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The New Neutral Secondary Goal based on Ideal DMU Evaluation in Cross-Efficiency

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Abstract

Cross-efficiency is a famous ranking method for data envelopment analysis (DEA) that deletes unrealistic weights pattern with no need to a priori information related to weights restrictions. This method analyzes each decision making unit (DMU) taking into account the best weights resulted from assessing other DMUs. In cross-efficiency evaluation, secondary goals such as aggressiveness, benevolence and neutrality are used because there are alternative optimal solutions. The neutral secondary goal makes the decision maker have no problem in selecting the aggressive and benevolent secondary goals. In the article at hand, a new secondary purpose is introduced which selects the optimal weight among the multiple optimal weights based on the evaluation of ideal virtual DMUs corresponding to each DMU. Since this kind of weight selection does not lead to any increase or decrease in other DMUs' cross-efficiency, the new secondary goal is neutral. The advantage of this method over other methods is that the selected weights are the best possible weights, because it maximizes the ideal virtual DMUs' efficiciency score corresponding to each DMU. For this purpose, some examples are used to illustrate its difference with other methods.

Keywords

Data envelopment analysis, Cross-efficiecny, Neutral secondary goal, Ideal virtual DMUs, Ranking

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Introduction

Data envelopment analysis (DEA) can be defined as a popular method in assessing the performance of decision making units (DMUs) that produce outputs based on inputs. For each DMU, the measure of the relative efficiency would be obtained as the weighted ratio of output to total weighted inputs. First, DEA was introduced by Farell (1957). Then Charnes et al. (1978) made it popular. DEA aims at estimating the unit under evaluation in its best possible state. DEA is an effective method in identifying the efficient DMUs which allocates a relative efficiency of one to efficient DMUs; this makes it impossible to distinguish the efficient DMUs from one another.

To solve the problem, there have been proposed different ranking methods that have been formed in accordance with a special criterion of DMU or the Production Possibility Set (PPS) to which DMUs belong (Nasseri et al., 2014; Amirteimoori et al., 2017; Aldamak & Zolfaghari, 2017; Kiaei et al., 2019; Ghasemi et al., 2019; Nasseri & Kiaei, 2019).

One of the ranking methods that has gained popularity among the researchers is cross efficiency evaluation. Sexton et al. (1986) are among the pioneers of this method. Doyle and Green (1994) generalized their idea. The main idea in cross-efficiency evaluation is the point that each DMU should be checked with whole DMUs' weights (instead of evaluation based on their own weights). In other words, peer-evaluation is used instead of self-evaluation. A specific DMU's cross-efficiency score, commonly computed as the average of cross-efficiency, results from the whole DMUs weights. Therefore, Contrary to DEA which uses self-evaluation, each DMU in this method would be evaluated by the whole DMUs' selected weights, to the extent made possible by peer-evaluation. This measure gives in a ranking on the basis of the results derived from the DMUs' cross-efficiency, while the cross-efficiency avoids untrue weights pattern without the need for weight restrictions. Cross-efficiency evaluation has a good discrimination power among DMUs, and has come to be used in various applications such as Cui and Li (2015) Tan et al. (2017), and Liu et al. (2017).

Nevertheless, cross-efficiency evaluation faces some problems. The main problem is that multiple optimal solutions exist for the weights resulting from the DEA model, which leads to different efficiency scores (that depend on weights selection). Sexton et al. (1986) were the first theorists who introduced secondary goal for solving this problem, which was then generalized by Doyle and Green (1994). This purpose is considered as a potential adjustment which avoids the deduction of the cross-efficiency advantages. In most papers, this idea is used for all DMUs with the exaction of some conditions on cross-efficiency results. These conditions are related to aggressive and benevolent formulas. An aggressive (benevolent) model searches the optimal weights which preserve the efficiency score of the unit under evaluation, and decreases (increases) other DMUs' efficiency score. Liang et al. (2008b) tried to expand Doyle and Green's models (1994), and suggested three different secondary goals from a benevolent perspective. Wang and Chin (2010a) extended Liang et al's models (2008b) by describing the true ideal points. Liang et al. (2008a) expanded the cross-efficiency concepts into cross-efficiency of the game and achieved the convergence of the repetitive algorithm through deduction of the balanced point. Demonstrating the inefficiency of conventional aggressive and benevolent model, Wu et al. (2016) exhibited better secondary goals. Wu et al. (2016) suggested a DEA cross-efficiency evaluation which was based on Pareto improvement. Abolghasem et al. (2019) expanded aggressive and benevolent methods when flexible measures exist for cross-efficiency.

A more unusual interpretation of the aggressive and benevolent scenario is used in other cross-efficiency estimates. It is extended from the cross-efficiency scores called "optimistic" and "pessimistic", and establishes the best and worst (respectively) possible scores obtained by each DMU with any of the optimal weight sets of its peers (Oukil & Amin, 2015; Jahanshahloo et al., 2016; Khodabakhshi & Aryavash, 2017; Song et al., 2017). The averaged scores in these models do not necessarily include a common evaluation scheme; so, there is a difference between the traditional cross-evaluation paradigm and the scores.

The review of the related articles makes it clear that all available cross-efficiencies are calculated either as aggressive or benevolent. There is not any warranty that the aggressive and benevolent formulas would be able to have the same results in ranking. Therefore, a neutral model has been introduced. The neutral model determines the inputs and output weight's just from its own view for the unit under evaluation, without examining the aggressiveness and benevolence of other DMUs. This model makes the decision maker have no problem in selecting the aggressive and benevolent model. For cross-efficiency evaluation, Wang and Chin (2010b) suggested DEA neutral model, and proposed to generalize it for cross-weights evaluation. They maximized the minimum relative efficiency of each output and thereupon, they considerably decreased the number of zero weights for outputs. Ramon et al. (2010a) exhibited another model operating concurrent with assessment in such a way that its secondary goal was to select profiles weights that avoided high differences in the related weights to inputs and outputs while examining non-zero weights. Also, Ramon et al. (2010b) selected peer-restricted method disregarding the weights of a specific inefficient DMU in computing cross-efficiency by bounding peer-restricted method. Wang et al. (2011) suggested a neutral model whose goal was to decrease the number of zero weights both for inputs and outputs. Nasseri and Kiaei (2018) proposed that weights are selected in the evaluation of cross-efficiency from among multiple optimal weights based on decreasing the share of inputs and increasing the share of outputs simultaneously, and at the same time, avoidng the selection of zero weights.

The evaluation of DMUs using virtual units is one of the ranking methods that has gained popularity among the researchers. There has been suggested a number of methods in the DEA literature about ranking the units by using virtual units. Wang and Luo (2006) first proposed the concept of virtual units in DEA literature. They introduced two DMUs called ideal decision making unit (IDMU) and anti-ideal decision making unit (ADMU). IDMU considers the minimum input among units with the maximum output, while ADMU is the unit consuming the maximum input to produce the minimum output. Jahanshahloo et al. (2010) defined an ideal line and determined a common set of weights (CSWs) for the efficient DMUs and used the new efficiency scores obtained to rank them. Wang et al. (2011) exhibited four neutral models for cross-efficiency evaluation through IDMU and ADMU from the multiple criteria decision analysis

(MADA) view. Sun et al. (2013) presented two models from MADA viewpoint for searching CSWs, which is obtained based on applying ideal and anti-ideal units. Among the research papers written by those authors who have recently come up with this method, the following articles can be mentioned: Nasseri and Kiaei (2016), Kritikos (2017), Hou et al. (2018), Carrillo and Jorge (2018).

By defining the ideal point corresponding with each unit, Entani and Tanaka (2006) improved DEA interval efficiency based on optimistic and pessimistic viewpoints. They changed the inputs and outputs in the form of production probability set (PPS) using the ideal points corresponding with each unit. Rezaie and Khanmohammady (2010) formed the ideal PPS for the units with the help of these ideal points corresponding with each unit, measured the distance of each unit from its own corresponding ideal using Russell (1985) model, and ranked the units based on the size of this distance.

In this paper, we first describe the ideal DMUs corresponding to each DMU according to Entani and Tanaka's (2006) article. Then, by assessing the ideal DMUs with input-oriented CCR multiplier model, we search for the optimal weights to calculate cross-efficiency and its resultant cross-efficiency score.

The rest of this article is organized in the following order: Section 2 offers a short introduction of the cross-efficiency evaluation and its major formulations. Section 3 develops a new DEA neutral model for cross-efficiency evaluation, while Section 4 illustrates the proposed method by some numerical examples. Finally, Section 5 is dedicated to conclusions.

Cross-efficiency

Cross-efficiency evaluation

Suppose there is a set of n DMUs, and each DMU_j (j=1, …, n) produces different s outputs using different m inputs, being respectively determined by x_{ij} (i=1, …, m) and y_{rj} (r=1, …, s). To assess each DMU_k (k=1, …, n) , the efficiency score E_{kk} performance can be calculated by the input-oriented CCR multiplier model as follows:

$$\begin{aligned} &Max \quad E_{kk} = \frac{\sum_{r=1}^{s} u_{rk} y_{rk}}{\sum_{i=1}^{m} v_{ik} x_{ik}}, \\ &s.t. \quad E_{kj} = \frac{\sum_{r=1}^{s} u_{rk} y_{rj}}{\sum_{i=1}^{m} v_{ik} x_{ij}} \le 1, \quad j = 1,\dots,n, \\ &\qquad v_{ik} \ge 0, \qquad \mathrm{i} = 1,\dots,m, \\ &\qquad u_{rk} \ge 0, \qquad r = 1,\dots,s. \end{aligned} \tag{1}$$

where v_{ik} and u_{rk} stand for *i*th input and *r*th output weights for DMU_k. Using Charnes and Cooper (1962) Conversion, we changed this to a linear model as follows:

$$\begin{aligned} &Max \quad E_{kk} = \sum_{r=1}^{s} u_{rk} y_{rk}, \\ &s.t. \quad \sum_{i=1}^{m} v_{ik} x_{ik} = 1, \\ &\qquad \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0, \quad j = 1,\dots,n, \\ &\qquad v_{ik} \ge 0, \qquad \mathrm{i} = 1,\dots,m, \\ &\qquad u_{rk} \ge 0, \qquad r = 1,\dots,s. \end{aligned} \tag{2}$$

Suppose $v_{ik}^*(i = 1,...,m)$ and $u_{rk}^*(r = 1,...,s)$ are the above LP model's optimal solutions. Then, $E_{kk}^* = \sum_{r=1}^{s} u_{rk}^* y_{rk}$ shows DMU_k's CCR-efficiency resulted from self-evaluation. However, $E_{kj}^* = \sum_{r=1}^{s} u_{rk}^* y_{rj} \Big/ \sum_{i=1}^{m} v_{ik}^* x_{ij}$ is DMU_j's cross-efficiency resulted from peer-evaluation using $DMU_k (k = 1,...,n, k \ne j)$.

The corressponding model for each DMU is solved and thereupon n series of input and output's weights are computed for *n* DMUs. Each DMU includes *(n-1)* cross-efficiencies added to one CCR efficiency. The efficiencies constitute an $n \times n$ matrix where E_{kj} is an entry in row *k* and in column *j*. This is called the cross-efficiency matrix.

Doyle and Green (1994) described the cross-efficiency score as the average of cross efficiencies DMU_j with other DMUs' optimal weights, as follows:

$$E_j^* = \frac{1}{\text{n}} \sum_{k=1}^{n} E_{kj}^*$$

On the basis of the above definitions, the cross efficiency matrix in DEA can be provided, as follows (Figure 1):

$$\begin{array}{cccc}
E_{11}^* & E_{12}^* & E_{1j}^* & E_{1n}^* \\
E_{21}^* & E_{22}^* & E_{2j}^* & E_{2n}^* \\
E_{k1}^* & E_{k2}^* & E_{kj}^* & E_{kn}^* \\
E_{n1}^* & E_{n2}^* & E_{nj}^* & E_{nn}^* \\
\hline
E_1^* & E_2^* & E_j^* & E_n^*
\end{array}$$

Fig. 1. Cross efficiency matrix in DEA

Note that the main diagonal elements on the matrix of the cross efficienciy are the efficiency scores.

Optimal solutions resulting from Model (2) are not often unique. Therefore, a desirable cross-efficiency was gained that is related to specific software which ideally selects optimal solutions. In order to overcome the problem, the secondary goals of cross-efficiency evaluation are exhibited.

Secondary goals

Secondary goals for solving the problem of multiple optimal weights were introduced to examine the solution among the multiple optimal solutions on the basis of a criterion. For the first time, Sexton et al. (1986) discussed the aggressive and benevolent models. Another form of aggressive and benevolent formulas is exhibited by Doyle and Green (1994) which are used more frequently in practice.

$$
\begin{aligned}
& Min \quad \sum_{r=1}^{s} u_{rk} \Big(\sum_{j=1, j\neq k}^{n} y_{rj} \Big) \\
& s.t. \\
& \qquad \sum_{r=1}^{s} v_{ik} \Big(\sum_{j=1, j\neq k}^{n} x_{ij} \Big) = 1, \\
& \qquad \sum_{r=1}^{s} u_{rk} y_{rk} - E_{kk}^{*} \sum_{i=1}^{m} v_{ik} x_{ik} = 0, \\
& \qquad \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \leq 0, \qquad j = 1, \ldots, n, \\
& \qquad v_{ik} \geq 0, \qquad \mathrm{i} = 1, \ldots, m, \\
& \qquad u_{rk} \geq 0, \qquad r = 1, \ldots, s.
\end{aligned}
\tag{3}
$$

and

$$
\begin{aligned}
& Max \quad \sum_{r=1}^{s} u_{rk} \Big(\sum_{j=1, j\neq k}^{n} y_{rj} \Big) \\
& s.t. \\
& \qquad \sum_{r=1}^{s} v_{ik} \Big(\sum_{j=1, j\neq k}^{n} x_{ij} \Big) = 1, \\
& \qquad \sum_{r=1}^{s} u_{rk} y_{rk} - E_{kk}^{*} \sum_{i=1}^{m} v_{ik} x_{ik} = 0, \\
& \qquad \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \leq 0, \qquad j = 1, \ldots, n, \\
& \qquad v_{ik} \geq 0, \qquad \mathrm{i} = 1, \ldots, m, \\
& \qquad u_{rk} \geq 0, \qquad r = 1, \ldots, s.
\end{aligned}
\tag{4}
$$

Model (3) and (4) intend to select optimal weights that besides preserving the efficiency of the unit under evaluation, successively decrease and increase the cross-efficiency of other DMUs. These two models select optimal weights from two different views so that two different ranking methods in cross-efficiency evaluation are accrued. However, it is possible that the decision maker selects a method which is neither aggressive nor benevolent. To overcome this problem of selection for the decision maker, Wang and Chin (2010b) introduced the neutral DEA model in cross-efficiency evaluation as follows:

$$
\begin{aligned}
&Max \quad \delta = \min_{r\in\{1,\dots,s\}} \frac{u_{rk} y_{rk}}{\sum_{i=1}^{m} v_{ik} x_{ik}} \\
&s.t. \\
&\qquad E_{kk}^{*} = \frac{\sum_{r=1}^{s} u_{rk} y_{rk}}{\sum_{i=1}^{m} v_{ik} x_{ik}}, \\
&\qquad E_{kj} = \frac{\sum_{r=1}^{s} u_{rk} y_{rj}}{\sum_{i=1}^{m} v_{ik} x_{ij}} \le 1, \qquad j = 1,\dots,n, \\
&\qquad v_{ik} \ge 0, \qquad \mathrm{i} = 1,\dots,m, \\
&\qquad u_{rk} \ge 0, \qquad r = 1,\dots,s.
\end{aligned} \tag{5}
$$

Among multiple optimal weights, Model (5) selects the optimal weights which maximize each output’s comparative efficiency. In this method, the number of zero weight output is decreased effectively. The weakness of this model is that when we just face one output, it does not necessarily yield a unique optimal solution, and indeed Model (5) is changed into Model (2). In order to solve the problem, Wang et al. (2011) suggested some neutral models based on the distance from the ideal and anti-ideal virtual DMUs by describing the ideal and anti-ideal virtual DMUs, as follows:

$$
\begin{aligned}
&Min \quad \mathrm{D}_k^{IDMU} = \sum_{r=1}^{s} u_{rk} (y_r^{\max} - y_{rk}) + \sum_{r=1}^{s} v_{ik} (x_{ik} - \mathrm{x}_i^{\min}) \\
&s.t. \\
&\qquad \sum_{i=1}^{m} v_{ik} x_{ik} = 1, \\
&\qquad \sum_{r=1}^{s} u_{rk} y_{rk} = E_{kk}^{*}, \\
&\qquad \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0, \qquad j = 1,\dots,n, \\
&\qquad v_{ik} \ge 0, \qquad \mathrm{i} = 1,\dots,m, \\
&\qquad u_{rk} \ge 0, \qquad r = 1,\dots,s.
\end{aligned} \tag{6}
$$

and

$$Max \quad D_k^{ADMU} = \sum_{r=1}^{s} u_{rk}(y_{rk} - y_r^{\min}) + \sum_{r=1}^{s} v_{ik}(x_i^{\max} - x_{ik})$$

$$s.t.$$

$$\sum_{i=1}^{m} v_{ik} x_{ik} = 1,$$

$$\sum_{r=1}^{s} u_{rk} y_{rk} = E_{kk}^*, \qquad (7)$$

$$\sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0, \qquad j = 1, \ldots, n,$$

$$v_{ik} \ge 0, \qquad \mathrm{i} = 1, \ldots, m,$$

$$u_{rk} \ge 0, \qquad r = 1, \ldots, s.$$

In Model (6), a series of input and output weights are needed in each DMU that besides preserving its efficiency, decrease the DMU's distance from the ideal DMU. However, each DMU in Model (7) selects a series of input and output weights that increase the DMU's distance from anti-ideal DMU as well as its efficiency.

The new neutral DEA model

Entani and Tanaka (2006) defined the ideal points corresponding to each unit on the basis of the improvement of DEA interval efficiency. Indeed, the ideal point of each unit was introduced on the basis of the improvement of the upper and lower bounds of the interval efficiency. Then, the adjustment of the inputs and outputs in the form of PPS – with the help of the ideal points corresponding toeach unit – improved the interval efficiency.

In the proposed method, we first describe the ideal DMU according to Entani and Tanaka's (2006) article. Then, on the basis of the evaluation of ideal DMUs, we introduce the secondary goal for the selection of the optimal weights from among the multiple optimal weights. To achieve this goal, we exhibit ideal inputs and outputs corresponding to each DMU such as DMU_k , as follows:

$$\bar{x}_{ik} = \min_{r} \left\{ \frac{y_{rk}}{\max_{j} \frac{y_{rj}}{x_{ij}}} \right\}, \; i = 1,...,m,$$

$$\bar{y}_{rk} = \max_{i} \left\{ \max_{j} \frac{y_{rj}}{x_{ij}} x_{ik} \right\}, \; r = 1,...,s.$$

Rezaie and Khanmohammady (2010) showed in a theorem that the ideal inputs of each unit are lower than or equal to its inputs, and the ideal outputs of each unit are upper than or equal to its outputs, that is:

$$\bar{x}_{ik} \le x_{ik}, \; i = 1,...,m, k = 1,...,n,$$

$$\bar{y}_{rk} \ge y_{rk}, \; r = 1,...,s, k = 1,...,n.$$

Ideal DMUs corresponding to each DMU are assessed using the input-oriented CCR multiplier model in the following form:

$$
\begin{aligned}
& Max \quad \frac{\sum_{r=1}^{s} u_{rk}\bar{y}_{rk}}{\sum_{r=1}^{s} v_{ik}\bar{x}_{ik}} \\
& s.t. \\
& \quad \sum_{r=1}^{s} u_{rk} y_{rk} - E_{kk}^{*} \sum_{i=1}^{m} v_{ik} x_{ik} = 0, \\
& \quad \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0, \qquad j = 1,\dots,n, \\
& \quad v_{ik} \ge 0, \qquad \mathrm{i} = 1,...,m, \\
& \quad u_{rk} \ge 0, \qquad r = 1,\dots,s.
\end{aligned} \tag{8}
$$

So that E_{kk}^{*} is the CCR efficiency score that is derived from the evaluation of *DMU*$_k$. In the objective function of modelabove , $\sum_{r=1}^{s} u_{rk}\bar{y}_{rk} \Big/ \sum_{r=1}^{s} v_{ik}\bar{x}_{ik}$ shows the efficiency of the ideal virtual DMU corresponding to *DMU*$_k$, which is not necessarily a number between $(0,1]$. The first constraint of Model (8) shows the optimal weights resulting from self-evaluation, so that the model above does not include an infinite optimal solution, and it selects a solution from among the multiple optimal solutions which is the best solution for the efficiency of the ideal DMU corresponding to DMU_k. Model (8) is nonlinear because of its objective function. Using Charnes and Cooper conversion, we change it into a linear model, as follows:

$$
\begin{aligned}
Max \quad & \sum_{r=1}^{s} u_{rk}\bar{y}_{rk} \\
s.t. \quad & \\
& \sum_{r=1}^{s} v_{ik}\bar{x}_{ik} = 1, \\
& \sum_{r=1}^{s} u_{rk}y_{rk} - E_{kk}^{*}\sum_{i=1}^{m} v_{ik}x_{ik} = 0, \\
& \sum_{r=1}^{s} u_{rk}y_{rj} - \sum_{i=1}^{m} v_{ik}x_{ij} \le 0, \qquad j = 1,\ldots,n, \\
& v_{ik} \ge 0, \qquad \mathrm{i} = 1,\ldots,m, \\
& u_{rk} \ge 0, \qquad r = 1,\ldots,s.
\end{aligned}
\tag{9}
$$

We solve Model (9) for n times, and then, with the purpose of calculating each DMU's cross-efficiency, we acquire the optimal weights resulting from it. In Model (9), from among the multiple optimal weights, we choose a weight which maximizes the ideal DMU's efficiency score corresponding to each DMU. Because this selection does not result in any increase or decrease in other DMUs' cross-efficiency, we introduce it as a neutral secondary goal. Therefore, the cross efficiency score of jth unit, based on the neutral approach, could be calculated via the equation $E_j = (1/n)\sum_{k=1}^{n} E_{kj}^{*} = (1/n)\sum_{k=1}^{n}(\sum_{r=1}^{s} u_{rk}^{*}y_{rj} / \sum_{r=1}^{s} v_{ik}^{*}x_{ij})$, where v_{ik}^{*} and u_{rk}^{*} is the unique optimal solution derived from Model (9) in the evaluation of kth unit. The advantage of this method in comparison with other models is that it selects weights that are the possible weights, since it maximizes the ideal virtual DMUs' efficiency score corresponding to each DMU.

In order to calrify the position of the ideal point corresponding to each unit in PPS, consider figure 2. Suppose we have seven units of A, B, C, D, E, F and G, each with two inputs and one output.

So, according to the definition of Entani and Tanaka (2006) for virtual ideal decision making unit (IDMU) corresponding to each unit, Figure 2 shows the ideal points corresponding to each unit in PPS. It can be seen that in new PPS, efficient units become inefficient. As a result, if these units are evaluated on the basis of CCR multiplier model in new PPS, an appropriate discrimination would be made to rank the units. Now, the abovementioned units are to be evaluated on the basis of cross-efficiency. In order to select the unique optimal solution among the multiple optimal solutions in cross-efficiency, Model (9) selects the unique optimal solution in a way that the ideal unit's efficiency corresponding to each unit is increased. This kind of unique optimal solution selection is performed without considering the increase or decrease of other units' cross-efficiency.

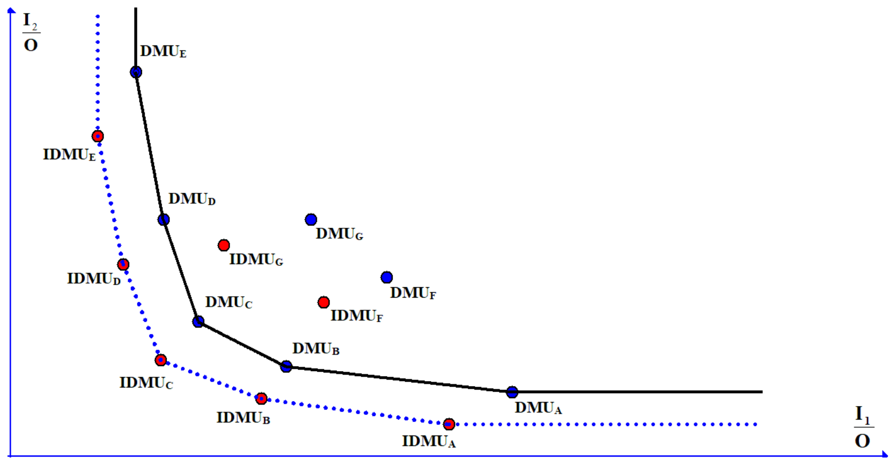

Fig. 2. The ideal virtual DMUs corresponding to each DMU in PPS

Remark 1: The virtual ideal unit defined by Entani and Tanaka (2006) was represented in each unit, not all units. Therefore, the ideal point corresponding to each unit leads to PPS expansion and better discrimination between the units to be ranked. On one hand, the selection of unique optimal solution from among the multiple optimal solutions is performed on the basis of the evaluation of ideal units corresponding with each unit, not based on the increase or decrease of other units' cross-efficiency (wich makes some problems for the decision maker in selecting the optimistic or pessimistic viewpoint). In other words, this kind of solution selection is based on the neutral viewpoint.

Remark 2: Based on the review of related literature, the available models which intended to evaluate cross-efficiency using the secondary goals based on certain criteria selected the unique optimal solution from among multiple optimal solutions as much as they could. Therefore, the unique cross-efficiency score was represented for the units as much as possible. The selection of unique optimal solution was proved in none of the applied models in the introduction of secondary goals by different works reviewed in this paper.

To clarify the suggested ranking method used in evaluating cross-efficiency and to compare it with other methods, we present two examples in the following section.

Numerical examples

We present two practical examples in this section which show the difference of aggressive and benevolent formulas in the evaluation of cross-efficiency. Then, the examples are used to introduce the results of neutral models and the proposed method.

Example 1

Wong and Beasley (1990) evaluated seven academic departments (DMUs) at a university. The inputs are the number of academic staff (x_1), academic staff salaries in thousands of pounds (x_2), and support staff salaries in thousands of pounds (x_3). The outputs are the number of undergraduate students (y_1), the number ofpostgraduate students (y_2)and the number of research papers (y_3). Data and the ideal point inputs and outputs corresponding to each DMU are shown in Table 1 along with the CCR efficiencies of seven academic departments.

It is observed that the CCR efficiency of six units equals one. Therefore, the performance of such units could not be discriminated using the common DEA model. So, cross-efficiency method is used for ranking the units. The obtained cross-efficiency scores from the Models (3), (4), (5), (6), (7) and the proposed method (Model (9)) have been shown in Table 2.

Table 1. Data for 7 academic departments

DMU	x_1	x_2	x_3	y_1	y_2	y_3	$\bar{x}_1$	$\bar{x}_2$	$\bar{x}_3$	$\bar{y}_1$	$\bar{y}_2$	$\bar{y}_3$	CCR Efficiency
1	12	400	20	60	35	17	5.88	261.54	15.87	89.27	46.54	34.67	1.0000
2	19	750	70	139	41	40	10.57	468.57	23.43	225.00	122.50	75.00	1.0000
3	42	1500	70	225	68	75	17.53	777.14	38.86	312.44	162.88	121.33	1.0000
4	15	600	100	90	12	17	3.09	137.14	6.86	321.43	175.00	107.14	0.8197
5	45	2000	250	253	145	130	34.01	1365.11	78.71	803.57	437.50	267.86	1.0000
6	19	730	50	132	45	45	11.60	514.29	25.71	160.71	87.50	54.89	1.0000
7	41	2350	600	305	159	97	33.58	1492.31	90.53	1928.57	1050.00	642.86	1.0000

Table 2. Cross-efficiencies score and the ranks obtained from Models (3), (4), (5), (6), (7) and (9)

Department (*DMU*)	Model(3)	Model(4)	Model(5)	Model(6)	Model(7)	Model(9)
1	0.8788 (1)	0.9442 (3)	0.9362 (2)	0.8295 (1)	0.8978 (1)	0.9352 (2)
2	0.7219 (4)	0.9486 (2)	0.9026 (3)	0.7440 (3)	0.7196 (4)	0.8704 (4)
3	0.7301 (3)	0.7827 (6)	0.7763 (6)	0.6861 (5)	0.7301 (3)	0.8047 (5)
4	0.4018 (7)	0.6160 (7)	0.5649 (7)	0.4501 (7)	0.3959 (7)	0.4483 (7)
5	0.6259 (5)	0.8534 (5)	0.8272 (5)	0.6734 (6)	0.6321 (5)	0.9087 (3)
6	0.8126 (2)	0.9801 (1)	0.9493 (1)	0.8050 (2)	0.8138 (2)	0.9929 (1)
7	0.5966 (6)	0.8992 (4)	0.8552 (4)	0.7110 (4)	0.5989 (6)	0.7238 (6)

As can be seen, the consequences of ranking the models show that DMU_6 in models (4), (5) and (9) has the first rank whereas in Models (3), (6) and (7) it has received the second rank. Also, DMU_1 in models (5) and (9) gained the second rank, whereas in model s (3), (6) and (7) it has achieved the first rank and in model (4) it has received the third rank. It can be seen that DMU_7. is the only CCR inefficient DMU and has the worst rank in cross-efficiency evaluation. Furthermore, it is observed that the cross-efficiency scores of the units obtained from Model (9) are different from the optimististic and pessimistic models of (3) and (4). For example, the cross-efficiency scores of DMU_5 and DMU_6 in Model (9) include maximum measure rather than two models of (3) and (4), whereas the cross-efficiency scores in Model (9) are located between the cross-efficiency scores obtained from the models of (3) and (4). So, Model (9) has a neutral – rather than aggressive and benevolent – viewpoint

Example 2

Fourteen major international passenger airlines (Tofallis, 1997) were assessed with respect to three inputs and two outputs, defined as follows: Aircraft capacity in ton kilometres (x_1), Operating cost (x_2), Non-flight assets like reservation systems, facilities, and current assets (x_3) Passenger kilometres (y_1) and Non-passenger revenue (y_2).

Table 3 shows the 14 passenger airlines' data as well as the inputs and outputs of the ideal point corresponding to each DMU along with their CCR-efficiencies that assess seven out of the 14 passenger airlines as DEA efficient but cannot distinguish them any further.

Table 3. Data for 14 passenger airlines

DMU	x_1	x_2	x_3	y_1	y_2	$\bar{x}_1$	$\bar{x}_2$	$\bar{x}_3$	$\bar{y}_1$	$\bar{y}_2$	*CCR* Efficiency
1	5723	3239	2003	26677	697	2576.11	1346.67	678.95	60381.62	2056.25	0.8684
2	5895	4225	4557	3081	539	463.64	176.21	102.20	137373.47	4678.15	0.3379
3	24099	9560	6267	124055	1266	4679.13	2446.04	1233.21	188922.44	6520.30	0.9475
4	13565	7499	3213	64734	1563	5776.84	3019.87	1522.52	131115.33	3881.27	0.9581
5	5183	1880	783	23604	513	1896.05	991.17	499.71	34441.94	1402.33	1.0000
6	19080	8032	3272	95011	572	2114.11	1105.16	557.19	140434.50	5162.34	0.9766
7	4603	3457	2360	22112	969	3327.53	1264.67	733.51	71143.60	2422.74	1.0000
8	12097	6779	6474	52363	2001	7395.69	2994.85	1737.00	195162.57	6646.12	0.8588
9	6587	3341	3581	26504	1297	3988.46	1515.87	879.20	107951.37	3676.20	0.9477
10	5654	1878	1916	19277	972	2900.90	1102.53	639.46	57758.96	1966.94	1.0000
11	12559	8098	3310	41925	3398	6309.09	2397.86	1390.75	141588.47	4191.30	1.0000
12	5728	2481	2254	27754	982	3629.47	1587.36	920.67	67948.17	2313.93	1.0000
13	4715	1792	2485	31332	543	2006.93	1049.13	528.94	74911.80	2551.07	1.0000
14	22793	9874	4145	122528	1404	5189.18	2712.67	1367.64	172640.72	6166.94	1.0000

Table 4 shows the efficiency scores resulted from cross evaluation of 14 passenger airlines in Models (3), (4), (5), (6), (7) and (9). By comparing the DMUs ranking in various foregoing models, DMU_{11}, for instance, gained the best rank in Models (4), (5) and (9), whereas it got the second rank in Models (3) and (6), and the forth rank in Model (7). Also, DMU_{13} recieved the first rank in model (6), the second rank in Models (4), (7) and (9), and the third rank in Models (3) and (5). It can be seen that among CCR inefficient DMUs, it has the worst rank.

According to Table 4, it is observed that the cross-efficiency scores of some units in the proposed method may be more than the obtained efficiency scores from Doyle and Green's benevolent model. For instance, the cross-efficiency scores of DMU_2, DMU_5 and DMU_{10} obtained from Model (9) are more than Model (4). Similarly, it is concluded that the obtained efficiency scores from Model (9) include a neutral viewpoint without considering the increase or decrease in the cross-efficiency of other units.

Table 4. Cross-efficiencies score and the ranks obtained from Models (3), (4), (5), (6), (7) and (9).

Airline (*DMU*)	Model(3)	Model(4)	Model(5)	Model(6)	Model(7)	Model(9)
1	0.5990 (12)	0.7543 (12)	0.7049 (11)	0.6287(12)	0.6214(12)	0.7369(11)
2	0.1652 (14)	0.1894 (14)	0.1912 (14)	0.1843(14)	0.1583(14)	0.1909(14)
3	0.6226 (11)	0.7678 (9)	0.7154 (10)	0.6381(10)	0.6730(9)	0.7515(10)
4	0.6734 (7)	0.8222 (6)	0.7733 (7)	0.6812(9)	0.7034(6)	0.8041(7)
5	0.7983 (1)	0.8912 (3)	0.8764 (2)	0.7693(4)	0.8446(1)	0.8961(3)
6	0.6385 (9)	0.7554 (11)	0.7024 (12)	0.6311(12)	0.7006(7)	0.7345(12)
7	0.6478 (8)	0.8214 (7)	0.7711 (8)	0.7027(7)	0.6550(10)	0.8003(6)
8	0.5855 (13)	0.7242 (13)	0.6906 (13)	0.6331(11)	0.5958(13)	0.7134(13)
9	0.6309 (10)	0.7590 (10)	0.7378 (9)	0.6814(8)	0.6345(11)	0.7560(9)
10	0.6813 (6)	0.7803 (8)	0.7813 (6)	0.7069(6)	0.6854(8)	0.7989(8)
11	0.7742 (2)	0.9193 (1)	0.9041 (1)	0.7847(2)	0.7637(4)	0.9193(1)
12	0.7314 (5)	0.8850 (4)	0.8541 (4)	0.7712(3)	0.7490(5)	0.8829(4)
13	0.7503 (3)	0.9190 (2)	0.8723 (3)	0.8044(1)	0.7950(2)	0.9030(2)
14	0.7316 (4)	0.8659 (5)	0.8140 (5)	0.7243(5)	0.7895(3)	0.8477(5)

Conclusions

The factor that might threat the advantage of cross-efficiency evaluation method is the presence of multiple optimal solutions in such a way that they make it difficult to obtain the cross-efficiency and so the unique cross-efficiency scores. Aggressive, benevolent and neutral secondary goals were suggested to solve the problem. Aggressive and benevolent formulas not only keep under evaluation unit efficiency, but also increase and decrease the cross-efficiency of other units, respectively. Meanwhile, the neutral formula selects weights on the basis of a special criterion without considering the increase or decrease in the cross-efficiency of other units. This model makes no problem for decision maker to select the aggressive and benevolent model.

To evaluate the cross-efficiency, the article at hand proposed a new model based on evaluating the efficiency of the ideal DMUs corresponding to each DMU according to Entani and Tanaka's (2006) article, so that in the evaluation of cross-efficiency, the unique optimal weight among the obtained optimal weights from evaluating the unit under evaluation is selected in a way that leads to increased virtual ideal unit efficiency corresponding to that unit. In the proposed method, the selection of unique optimal weight is performed without considering the increase or decrease in the cross-efficiency of other units. As a result, this kind of secondary goal is called neutral secondary goal.

We present a novel neutral model that is more intellectual than aggressive and benevolent methods, that is to say, because the efficiency score of ideal DMUs corresponding to each DMU is made maximum, it introduces the best weights in evaluating the cross-efficiency that further shows the advantage of this method over the neutral methods. This method can give a distinguished ranking in assessing the DMUs efficiency.

This article discussed the comparison of the results of cross-efficiency rankings of the foregoing models with the proposed model in the instances mentioned above. Besides, as Wang and Chin (2010b) claimed, the neutral model in the cross-efficiency evaluation mainly tries to avoid the problem of selecting between two aggressive and benevolent formulas. Therefore, the use of the proposed method in evaluating the cross-efficiency in various applications seems to be justified.

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