in Eq. (16). Based on Procedure 1, optimize Eq. (16) and calculate the optimal values of n_3 and q_3.

3. Based on the corresponding range in Eq. (9), specify which values of q_3 and n_3, which are obtained in Step 2, are acceptable.

4. For the acceptable values of q_3 and n_3, calculate Eq. (16)

5. The minimum values of TSC_3 in Step 4 comprise the optimal value of the objective function and the corresponding values of n_3 and q_3 are the optimum values of n_3, q_3.

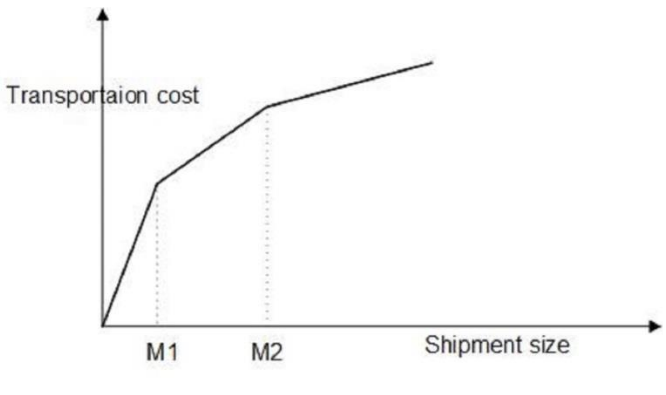

Fig. 3. Incremental quantity discount in transportation cost

3. Numerical example

In this subsection, to clarify the performance of the models and procedures described in the previous subsections, an illustrative example is presented. A similar example is also used by Ertogral et al. (2007). The parameters of the models are as follows:

A_v=200; A_b=15; h_v=4; h_b=5; P=3200; D=1000.

Range	Unit transportation cost
$0 \le q_3 < 100,$	0.4
$100 \le q_3 < 200,$	0.25
$200 \le q_3 < 300,$	0.17
$300 \le q_3,$	0.14

(1): System with fixed transportation cost

Procedure 1.

1. The continuous optimal value of n_1*is 4.119
2. $n_1 = \lceil n^* \rceil$ *=5; q_1=79.85; TSC_1=1377*
3. $n_1 = \lfloor n^* \rfloor$ *=4; q_1=94.68; TSC_1=1373*
4. The optimal solution is: *n_1*=4; q_1*=94.68; TSC_1*=1373*

(2): System under all-unit quantity discount

1. $n^*_{1=}$4; q_1*= 94.68
2. since $q_1^* < 300$, go to the next step.
3. n_{2upper}=4; n_{2lower}=1
4. a1. n_2=1; q_2=262.30; r=2

b1. TSC_2(262.30,1)=1809; TSC $_2$(300,1)=1794; Optimal is n_2=1; q_2=300

a2. n_2=2; q_2=159.86, r=1

b2. TSC_2(159.86,2)=1688; TSC_2(200,2)=1645; TSC_2(300,2)=1873; optimal is n_2=2; q_2=200

a3. n_2=3; q_2=117.90; r=1

b3. TSC_2 (117.90,3)=1635; TSC_2(200,3)=1753; TSC_2(300,3)=2174; optimal is n_2=3, q_2=117.90

a4. n_2=4; q_2= 94.68; r=0

b4. TSC_2 (94.68,4)=1773; TSC_2(100,4)=1625; TSC_2(200,4)=1945; TSC_2(300,4)=2531; optimal is $n_2=4$; $q_2=100$

thus the optimal solution is $n_2{*}=4$; $q_2{*}=100$; $TSC_2{*}(100,4)=1625$

(3): System under incremental quantity discount

1. $A'_{b1}=15; A'_{b2}=29.85; A'_{b3}=45.77; A'_{b4}=54.74$
2. $n_{3,1}=4$; $q_{3,1}=94.7$;
   $n_{3,2}=3$; $q_{3,2}=128.2$;
   $n_{3,3}=2$; $q_{3,3}=180$;
   $n_{3,4}=2$; $q_{3,4}=185.4$;
3. thus $n_{3,1}=4$; $q_{3,1}=94.7$ and $n_{3,2}=3$;$q_{3,2}=128.2$; are acceptable
4. $TSC_{3,1}(94.7;\ 4)=1773$; $TSC_{3,2}(128.3,3)=1756$
5. The optimal solution is $n_3{*}=3$; $q_3{*}=128.3$; $TSC_3{*}=1756$

Numerical analysis

In this section, numerical analyses of the system are carried out. The analyses are performed in two subsections. First, for each of the aforementioned states, the impact of the transportation cost on the performance of the system is analyzed. Second, the impacts of the inventory system parameters are analyzed.

1. Transportation cost

In this subsection, the impact of the transportation costs is elaborated. For this purpose, transportation costs for each unit are increased by a factor. Table 1 displays the results of our analyses. The first observation obtained from this table is that for all the values of the transportation factor, the value of total system cost in State 2 is less than the value of total system cost in State 3. Also, the value of total system cost in State 3 is less than the value of total system cost in State 1. The second result that can be derived from this table is that by increasing the value of transportation cost in State 2 and 3, the value of *n* decreases and the value of *q* increases. This observation is intuitive to some extent because increasing the shipment quantity enables the system to obtain more saving from discount. The third result concluded from Table 1 is related to the value of saving obtained using quantity discount. The saving is calculated based on the following formula:

Saving in state i= $\frac{TSC_1 - TSC_i}{TSC_1}, i = 2,3$ (17)

As can be seen from Table 2, increasing the value of transportation cost leads to an increase in the value of saving in both States, i.e. 2 and 3. Also, for all the values of the factor, the value of saving obtained from all-unit quantity discount is greater than the corresponding value from incremental quantity discount.

Figures 4 and 5 show the impact of the transportation factor on the value of total system cost and the value of saving, respectively. As discussed above, increasing the value of transportation factor leads to an increase in the value of *TSC* and saving in both States 2 and 3. Also, figure 4 displays that by increasing the value of the factor, the difference between the obtained saving in States 2 and 3 is widen. This means that all-unit quantity discount is more profitable than incremental discount and also in the larger values of transportation cost, this profitability becomes more significant. Thus, in the greater values of transportation costs, it is more important to discern which kind of discount should be applied.

Finally, regarding the differences between the aforementioned scenarios of transportation costs, the following points and managerial implications can be inferred from the analyses of this section:

- Either incremental or total discount in transportation cost leads to a significant saving in the total costs of a SC.
- All-unit quantity discount yields more decrease in the total costs of the SC in comparison with the incremental quantity discount.
- For the larger values of transportation costs, the more saving is derived from discount.
- As the transportation cost increases, the difference between the performance of the incremental and all-unit quantity discounts becomes more noticeable. It means that in the larger values of transportation costs, applying discounts strategy, either incremental or all-unit, yields more saving in the SC costs.

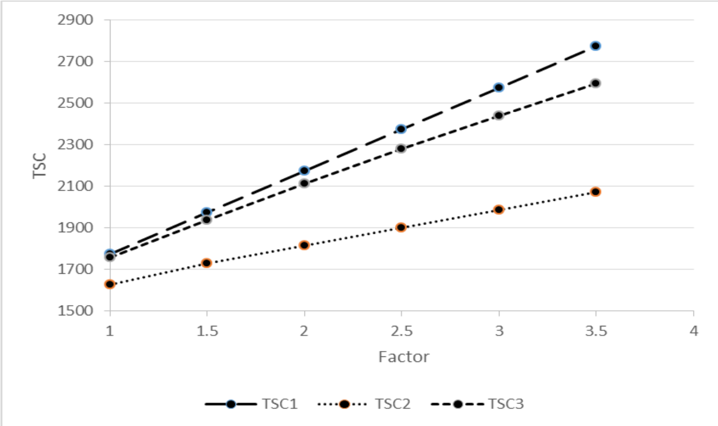

Fig. 4. The effect of transportation factor on the value of total system cost

Table 2. The impact of transportation factor on the performance of the system

Factor	Fixed transportation cost(Sate1)			All unit quantity discount (satete2)				Incremental quantity discount (state3)			
	n	***q***	***TSC***	***n***	***q***	***TSC***	***Saving***	***n***	***q***	***TSC***	***Saving***
1	4	94	1773	4	100	1625	8.34	3	128	1756	0.95
1.5	4	94	1973	2	200	1730	12.31	3	133	1937	1.82
2	4	94	2173	2	200	1815	16.47	2	179	2114	2.71
2.5	4	94	2373	3	200	1900	19.93	2	184	2280	3.91
3	4	94	2573	3	200	1985	22.85	2	214	2441	5.13
3.5	4	94	2773	3	200	2070	25.35	1	336	2594	6.45

2. Parameters of the inventory system

In this subsection, the effects of the parameters of the inventory system are analyzed. First, setup costs in the vendor and buyer site are changed. The impact of these changes is shown in Table 3. In State 1 and State 3, by increasing the value of A_v/A_b , it can be seen that the value of n increases and the value of q decreases, but in State 2, the values of n and q remain unchanged. In all three states, an increase in the value of A_v/A_b leads to a decrease in the value of total system cost. Moreover, increasing the value of A_v/A_b leads to a decrease in the value of saving in States 2 and 3. It can be explained because in the greater values of A_v/A_b , the cost of inventory system leads to a decrease in the value of q and an increase in the value of n, as Table 2 displays for State 1. This effect is in an opposite direction of the impact of discount, i.e., the discount leads to an increase in the value of *q*. These two opposite effects lessen the impact of discount and its corresponding saving.

The similar analysis about ordering cost is performed by Ertogral et al. (2007) for a JELS problem under all-unit quantity discount. The Results of Ertogral et al. (2007) model are comparable with the results of present study, but the novelty of this paper is the development of Ertogral et al. (2007) model for the incremental state. Also, Figure 5 shows the effect of ordering costs of the vendor and buyer on the total system costs.

Table 3. The impact of ordering cost on the system performance

A_v/A_b	Fixed transportation cos(state1)			All unit quantity discount (state2)				Incremental quantity discount (state3)			
	n	q	TSC	n	q	TSC	Saving	n	q	TSC	Saving
13	4	94	1773	4	100	1625	8.34	3	128	1756	0.95
15	4	93	1751	4	100	1605	8.33	3	126	1740	0.62
18	5	77	1726	4	100	1585	8.16	3	125	1724	0.11
22	5	75	1700	4	100	1565	7.94	5	75	1700	0
28	6	63	1670	4	100	1545	7.48	6	63	1670	0

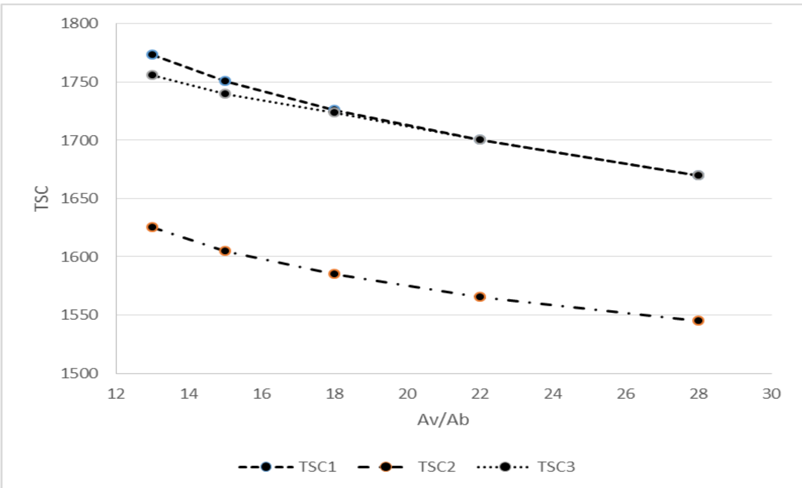

Fig. 5. Effect of ordering costs on the total system costs

In the second step, the impact of holding cost in the buyer and vendor site is analyzed. The difference between the values of h_b and h_v is increased, and the results obtained are analyzed. As indicated in Table 4, this increase leads to an increase in the value of n and a decrease in the value of q except for State 2 that the values of q and n are almost unchanged. Also, the increase in the value of h_b-h_v leads to an increase in the value of TSC in all three states. Moreover, the value of saving obtained in States 2 and 3, compared to State 1, decreases by an increase in the value of h_b-h_v. The justification for this trend is similar to the effect of A_v/A_b. The effect of holding costs on the total system costs is illustrated in Figure 6.

It is worth noting that the similar analyses about holding cost are also performed by Ertogral et al. (2007) in a JELS problem. The results obtained in the present study are consistent with the results of Ertogral at al. (2007) model.

Table 4. The impact of holding cost on the performance of the system

h_b-h_v	Fixed transportation cost (state1)			All unit quantity discount (state2)				Incremental quantity discount (state3)			
	n	q	TSC	n	q	TSC	Saving	n	q	TSC	Saving
1	4	94	1773	4	100	1624	8.40	3	128	1756	0.95
2	5	77	1816	4	100	1675	7.76	5	77	1816	0.00
3	5	75	1855	4	100	1725	7.00	5	75	1855	0.00
4	6	65	1891	4	100	1775	6.13	6	65	1891	0.00
5	6	63	1923	4	100	1825	5.09	6	63	1923	0.00

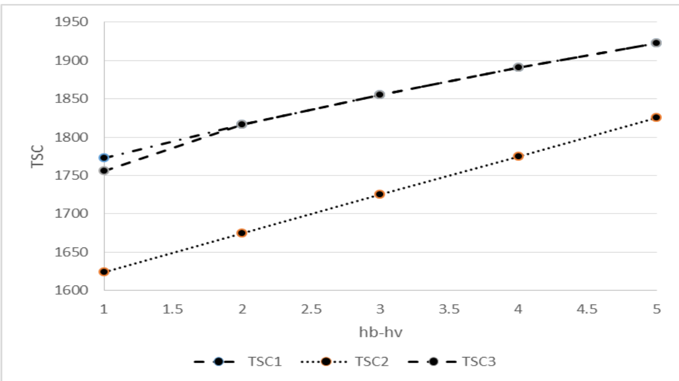

Fig. 6. Effect of holding costs on the total system costs

Finally, the effect of production and demand rate is illustrated in Table 5. As it is expected, an increase in the value of D/P leads to an increase in the TSC for all the three scenarios. This change also leads to an increase in the value of *n* and *q* which is intuitive to some extent. Also, the table shows that for the larger values of demand, applying discounts leads to more saving.

Table 5. The effect of demand and production rates

D/P	Fixed transportation cost (state1)			All unit quantity discount (state2)				Incremental quantity discount (state3)			
	n	q	TSC	n	q	TSC	Saving	n	q	TSC	Saving
0.3125	4	94	1773	4	100	1624	8.4%	3	128	1756	0.095%
0.5	6	95	2261	6	100	2023	10%	4	140	2222	1.7%
0.7	9	98	2599	4	200	2248	13.5%	6	143	2536	2.4%
0.9	17	101	2672	8	200	2056	23%	12	143	2586	3.2%

Finally, the following points can be inferred from the analyses of this section:

- For the larger values of demand rate, applying discounts, either incremental or all-unit, leads to more saving in total costs of SC.
- As the difference between the holding costs of inventory in the buyer and vendor sites decreases, more saving is expected from the discounts of transportation costs.
- The saving obtained from discount decreases as the difference between vendor's set-up costs and buyer's ordering costs increases.

Conclusion

In this paper, transportation costs in a joint economic lot sizing problem (JELS) were investigated where the supply chain consisted of one vendor and one buyer. The transportation costs were studied in three states, including 1) fixed transportation cost, 2) all-unit quantity discount, and 3) incremental quantity discount. First, appropriate equations were derived to compute the costs related to the inventory systems in the buyer and vendor sites. Three procedures, each corresponding to each state, were proposed to optimize the performance of the supply chain. The procedures determined the parameters of the system so that the total system cost per time unit could be minimized. Numerical examples were provided to clarify the models and procedures performance. In addition, some sensitivity analyses were carried out to show the impact of transportation costs and inventory parameters on the system performance. From a managerial perspective, our analyses showed that all-unit quantity discount led to more saving in comparison with the incremental discount. Moreover, for the larger values of transportation costs, all-unit discount policy yielded more saving.

The main novelty of this study is the development of a model that takes into account the incremental discount in transportation costs in a JELS problem. The paper can be extended in several directions, including modeling quantity discount of transportation costs in inventory routing and production routing problems, and extension of the models while demand is price sensitive.

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