journal homepage: http://jac.ut.ac.ir

# 4-Total prime cordial labeling of some cycle related graphs 

R. Ponraj* ${ }^{* 1}$ and J. Maruthamani ${ }^{\dagger 2}$<br>${ }^{1}$ Department of Mathematics, Sri Paramakalyani College, Alwarkurichi-627412, Tamilnadu, India.<br>${ }^{2}$ Research Scholar, Register number: 18124012091054, Department of Mathematics, Manonmaniam sundarnar university, Abishekapatti, Tirunelveli-627012, Tamilnadu, India.

## ABSTRACT

Let $G$ be a $(p, q)$ graph. Let $f: V(G) \rightarrow\{1,2, \ldots, k\}$ be a map where $k \in \mathbb{N}$ and $k>1$. For each edge $u v$, assign the label $\operatorname{gcd}(f(u), f(v))$. $f$ is called $k$ Total prime cordial labeling of $G$ if $\left|t_{f}(i)-t_{f}(j)\right| \leq 1$, $i, j \in\{1,2, \cdots, k\}$ where $t_{f}(x)$ denotes the total number of vertices and the edges labelled with $x$. A graph with a $k$-total prime cordial labeling is called $k$-total prime cordial graph. In this paper we investigate the 4 -total prime cordial labeling of some graphs like Prism, Helm, Dumbbell graph, Sun flower graph.

## ARTICLE INFO

Article history:
Received 15, March 2018
Received in revised form 14, october 2018
Accepted 18 November 2018
Available online 30, December 2018

Keyword: Prism, Helm, Dumbbell graph, Sun flower graph
AMS subject Classification: 05C78.

## 1 Introduction

Graphs considered here are finite, simple and undirected. Ponraj et al. [4], have been introduced the concept of $k$-total prime cordial labeling and the $k$-total prime cordial labeling of certain graphs have been investigated. Also in [4, 5, 6, 7, 8], the 4 -total prime

[^0]cordial labeling behaviour of path, cycle, star, bistar, some complete graphs, comb, double comb, triangular snake, double triangular snake, ladder, friendship graph, flower graph, gear graph, Jelly fish, book, irregular triangular snake, corona of irregular triangular snake, corona of some graphs and subdivision of some graphs and also the 3-total prime cordial labeling behaviour of path, cycle, star, comb, wheel, fan have been investigated [7]. In this paper we investigate the 4 -total prime cordial labeling of few graphs like Prism, Helm, Dumbbell graph, Sun flower graph.

## $2 k$-total prime cordial labeling

Definition 2.1 Let $G$ be a $(p, q)$ graph. Let $f: V(G) \rightarrow\{1,2, \ldots, k\}$ be a function where $k \in \mathbb{N}$ and $k>1$. For each edge $u v$, assign the label $\operatorname{gcd}(f(u), f(v)) . f$ is called $k$-Total prime cordial labeling of $G$ if $\left|t_{f}(i)-t_{f}(j)\right| \leq 1, i, j \in\{1,2, \cdots, k\}$ where $t_{f}(x)$ denotes the total number of vertices and the edges labelled with $x$. A graph with a $k$-total prime cordial labeling is called $k$-total prime cordial graph.

## 3 Preliminaries

Definition 3.1 An $n$-sided prism $P r_{n}$ is a planar graph having 2 faces viz., an inner face and outer face with n sides and every other face is a 4 -cycle. In other words, it is $C_{n} \times K_{2}$.
Definition 3.2 The graph obtained by joining two disjoint cycles $u_{1} u_{2} \ldots u_{n} u_{1}$ and $v_{1} v_{2} \ldots v_{n} v_{1}$ with an edge $u_{1} v_{1}$ is called dumbbell graph $D b_{n}$.
Definition 3.3 The graph $W_{n}=C_{n}+K_{1}$ is called a wheel. In a Wheel, a vertex of degree 3 on the cycle is called a rim vertex. A vertex which is adjacent to all the rim vertices is called the central vertex. The edges with one end incident with a rim vertex and the other incident with the central vertex are called spokes.
Definition 3.4 The Helm $H_{n}$ is obtained from a wheel $W_{n}$ by attaching a pendent edge at each vertex of the cycle $C_{n}$.
Definition 3.5 The Sunflower graph $S F_{n}$ is obtained from a Wheel with the central vertex $v_{0}$, the cycle $C_{n}: v_{1} v_{2} \ldots v_{n} v_{1}$ and additional vertices $w_{1} w_{2} \ldots w_{n}$ where $w_{i}$ is joined by edges to $v_{i}, v_{i+1}$ where $v_{i+1}$ is taken modulo $n$.
remark. 2 - total prime cordial graph is 2 -total product cordial graph.

## 4 Main Results

Theorem 4.1 The graph prisms $C_{n} \times P_{2}$, is 4 -total prime cordial for all $n \geq 3$.
Proof. Let $V\left(C_{n} \times P_{2}\right)=\left\{u_{i}, v_{i}: 1 \leq i \leq n\right\}$ and $E\left(C_{n} \times P_{2}\right)=\left\{u, u_{n}, v, v_{n}\right\} \cup\left\{u_{i} v_{i}\right.$ : $1 \leq i \leq n\} \cup\left\{u_{i} u_{i+1}, v_{i} v_{i+1}: 1 \leq i \leq n-1\right\}$. It is easy to verift that $\left|V\left(C_{n} \times P_{2}\right)\right|+$ $\left|E\left(C_{n} \times P_{2}\right)\right|=5 n$.
Case 1. $n \equiv 0(\bmod 4)$.

Let $n=4 r, r>1$ and $r \in \mathbb{N}$. Assign the label 4 to the vertices $u_{1}, u_{2}, \ldots, u_{r}$ and assign the label 2 to the vertices $u_{r+1}, u_{r+2}, \ldots, u_{2 r}$. Next we assign the label 3 to the vertices $u_{2 r+1}, u_{2 r+2}, \ldots, u_{3 r+1}$ then we assign 1 to the vertices $u_{3 r+2}, u_{3 r+3}, \ldots, u_{4 r-1}$. Finally we assign the label 4 to the vertex $u_{4 r}$. Next we consider the vertices $v_{i}(1 \leq i \leq n)$. Assign the label 4 to the vertices $v_{1}, v_{2}, \ldots, v_{r}$ and assign the label 2 to the vertices $v_{r+1}, v_{r+2}, \ldots, v_{2 r}$ and next we assign the label 3 to the vertices $v_{2 r+1}, v_{2 r+2}, \ldots, v_{3 r}$. Finally we assign the label 1 to the vertices $v_{3 r+1}, v_{3 r+2}, \ldots, v_{4 r}$. Clearly $t_{f}(1)=t_{f}(2)=$ $t_{f}(3)=t_{f}(4)=5 r$.
Case 2. $n \equiv 1(\bmod 4)$.
Let $n=4 r+1, r>1$ and $r \in \mathbb{N}$. As in case 1, assign the label to the vertices $u_{i}(1 \leq$ $i \leq n-3)$ and $v_{i}(1 \leq i \leq n-3)$. Finally we assign the labels $1,2,4,4,3,1$ respectively to the vertices $u_{4 r-2}, u_{4 r-1}, u_{4 r}, v_{4 r-2}, v_{4 r-1}$ and $v_{4 r}$. Here $t_{f}(1)=t_{f}(3)=t_{f}(4)=5 r+1$ and $t_{f}(2)=5 r+2$.
Case 3. $n \equiv 2(\bmod 4)$.
Let $n=4 r+2, r>1$ and $r \in \mathbb{N}$. Assign the label the vertices to $u_{i}(1 \leq i \leq n-1)$ and $v_{i}(1 \leq i \leq n-1)$ by case 2 . Finally we assign the labels 4,3 to the vertices $u_{4 r}$ and $v_{4 r}$ respectively. It is easy to verify that $t_{f}(1)=t_{f}(4)=5 r+3$ and $t_{f}(2)=t_{f}(3)=5 r+2$.
Case 4. $n \equiv 4(\bmod 4)$.
Let $n=4 r+3, r>1$ and $r \in \mathbb{N}$. As in case 1, assign the label the vertices to $u_{i}(1 \leq$ $i \leq n-3)$ and $v_{i}(1 \leq i \leq n-3)$. Finally we assign the labels $3,3,4,2,2,4$ respectively to the vertices $u_{4 r-2}, u_{4 r-1}, u_{4 r}, v_{4 r-2}, v_{4 r-1}$ and $v_{4 r}$. Here $t_{f}(1)=t_{f}(2)=t_{f}(4)=5 r+4$ and $t_{f}(3)=5 r+3$.
Case 5. $n=3,4,5,6,7$.
A 4-total prime cordial labeling follows from Table 1.

| n | 3 | 4 | 5 | 6 | 7 |
| :---: | :--- | :--- | :--- | :--- | :--- |
| $u_{1}$ | 4 | 4 | 4 | 4 | 4 |
| $u_{2}$ | 4 | 4 | 4 | 4 | 4 |
| $u_{3}$ | 2 | 2 | 4 | 2 | 2 |
| $u_{4}$ |  | 3 | 3 | 2 | 2 |
| $u_{5}$ |  |  | 3 | 3 | 3 |
| $u_{6}$ |  |  |  | 3 | 1 |
| $u_{7}$ |  |  |  |  | 2 |
| $v_{1}$ | 3 | 3 | 2 | 4 | 4 |
| $v_{2}$ | 3 | 4 | 2 | 4 | 4 |
| $v_{3}$ | 4 | 2 | 3 | 2 | 2 |
| $v_{4}$ |  | 3 | 3 | 3 | 3 |
| $v_{5}$ |  |  | 4 | 3 | 3 |
| $v_{6}$ |  |  |  | 1 | 3 |
| $v_{7}$ |  |  |  |  | 3 |

Table 1:

Theorem 4.2 The dumbbell graph $D b_{n}$ is 4 -total prime cordial for all $n \geq 3$.
Proof. Let $u_{1} u_{2} \ldots u_{n} u_{1}$ and $v_{1} v_{2} \ldots v_{n} v_{1}$ be the two disjoint cycles joining with an edge $u_{1} v_{1}$. Clearly $\left|V\left(D b_{n}\right)\right|+\left|E\left(D b_{n}\right)\right|=4 n+1$.
Case 1. $n \equiv 0(\bmod 4)$.
Let $n=4 r, r>1$ and $r \in \mathbb{N}$. Assign the label 4 to the vertices $u_{1}, u_{2}, \ldots, u_{r}$ and assign 2 to the vertices $u_{r+1}, u_{r+2}, \ldots, u_{2 r}$. Then we assign the label 3 to the vertices $u_{2 r+1}, u_{2 r+2}, \ldots, u_{3 r}$. Next we assign the labels 4,3 respectively to the vertices $u_{3 r+1}$ and $u_{3 r+2}$. Finally we assign the label 1 to the vertices $u_{3 r+3}, u_{3 r+4}, \ldots, u_{4 r}$. Now we move to the vertices $v_{i}(1 \leq i \leq n)$. Assign the label 4 to the vertices $v_{1}, v_{2}, \ldots, v_{r}$ and assign 2 to the vertices $v_{r+1}, v_{r+2}, \ldots, v_{2 r}$. Then we assign the label 3 to the vertices $v_{2 r+1}, v_{2 r+2}, \ldots, v_{3 r}$. Next we assign the labels 4,3 respectively to the vertices $v_{3 r+1}$ and $v_{3 r+2}$. Finally we assign the label 1 to the vertices $v_{3 r+3}, v_{3 r+4}, \ldots, v_{4 r}$. Here $t_{f}(1)=$ $t_{f}(2)=t_{f}(3)=4 r$ and $t_{f}(4)=4 r+1$.
Case 2. $n \equiv 1(\bmod 4)$.
Let $n=4 r+1, r>1$ and $r \in \mathbb{N}$. As in case 1 , assign the label to the vertices $u_{i}$ $(1 \leq i \leq n-1)$ and $v_{i}(1 \leq i \leq n-1)$. Finally we assign the labels 2,3 to the vertices $u_{4 r}$ and $v_{4 r}$ respectively. Clearly $t_{f}(1)=t_{f}(3)=t_{f}(4)=4 r+1$ and $t_{f}(2)=4 r+2$.
Case 3. $n \equiv 2(\bmod 4)$.
Let $n=4 r+2, r>1$ and $r \in \mathbb{N}$. As in case 2 , assign the label to the vertices $u_{i}$ ( $1 \leq i \leq n-1$ ) and $v_{i}(1 \leq i \leq n-3)$. Finally we assign the labels $4,3,1,1$ respectively to the vertices $u_{4 r}, v_{4 r-2}, v_{4 r-1}$ and $v_{4 r}$. It is easy to verify that $t_{f}(1)=t_{f}(2)=t_{f}(3)=4 r+2$ and $t_{f}(4)=4 r+3$.
Case 4. $n \equiv 3(\bmod 4)$.
Let $n=4 r+3, r>1$ and $r \in \mathbb{N}$. Assign the label to the vertices $u_{i}(1 \leq i \leq n-2)$ and $v_{i}(1 \leq i \leq n-1)$ as in case 3 . Finally we assign the labels $2,4,3$ respectively to the vertices $u_{4 r-1}, u_{4 r}$ and $v_{4 r}$. Here $t_{f}(1)=t_{f}(3)=t_{f}(4)=4 r+3$ and $t_{f}(2)=4 r+4$.
Case 5. $n=3,4,5,6,7$.
A 4-total prime cordial labeling follows from Table 2 and 3.

| n | 3 | 4 | 5 | 6 | 7 |
| :---: | :--- | :--- | :--- | :--- | :--- |
| $u_{1}$ | 4 | 4 | 4 | 4 | 4 |
| $u_{2}$ | 4 | 2 | 4 | 4 | 4 |
| $u_{3}$ | 2 | 4 | 3 | 2 | 2 |
| $u_{4}$ |  | 3 | 2 | 3 | 2 |
| $u_{5}$ |  |  | 3 | 3 | 3 |
| $u_{6}$ |  |  |  | 3 | 1 |
| $u_{7}$ |  |  |  |  | 3 |
| $v_{1}$ | 3 | 4 | 4 | 4 | 4 |
| $v_{2}$ | 3 | 2 | 2 | 4 | 4 |
| $v_{3}$ | 2 | 3 | 2 | 2 | 2 |

Table 2:

| $v_{4}$ |  | 3 | 3 | 2 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $v_{5}$ |  |  | 3 | 3 | 3 |
| $v_{6}$ |  |  | 1 | 3 |  |
| $v_{7}$ |  |  |  | 3 |  |

Table 3:

Theorem 4.3 The graph helm $H_{n}$ is 4 -total prime cordial for all the values of $n \geq 3$.
Proof. Let $C_{n}$ be the cycle $u_{1} u_{2} \ldots u_{n} u_{1}$. Let $v_{i}(1 \leq i \leq n)$ be the vertices adjacent to each vertex of the cycle $C_{n}$. Clearly $\left|V\left(H_{n}\right)\right|+\left|E\left(H_{n}\right)\right|=5 n+1$.
Case 1. $n \equiv 0(\bmod 4)$.
Let $n=4 r, r>1$ and $r \in \mathbb{N}$.
Subcase 1. $r$ is even.
Assign the label 4 to the central vertex $u$. Next assign the label 4 to the vertices $u_{1}, u_{2}, \ldots, u_{r}$ and assign 2 to the vertices $u_{r+1}, u_{r+2}, \ldots, u_{2 r}$ then we assign the label 3 to the vertices $u_{2 r+1}, u_{2 r+2}, \ldots, u_{\frac{7 r}{2}}$. Finally we assign the label 1 to the vertices $u_{\frac{7 r}{2}+1}, \ldots, u_{4 r}$. Now we consider the vertices $v_{i}(1 \leq i \leq n)$. Assign the label 4 to the vertices $v_{1}, v_{2}, \ldots, v_{r}$ and assign 2 to the vertices $v_{r+1}, v_{r+2}, \ldots, v_{2 r}$ then we assign the label 3 to the vertices $v_{2 r+1}, v_{2 r+2}, \ldots, v_{\frac{6 r}{2}+1}$. Finally we assign the label 1 to the vertices $v_{\frac{6 r}{2}+2}, \ldots, v_{4 r}$. Here $t_{f}(1)=t_{f}(2)=t_{f}(4)=5 r$ and $t_{f}(3)=5 r+1$.
Subcase 2. r is odd.
Assign the label 4 to the central vertex $u$. Next assign the label 4 to the vertices $u_{1}, u_{2}, \ldots, u_{r}$ and assign 2 to the vertices $u_{r+1}, u_{r+2}, \ldots, u_{2 r}$ then we assign the label 3 to the vertices $u_{2 r+1}, u_{2 r+2}, \ldots, u_{\frac{7 r-1}{2}}$. Finally we assign the label 1 to the vertices $u_{\frac{\eta_{r-1}^{2}+1}{2}}, \ldots, u_{4 r}$. Now we consider the vertices $v_{i}(1 \leq i \leq n)$. Assign the label 4 to the vertices $v_{1}, v_{2}, \ldots, v_{r}$ and assign 2 to the vertices $v_{r+1}, v_{r+2}, \ldots, v_{2 r}$ then we assign the label 3 to the vertices $v_{2 r+1}, v_{2 r+2}, \ldots, v_{\frac{6 r}{2}+1}$. Finally we assign the label 1 to the vertices $v_{\frac{6 r}{2}+2}, \ldots, v_{4 r}$. Clearly $t_{f}(1)=5 r+1$ and $t_{f}(2)=t_{f}(3)=t_{f}(4)=5 r$.
Case 2. $n \equiv 1(\bmod 4)$.
Let $n=4 r+1, r>1$ and $r \in \mathbb{N}$.
Subcase 1. $r$ is even.
As in subcase(1) of case 1, assign the label to the vertices $u, u_{i}(1 \leq i \leq n-2)$ and $v_{i}$ $(1 \leq i \leq n-2)$. Finally we assign the labels $4,1,2,1$ to the vertices $u_{4 r-1}, u_{4 r}, v_{4 r-1}$ and $v_{4 r}$ respectively. Clearly $t_{f}(1)=t_{f}(3)=5 r+1$ and $t_{f}(2)=t_{f}(4)=5 r+2$.
Subcase 2. r is odd.
As in subcase(2) of case 1, assign the label to the vertices $u, u_{i}(1 \leq i \leq n-2)$ and $v_{i}$ $(1 \leq i \leq n-2)$. Finally we assign the labels $4,1,2,3$ to the vertices $u_{4 r-1}, u_{4 r}, v_{4 r-1}$ and $v_{4 r}$ respectively. Obviously $t_{f}(1)=t_{f}(3)=5 r+1$ and $t_{f}(2)=t_{f}(4)=5 r+2$.
Case 3. $n \equiv 2(\bmod 4)$.
Let $n=4 r+2, r>1$ and $r \in \mathbb{N}$.

Subcase 1. r is even.
As in subcase(1) of case 2, assign the label to the vertices $u, u_{i}(1 \leq i \leq n-2)$ and $v_{i}$ $(1 \leq i \leq n-2)$. Finally we assign the labels $1,3,3,2$ respectively to the vertices $u_{4 r-1}$, $u_{4 r}, v_{4 r-1}$ and $v_{4 r}$. Clearly $t_{f}(1)=t_{f}(2)=t_{f}(3)=5 r+3$ and $t_{f}(4)=5 r+2$.
Subcase 2. r is odd.
As in subcase(2) of case 2, assign the label to the vertices $u, u_{i}(1 \leq i \leq n-2)$ and $v_{i}$ $(1 \leq i \leq n-2)$. Finally we assign the labels $3,1,3,2$ to the vertices $u_{4 r-1}, u_{4 r}, v_{4 r-1}$ and $v_{4 r}$ respectively. It is easy to verify that $t_{f}(1)=t_{f}(2)=t_{f}(3)=5 r+3$ and $t_{f}(4)=5 r+2$.
Case 4. $n \equiv 3(\bmod 4)$.
Let $n=4 r+3, r>1$ and $r \in \mathbb{N}$.
Subcase 1. $r$ is even.
As in subcase(1) of case 2, assign the label to the vertices $u, u_{i}(1 \leq i \leq n-2)$ and $v_{i}$ $(1 \leq i \leq n-2)$. Finally we assign the labels $4,3,2,3$ respectively to the vertices $u_{4 r-1}$, $u_{4 r}, v_{4 r-1}$ and $v_{4 r}$. Here $t_{f}(1)=t_{f}(2)=t_{f}(3)=t_{f}(4)=5 r+4$.
Subcase 2. r is odd.
As in subcase (2) of case 2, assign the label to the vertices $u, u_{i}(1 \leq i \leq n-2)$ and $v_{i}$ $(1 \leq i \leq n-2)$. Finally we assign the labels $4,3,2,3$ to the vertices $u_{4 r-1}, u_{4 r}, v_{4 r-1}$ and $v_{4 r}$ respectively. It is easy to verify that $t_{f}(1)=t_{f}(2)=t_{f}(3)=t_{f}(4)=5 r+4$.
Case 5. $n=3,4,5,6,7$.
A 4-total prime cordial labeling follows from Table 4 and 5.

| n | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $u_{1}$ | 4 | 4 | 4 | 4 | 4 |
| $u_{2}$ | 3 | 2 | 2 | 4 | 4 |
| $u_{3}$ | 2 | 3 | 3 | 2 | 2 |

Table 4:

| $u_{4}$ |  | 3 | 3 | 3 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $u_{5}$ |  |  | 3 | 3 | 3 |
| $u_{6}$ |  |  |  | 1 | 3 |
| $u_{7}$ |  |  |  |  | 3 |
| $v_{1}$ | 3 | 4 | 4 | 4 | 4 |
| $v_{2}$ | 3 | 2 | 2 | 2 | 1 |
| $v_{3}$ | 4 | 3 | 3 | 2 | 2 |
| $v_{4}$ |  | 1 | 2 | 3 | 4 |
| $v_{5}$ |  |  | 4 | 3 | 1 |
| $v_{6}$ |  |  |  | 3 | 3 |
| $v_{7}$ |  |  |  |  | 3 |

Table 5:

Theorem 4.4The sunflower graph $S F_{n}$ is 4 -total prime cordial for all $n \geq 3$.
Proof. Let $u$ be the central vertex of the cycle $C_{n}: u_{1} u_{2} \ldots u_{n} u_{1}$ and additional vertices $v_{1} v_{2} \ldots v_{n}$ where $v_{i}$ is joined by edges to $u_{i}, u_{i+1}$. Clearly $\left|V\left(S F_{n}\right)\right|+\left|E\left(S F_{n}\right)\right|=5 n+1$.
Case 1. $n \equiv 0(\bmod 4)$.
Let $n=4 r, r>1$ and $r \in \mathbb{N}$.
Subcase 1. $r$ is even.
Assign the label 4 to the central vertex $u$. Next assign the label 4 to the vertices $u_{1}, u_{2}, \ldots, u_{r}$ and assign 2 to the vertices $u_{r+1}, u_{r+2}, \ldots, u_{2 r}$ then we assign the label 3 to the vertices $u_{2 r+1}, u_{2 r+2}, \ldots, u_{\frac{7 r}{2}}$. Finally we assign the label 1 to the vertices $u_{\frac{7 r}{2}+1}, \ldots, u_{4 r}$. Now we consider the vertices $v_{i}(1 \leq i \leq n)$. Assign the label 4 to the vertices $v_{1}, v_{2}, \ldots, v_{r}$ and assign 2 to the vertices $v_{r+1}, v_{r+2}, \ldots, v_{2 r}$ then we assign the label 3 to the vertices $v_{2 r+1}, v_{2 r+2}, \ldots, v_{3 r}$. Next we assign the labels 4,3 to the vertices $u_{\frac{7 r}{2}}$ and $u_{\frac{7 r}{2}+1}$ respectively. Finally we assign the label 1 to the vertices $v_{\frac{7 r}{2}+2}, \ldots, v_{4 r}$. Here $t_{f}(1)=6 r+1$ and $t_{f}(2)=t_{f}(3)=t_{f}(4)=6 r$.
Subcase 2. r is odd.
Assign the label 4 to the central vertex $u$. Next assign the label 4 to the vertices $u_{1}, u_{2}, \ldots, u_{r}$ and assign 2 to the vertices $u_{r+1}, u_{r+2}, \ldots, u_{2 r}$ then we assign the label 3 to the vertices $u_{2 r+1}, u_{2 r+2}, \ldots, u_{\frac{\pi r+1}{2}}$. Finally we assign the label 1 to the vertices $u_{\frac{7 r+1}{2}+1}, \ldots, u_{4 r}$. Now we consider the vertices $v_{i}(1 \leq i \leq n)$. Assign the label 4 to the vertices $v_{1}, v_{2}, \ldots, v_{r}$ and assign 2 to the vertices $v_{r+1}, v_{r+2}, \ldots, v_{2 r}$ then we assign the label 3 to the vertices $v_{2 r+1}, v_{2 r+2}, \ldots, v_{3 r}$. Next we assign the label 4 to the vertex $v_{3 r+1}$. Finally we assign the label 1 to the vertices $v_{3 r+2}, v_{3 r+3}, \ldots, v_{4 r}$. Clearly $t_{f}(1)=6 r+1$ and $t_{f}(2)=t_{f}(3)=t_{f}(4)=6 r$.
Case 2. $n \equiv 1(\bmod 4)$.
Let $n=4 r+1, r>1$ and $r \in \mathbb{N}$.
Subcase 1. $r$ is even.
As in subcase(1) of case 1, assign the label to the vertices $u, u_{i}(1 \leq i \leq n-2)$ and $v_{i}$ $(1 \leq i \leq n-1)$. Finally we assign the labels $4,3,2$ to the vertices $u_{4 r-1}, u_{4 r}$ and $v_{4 r}$ respectively. Clearly $t_{f}(1)=t_{f}(2)=t_{f}(4)=6 r+2$ and $t_{f}(3)=6 r+1$.
Subcase 2. r is odd.
As in subcase(2) of case 1, assign the label to the vertices $u, u_{i}(1 \leq i \leq n-2)$ and $v_{i}$ $(1 \leq i \leq n-2)$. Finally we assign the labels $4,1,2,3$ to the vertices $u_{4 r-1}, u_{4 r}, v_{4 r-1}$ and $v_{4 r}$ respectively. Obviously $t_{f}(1)=t_{f}(2)=t_{f}(4)=6 r+2$ and $t_{f}(3)=6 r+1$.
Case 3. $n \equiv 2(\bmod 4)$.
Let $n=4 r+2, r>1$ and $r \in \mathbb{N}$.
Subcase 1. $r$ is even.
As in subcase(1) of case 1, assign the label to the vertices $u, u_{i}(1 \leq i \leq n-2)$ and $v_{i}$ $(1 \leq i \leq n-2)$. Finally we assign the labels $3,4,3,2$ respectively to the vertices $u_{4 r-1}$, $u_{4 r}, v_{4 r-1}$ and $v_{4 r}$. Clearly $t_{f}(1)=6 r+4$ and $t_{f}(2)=t_{f}(3)=t_{f}(4)=6 r+3$.
Subcase 2. r is odd.
As in subcase(2) of case 1 , assign the label to the vertices $u, u_{i}(1 \leq i \leq n-2)$ and $v_{i}$ $(1 \leq i \leq n-2)$. Finally we assign the labels $3,4,3,2$ to the vertices $u_{4 r-1}, u_{4 r}, v_{4 r-1}$ and
$v_{4 r}$ respectively. It is easy to verify that $t_{f}(1)=6 r+4$ and $t_{f}(2)=t_{f}(3)=t_{f}(4)=6 r+3$.
Case 4. $n \equiv 3(\bmod 4)$.
Let $n=4 r+3, r>1$ and $r \in \mathbb{N}$.
Subcase 1. $r$ is even.
As in subcase(1) of case 1, assign the label to the vertices $u, u_{i}(1 \leq i \leq n-3)$ and $v_{i}$ $(1 \leq i \leq n-4)$. Finally we assign the labels $2,3,4,3,3,4,2$ respectively to the vertices $u_{4 r-2}, u_{4 r-1}, u_{4 r}, v_{4 r-3}, v_{4 r-2}, v_{4 r-1}$ and $v_{4 r}$. Here $t_{f}(1)=t_{f}(2)=t_{f}(4)=6 r+5$ and $t_{f}(3)=6 r+4$.
Subcase 2. r is odd.
As in subcase(2) of case 2, assign the label to the vertices $u, u_{i}(1 \leq i \leq n-3)$ and $v_{i}$ $(1 \leq i \leq n-4)$. Finally we assign the labels $4,3,3,3,2,4,2$ to the vertices $u_{4 r-2}$, $u_{4 r-1}, u_{4 r}, v_{4 r-3}, v_{4 r-2}, v_{4 r-1}$ and $v_{4 r}$ respectively. It is easy to verify that $t_{f}(1)=t_{f}(2)=$ $t_{f}(4)=6 r+5$ and $t_{f}(3)=6 r+4$.
Case 5. $n=3,4,5,6,7$.
A 4-total prime cordial labeling follows from Table 6.

| n | 3 | 4 | 5 | 6 | 7 |
| :---: | :--- | :--- | :--- | :--- | :--- |
| $u$ | 4 | 2 | 4 | 4 | 4 |
| $u_{1}$ | 4 | 4 | 4 | 4 | 4 |
| $u_{2}$ | 2 | 4 | 2 | 4 | 4 |
| $u_{3}$ | 3 | 3 | 4 | 2 | 2 |
| $u_{4}$ |  | 3 | 3 | 3 | 3 |
| $u_{5}$ |  |  | 3 | 3 | 3 |
| $u_{6}$ |  |  |  | 3 | 3 |
| $u_{7}$ |  |  |  |  | 2 |
| $v_{1}$ | 4 | 4 | 4 | 4 | 4 |
| $v_{2}$ | 3 | 2 | 2 | 2 | 2 |
| $v_{3}$ | 3 | 3 | 3 | 2 | 1 |
| $v_{4}$ |  | 2 | 3 | 3 | 3 |
| $v_{5}$ |  |  | 1 | 2 | 3 |
| $v_{6}$ |  |  |  | 3 | 1 |
| $v_{7}$ |  |  |  |  | 4 |

Table 6:

## References

[1] Cahit, I., Cordial graphs:A weaker version of graceful and harmonious graphs, Ars Combinatoria, 23(1987), 201-207.
[2] Gallian, J.A., A Dynamic survey of graph labeling, The Electronic Journal of Combinatorics, 19 (2017) \#Ds6.
[3] Harary, F., Graph theory, Addision wesley, New Delhi (1969).
[4] Ponraj, R., Maruthamani J., and Kala, R., $k$-Total prime cordial labeling of graphs, Journal of Algorithms and Computation, 50(1), 143-149.
[5] Ponraj, R., Maruthamani J., and Kala, R., Some classes of 4-Total prime cordial labeling of graphs, Global Journal of Engineering science and Researches, 5(11), 319327.
[6] Ponraj, R., Maruthamani J., and Kala, R., Some 4-total prime cordial labeling of graphs, Journal of Applied Mathematics and Informatics, 37(1-2) (2019), 149-156.
[7] Ponraj, R., Maruthamani J., and Kala, R., 4-total prime cordial labeling of some special graphs, Jordan Journal of Mathematics and Statistics, (Accepted for Publication).
[8] Ponraj, R., Maruthamani J., and Kala, R., Some results on 3-Total prime cordial labeling of graphs.(communicated)


[^0]:    *Corresponding author: R. Ponraj. Email: ponrajmaths@gmail.com
    ${ }^{\dagger}$ mmani2011@gmail.com

