# Three-Dimensional Finite Element Modeling of Stone Column-Improved Soft Saturated Ground

Shahraki, M.<sup>1</sup>, Rafiee-Dehkharghani, R.<sup>2\*</sup> and Behnia, C.<sup>3</sup>

<sup>1</sup> M.Sc., School of Civil Engineering, College of Engineering, University of Tehran, Tehran, Iran.

<sup>2</sup> Assistant Professor, School of Civil Engineering, College of Engineering, University of Tehran, Tehran, Iran.

<sup>3</sup> Associate Professor, School of Civil Engineering, College of Engineering, University of Tehran, Tehran, Iran.

Received: 16 Feb. 2018; Revised: 13 Aug. 2018; Accepted: 14 Aug. 2018

**ABSTRACT:** Installing stone columns in the ground is an effective improvement technique to increase the load bearing capacity and reduce the consolidation settlement of the loose or weak cohesive soils. In addition to the increase in the bearing capacity and reduction in the settlement, stone columns can accelerate the dissipation rate of the excess pore water pressure generated by the surcharge, which expedites the ground improvement procedure. Due to these advantages, this technique has been widely used to improve the mechanical properties of the soft and problematic soils. In this study, the behavior of stone columns in saturated soft grounds are studied using Finite Element (FE) numerical method. For this purpose, a three-dimensional (3D) FE model of the stone column-improved ground is built considering the nonlinear behavior of the soil and stone columns. This model considers the effect of consolidation, and its accuracy is verified using the unit cell concept and the results of a real ground improvement project in Iran. In addition, a parametric study is performed using the verified FE model to investigate the effect of different material and geometric characteristics of the stone columns on the behavior of improved ground. At the end, the efficacy of the stone column method is compared to that of deep soil mixing (DSM) ground improvement technic.

Keywords: Consolidation, Finite Element, Ground Improvement, Settlement, Stone Column.

### **INTRODUCTION**

The loadbearing capacity of the ground has always been a crucial question for the construction of engineering structures. In recent years, industrial development, increase in the population, and the insufficiency of suitable ground for construction has given rise to a demand to utilize sites of inferior quality. However, for the construction purposes, these sites need to be improved. To this end, the use of deep foundations or ground improvement techniques has attracted significant attention. To select an appropriate ground improvement technic, one has to consider three important characteristics: 1)

<sup>\*</sup> Corresponding author E-mail: rezarafiee@ut.ac.ir

design criteria and requirements, 2) type of soil, and 3) optimal design considering the economic and practical limitations.

the techniques Among of soil improvement, stone columns have been applied widely and successfully for increasing the loadbearing capacity and reducing the consolidation settlements of the foundations of various types of structures such as large storage tanks and industrial factories that should be built on mat foundations. Due to the complicated behavior of stone columns and lack of advanced theoretical formulations, Finite Element (FE) has widely been used analysis for determining the mechanical characteristics of stone column vertical elements. The amount of soil improvement depends on the distribution of stress at the interface of soil and columns, yielding strength, friction angle, and dilation angle of the stone columns. Hence, it is required to use advanced numerical analysis in order to consider the nonlinear behavior of stone columns and its surrounding soil. Within this context, many researchers have performed numerical studies to investigate the behavior of stone column-improved grounds and have tried to enhance the accuracy and preciseness of their models considering the real physics behind the problem. Generally, stone columns behave as an assemblage of rigid inclusions with larger shear strength and permeability comparing to the unimproved ground.

Many theoretical and analytical solutions are provided in the literature for estimating the bearing capacity and settlement of the stone-column reinforced foundations. Barron (1948) and Han and Ye (2001) developed a simplified formulation for calculating the rate of consolidation in the grounds reinforced with stone columns. Barron (1948) provided an analytical solution for the radial consolidation in vertical drains with the assumption of negligible drain stiffness.

Besides, the solution provided by Han and Ye one-dimensional (2001)assumes deformation theory, which cannot be used for general three-dimensional problems. Some studies which consider the settlement of foundations on stone columns, is performed by Balaam and Booker (1981) and Priebe (1995). Balaam and Booker (1981) have presented solutions for evaluating the magnitude and rate of the settlement of rigid foundations supported by stone columnimproved ground. Moreover, some research has been conducted for evaluating the bearing capacity (Hughes et al., 1974; Barksdale and Bachus, 1983).

In recent decades, advanced techniques for bearing capacity, studying radial consolidation, and settlement has been proposed by (Ambily and Gandhi, 2007; Pulko et al., 2011; Castro et al., 2011, 2013; Tan et al. 2008, 2014; Indraratna et al., 2015). Ambily and Gandhi (2007) measured axial stress on columns experimentally and verified their results with finite element simulations. In order to investigate the consolidation and deformation around stone columns, Castro et al. (2013) compared the results of laboratory tests by analytical solution and numerical modeling of unit cell in PLAXIS software. It was observed that analytical solutions, due to the elastic behavior of the soil, give rise to higher settlement compared to reality; therefore, it is recommended to use a decay factor for the typical design problems. However, the settlement was compatible with numerical analyses that considered elasto-plastic behavior for the stone columns.

The stone column ground improvement method has widely been used in Iran for both increasing the general mechanical properties of the soft grounds and preventing liquefaction in saturated coarse-grained soils. To the best of the authors' knowledge, the stone columns were first installed under the foundation of Parsian Azadi Khazar Hotel in Namakabrud, Mazandaran province of Iran in 1970s. This hotel is one of the oldest hotels in northern Iran, and no significant damage was observed in its structure after the 1990 Manjil–Rudbar earthquake with the magnitude of 7.4 M<sub>w</sub>.

Many other stone column ground improvement projects are performed in Iran since 1970s (e.g. Moayedi et al., 2010); however, because of poor documentation, the information of these projects are not publicly available. In this paper, the stone column construction in Khayyam steel mill project in Neyshabur, Razavi Khorasan province of Iran is provided as a case study. This project is performed by Reinforced Earth Iran Company in 2013.

In this paper, ABAQUS commercial FE software (Hibbitt et al., 2001) is used for modeling the behavior of stone columnimproved grounds. ABAQUS is a robust software and it is possible to build advanced three-dimensional (3D) FE models in its environment considering the nonlinear behavior of materials and 3D consolidation. In the following section, the details of the 3D-FE modeling of a group of stone columns in a soft ground, and its verification with Khayyam steel mill project case study results are explained comprehensively. In addition, at the end of the paper, the performance of the stone column ground improving method is compared to that of deep soil mixing (DSM) technic. It should be noted that the majority of the research in this field have mainly focused on 2D plane strain simulation and 3D single column modeling. Due to this reason, the major purpose in this paper is modeling the behavior of stone column groups using elaborate 3D-FE simulations.

# NUMERICAL MODELING

In this section, the details of the 3D-FE modeling of the stone column-improved grounds is explained for investigating their

influence on the magnitude of settlement and consolidation rate. For this purpose, first the geometry of the model is built as shown in Figure 1. The model is a soil cylinder with the diameter and height of 9 m and 45 m, respectively. The soil contains seven stone columns with the height of 12 m, and measurements have been performed on the center column. It is assumed that the bottom boundary of the soil medium reaches a hard stiff layer.

One of the major difficulties associated with running large 3D Finite Element models complex loading and boundary with conditions is the computational cost. Generally, more accurate results can be obtained from Finite Element models by reducing the mesh size; however, the cost of computation increases significantly in this case. Therefore, it is a common practice to perform a mesh convergence analysis for finding the optimal size of the mesh, which ensures obtaining precise results within an appropriate computation time. Such an analysis for finding the optimal mesh size in the vertical direction of the model is shown in Figure 2. Considering this figure, the minimum mesh size (in the vertical direction) in the model is considered to be 1 m. To save more analysis time, the size of the mesh is increased slightly from 1 m at the top to 3 m at the bottom of the model. It should be noted that the mesh size in the horizontal plane is smaller (approximately 0.25 m) for the stone column regions because the columns have limited diameter values.

To mesh the model, 20-node brick with pore pressure elements (element C3D20RP from ABAQUS Library) is used. The soil and stone column media are considered to be completely saturated; hence, it is assumed that they have fluid/stress behavior. The nonlinear behavior of the soil and stone columns is modeled using Mohr-Coulomb constitutive law. The material properties of soil and stone columns are presented in table 1. One of the major steps in FE simulations is implementing the boundary conditions in the model. This becomes more challenging when there exists pore pressure boundary conditions in saturated media, such as the problem in this paper. Therefore, two kinds of boundary conditions are specified in the 3D-FE model: displacement boundary conditions and hydraulic boundary conditions. The bottom of the model is completely fixed ( $u_z =$   $u_x = u_y = 0$ ) and the displacement around the stone columns' cylindrical surface, along x and y directions, are set to be zero ( $u_x = u_y =$ 0), and it is subjected to a vertical abrupt loading. During loading stage, the top surface of the model is made impervious by preventing the drainage. Thereafter, the surface is subjected to a full drainage boundary condition by setting the pore water pressure equal to zero.



Fig. 1. Model geometry and finite element mesh



Fig. 2. Mesh convergence analysis for the FE model

The initial and boundary conditions of the model (shown in Figure 1) are modeled using the following steps:

a) Applying the soil weight: At this stage, the specific weight of the soil is applied and atrest horizontal stresses are created using the geo-static step in order to reach a stable state condition.

b) Applying the load: At this stage, surface pressure is applied to the model instantaneously through a rigid plate. The reason for this kind of quick loading is to transform the applied stresses to excess pore water pressure in order to simulate the process of consolidation.

c) Drainage: At this stage, while keeping the intensity of the load constant, the model is allowed to be drained through the permeable boundaries. In order to obtain the ground settlements in different times, the drainage stage is divided into appropriate time-steps and this process is continued until no variation is observed in the pore water pressure, i.e. a full consolidation state is reached. The analysis of consolidation needs the solution of coupled stress-diffusion equations. In this finite element analysis, the effective-stress principle is applied. Each point of the saturated soil is subjected to the sum of the effective stress and the pore pressure (u). By applying the pressure at the ground surface, the pore water pressure increases abruptly (i.e. at t = 0); then, a hydraulic gradient of pore pressure will create between two points inside the soil which causes the water to flow. Considering Darcy's law, v = ki (with k = soilpermeability), the flow velocity v is proportional to the hydraulic gradient *i*. This means that by absorbing extra stress by the soil skeleton, the soil consolidates as the pore water pressure decreases.

# Verification of FE Simulation

In order to evaluate the accuracy of the FE modeling of the stone-column improved

ground, two comprehensive verifications are performed as follows:

a) Verification using the results obtained by Tan et al. (2008), Han and Ye (2001) and Balaam and Booker (1981).

b) Comparison of available experimental data from the plate load test on stone columns, which was conducted on the ground of Khayyam steel mill project in Neyshabur, Iran (performed by Reinforced Earth Iran Company, 2013).

# Verification with the Results of Tan et al. (2008)

Tan et al. (2008) analyzed the unit cell of the stone column (a cylindrical soil body around a column) for their study. When the number of stone columns and the area of the ground is very large, the behavior of the improved ground can be modeled using unit cell concept. Unit cell is a cylinder containing a single column and the soil around it Bachus. 1983). (Barksdale and The consolidation and settlement of the unit cell investigated through exerting the vertical pressure of 100 kPa. Table 2 summarizes the material properties used in the numerical model of the soft foundation improved by stone column.

Figures 3 and 4 present the elastic and elasto-plastic response of the 3D-FE model built in this paper and the mentioned studies in the literature, respectively. The results are presented in terms of the ground surface settlement and the excess pore water pressure during the consolidation process. The diagrams in figures 3 and 4 reveal that there is a very good agreement between the results of the FE simulation in this paper with the analytical results obtained by Han and Ye (2001) and the numerical solution of Balaam and Booker (1981) and Tan et al. (2008).

The reason for the slight difference in the results (which is less than 10%), could be attributed to the difference in the type of meshing and modeling procedures in Tan et al. (2008) and 3D ABAQUS simulations in this paper.

#### Verification Using the Plate Load Tests in Khayyam Steel Mill Project

As a second verification, the results of the 3D-FE model is verified with the results of the field data obtained in Khayyam steel mill project in Neyshabur, Iran (performed by Reinforced Earth Iran Company in 2013). The characteristics and arrangements of the stone columns in this project are presented in Figure 5. It should be noted that the contractor

constructed the stone column groups with a triangular pattern in two stages. At the first stage, they created a rectangular pattern, thereafter, in the second stage, they completed the pattern by constructing another stone column in the middle of the previous pattern. The first and second stages shown in Figure 5 are related to these construction stages. Plate load tests are performed on some of the columns after the construction and the results are compared with the FE simulations in this section.

Table 1. Geotechnical parameters based on the results of field and laboratory tes
---

Layer Type	Depth (m)	Dry Density (kN/m <sup>3</sup> )	Void Ratio	Cohesion (kg/cm²)	Angle of Internal Friction (Φ)	Modulus of Elasticity (kg/cm <sup>2</sup> )	Poisson's Ratio	Kh (m/day)	Kv (m/day)
CL/ML	0-12	14.56	0.72	0.45	0.0	50~160	0.35~0.45	0.00864	0.00864
	12-33	16	0.67	0.8	0.0	160~270	0.40~0.45	0.00864	0.00864
	33~45	17.25	0.67	1.5	0.0	270~540	0.40~0.45	0.00864	0.00864
Stone column	0-12	19	0.7	0	40	80~100	0.3	8.64	8.64

<b>Table 2.</b> Material properties (Tan et al., 2008)								
	Unit Weight (y)	Poisson's Ratio (v)	Elastic Modulus (E)	Cohesion (c)	Angle of Friction (Φ)	Radial Permeability (k <sub>h(s)</sub> )	Vertical Permeability (k <sub>v(s)</sub> )	
	(kN/m <sup>3</sup> )		(kPa)	(kPa)	(deg)	(m/s)	(m/s)	
Soil	15	0.3	3,000	0.1	22	3.47×10-9	1.16×10 <sup>-9</sup>	
Stone column	15	0.3	30,000	1	40	3.47×10 <sup>-5</sup>	1.16×10 <sup>-5</sup>	







Fig. 4. 3D elastoplastic modeling: a) settlement, b) excess pore water pressure



Fig. 5. Arrangement of stone columns in Khayyam Neyshabur (in curtsey of Reinforced earth Iran Company)

The mechanical characteristics of the existing soil at the project site are shown in Figure 6. The investigations by Reinforced Earth Iran Company indicated the necessity of ground improvement. As shown in Figure 6, the ground water table is high and the ground is mainly composed of silty clay.

Reinforced Earth Iran Company has proposed the use of stone columns for soil improvement of the ground. By transmission of load to the depth of ground, these columns not only increase the loadbearing capacity of the soil, but also decrease the settlement. Moreover, because of high value of permeability, the excess pore water pressure is dissipated faster.

The plate load test consists of controlling the loadbearing capacity of the stone columns by exerting an axial load of approximately 1.5 to 2 times the stress due to service loads, which is about 260 kPa, and measuring the amount of column settlement. This test conforms to ASTM D1194 standard and is conducted on the stone columns with the goal of measuring the deformability of ground. The parameters of soil and stone columns are selected based on the geotechnical investigation report of Khayyam steel mill project (the values are presented in table 1). It should be noted that the drilling of the stone columns is carried out with auger, and the columns have the diameter and depth of 1m

and 12m, respectively. Figure 7 shows the plate load test results for one of the middle stone columns in the group and its comparison with 3D-FE simulation. It is obvious that the 3D-FE model can capture the real settlement behavior of the columns accurately.

Furthermore. the numerical results indicate that bulging occurs in the upper part of the column, which shows the fact that the column is not able to bear larger loads. This condition was observed under the pressure exceeding 450 kPa. It is observed in the simulations that the depth at which bulging might occur in the column is approximately two to three times the diameter of the column (i.e., Bulging depth =  $2 - 3 D_c$  with  $D_c$ representing the column diameter). This observation is in agreement with the findings of Hu et al. (1974). The bulging is vividly manifested in Figure 8 using the 3D-FE model. The results show that the bulging occurs within the depth of 1 m to 2.5 m.

Considering the very good agreement between the results, the validity of the assumptions and the analytical method implemented in numerical analysis can be ensured. Therefore, the generated 3D-FE model is used for a parametric study in the next section.



Fig. 6. Soil profile in Khayyam steel mill project in Neyshabur, Iran (in curtsey of Reinforced Earth Iran Company)



Fig. 7. Comparison of settlement result between the 3D-FE simulation and field data



contours (kPa)

#### PARAMETRIC STUDY

In this part, a comprehensive parametric study is conducted for investigating the effect of various geometric and material properties. These parameters and their corresponding ranges are presented in Table 3.

#### **Geometric Characteristics**

The results of the parametric studies related to the length, diameter, and distance of the columns are presented in terms of settlement and pore water dissipation in Figure 9. Considering the height of the columns and soil layer, the columns with the lengths of 7 and 10 m are floating columns,

while the 12 m long column is placed on a stiff layer, and the 15 m long column enters the hard layer for 3 m. The results of the parametric study about the effect of the columns' length are presented in Figure 9a,b. These results exhibit that the length of a column has a significant effect on the behavior of the stone columns. By increasing the length of a column, the ultimate settlement on the ground surface is decreased, and the rate of consolidation is increased. The settlement decrease by increasing the length of the columns from 7 to 12 m; however, by increasing the length from 12 to 15 m no significant reduction is observed in the amount of settlement.

#### Shahraki, M. et al.

Table 5. The variation range of the parameters					
Parameter	Variation Range				
The Length of the stone columns (L)	7-15 m				
The diameter of the stone columns (D)	0.6-1.2 m				
The space between the stone columns (S)	1.5-3 m				
Elastic Modulus of clay (Ec)	5-20 MPa				
Elastic Modulus of stone columns (Es)	50-150 MPa				
Poisson's ratio of clay (vc)	0.35-0.45				
Poisson's ratio of stone columns (vs)	0.20-0.35				

Table 3. The variation range of the parameters

To explore the effect of column diameter, 12 m long columns with the constant spacing of 3 m is considered. The parametric studies for column diameter are shown in Figure 9c,d. According to these figures by increasing the diameter, the settlement is reduced; however, because of the limited area of diameter, its effect on pore water pressure is not noticeable. Since further increase in the diameter is not possible due to the practical limitations, the process can be progressed by reducing the spacing between the columns. Decreasing distances between the columns causes further confinement and as a result, the settlement would be decreased (see Figure 9e,f. It should be noted that, due to practical and construction limitations, the diameter of stone column are generally limited to at most 1.2 m; Therefore, for increasing the replacement ratio of the stone columns (i.e. the area/volume occupied by the stone columns in the improved ground), it is usually practical to decrease the distance (spacing) between the columns.

## **Material Properties**

In this section, the effect of material properties of soil and stone columns such as elastic modulus and Poisson's ratio are investigated according to the values shown in Table 3. The results are presented in terms of settlement versus time as shown in Figure 10. The geometric characteristics are kept to be constant in all of the analyses and it is assumed that the columns have the height and diameter of 12 and 1m, respectively and they are arranged as shown in Figure 1 with the

constant spacing of 3 m. Three different values are assumed for the modulus of elasticity of the columns which are 50, 100 and 150 MPa, and also for the soft clay surrounding the stone column (5, 10, and 20 MPa according to Barksdale and Bachus, 1983). The results in Figure 10a show that increasing the stone columns' modulus of elasticity reduces the settlement to a certain amount. Figure 10b demonstrates the effect of Poisson's ratio on the amount of settlement. According to this figure, the Poisson's ratio of the surrounding soil has more significant effect in comparison to the stone column. In other words, the change in the settlement is trivial for different values of stone column's Poisson ratio, while it is salient for the surrounding soil.

## COMPARISON TO DEEP SOIL MIXING METHOD

Varieties of methods exist for solving the issue of soft grounds. The goal of soil improvement is, in fact, controlling the geotechnical behavior of soil so as to enhance its functionality in a typical project. To the best technique of choose soil improvement, different criteria have to be taken into account, such as the scarcity of construction time, costs, importance of the structure, the site conditions and soil type. For the clayey soil in the aforementioned project, it was possible to use Deep Soil Mixing (DSM) method as an alternative ground improvement technic.



**Fig. 9.** Parametric studies for geometric characteristics: a) and b) The effect of length of column, c) and d) The effect of column diameter, e) and f) The effect of spacing between the stone columns

Shahraki, M. et al.



Fig. 10. Parametric studies for material properties: a) effect of elastic modulus, b) effect of Poisson's ratio

In fact, using stone columns and DSM methods can better fit the problems in urban areas comparing to preloading and dynamic compaction ground improvement technics (Han, 2015). Generally, stone columns and DSM methods have shorter construction time and lower expenses in comparison with the preloading method, and they induce much lower amount of vibrations and sound pollutions as compared with dynamic compaction method. According to the Federal Highway Administration, FHWA, (Bruce et al., 2013), the columns of deep soil mixing behave similarly to the site-cast piles. Piles

are used beneath the shallow foundation. where the foundation cannot withstand the applied loads or the settlements exceed the allowable limit (Azizkandi et al., 2014). However, DSM can provide higher lateral strength with minimum costs and time in specific projects. Moreover, piles should reach the loadbearing layers of the ground in order to prevent the negative friction phenomenon. Due to these facts, the functionality of DSM method, as an alternative ground improvement technic, is compared to stone columns.

The material properties of the DSM

material in dry mode are presented in Table 4. In this kind of improvement method with cement, chemical interactions between soil, cement and water content reduce voids in soil (Mousavi et al., 2016). In order to compare the behavior of stone columns to deep soil mixing, all of the loading, boundary conditions, and geometric properties are assumed similar to that of mentioned stone columns. Therefore, the DSM columns are 12 m long with the diameter of 1 m and subjected to the service load of 260 kPa.

To compare stone column and DSM methods, the settlement versus time curve is measured using 3D-FE simulations and their diagram are presented in Figure 11. As illustrated in this figure, the stone columns let the soil consolidate with a higher rate. As a result, the stone column consolidated and reached its ultimate settlement in a period of 7 days. The comparable period for DSM is estimated to be about 64 days. On the other hand, it is observed that the DSM columns have a larger strength because they have higher values of rigidity and cohesion due to

the high amount of cement in the composition of their materials. Hence, they reduce the amount of settlement somewhat more; however, this is not very significant in this case. In addition, as depicted in Figure 12 the stone columns dissipate the excess pore water pressure with a higher rate and let the loose soil consolidate more quickly.

Accordingly, for such clay that needs controlling the consolidation settlements and improvement of the loadbearing capacity up to a specific extent, utilizing stone columns, because of their suitable properties, are advantageous as compared to DSM columns. Using DSM method is of primary interest when one has to deal with silty and loose soils and where the risk of bulging of the stone columns is high. In these circumstances, the DSM columns can provide higher bearing capacity (especially when it is required to transfer the load to larger depths, i.e. more than 15 m) since bulging does not occur in DSM columns because of having better integrity and larger strength comparing to that of stone columns.



Fig. 11. Comparison the amount of settlement and the consolidation rate of stone columns to DSM columns

Shahraki, M. et al.



Fig. 12. Comparison the dissipation of excess pore water pressure of stone columns to DSM columns

## CONCLUSIONS

In this paper, a robust 3D-FE model is built to study the behavior of a stone columnimproved ground. The 3D-FE model is verified using the available data in the literature and a real ground improvement project at Iran. Thereafter, comprehensive parametric studies are conducted to better understand the effect of various geometric characteristics and material properties on the behavior of stone columns. At the end, a quantitative comparison is provided between stone column and DSM improvement methods in order to give the engineers a better view for the choice of ground improvement methods in similar soils. The most important conclusions of the present paper can be itemized as follows:

- The results exhibit that the consolidation rate increases significantly by elongating the columns. Reaching the hard layer has a remarkable effect in decreasing the settlement of soil. However, penetration in hard layer does not change the amount of settlement significantly. In addition, the effect of diameter and area ratio on consolidation rate is not very impressive.

- If the time of ground improvement is not

crucial, The DSM method can be utilized as an alternative method. In other words, it is observed that both stone columns and DSM columns can improve the mechanical properties of the ground significantly; however, the major difference between these methods is the consolidation time. Because of providing radial drainage paths for water, stone columns can enhance the ground properties in a much shorter time. On the other hand, DSM columns can provide larger bearing capacities and settlement reduction in deep soft soils because of having more integrity, which prevents the bulging of the column at lower depths. Therefore, based on the nature of the project and its priorities, one appropriate select an ground can improvement method.

- Due to the property of the stone columns' coarse aggregates, the columns admit the soil to have a high rate of consolidation. They not only reduce the amount of final settlement, but also dissipate the excess pore water pressure more quickly. According to the results, the stone columns can be utilized as effective vertical drains that can strengthen the soft clay grounds and increase the bearing capacity to a certain extent.

#### REFERENCES

- Ambily, A.P. and Gandhi, S.R. (2007). "Behavior of stone columns based on experimental and FEM analysis", *Journal of Geotechnical and Geoenvironmental Engineering*, 133(4), 405-415.
- Balaam, N.P. and Booker, J.R. (1981). "Analysis of rigid rafts supported by granular piles", *International Journal for Numerical and Analytical Methods in Geomechanics*, 5(4), 379-403.
- Barksdale, R.D. and Bachus, R.C. (1983). "Design and construction of stone columns Volume I, Appendixes", Washington, D.C., USA: Federal Highway Administration.
- Barron, R.A. (1948). "Consolidation of fine-grained soils by drain wells", *Transactions ASCE*, Vol. 113, Paper No. 2346.
- Bruce, M.E.C., Berg, R.R., Collin, J.G., Filz, G.M., Terashi, M. and Yang, D.S. (2013). "Federal Highway Administration design manual: Deep mixing for embankment and foundation support", *Publication No. FHWA-HRT-13-046*.
- Castro, J. and Sagaseta, C. (2011). "Consolidation and deformation around stone columns: Numerical evaluation of analytical solutions", *Computers and Geotechnics*, 38(3), 354-362.
- Castro, J., Cimentada, A., da Costa, A., Cañizal, J. and Sagaseta, C. (2013). "Consolidation and deformation around stone columns: Comparison of theoretical and laboratory results", *Computers and Geotechnics*, 49, 326-337.
- Han, J. (2015). *Principles and practice of ground improvement*, John Wiley & Sons.
- Han, J. and Ye, S.L. (2001). "Simplified method for consolidation rate of stone column reinforced foundations", *Journal of Geotechnical and Geoenvironmental Engineering*, 127(7), 597-603.
- Hibbitt, Karlsson and Sorensen. (2001). "ABAQUS/Explicit: User's manual (Vol. 1)", *Hibbitt, Karlsson and Sorenson.*
- Hughes, J.M.O. and Withers, N.J. (1974). "Reinforcing of soft cohesive soils with stone columns", *Ground Engineering*, 7(3), 42-49.
- Indraratna, B., Ngo, N.T., Rujikiatkamjorn, C. and Sloan, S.W. (2015). "Coupled Discrete Element– Finite Difference method for analysing the loaddeformation behaviour of a single stone column in soft soil", *Computers and Geotechnics*, 63, 267-278.
- Moayedi, H., Huat, B.B., Mokhberi, M., Moghaddam, A.A. and Moghaddam, S.A. (2010). "Using stone column as a suitable liquefaction remediation in Persian Gulf coast", *Electronic Journal of Geotechnical Engineering*, 15(P), 1757-1767.

- Mousavi, S.E. and Wong, L.S. (2016). "Permeability characteristics of compacted and stabilized clay with cement, peat ash and silica sand", *Civil Engineering Infrastructures Journal*, 49(1), 149-164.
- Priebe, H.J. (1995). "The design of vibro replacement", *Ground Engineering*, 28(10), 31.
- Pulko, B., Majes, B. and Logar, J. (2011). "Geosynthetic-encased stone columns: Analytical calculation model", *Geotextiles and Geomembranes*, 29(1), 29-39.
- Reinforced Earth Iran Company, (2013), "Designing report for constructing the foundation of storage building of Khayyam steel mill project in Neyshabur, Iran",

http://khakemosalah.ir/content/573.

- Saeedi Azizkandi, A. and Fakher, A. (2014). "A simple algorithm for analyzing a piled raft by considering stress distribution", *Civil Engineering Infrastructures Journal*, 47(2), 215-227.
- Tan, S.A., Ng, K.S. and Sun, J. (2014). "Column groups analyses for stone column reinforced foundation", *Geotechnical Special Publication*, 233, 597-608.
- Tan, S.A., Tjahyono, S. and Oo, K.K. (2008). "Simplified plane-strain modeling of stone-column reinforced ground", *Journal of Geotechnical and Geoenvironmental Engineering*, 134(2), 185-194.