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Optimization of profit and customer satisfaction in combinatorial production and purchase model by genetic algorithm

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ABSTRACT

Optimization of inventory costs is the most important goal in industries. But in many models, the constraints are considered simple and relaxed. Some actual constraints are to consider the combinatorial production and purchase models in multi-products environment. The purpose of this article is to improve the efficiency of inventory management and find the economic order quantity and economic production quantity that can minimize the cost of inventory and customer satisfaction. In this study, the models with these targets in combinatorial production and purchase systems with the assumption the warehouse and budget constraints are proposed. Since a long time for solving the problem with an exact method is required, we develop a genetic algorithm. To evaluate the efficiency of the proposed

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1 Abstract continued

algorithm, test problems with different sizes of the problem in the range from 1 to 2000 jobs, are generated. The results show that the genetic method is efficient to determine economic order quantity and economic production quantities. The computational results demonstrate that the average error of the solution is 10.93%. (FITNESS AND TAGUCHI)

2 Introduction

Inventory is accumulated materials for supporting manufacturing system fluctuations that occur probably and includes production horizon, distribution of demand, inflation, fashion, and operating support [12]. Hill [6] states that inventory management has wide application in manufacturing systems. Therefore, the company accumulates the appropriate inventory to meet the fluctuations and to avoid material shortages. Stock and Lambert [14] mention the advantages of holding inventory. Inventory cost is one of the main components of a manufacturing companys total cost. Heizer and Render [5] point out that inventory includes 40% of the total manufacturing cost. Therefore, to reduce the total cost significantly, one of the Solutions is minimizing the inventory cost without reducing the quality of its output like customer satisfaction. To reduce the inventory cost, the company needs to carry materials in the appropriate inventory level. Maintenance the inventory at a low level can reduce the holding cost. However, The risk of material shortage will be increased. On the other hand, carrying inventory at a high level decreases the risk of shortages and increases customer satisfaction but the cost of carrying inventory will be high. Therefore, the suitable policy that brings the company to the optimal level of inventory is very important. Finding the best solution of the mathematical model is obtained by an optimization method.
Optimization is a mathematical tool to find the best solution by using historical data and a combination of the related variable for achieving the main objective [4]. There are many optimization methods that have been developed by researchers based on different assumptions and conditions. Kumar [8] studied the assumption and limitations of the EOQ model in the actual environment. Harris [3] studied the classic economic order quantity (EOQ) model that forms the basis for many other models that relax one or more of its assumptions. One assumption, instantaneous delivery, was relaxed by Taft [15], who used a finite production rate, leading to the basic economic production quantity (EPQ) model. An assumption of both of these models is that shortage is not allowed. Relaxing this assumption led to models for the two basic cases: Backorders and lost sales. Montgomery et al. [10] considered a model for the basic EOQ with partial backordering (EOQ-PBO) firstly. Mak

[9] added partial backordering to the basic EPQ model (EPQ-PBO). Although a comprehensive review of deterministic partial backordering models is proposed by Pentico and Drake [11]. They studied more complicated model structures, partial backordering models that include either time-based backordering rate functions or additional considerations. A model for the EPQ-PBO in which the constant backordering rate changes when production starts is studied by them [11].
Stockton and Quinn [13] considered the basic EOQ model using Genetic Algorithms to solve economic lot size. This model is based on a deterministic policy such as constant demand and repetitive replenishment, in which backorder is not allowed. Besides, Hou et al. [7] also applied the periodic review method in production inventory management using Genetic Algorithms approach. In that model, they assume that backorders are allowed and the demand is constant. Furthermore, [1] demonstrated the use of a periodic review model by determining the fixed order quantity in periodic review approach. Ghodsypour and O'Brien [2] developed an integer non-linear programming model which consider the total cost of inventory, including total price, shortage, and transportation and ordering cost. Yokoyama [16] proposed a model for a multiple sourcing inventory and distribution system. That model focused on finding the target inventory and transportation quantity that minimizing the total cost of the system by using a random local search method combined with GAs.
In the present work, Genetic Algorithms are applied to find out optimal solutions through the optimization process. Our aim is to find an optimal ordering quantity for various inventory items stored.
Also, the single machine scheduling problem with one flexible maintenance period and non-resumable jobs are considered by minimizing the weighted number of tardy jobs. It is assumed that there is a flexible maintenance period that its starting time is as the decision variable. Moreover, the idle time is not allowed.
The rest of this paper is organized as follows: the problem is described in Section 3. Notation and mathematical model is presented in Section 4. Section 5 is aimed at proposing a heuristic algorithm. Computational experiments are then given in Section 6 to demonstrate the effectiveness of algorithm, followed by the conclusion in Section 7.

3 Preliminary

- Holding Costs: Holding costs are those associated with storing inventory that remains unsold. These costs are one component of total inventory costs, along with ordering and shortage costs. A firm's holding costs include the price of goods damaged or spoiled, as well as that of storage space, labor, and insurance. Minimizing inventory costs is an important supply-chain management strategy.

- Perpetual Inventory: Perpetual inventory is a method of accounting for inventory that records the sale or purchase of inventory immediately through the use of computerized point-of-sale systems and enterprise asset management software. The perpetual inventory provides a highly detailed view of changes in inventory with immediate reporting of the amount of inventory in stock, and accurately reflects the level of goods on hand.

- Purchase Order Lead Time: Purchase order lead time is the number of days from when a company places an order for production inputs it needs to when those items arrive at the manufacturing plant. Purchase order lead times vary from company to company and from industry to industry, depending on the types of goods or materials being ordered, their relative abundance or scarcity, where the suppliers are located, and even the time of year.

- Back Order: A backorder is a customer order that has not been fulfilled. A backorder generally indicates that customer demand for a product or service exceeds a company's capacity to supply it.

In many studies in inventory control purchase models and production, models are considered separately with regard to special assumption, but in a real environment, it is important to consider two scenarios together. In this article the combinatorial EOQ and EPQ model considering multi products and budget and warehouse space constraints. The shortage is permitted such as backordering. The aim of this research is to minimize the bi-criteria targets, inventory cost, and customers satisfaction. The purpose of this article is to improve the efficiency of inventory management and find the economic order quantity and economic production quantity that can minimize the cost of inventory and customer satisfaction. the model with these targets in combinatorial production and purchase systems with the assumption the warehouse and budget constraints in multi products environment is developed. It is assumed that if the shortage gets to a specific level, the new order is performed. Before the model's description, there are some assumptions that we need to define in this research.

- Annual demand is constant
- Lead times are known and constant
- Backorders are allowed
- On hand inventory at the end of the starting period is zero.
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- Purchasing cost is less than the shortage cost.

4 Notations and mathematical modeling

In this section, we define notations and decision variables to model the problem. Then a mathematical model is proposed. The following notations are used in this section:

Problem parameters:

- D_i: demand rate of product i
- P_i: production rate of product i
- S_i: shortage value of product i
- h_i: holding cost of product i per time unit
- f_i: required space for product i
- X: maximum budget
- $A1_i$: ordering cost of product i per order
- $A2_i$: production cost of product i per order
- $\pi 1_i$: shortage cost per unit product i independent unit time
- $\pi 2_i$: shortage cost per unit product i per unit time
- F: total space of the warehouse
- $C1_i$: purchase cost of product i
- $C2_i$: production cost of product i
- R_i: sales price of product i
- N: number of products

Decision variables:

- TIC: inventory cost
- THC_i: total holding cost of product i
- TSC_i: total shortage cost of product i
- TMC_i: total purchasing cost of product i
- TOC_i: total ordering cost of product i
- T_i: cycle length of product i
- $Q1_i$: optimal order value of product i
- $Q2_i$: optimal production value of product i
- E_i: start point of product i

The mathematical model

The inventories costs are obtained from the summation of holding, ordering, purchasing, and production costs and cost of shortage. By considering the assumptions mentioned in the previous section, the component of inventory costs are calculated as follows:

$$TIC = \sum_{i=1}^{N}(THC_i + TSC_i + TMC_i + TOC_i).$$

Calculation of total holding cost: total holding costs are calculated by the following relation:

$$THC_i = \frac{h_i}{2}[\frac{(Q1_i + E_i)^2}{D_i} + \frac{E_i^2}{D_i - P_i}]\frac{1}{T_i}.$$

Calculation of total cost of shortage: for calculating the total cost of shortage, we must sum two related costs such as dependent to unit time and independent one. Therefore, it is obtained by the following equation:

$$TSC_i = [\pi 1_i S_i + \pi 2_i(\frac{S_i^2}{(D_i - P_i)^2})]\frac{1}{T_i}.$$

Calculation of total purchasing and production cost: total purchasing and production costs is calculated by the following relation:

$$TMC_i = (C1_iQ1_i + C2_i + Q2_i)\frac{1}{T_i}.$$

Calculation of total ordering cost: total ordering costs is calculated by the following relation:

$$TOC_i = (A1_i + A2_i)\frac{1}{T_i}.$$

Calculation of total revenue: total revenue of products sales is obtained as follows:

$$\sum_{i=1}^{N} R_iD_i.$$

Constraints: the constraints of budget and warehouse space are calculated as the following relations respectively:

$$\sum_{i=1}^{N} C1_iQ1_i + C2_iQ2_i \leq X \quad \text{and} \quad \sum_{i=1}^{N} f_iQ1_i \leq F.$$

Satisfaction of customer: the customer's satisfaction in this study, is calculated as follows:

$$\sum_{i=1}^{N} \frac{D_i}{Q1_i + Q2_i}.$$

According to above explanation about component of the mathematical model, the inventory model is obtained as follows:

$$\max Z1 : \sum_{j=1}^{N} R_iD_i - \frac{D_i(D_i - P_i)}{P_i(E_i - Q1_i) + z_iD_i}[(A1_i + A2_i) + (C1_iQ1_i + C2_iQ2_i) +$$

$$\frac{h_i}{2}[\frac{(Q1_i + E_i)^2}{D_i} + \frac{E_i^2}{D_i - P_i}]\frac{1}{T_i} + [\pi 1_iS_i + \pi 2_i(\frac{S_i^2}{(D_i - P_i)^2})]\frac{1}{T_i}] \tag{1}$$

$$\max Z2 : \sum_{i=1}^{N} \frac{D_i}{Q1_i + Q2_i} \tag{2}$$

$$st:$$

$$\sum_{i=1}^{N} C1_i Q1_i + C2_i Q2_i \leq X \qquad i = 1, 2, \ldots, N \tag{3}$$

$$\sum_{i=1}^{N} f_i Q1_i \leq F \qquad i = 1, 2, \ldots, N \tag{4}$$

$$Q1_i \leq E_i \qquad i = 1, 2, \ldots, N \tag{5}$$

The objective function (1) maximizes the total benefit of inventory. The objective function in relation (2) maximizes the satisfaction of the customer. Constraint (3) ensure that the total costs of purchase and production of all products are not greater than the total budget in hand. Constraint (4) ensure that the occupied space by-products are not greater than the total space of the warehouse. Constraint (5) shows that the order quantity of product i is not greater than the point of production of it.

5 solution method

In this study, regarding the bi-criteria objective function, the ϵ constraint method is proposed. In this procedure, by recursive stages, the optimal solution is obtained in a long time commonly. Therefore, in this research, the ϵ constraints method is applied and after passing a long time (with 3600 seconds constraint), two values are obtained for Z2 as 0.1 and 0.61. For calculating the optimum value of Z1, this problem must be solved in GAMS software with one objective and the obtained values for Z2 as constraints, again. Because of a long time for solving the problem with an exact method and regarding NP-hardness of the problem, in this research, the metaheuristic method, a genetic algorithm, is developed. That model concentrate on finding the target reorder quantities that minimizing the total cost of inventory and satisfaction of the customer, by using random local search method combined with GAs.

Genetic Algorithm

The genetic procedure begins with an initial set of solution which is called chromosome. One of them has some gens which specifies their characteristics. The chromosome forms a matrix with i rows and 6 columns as below shape:

R_i	z_i	$Q1_i$	$Q2_i$	S_i	E_i

The values of genes demonstrate values of decision variables that getting amount randomly and by these values, the objective function which is called fitness in GAs, is calculated.
population parameters:

The population size of GAs is considered 20 and the iteration as stop condition is 20 after controlling the feasibility of solutions. Furthermore, the mechanism of chromosome selection is a roulette wheel, where the selection was done randomly. The probability of mutation is 0.05 and the strategy of doing mutation is one point. The probability of crossover is 0.7 and its process is replacing two genes together randomly. It is worth noting that one chromosome is not feasible from the point of view constraints, that chromosome is eliminated and a new one is generated.

6 Computational results

The proposed model is illustrated by considering the following 20 examples(displayed in table 2). In each of examples, 100 independent runs have been performed by the genetic algorithm. So, the best-found values $Z1$, $Z2$, $Q1$, $Q2$, S_i and E_i have been obtained and showed in table 2. The GA parameters that are applied including the population of GA is 20, the probability of crossover is 0.7 and the probability of mutation is 0.05.

7 Conclusion

This article addressed the inventory problem for multi-item by warehouse capacity and budget constraint. Moreover, this inventory model maximizes the bi-criteria objective, the benefit of inventory and satisfaction of the customer. this article used a genetic algorithm to determine the order quantities of various products. The result obtained from the proposed method is quite satisfactory and efficient. this approach has achieved the objectives which are obtained a better control of inventory and reduce costs. To evaluate the efficiency of the proposed algorithms, 696 test problems with different sizes of the problem in the range from 1 to 2000 jobs, are generated. The computational results demonstrate that the number of problems that are solved optimally using GAMS isn't up to 10 jobs but the heuristic algorithm can solve the problems up to 1000 jobs with maximum error bound 10.93% in optimality.

Table 1: Specification of series and error bound of WSFTA

$series$	R_1	Z_1	$Q1_1$	$Q2_1$	S_1	E_1	R_2	Z_2	$Q1_2$	$Q2_2$	S_2	E_2	R_3	Z_3	$Q1_3$	$Q2_3$	S_3	E_3	$fitness1$	$fitness2$
1	3778	3694	5553	7747	6019	5555	6333	7010	2407	4443	4255	4010	8900	7950	2000	2219	2219	3000	$2.25 * 10^8$	0.302433
2	2098	3961	2555	7353	4877	6804	3736	4156	3656	3405	2580	3584	9752	1948	2800	7353	9640	5020	$1.75 * 10^8$	0.271735
3	2539	2006	6840	4718	8858	9200	6908	2485	9816	2994	2671	9987	9967	989	8858	8925	8637	9853	$5.56 * 10^8$	0.174848
4	8785	6723	5277	7894	1570	8542	1772	9965	7086	2049	258	8457	9671	700	6723	8279	9965	9444	$3.62 * 10^9$	0.197545
5	4386	5711	6600	6496	6527	6783	8005	8998	4510	356	4386	5210	8999	1040	8279	8999	8999	9570	$5.06 * 10^9$	0.209137
6	897	4068	9840	3973	8200	9900	5950	797	9065	978	4521	9158	9877	1458	9313	9840	9588	9500	$1.16 * 10^8$	0.171359
7	5472	1693	1693	5846	9425	5133	8052	844	9025	5421	6617	9801	9982	204	9318	9005	9801	9585	$2.55 * 10^8$	0.182842
8	1968	4971	8066	1326	473	9486	1968	2033	1002	5627	4983	1968	9984	630	9906	9447	9984	9978	$1.3 * 10^8$	0.208345
9	5408	8156	7926	1365	5182	8012	7077	1882	4635	6326	5921	4787	9998	1520	9903	852	5472	9987	$1.09 * 10^8$	0.237688
10	9058	2612	4626	6229	3758	5014	6039	4180	9058	4688	5111	9875	9276	474	9265	6684	9276	9856	$3.95 * 10^8$	0.181751
11	9551	7075	4923	457	3169	8234	1020	706	3308	4036	8464	5014	9671	706	9041	9551	9671	9787	$2.46 * 10^8$	0.235343
12	6379	7235	7827	2577	4425	8017	3684	5180	3582	4110	6483	4747	9860	385	9215	9860	9813	9888	$6.86 * 10^8$	0.198273
13	1010	4661	5049	4120	5558	5171	254	1175	9719	2678	325	9797	9901	1205	6807	9901	9901	9814	$1.13 * 10^9$	0.192559
14	6351	5660	7485	5485	7296	8452	9789	1176	5660	2722	8390	5859	9811	232	9789	4758	9789	9830	$2.1 * 10^8$	0.205298
15	8873	2197	1488	9432	3227	9587	7404	3672	2164	3147	8435	3672	9749	1488	8937	1478	8873	9785	$1.4 * 10^8$	0.276589
16	9502	9477	6981	6303	442	9477	3821	208	3565	2376	6981	8648	9985	564	8948	9700	9700	9896	$2.12 * 10^8$	0.201208
17	7289	6966	6584	963	5479	7755	789	5479	6382	4718	991	7852	8867	1464	8827	8829	8829	9785	$1.92 * 10^8$	0.208247
18	452	753	3894	3210	4502	4213	7796	8818	4213	2577	4502	5428	9673	195	9353	9673	9673	9758	$1.68 * 10^9$	0.229648
19	5071	3515	950	4158	2648	4909	7503	3224	9201	652	3553	9532	9980	569	8964	9980	9980	10152	$7.61 * 10^8$	0.222976
20	1441	790	5769	5587	5629	8690	4948	8613	1441	5201	4500	5587	9521	7831	9787	9026	9521	9823	$1.4 * 10^9$	0.205373
21	754	6236	9707	4176	640	9850	9207	8195	6218	3113	8995	8325	9924	640	9207	8546	9924	9521	$2.58 * 10^8$	0.184539
22	1331	1485	9427	4868	6509	9885	8047	5643	6496	905	3465	8710	9994	405	9509	8285	9994	9625	$2.93 * 10^8$	0.191441
23	1052	4134	2585	1052	1709	8692	4203	7758	2209	2001	5351	3589	9538	1283	9502	8881	8881	9563	$1.17 * 10^8$	0.28822
24	2683	775	8986	724	6365	9652	724	1115	2986	1359	7159	7310	9730	1115	8986	9300	8986	9478	$1.11 * 0^8$	0.233759
25	6521	9455	9368	5217	3755	9458	9455	7639	5064	7685	3755	7959	9962	900	7639	9200	9962	9885	$4.52 * 10^9$	0.171145
26	5050	9646	2582	2704	6202	3029	8645	1861	1532	5489	6494	3395	9807	1861	9646	7520	8734	9853	$2.66 * 10^8$	0.201098
27	4888	8975	8148	456	7235	9856	859	430	6371	2748	8148	8858	9978	456	9582	9294	9294	9991	$5.16 * 10^8$	0.201098

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