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Optimum Structural Design with Discrete Variables Using League Championship Algorithm

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ABSTRACT: In this paper a league championship algorithm (LCA) is developed for structural optimization where the optimization variables are of discrete type and the set of the values possibly obtained by each variable is also given. LCA is a relatively new metaheuristic algorithm inspired from sport championship process. In LCA, each individual can choose to approach to or retreat from other individuals in the population. This makes it able to provide a good balance between exploration and exploitation tasks in course of the search. To check the suitability and effectiveness of LCA for structural optimization, five benchmark problems are adopted and the performance of LCA is investigated and deeply compared with other approaches. Numerical results indicate that the proposed LCA method is very promising for solving structural optimization problems with discrete variables.

Keywords: Discrete Variables, League Championship Algorithm, Structural Optimization.

INTRODUCTION

Optimal design of structures, a fundamental problem in structural engineering, has attracted increasing interest from researchers in recent decades. It generally aims to achieve minimum structural weights by different optimization methods across a number of design constraints. Based on the type of the design variables, three major types of structural optimum design problems include: i) Size optimization that considers only the size variables of structural elements as design variables, which is suitable for optimal design of skeletal structures with fixed shape and connectivity (Jalili and Hosseinzadeh, 2015); ii) Layout optimization that aims to minimize the weight of the structure with considering size and shape variables together (Hosseinzadeh et al., 2016; Jalili and Talatahari, 2017); and iii) Topology optimization that tries to find optimal connectivity of structural elements by considering stability requirements of the structure (Xu et al., 2003). This paper will focus on the first class of the optimum structural design problems.

In recent years, meta-heuristic optimization methods have been successfully applied to solve various problems in civil engineering (Meshkat Razavi and Shariatmadar, 2015; Moosavian and

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Jaefarzadeh, 2015). These methods have shown great potential in solving structural optimization problems, such as Genetic Algorithms (GAs) (Pezeshk et al., 2000), Particle Swarm Optimizer (PSO) (Doğan and Saka, 2012), Ant Colony Optimization (ACO) (Camp et al., 2005), Big Bang-Big Crunch (BB-BC) (Camp and Huq, 2013) algorithm, Biogeography-Based Optimization (BBO) (Jalili et al., 2016), and Harmony Search (HS) (Lee et al., 2005, Degertekin 2008; Saka et al., 2011) algorithm.

The advantages of using the meta-heuristic search methods for attaining optimal structural designs are the finding global solutions with the high quality, simple but powerful search capability, easy to understand, simple framework, and ease of use. However, it has been experimentally observed that the construction of a perfect optimizer to solve all types of structural optimization problems, using a specific heuristic search method, is often impossible. In another word, most of the meta-heuristic optimization algorithms only give a better solution for some particular problems than others. Therefore, researchers have been developed novel optimization methods for different structural optimum design problems. The Colliding Bodies Optimization (CBO) developed by Kaveh and Mahdavi (2014), League Championship Algorithm (LCA) introduced by Jalili et al. (2016), Optics Inspired Optimization (OIO) developed by Jalili and Husseinzadeh Kashan (2018), Search Group Algorithm (SGA) proposed by Gonçalves et al. (2015), and Social Spider Algorithm (SSA) utilized by Aydogdu et al. (2017) are examples of these methods. In addition, a series of improved/hybridized versions of the standard meta-heuristic methods have been developed for solving structural optimum design problems more efficiently (Jalili and Hosseinzadeh, 2018a; Baghlani et al., 2014; Jalili et al., 2014; Kaveh et al., 2015; Aydoğdu et al., 2016; Taheri and Jalili, 2016; Aydogdu et al., 2017; Jalili and Hosseinzadeh, 2017; Jalili and Hosseinzadeh, 2018b)

In relatively recent years, more and more modern meta-heuristics inspired by nature are introducing by researchers. The power of most these algorithms comes from the fact that they mimic the successful characteristics of natural evolvable systems, e.g., selection of the fittest and adaptation to the environment. Among these algorithms is the League Championship Algorithm (LCA) which is an evolutionary stochastic search algorithm. LCA follows the concept of championship in sport. In this sense it is one of the socio-inspired algorithms. The idea of using sport as a social phenomenon to develop a modern meta-heuristic has been employed for the first time in LCA. In LCA each individual solution in the population is regarded as the team formation adopted by a sport team. These artificial teams compete according to a given schedule generated based on a single round-robin logic. Using a stochastic method, the result of the game between pair of teams is determined based on the fitness value associated to the team's formation in such a way that the fitter one has a more chance to win. Given the result of the games in the current iteration, each team preserves changes its formation (a new solution is generated) following a SWOT type analysis and the championship continues for several iterations. In this paper, LCA is used to solve structural optimization problems with discrete variables. Effectiveness of the method is verified by solving five benchmark structural design examples. The results demonstrate the surpassing ability of the proposed algorithm compared with existing techniques available in literature.

The remaining contents of the paper are organized as follows. Next section formulates the problem of optimum design of structures with discrete variables. Then, the basic concepts of the league championship algorithm are explained in detail. Numerical results and comparison are provided by using five benchmark design examples. Finally, concluding remarks are summarized.

PROBLEM DEFINITION

The main target of the optimum structural design problem is the minimization of a structure's weight, while enforcing a number of constraints on deflections and stresses. The optimum discrete design of structures can be formulated as:

$$\begin{aligned} &\text{Find: } X = \{x_1, x_2, \dots, x_{eg}\} \\ &\text{To minimize: } W(X) = \textstyle\sum_{i=1}^{m} \gamma x_i l_i \\ &x_i \in \{x_1, x_2, \dots, x_k\} \end{aligned} \quad (1)$$

where X: is the vector containing cross-sectional areas; eg: is the number of element groups; *m*: is the number of structural members; *W(.)*: is the structural weight; γ: is the material density; x_i and l_i: are the cross-sectional area and length of member *i*, respectively; $\{x_1, x_2, \dots, x_k\}$: represents the discrete set of cross-sectional areas, and *k*: is the number of available cross-sectional areas.

When applying a meta-heuristic method to the structural optimum design problem, a key issue is how the method handles the constraints relating to the problem. The literature proposes several approaches for constraint handling in the meta-heuristic methods (Mezura-Montes and Coello, 2011). However, the penalty function method is one of the simplest and very widely utilized constraint handling approaches in the field of the structural optimization. In this study, in order to consider the constraints of the problem during search process, following penalized weight is defined:

$$W^P(X) = W(X)(1 + \varphi(X))^{\varepsilon} \quad (2)$$

where $W^P(.)$: is the penalized structural weight; $\varphi(.)$: is the penalty function, and ε: is a constant positive value. The value of the penalty function is calculated based on the constraints of the problem. It has a positive value when the design constraints are violated and it is zero when the constraints of the problem are satisfied. Based on the type of the structure (truss or frame), the problem of the structural optimum design is subjected to the following inequality constraints.

Truss Structures

For a truss structure, it is assumed that the members are subjected to the axial loads. Therefore, the axial stresses caused by these axial forces should not exceed from the allowable compression or tension stresses. In addition, the displacements of all free nodes in all directions should be less than a given allowable value. Thus, by considering stress and displacement constraints, following penalty function is defined for each candidate solution:

$$\varphi(X) = \sum_{i=1}^{m}\left(max\left(g_{\sigma_t^i}(X), 0\right) + max\left(g_{\sigma_c^i}(X), 0\right)\right) + \sum_{j=1}^{n_d} max\left(g_{\delta^j}(X), 0\right) \quad (3)$$

where:

$$g_{\sigma_t^i}(X) = \frac{\sigma^i}{\sigma_t^i} - 1 \le 0 \quad (4)$$

$$g_{\sigma_c^i}(X) = 1 - \frac{\sigma^i}{\sigma_c^i} \le 0 \quad (5)$$

$$g_{\delta^j}(X) = \frac{\delta^j}{\delta_{all}^j} - 1 \le 0 \quad (6)$$

where $g_{\sigma_t^i}(.)$ and $g_{\sigma_c^i}(.)$: are the tension and compressive stress constraints for the *i*th member; $g_{\delta^j}(.)$: is the deflection constraint for *j*th node; σ^i, σ_t^i, and σ_c^i: are the existing, allowable tension, and compressive stresses for the *i*th member, respectively; δ^j: is the displacement of the *j*th node and δ_{all}^{j}: denotes its allowable value; and n_d: is the number of free nodes.

Frame Structures

In the frame structures, the members are subjected to the combined axial force and bending moment. According to LRFD (1994), interaction formula given in Eq. (7) should be checked for each member:

$$g_{\sigma^i}(X) = \begin{cases} \frac{P_u}{2\phi_c P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) - 1 \le 0 \\ \quad for: \quad \frac{P_u}{\phi_c P_n} < 0.2 \\ \frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) - 1 \le 0 \\ \quad for: \quad \frac{P_u}{\phi_c P_n} \ge 0.2 \end{cases} \quad (7)$$

where $g_{\sigma^i}(.)$: is the interaction constraint for *i*th member of the frame structure; P_u: is the required axial tension or compressive strength; P_n: is the nominal axial tension or compressive strength; ϕ_c: denotes the resistance factor (0.9 for tension and 0.85 for compression); ϕ_b: is the flexural strength factor, which is equal to 0.9; M_{ux} and M_{uy}: are the required flexural strengths in x and y directions of the section (for two dimensional frame structures: $M_{uy} = 0$) and M_{nx} and M_{ny}: represent the nominal flexural strength in the x and y directions of the section. It should be noted that the second order effect (P-delta effect) is not considered in calculation of M_{ux} and M_{uy}.

In the frame structures, the maximum lateral and inter-story displacements of the structure are regarded as displacement constraints as follows:

$$g_{\Delta}(X) = \frac{\Delta}{H} - \Delta^{*} \le 0 \quad (8)$$

$$g_{\delta_r}(X) = \frac{\delta_r}{h_r \delta^{*}} - 1 \le 0 \; for \; r = 1, \dots, ns \quad (9)$$

where $g_{\Delta}(.)$ and $g_{\delta_r}(.)$: are the lateral drift and inter-story drift constraints, respectively; Δ: is the maximum lateral displacement; Δ^{*}: is the maximum drift index; *H*: is the height of the structure; δ_r: denotes the inter-story displacement for the *r*th story; h_r: is the height of the *r*th story; *ns*: is the total number of stories in the structure; and δ^{*}: is the maximum inter-story drift index, which is considered as 1/300 according to LRFD (1994). Finally, the penalty function for a frame structure is calculated as follows:

$$\varphi(X) = \left(\sum_{i=1}^{m} \max\left(g_{\sigma^i}(X), 0\right) + \sum_{r=1}^{ns} \max\left(g_{\delta_r}(X), 0\right)\right) + \max(g_{\Delta}(X), 0) \quad (10)$$

The positive constant of ε in Eq. (2) should be selected based on the optimization problem on hand and its value is in fact problem dependent. This parameter controls the penalization of infeasible solutions and helps algorithm to focus on the feasible regions of the search space. At the initial stages of the optimization process, this value should be small enough to increase exploration ability of algorithm. But by lapse of iterations, solutions may get very close to infeasible areas of the search space. Therefore, the value of ε should be increased for more focus on feasible domain of the search space. In this study, the value of ε starts from 2 and linearly increases to 4 by lapse of the iteration.

THE LEAGUE CHAMPIONSHIP ALGORITHM (LCA)

As a socio-inspired algorithm, the league championship algorithm (LCA) is the first meta-heuristic algorithm founded on the basis of championship process followed in sport. LCA was introduced first by Husseinzadeh Kashan (Kashan, 2009; Kashan and Karimi, 2010) as an evolutionary algorithm and has gained succeed on a number of well-known optimization problems in various disciplines. For detailed reviews on this algorithm, the interested reader may refer to (Kashan and Karimi, 2010; Kashan, 2011; Kashan, 2014; Alatas 2017).

There is a unique mapping between LCA and a typical evolutionary algorithm. Just similar to population based evolutionary algorithms, a set of L random solutions form the initial population of LCA. The population may referred to as "league". The ith solution in the population is treated as the team formation associated to agent i in the population. The fitness value along with each solution is referred to as "playing strength" of the relevant team formation, in LCA terminology.

At the core of LCA is the artificial match analysis process which is responsible for the generation of new solutions within the search space. Such an analysis is followed by the coachers when they are trying to set a suitable arrangement/formation for their upcoming match.

Selection in LCA is the simple greedy selection. As output of the match analysis process, whenever a new better solution (or formation), in terms of the fitness function, has been produced for team i, which its quality exceeds the current solution, since after it enters into population as the best formation for team i. The algorithm continues for a number of seasons (S), where each season has L-1 weeks (or iterations), yielding $S\times(L-1)$ weeks of contests which is the maximum number of iterations. Remember that based on a single round-robin tournament the number of matches for each team in each season is L-1.

LCA imitates the championship process followed in sport leagues to attain a repetitive method for optimization. That is, based on the league schedule at each week, teams play in pairs and the outcome is determined based on each team playing strength resultant from a particular team formation. In the recovery period, keeping track of the previous week events, each team devises the required changes in its formation to set up a new formation for the next week contest and the championship goes on for a number of seasons. Figure 1 depicts the entire process of LCA.

LCA maintains an idealized league with its governing rules. The list of these rules that form the building blocks of the different steps of LCA can be found in (Kashan, 2009; Kashan, 2011). Given the flowchart of Figure 1, in the following, a brief introduction is given on the main modules of LCA.

Generating the League Schedule

In LCA a single round-robin (SRR) schedule is used by which each participant plays every other participant once in a season. For a league composed of L teams the SRR tournament conducts $L\times(L-1)/2$ matches for the reason that in each of (L-1) weeks, L/2 matches will be run between all teams.

Figure 2 shows the single round-robin scheduling algorithm for the case of $L = 8$. In Figure 2a the schedule of matches for the first week has been depicted, where team 8 plays with 1; team 7 plays with 2 and so on. Based on Figure 2b, in the second week, one team, say team 1, is fixed and the order is rotated clockwise. So, team 1 plays with team 7, team 8 plays with team 6 and so on. In the third week, the order is rotated once again clockwise. The process proceeds until reaching the initial state again. Typically we assume that L is even. In LCA the same schedule is used for all of the S seasons.

Determining the Winner/Loser

Given the league schedule, let us assume that teams i and j will play at week t. The formation associated to teams i and j is represented by $X_i^t=(x_{i1}^t,x_{i2}^t,...,x_{in}^t)$, and $X_j^t=(x_{j1}^t,x_{j2}^t,...,x_{jn}^t)$, and their associated playing strengths is $f(X_i^t)$ and $f(X_j^t)$, respectively. Recall that $f(X = (x_1, x_2, ..., x_n))$ is a n variables numerical function that should be minimized over decision space defined by $x_d^{\min} \leq x_d \leq x_d^{\max}, \forall d = 1,..,n$. Then Eq. (11) is expressed as follows:

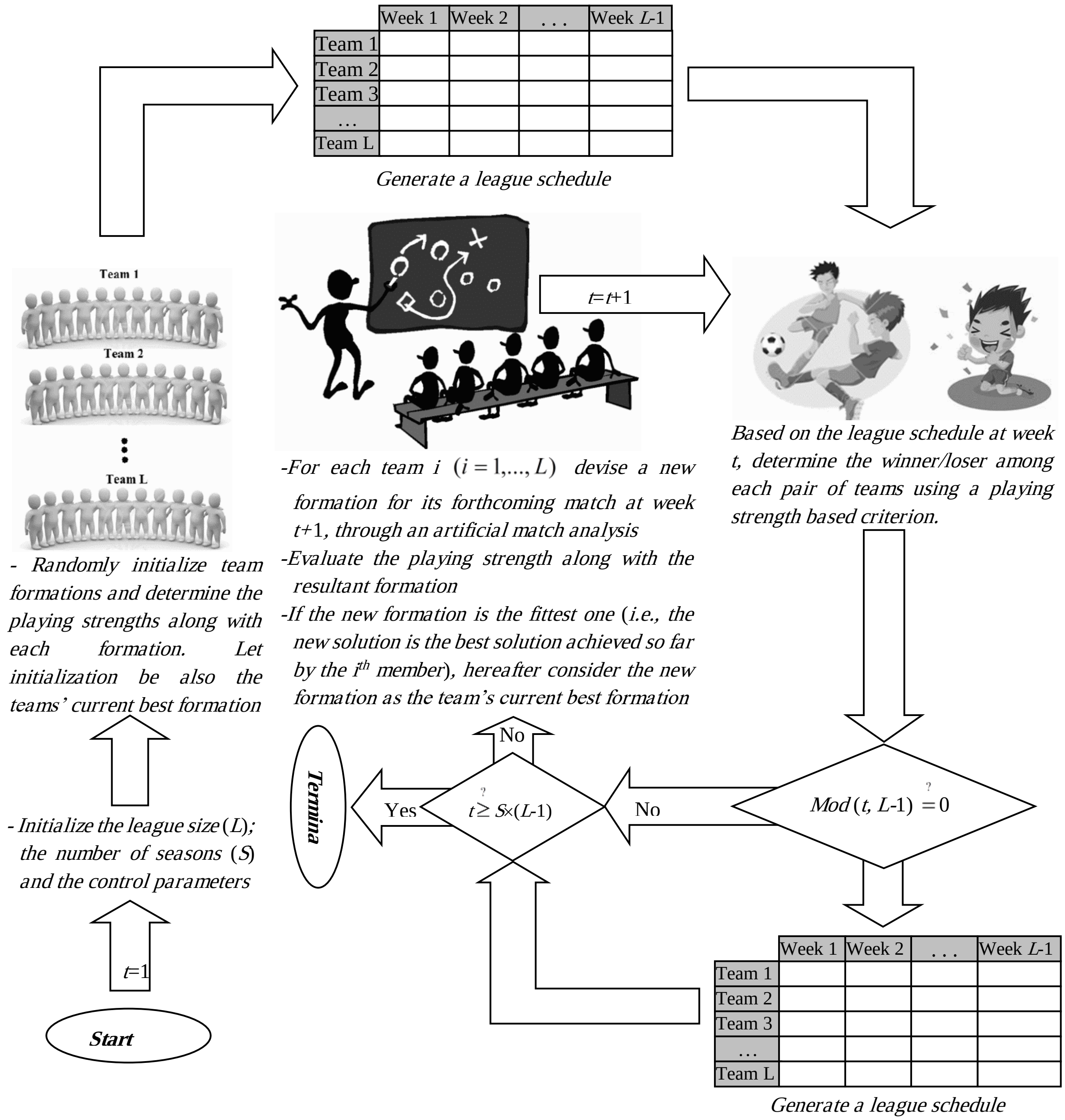

Fig. 1. Flowchart of LCA

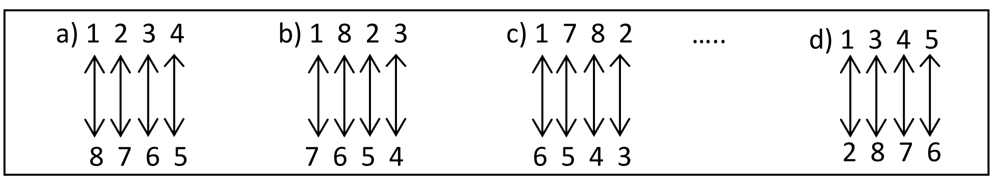

Fig. 2. An illustrative example of the league scheduling algorithm

$$p_i^t = \frac{f(X_j^t) - \hat{f}}{f(X_j^t) + f(X_i^t) - 2\hat{f}}. \tag{11}$$

The probability of beating team *j* is addressed by team *i* at week *t*. $\hat{f}^t = \min_{i=1,\dots,L}\{f(B_i^t)\}$ is an ideal value, where $B_i^t = (b_{i1}^t, b_{i2}^t, \dots, b_{in}^t)$ is the best experienced formation by team *i* until week *t*. To determine B_i^t, a selection based on fitness values is conducted between X_i^t and B_i^{t-1}. After computing p_i^t, a random value is generated and team *i* wins and team *j* loses if the random value is less than or equal to p_i^t.

Setting Up a New Team Formation

There are typically two types of learning sources available for coachers during the post-match analysis; internal learning and external learning. Similarly in LCA, there are both internal and external learning for generating possibly better solutions. By internal learning we address the artificial analysis of the previous performance at week *t* in terms of strengths or weaknesses. By external learning we address the artificial analysis of the opponent's previous performance at week *t* in terms of opportunities or threats. Such an analysis is known as SWOT analysis.

To model an artificial match analysis for team *i* for generating a new solution correspond to it at iteration (week) *t*+1, if it had already won/lost the match from/to team *j* at iteration *t*, we can assume this success/loss had been directly the result of strengths/weaknesses along with team *i* or alternately it had been directly the result of weaknesses/strengths along with team *j*. Based on the league timetable, if the upcoming game of team *i* at iteration $t+1$ is with *l*, then if it already had won/lost the match from/to team *k* at iteration *t*, then the victory/fail and the formation supporting it may be sensed as a direct threat/opportunity by team *i*. Obviously, such victory/fail has been attained via some strengths/weaknesses. Concentrating on the strengths/weaknesses of team *l*, can provide a way to avoid from the possible threats.

The above rational is modelled mathematically to obtain the updating equations for generating new solutions by LCA. The new formation $X_i^{t+1} = (x_{i1}^{t+1}, x_{i2}^{t+1}, \dots, x_{in}^{t+1})$ for team *i* $(i = 1, \dots, L)$ at iteration *t*+1 can be set based on one of the following equations, and is determined based the result of its previous game and its opponent previous game (for more details on the rationale of these equations please refer to Kashan, 2011, 2014).

Case 1: *i had won and l had won too, then*

$$x_{id}^{t+1} = b_{id}^t + (\psi_1 r_1 (b_{id}^t - b_{kd}^t) + \psi_1 r_2 (b_{id}^t - b_{jd}^t)) \quad \forall d = 1, \dots, n \tag{12}$$

Case 2: *i had won and l had lost, then*

$$x_{id}^{t+1} = b_{id}^t + (\psi_2 r_1 (b_{kd}^t - b_{id}^t) + \psi_1 r_2 (b_{id}^t - b_{jd}^t)) \quad \forall d = 1, \dots, n \tag{13}$$

Case 3: *i had lost and l had won, then*

$$x_{id}^{t+1} = b_{id}^t + (\psi_1 r_1 (b_{id}^t - b_{kd}^t) + \psi_2 r_2 (b_{jd}^t - b_{id}^t)) \quad \forall d = 1, \dots, n \tag{14}$$

Case 4: *i had lost and l had lost too, then*

$$x_{id}^{t+1} = b_{id}^t + (\psi_2 r_1 (b_{kd}^t - b_{id}^t) + \psi_2 r_2 (b_{jd}^t - b_{id}^t)) \quad \forall d = 1, \dots, n \tag{15}$$

In Eqs. (12) and (13) r_1 and r_2: are uniform random numbers. ψ_1 and ψ_2 : are scale coefficients.

The feasible X_i^t differs from B_i^t in all dimensions. However on many functions, due to the early convergence of the algorithm the number of dimension changes should be less than *n*. Eq. (16) simulates the number of changes made in B_i^t randomly via inserting the randomly selected elements of X_i^{t+1}.

$$q_i^t = \left\lceil \frac{\ln(1-(1-(1-p_c)^n)r)}{\ln(1-p_c)} \right\rceil + q_0 - 1 \quad : q_i^t \in \{1,...,n\} \tag{16}$$

Again *r*: is a random number and $p_c < 1$, $p_c \neq 0$ is a control parameter. It is expected that the larger values for p_c, enforce a smaller number of changes are recommended.

NUMERICAL EXAMPLES

The applicability of the LCA method has been investigated on five benchmark design examples namely; 52-bar planar truss, 47-bar transmission tower, one-bay 8-story frame, three-bay 24-story frame, and 582-bar tower structures, and results are compared with the results reported by a number of existing meta-heuristic search techniques. The maximum number of structural analysis is considered as follows: 12,500 for first example, 30,000 for second example, 12,000 for third and fifth example, and 15,000 for the fourth example. Moreover, the parameters used to run LCA on all design examples considered in this sections are as follows. The league size L is set equal to 8 teams. The retreat scale coefficient ψ_2 is set equal to 1.5 and the approach scale coefficient ψ_1 is considered equal to 0.5. The value of p_c is decreased in quadratic way from 1 to -1 to enforce a small number of changes made in a team's solution at the start of search and preserve many number changes made in a team's solution at the final stages of the search. In addition, LCA and the structural analysis are coded in Matlab platform and run 30 independent trials for each design example on a Dell Vostro 1520 with Intel CoreDuo2 2.66 GHz processor and 4 GB RAM memory.

A 52-Bar Planar Truss Structure

The 52-bar planar truss structure shown in Figure 3 is our first design example. All members are made of steel: the material density and modulus of elasticity are 207 GPa and 7860 kg/m^3, respectively. The structure members are classified in 12 groups as follows: (1) A_1-A_4, (2) $A_5 – A_{10}$, (3) $A_{11} – A_{13}$, (4) $A_{14} – A_{17}$, (5) $A_{18} – A_{23}$, (6) $A_{24} – A_{26}$, (7) $A_{27} – A_{30}$, (8) $A_{31} – A_{36}$, (9) $A_{37} – A_{39}$, (10) $A_{40} – A_{43}$, (11) $A_{44} – A_{49}$ and (12) $A_{50} – A_{52}$. The nodes 17, 18, 19 and 20 at top of the structure bear the loads P_x = 100 kN and P_y = 200 kN in the *x* and *y* directions, respectively. Moreover, the allowed compressive and tension stresses in each member is considered as ±180 MPa. In addition, the cross-sectional area values for the members should be selected from the discrete set listed in the Table 1.

The optimal designs obtained through LCA and other related optimization techniques in the literature are recorded in Table 2. From the results of Table 2, it can be concluded that LCA finds a better design than HPSO (Li et al., 2009), DHPSACO (Kaveh and Talatahari, 2009), SOS (Cheng and Prayogo, 2014), AFA (Baghlani et al., 2014), WOA (Mirjalili and Lewis, 2016), MCSS (Kaveh et al., 2015), and IMCSS (Kaveh al., 2015) methods, and the same design as compared with the CBO (Kaveh and Mahdavi, 2014) method. However, it should be noted that LCA is more efficient than the CBO (Kaveh and Mahdavi, 2014) method in terms performance statistics. The average weight, the standard deviation, and the worst weight obtained by LCA are 1949.06 lb, 60.85 lb, and 2135.96 lb, respectively, while these values for the CBO (Kaveh and Mahdavi, 2014) method are 1963.12 lb, 106.01 lb, and 2262.8 lb, respectively. Although LCA requires slightly more structural analyses than CBO (Kaveh and Mahdavi, 2014) method, the required structural analyses to reach the optimal design for LCA is significantly less than HPSO (Li et al., 2009), DHPSACO (Kaveh and Talatahari, 2009), AFA (Baghlani et al., 2014), MCSS (Kaveh et al., 2015), and IMCSS (Kaveh et al., 2015) methods. Moreover, Figure 4 compares the existing values of axial stresses in the members of the structure with the corresponding allowable values. As can be seen, the axial stresses in some members are very close to the allowable tension stress. In addition, Figure 5 shows the convergence curves of LCA for the 52-bar planar truss structure.

A 47-Bar Planar Power Line Tower Structure

Figure 6 shows the 47-bar planar power line tower structure as the second design example. This structure consists of 47 members and 22 nodes. Using symmetry about the y-axis, the members are classified into 27 design groups. The Young's modulus and material density of members are 0.3 lb/in^2 and 30,000 ksi, respectively. The structure is subjected to the three different loading conditions as follows: i) 6.0 kips acting in the positive x-direction and 14.0 acting in the negative y-direction at nodes 17 and 22, ii) 6.0 kips acting in the positive x-direction and 14.0 kips acting in the negative y-direction at node 17, and iii) 6.0 kips acting in the positive x-direction and 14.0 kips acting in the negative y-direction at node 22. In fact, the first loading condition demonstrates the applied load by the two power lines to the tower at an angel and the rest of the conditions occur when one of the two lines snaps. As design constraints, both stress and buckling constraints are considered in this design example. The stress constraint is considered as 20 ksi in tension and 15.0 ksi in compression. In addition, the Euler buckling compressive stress for each member of the structure is calculated as follows:

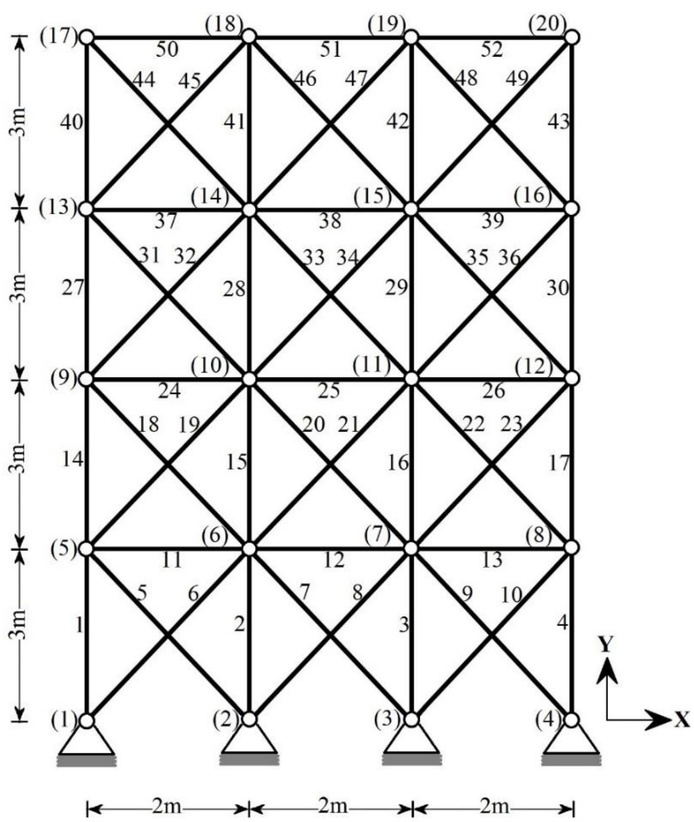

Fig. 3. Schematic of 52-bar planar truss structure

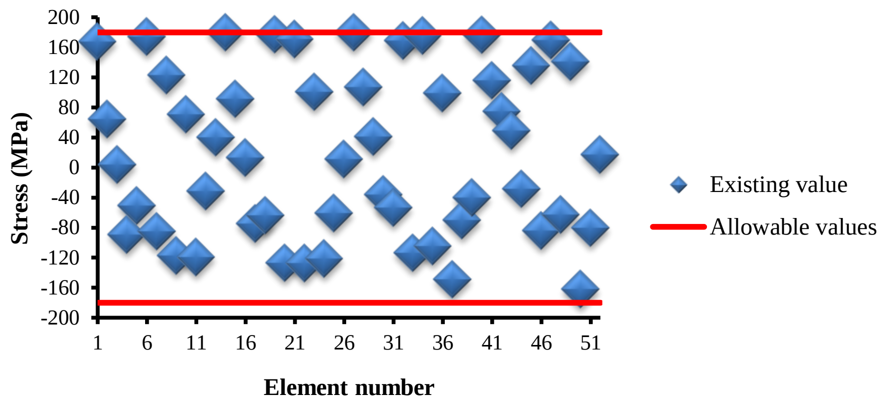

Fig. 4. Comparison of the existing axial stresses with the allowable values

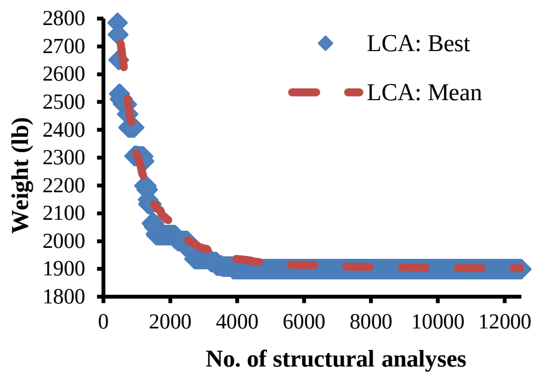

Fig. 5. Convergence curves of LCA for the 52-bar planar truss structure

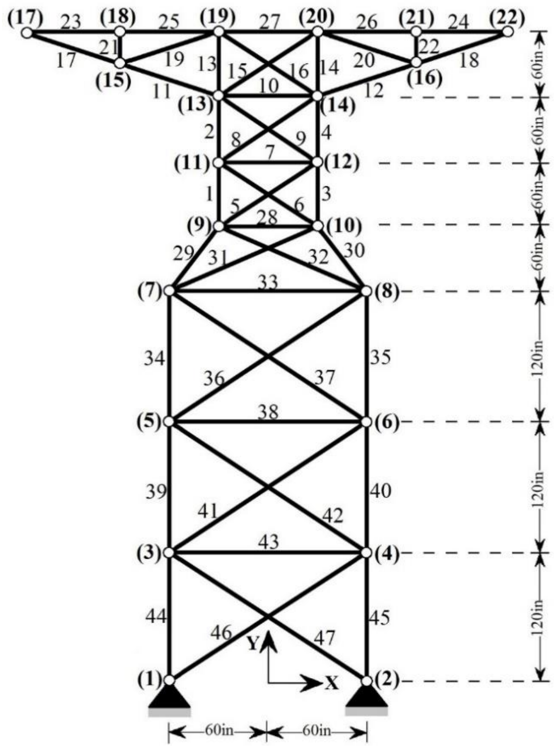

Fig. 6. Schematic of 47-bar planar power line tower structure

Table 1. The list of available cross-sectional areas from the AISC code

	A (mm^2)		A (mm^2)		A (mm^2)		A (mm^2)
1	71.613	17	1008.385	33	2477.414	49	7419.43
2	90.968	18	1045.159	34	2496.769	50	8709.66
3	126.451	19	1161.288	35	2503.221	51	8967.724
4	161.29	20	1283.868	36	2696.769	52	9161.272
5	198.064	21	1374.191	37	2722.575	53	9999.98
6	252.258	22	1535.481	38	2896.768	54	10322.56
7	285.161	23	1690.319	39	2961.248	55	10903.2
8	363.225	24	1696.771	40	3096.768	56	12129.01
9	388.386	25	1858.061	41	3206.445	57	12838.68
10	494.193	26	1890.319	42	3303.219	58	14193.52
11	506.451	27	1993.544	43	3703.218	59	14774.16
12	641.289	28	729.031	44	4658.055	60	15806.42
13	645.16	29	2180.641	45	5141.925	61	17096.74
14	792.256	30	2238.705	46	5503.215	62	18064.48
15	816.773	31	2290.318	47	5999.988	63	19354.8
16	939.998	32	2341.931	48	6999.986	64	21612.86

Table 2. Comparison of the optimal designs obtained by different methods for the 52-bar planar truss structure

Element Group	Li et al. (2009)	Kaveh and Talatahari (2009)	Kaveh and Mahdavi (2014)	Cheng and Prayogo (2014)	Baghlani et al. (2014)	Mirjalili and Lewis (2016)	Kaveh et al. (2015)		Present Work
	HPSO	**DHPSACO**	**CBO**	**SOS**	**AFA**	**WOA**	**MCSS**	**IMCSS**	**LCA**
A_1-A_4	4658.055	4658.055	4658.055	4658.055	4658.055	4658 .055	4658.055	4658.055	4658.055
A_5-A_{10}	1161.288	1161.288	1161.288	1161.288	1161.288	1161 .288	1161.288	1161.288	1161.288
A_{11}-A_{13}	363.225	494.193	388.386	494.193	363.225	494 .193	363.225	494.193	506.451
A_{14}-A_{17}	3303.219	3303.219	3303.219	3303.219	3303.219	3303 .219	3303.219	3303.219	3303.219
A_{18}-A_{23}	940.000	1008.385	939.998	940.000	939.998	940.000	939.998	939.998	939.998
A_{24}-A_{26}	494.193	285.161	506.451	494.193	494.193	494 .193	506.451	494.193	506.451
A_{27}-A_{30}	2238.705	2290.318	2238.705	2238.705	2238.705	2238 .705	2238.705	2238.705	2238.705
A_{31}-A_{36}	1008.385	1008.385	1008.385	1008.385	1008.385	1008 .385	1008.385	1008.385	1008.385
A_{37}-A_{39}	388.386	388.386	506.451	494.193	641.289	494 .193	388.386	494.193	388.386
A_{40}-A_{43}	1283.868	1283.868	1283.868	1283.868	1283.868	1283 .868	1283.868	1283.868	1283.868
A_{44}-A_{49}	1161.288	1161.288	1161.288	1161.288	1161.288	1161 .288	1161.288	1161.288	1161.288
A_{50}-A_{52}	792.256	506.451	506.451	494.193	494.193	494 .193	729.031	494.193	506.451
Best Weight (lb)	1905.49	1904.83	1899.35	1902.605	1903.37	1902 .605	1904.05	1902.61	1899.35
Average weight (lb)	N/A	N/A	1963.12	N/A	N/A	N/A	N/A	N/A	1949.06
Standard deviation (lb)	N/A	N/A	106.01	N/A	N/A	N/A	N/A	N/A	60.85
No. of structural analyses	100,000	5300	3840	N/A	52,600	2250	4225	4075	3920
Worst weight (lb)	N/A	N/A	2262.8	N/A	N/A	N/A	N/A	N/A	2135.96
CPU time (s)	-	-	-	-	-	-	-	-	136.06

$$\sigma_i^{cr} = \frac{-KEA_i}{L_i^2} \text{ (i=1,2,3,…, 47)} \quad (17)$$

where K: is a constant parameter which depends on the type of the cross-sectional geometry; E: is the Young's modulus of the material; and L_i: is the length of ith member. The buckling constant K is set to 3.96 as in Lee et al. (2005).

Table 3 compares the designs parameters reported by LCA with the results of other methods taken from literature. From Table 3, it is obviously that LCA can obtain better design than both HS (Lee et al., 2005) and CBO (Kaveh and Mahdavi, 2014) methods. On the other hand, LCA requires 18,720 structural analyses to reach optimum solution, which is significantly less than those required by the other methods. In this way, LCA saves more than 60% and 25% computational effort than HS (Lee, Geem et al. 2005) and CBO (Kaveh and Mahdavi 2014) methods in this design example. The average and standard deviation of the results obtained by the CBO (Kaveh and Mahdavi 2014) method are 2405.91 lb and 19.61 lb, respectively, while the corresponding values for LCA are 2421 lb and 18.11 lb, respectively.

Moreover, in order to check the feasibility of the best design obtained by LCA, Figure 7 compares the existing values of axial stresses in the members of the structure with the allowable values for three different loading conditions. From Figure 7, it is clearly seen that LCA yields a better design compared to other methods while satisfying all the constraints considered. In addition, Figure 8 depicts the convergence curves of LCA for this design example. From this figure, it can be seen that LCA reaches gradually to the vicinity of the optimum solutions after about 17,000 analyses without any abrupt changes.

Table 3. Comparison of the optimal designs obtained by different methods for the 47-bar planar power line tower structure

Design Variables	**Lee et al. (2005) HS**	**Kaveh and Mahdavi (2014) CBO**	**Present Work LCA**
A_1,A_3	3.840	3.84	3.840
A_2,A_4	3.380	3.38	3.380
A_5,A_6	0.766	0.785	0.766
A_7	0.141	0.196	0.111
A_8,A_9	0.785	0.994	0.785
A_{10}	1.990	1.8	2.130
A_{11},A_{12}	2.130	2.130	2.130
A_{13},A_{14}	1.228	1.228	1.228
A_{15},A_{16}	1.563	1.563	1.563
A_{17},A_{18}	2.130	2.130	2.130
A_{19},A_{20}	0.111	0.111	0.111
A_{21},A_{22}	0.111	0.111	0.111
A_{23},A_{24}	1.800	1.800	1.800
A_{25},A_{26}	1.800	1.800	1.800
A_{27}	1.457	1.563	1.457
A_{28}	0.442	0.442	0.602
A_{29},A_{30}	3.630	3.630	3.630
A_{31},A_{32}	1.457	1.457	1.563
A_{33}	0.442	0.307	0.250
A_{34},A_{35}	3.630	3.090	3.090
A_{36},A_{37}	1.457	1.266	1.266
A_{38}	0.196	0.307	0.307
A_{39},A_{40}	3.840	3.840	3.840
A_{41},A_{42}	1.563	1.563	1.563
A_{43}	0.196	0.111	0.111
A_{44},A_{45}	4.590	4.590	4.590
A_{46},A_{47}	1.457	1.457	1.457
Weight (lb)	2396.8	2386.0	2385.04
Average weight (lb)	N/A	2405.91	2421.61
Standard deviation (lb)	N/A	19.61	18.11
No. of structural analyses	45,557	25,000	18,720
Worst weight (lb)	N/A	2467.73	2421.61
CPU time (s)	-	-	293.78

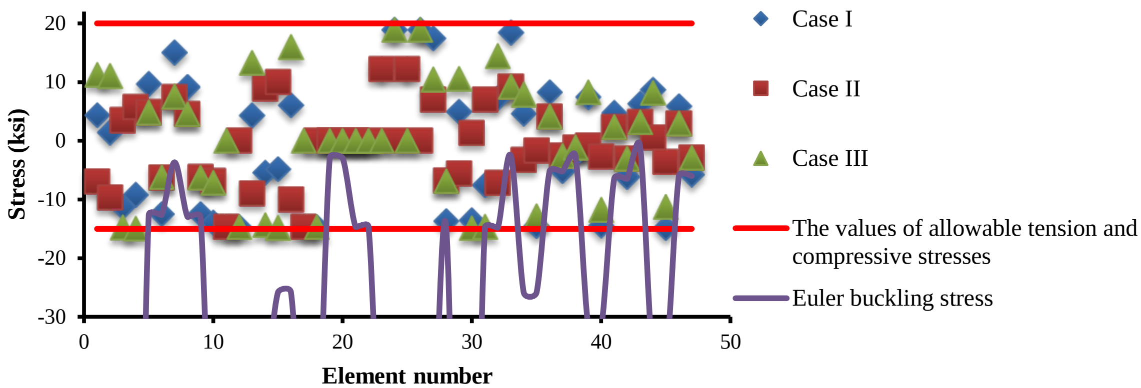

Fig. 7. Comparison of existing axial stresses with the allowable values for the 47-bar planar power line tower structure

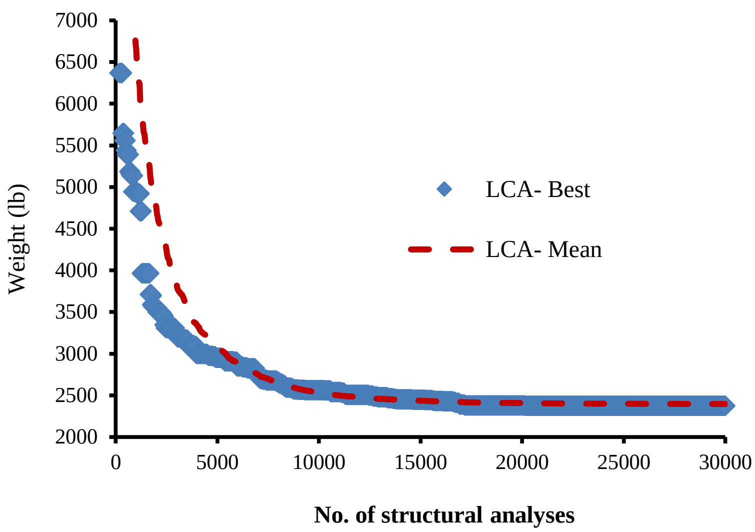

Fig. 8. Convergence curves of LCA for the 47-bar planar power line tower structure

A One-Bay 8-Story Frame Structure

The third design example is the size optimization of a one-bay eight-story frame structure shown in Figure 9. The Young's modulus is taken as 200 GPa. Due to fabrication conditions, the members of the frame structure are categorized into eight design group as depicted in Figure 9. The lateral drift at the top of the structure is considered as design constraint, which must be less than 5.08 cm. Also, the cross-sectional areas for the members of the structure must be selected from 267 W-shaped sections of the AISC (LRFD 1994) database.

Table 4 provides comparison of the optimal designs obtained using LCA with that of other techniques in the literature including OC (Khot et al., 1976, Camp et al., 1998), GA (Camp et al., 1998), ACO (Kaveh and Shojaee, 2007), and IACO (Kaveh and Talatahari, 2010) methods. Again, from Table 4, it can be checked that the design yielded by LCA is lighter than other methods. Also, LCA needs significantly fewer amount of structural analyses than ACO (Kaveh and Shojaee, 2007) method. However, LCA needs a little more structural analyses than IACO (Kaveh and Talatahari, 2010) method. In addition, Figure 10 illustrates the convergence diagrams of LCA for this design example.

A Three-Bay 24-Story Frame Structure

The fourth design example is a three-bay 24-story frame structure shown in Figure 11. The loads demonstrated in Figure 11 are as follows: W = 5761.85 lb, w_1 = 300 lb/ft, w_2 = 436 lb/ft, w_3 = 474 lb/ft, and w_4 = 408 lb/ft.

The frame structure is composed of 168 members. In order to impose the fabrication condition on the construction process, the members are divided into 20 design groups as shown in Figure 11. Each of the four beam element groups are selected from all of the 267W-sections, while 16 column member groups should be selected from only W14 sections. The Young's modulus and yield stress of frame members are 29,732 ksi and 33.4 ksi, respectively. The frame is designed based on the LRFD (1994) specification. Moreover, the inter-story drift displacement is considered as a deflection constraint, which should not be exceeds from 1/300 of story height.

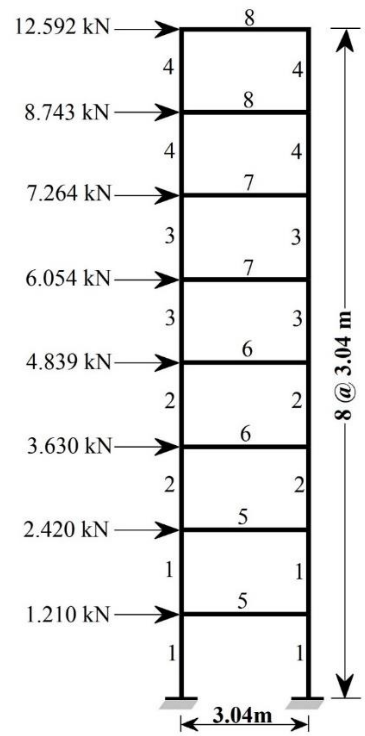

Fig. 9. Schematic of one-bay 8-story frame structure

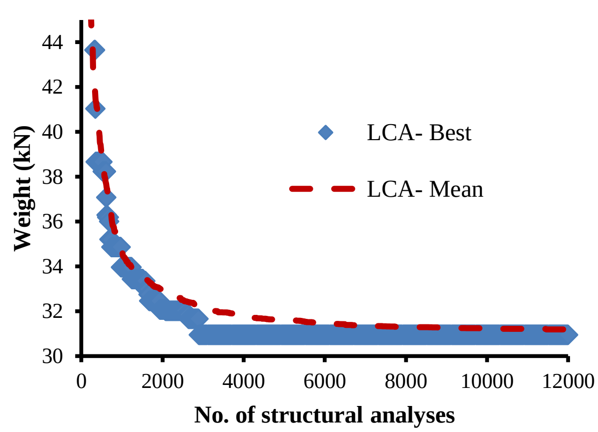

Fig. 10. Convergence curves of LCA for the one-bay 8-story frame structure

Table 4. Comparison of the optimal designs obtained by different methods for the one-bay eight-story frame structure

Design Variables		**Khot et al. (1976)**	**Camp et al. (1998)**	**Kaveh and Shojaee (2007)**	**Kaveh and Talatahari (2010)**	**Present Work**
Type	**Story**	**OC**	**GA**	**ACO**	**IACO**	**LCA**
Beam	1-2	W21×68	W18×35	W16×26	W21×44	W21×44
Beam	3-4	W24×55	W18×35	W18×40	W18×35	W18×35
Beam	5-6	W21×50	W18×35	W18×35	W18×35	W16×26
Beam	7-8	W12×40	W18×26	W14×22	W12×22	W14×22
Column	1-2	W14×34	W18×46	W21×50	W18×40	W21×44
Column	3-4	W10×39	W16×31	W16×26	W16×26	W16×26
Column	5-6	W10×33	W16×26	W16×26	W16×26	W16×26
Column	7-8	W8×18	W12×16	W12×14	W12×14	W12×14
Weight (kN)		41.02	32.83	31.68	31.05	30.8497
No. of structural analyses		N/A	N/A	4500	2440	4600
CPU time (s)		-	-	-	-	101.60

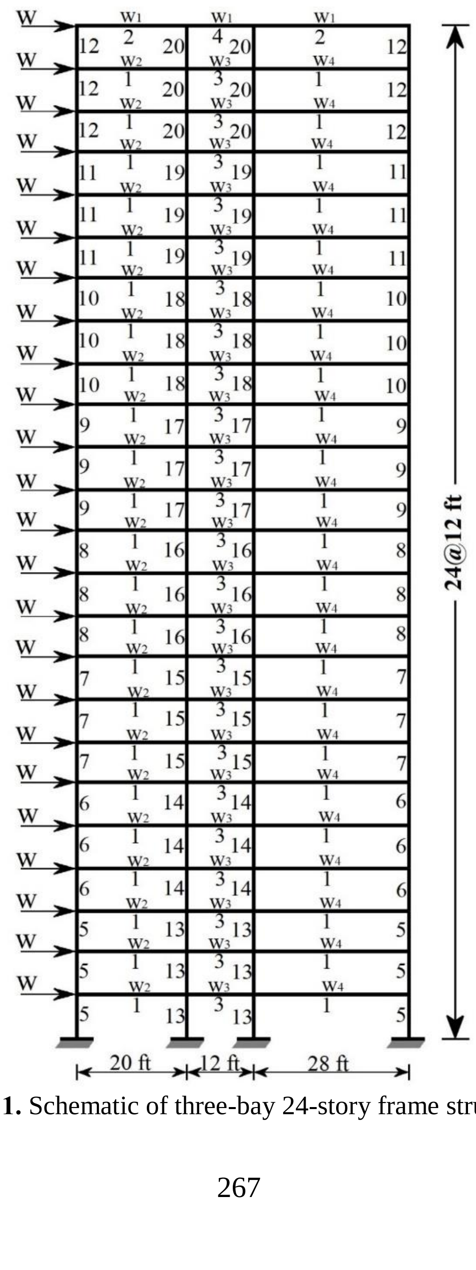

Fig. 11. Schematic of three-bay 24-story frame structure

The values of effective length factor (K_x) for members of frame structure are calculated by the following approximate equation proposed by Dumonteil (1992), which is accurate within about -1.0 and +2.0% of the exact value (Hellesland 1994):

$$K_x = \sqrt{\frac{1.6G_AG_B + 4(G_A + G_B) + 7.5}{G_A + G_B + 7.5}} \qquad (18)$$

where G_A and G_B: are the relative stiffness of a column at its two ends. Also, the out-of-plane effective length factor (K_y) is considered as 1.0 and all members are considered as unbraced along their lengths.

The effectiveness and robustness of LCA are verified via the comparison of the best weight and structural analyses with HS (Degertekin 2008), IACO (Kaveh and Talatahari 2010), ICA (Kaveh and Talatahari 2010), TLBO (Toğan 2012), and DE (Kaveh and Farhoudi 2013) methods as given in Table 5. As observed from the table, LCA is capable to find lighter structural weight than all other methods. The structural weight obtained by LCA is 202,410 lb which is 6%, 7% and 5% lighter than those yielded by HS (Degertekin 2008), IACO (Kaveh and Talatahari 2010), and ICA (Kaveh and Talatahari 2010) methods, respectively. Also, not only the design obtained by LCA is slightly lighter than TLBO (Toğan 2012) method, but it also requires fewer amount of structural analyses than TLBO (Toğan 2012) method. In order to check the feasibility of the optimum design obtained by LCA, Figures. 12 and 13 compare the value of inter-action ratios, Eq. (7), in the members and inter-story drifts with the corresponding allowable values. The maximum value of the inter-action formula is 0.87. Moreover, from Fig.13, it can be seen that the inter-story drifts in seven stories of the structure approach to allowable value.

Finally, Figure 14 illustrates the convergence diagrams of LCA for the three-bay 24-story frame structure. The values of the average and standard deviation during 30 independent runs are 209,255.37 lb and 4933 lb, respectively.

A 582-Bar Tower Structure

Figure 15 shows the last investigated design example. This is a tower structure with pin-jointed connections that consists of 582 members and 153 nodes. The members of the structure are classified into 32 independent design groups as displayed in Figure 15. The cross-sectional areas should be selected from the discrete set of 137 standard steel W-shaped sections based on the area and radii of gyration of the section (Hasançebi et al. 2009). The range of cross-sectional areas varies from 39.74 cm^2 to 1387.09 cm^2. The utilized steel for the members of the structure has a Young's modulus of 29,000 ksi and a yield stress of 36 ksi. At the nodes of the structure, a load of 1.12 kips acts in the X and Y directions, and a load of -6.74 kips acts in the Z direction. For this design example, the design constraints consist of the displacement and stress constraints. For all of the free nodes, the displacement should not exceed from ±3.15 in. In addition, the stress constraint is calculated as follows (AISC (1989) code):

$$\begin{array}{ll} \sigma_i^+ = 0.6Fy & \text{for } \sigma_i \geq 0 \\ \sigma_i^- & \text{for } \sigma_i < 0 \end{array} \qquad (19)$$

where:

$$\sigma_i^- = \begin{cases} [(1 - \frac{\lambda_i^2}{2C_c^2})F_y]/(\frac{5}{3} + \frac{3\lambda_i}{C_c} - \frac{\lambda_i{}^3}{8C_c{}^3}) & \text{for } \lambda_i < C_c \\ \frac{12\pi^2 E}{23\lambda_i^2} & \text{for } \lambda_i \geq C_c \end{cases} \qquad (20)$$

where E: is the modulus of elasticity; F_y: is the yield stress of steel; C_C: is the slenderness ratio (λ_i) dividing the elastic and inelastic buckling regions ($C_C = \sqrt{2\pi^2 E/F_y}$) ; λ_i: is the slenderness ratio ($\lambda_i = kL_i/r_i$); k: is the effective length factor; L_i: is the member length; and r_i: is the radius of gyration.

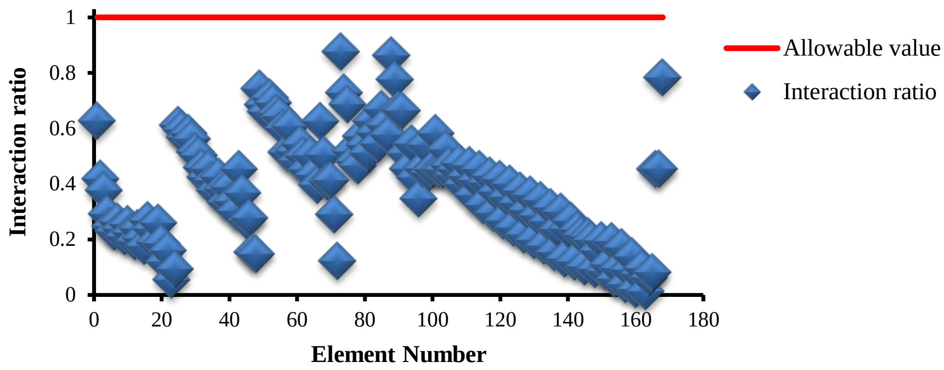

Fig. 12. The values of inter-action formula for member of the three-bay 24-story frame structure

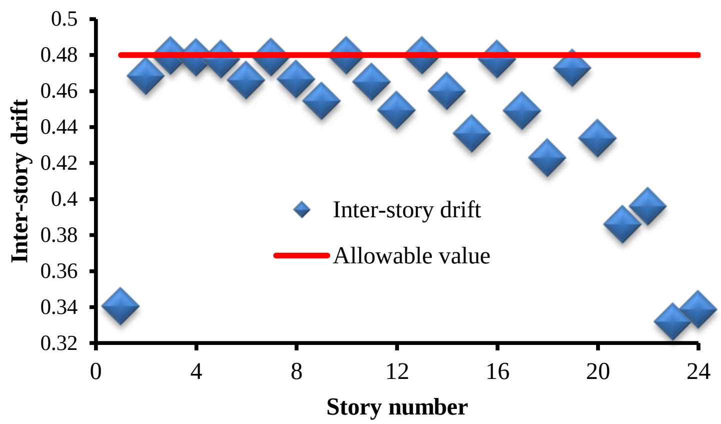

Fig. 13. Comparison of the inter-story drifts with the allowable value for the three-bay 24-story frame structure

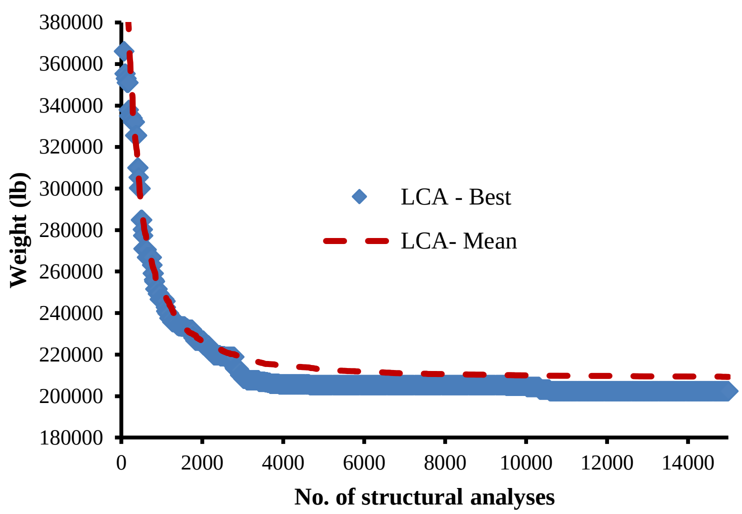

Fig. 14. Convergence curves of LCA for the three-bay 24-story frame structure

Table 5. Comparison of the optimal designs obtained by different methods for the three-bay 24-story frame structure

Element Group			Degertekin (2008)	Kaveh and Talatahari (2010)	Kaveh and Talatahari (2010)	Toğan (2012)	Kaveh and Farhoudi (2013)	Present Work
Type	**Bay**	**Story**	**HS**	**IACO**	**ICA**	**TLBO**	**DE**	**LCA**
Beam	1,3	1-23	W30×90	W30×99	W30×90	W30×90	W30X90	W30×90
Beam	1,3	24	W10×22	W16×26	W21×50	W8×18	W6X20	W10×12
Beam	2	1-23	W18×40	W18×35	W24×55	W24×62	W21X44	W24×55
Beam	2	24	W12×16	W14×22	W8×28	W6×9	W6X9	W6×8.5
Column-E	-	1-3	W14×176	W14×145	W14×109	W14×132	W14X159	W14×120
Column-E	-	4-6	W14×176	W14×132	W14×159	W14×120	W14X145	W14×159
Column-E	-	7-9	W14×132	W14×120	W14×120	W14×99	W14X132	W14×120
Column-E	-	10-12	W14×109	W14×109	W14×90	W14×82	W14X99	W14×90
Column-E	-	13-15	W14×82	W14×48	W14×74	W14×74	W14X68	W14×68
Column-E	-	16-18	W14×74	W14×48	W14×68	W14×53	W14X61	W14×38
Column-E	-	19-21	W14×34	W14×34	W14×30	W14×34	W14X43	W14×38
Column-E	-	22-24	W14×22	W14×30	W14×38	W14×22	W14X22	W14×22
Column-I	-	1-3	W14×145	W14×159	W14×159	W14×109	W14X109	W14×109
Column-I	-	4-6	W14×132	W14×120	W14×132	W14×99	W14X109	W14×90
Column-I	-	7-9	W14×109	W14×109	W14×99	W14×99	W14X90	W14×90
Column-I	-	10-12	W14×82	W14×99	W14×82	W14×90	W14X82	W14×82
Column-I	-	13-15	W14×61	W14×82	W14×68	W14×68	W14X74	W14×68
Column-I	-	16-18	W14×48	W14×53	W14×48	W14×53	W14X43	W14×61
Column-I	-	19-21	W14×30	W14×38	W14×34	W14×34	W14X30	W14×30
Column-I	-	22-24	W14×22	W14×26	W14×22	W14×22	W14X26	W14×26
Weight (lb)			214,860	217,464	212,725	203,008	205,084.206	202,410
No. of structural analyses			13,942	3500	7500	12,000	N/A	10,640
CPU time (s)			-	-	-	-	-	670.87

Column-E: exterior column; Column-I: interior column

Optimization results obtained from PSO (Hasançebi, Çarbaş et al., 2009), CBO (Kaveh and Mahdavi, 2014), and LCA have been summarized in Table 6. When Table 6 has been examined, it is seen that LCA gives a better design than the PSO (Hasançebi, Çarbaş et al., 2009) and CBO (Kaveh and Mahdavi 2014) methods. LCA obtains a structural volume of 21.5661 m^3, while it is 22.3958 m^3 and 21.8376 m^3 for the PSO (Hasançebi, Çarbaş et al., 2009) and CBO (Kaveh and Mahdavi, 2014) methods, respectively. Moreover, the statistical results give a vision on the general behavior of the algorithms during the solution finding process. According to Table 6, LCA is also better than the PSO (Hasançebi, Çarbaş et al., 2009) and CBO (Kaveh and Mahdavi, 2014) methods in terms of the average weight, the standard deviation, and the worst weight. Moreover, LCA requires significantly fewer amount of structural analyses than PSO (Hasançebi et al., 2009) method. The graphics showing the change of the minimum structural volume according to the number of structural analyses have been given in Figure 16.

According to the investigated numerical tests, it can be seen that the exploration ability of LCA is managed well since it allows getting away from loser solutions in the population to escape from local optima traps. At the same time the exploitation ability of algorithm is managed by getting approach to winner solutions. So there is a balance between exploration and exploitation tasks during the search process by LCA. Moreover, as our experimental results indicate, the optimum structural weights generated and evaluated by the algorithm is competitive and on some cases is the smallest among rivals. This implies that the convergence speed of LCA is acceptable.

Table 6. Comparison of optimum designs obtained by various methods for 582-bar tower truss structure

Design Variables	Hasançebi et al. (2009)		Kaveh and Mahdavi (2014)		Present Work	
	PSO		CBO		LCA	
	Ready Section	Area (cm^2)	Ready Section	Area (cm^2)	Ready Section	Area (cm^2)
1	W8X21	39.74	W8X21	39.74	W12X22	41.81
2	W12X79	149.68	W12X79	149.68	W24X76	144.52
3	W8X24	45.68	W8X28	53.22	W8X28	53.16
4	W10X60	113.55	W10X60	90.96	W21X62	118.06
5	W8X24	45.68	W8X24	45.68	W8X24	45.68
6	W8X21	39.74	W8X21	39.74	W10X22	41.87
7	W8X48	90.97	W10X68	128.38	W8X48	90.97
8	W8X24	45.68	W8X24	45.68	W8X24	45.68
9	W8X21	39.74	W8X21	39.74	W14X22	41.87
10	W10X45	85.81	W14X48	90.96	W21X57	107.74
11	W8X24	45.68	W12X26	49.35	W10X22	41.87
12	W10X68	129.03	W21X62	118.06	W21X62	118.06
13	W14X74	140.65	W18X76	143.87	W12X65	123.23
14	W8X48	90.97	W12X53	100.64	W8X67	127.10
15	W18X76	143.87	W14X61	115.48	W10X77	145.81
16	W8X31	55.9	W8X40	75.48	W8X35	66.45
17	W8X21	39.74	W10X54	101.93	W10X54	101.94
18	W16X67	127.1	W12X26	49.35	W8X24	45.68
19	W8X24	45.68	W8X21	39.74	W12X22	41.81
20	W8X21	39.74	W14X43	81.29	W16X45	85.81
21	W8X40	75.48	W8X24	45.68	W10X22	41.87
22	W8X24	45.68	W8X21	39.74	W12X22	41.81
23	W8X21	39.74	W10X22	41.87	W12X30	56.71
24	W10X22	41.87	W8X24	45.68	W10X22	41.87
25	W8X24	45.68	W8X21	39.74	W12X22	41.81
26	W8X21	39.74	W8X21	39.74	W8X24	45.68
27	W8X21	39.74	W8X24	45.68	W12X22	41.81
28	W8X24	45.68	W8X21	39.74	W14X22	41.87
29	W8X21	39.74	W8X21	39.74	W12X22	41.81
30	W8X21	39.74	W6X25	47.35	W10X22	41.87
31	W8X24	45.68	W10X33	62.64	W12X22	41.81
32	W8X24	45.68	W8X28	53.22	W14X22	41.87
Volume (m^3)	22.3958		21.8376		21.5661	
Mean (m^3)	22.48		23.41		22.0676	
Standard Deviation (m^3)	N/A		1.67		0.2442	
No. of analyses	50,000		6400		6400	
Worst (m^3)	22.78		26.82		22.4021	
CPU time (s)	-		-		1282.31	

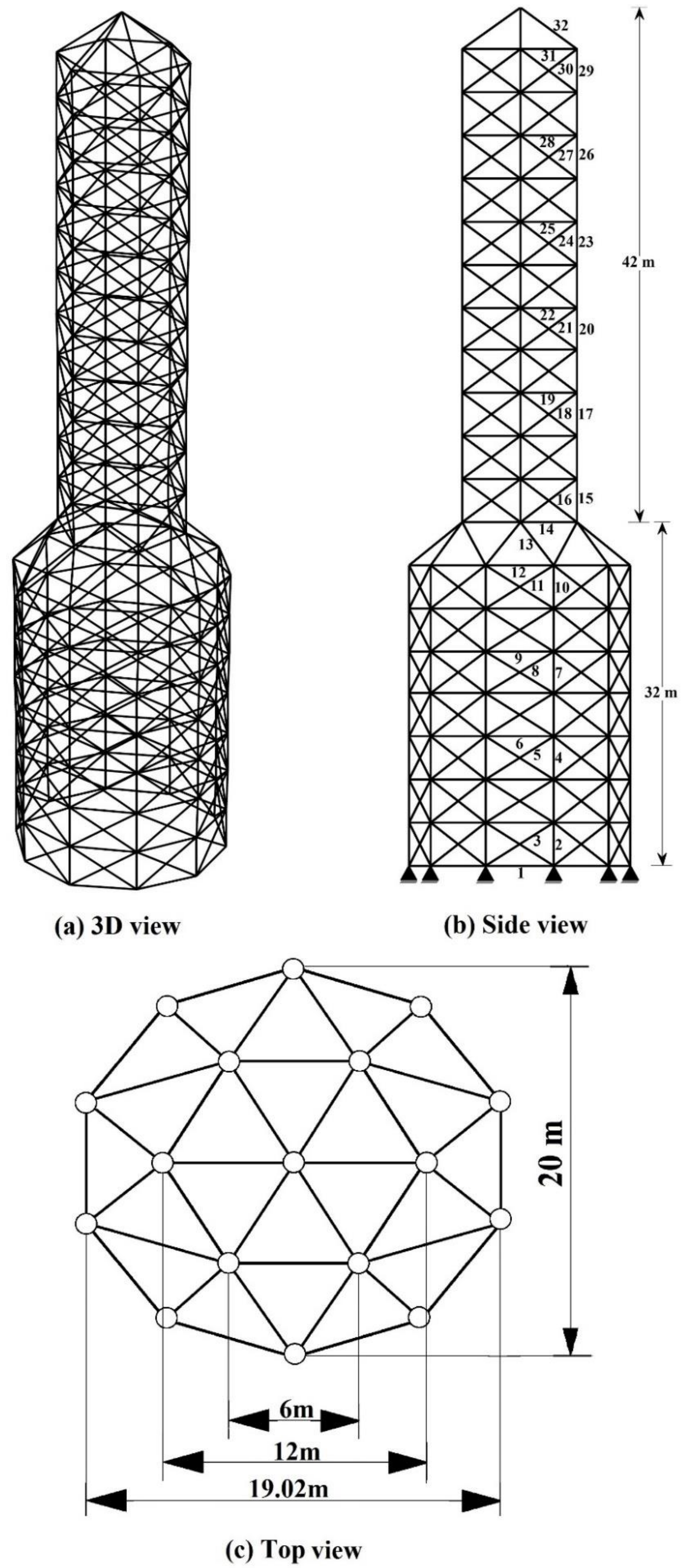

Fig. 15. Schematic of 582-bar tower structure

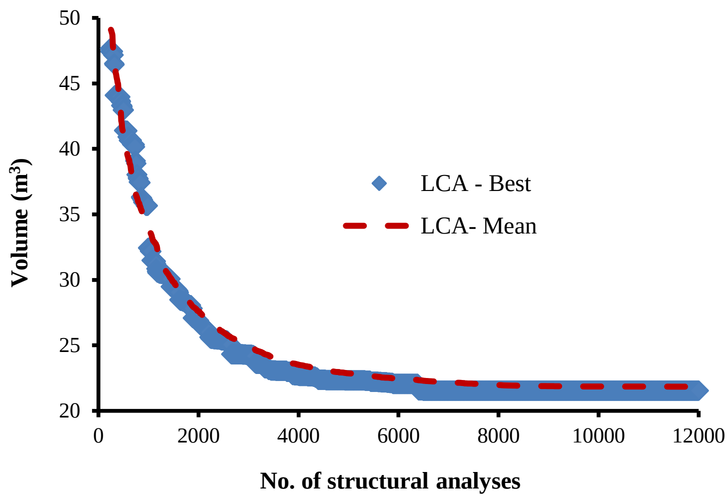

Fig. 16. Convergence curves of LCA for 582-bar tower structure

CONCLUSIONS

In this paper, the proposed league championship algorithm (LCA) was successfully implemented to solve structural optimization problems with discrete variables. LCA is a new, robust, and strong algorithm to solve global numerical optimization problems. The main idea of this method is inspired by the championship process followed by sport teams in a sport league. In LCA, a number of individuals as sport teams compete in an artificial league for several weeks (iterations). Based on the league schedule in each week, teams play in pairs and their game outcome is determined in terms of win or loss, given known the playing strength (fitness value) along with the particular team formation/arrangement (solution) followed by each team. Keeping track of the previous week events, each team devises the required changes in its formation/playing style (a new solution is generated) for the next week contest and the championship goes on for a number of seasons (stopping condition). In order to show the abilities of the new approach in finding optimal designs for structures, LCA has been implemented on five benchmark structural design examples with discrete design variables. For the all design examples, the same internal parameters are used in LCA. The results have been compared with those obtained by the other available optimization techniques in the literature. It is seen from the comparisons that the proposed LCA method performs better than other methods in the literature in terms of obtained optimum designs and required computational effort. The performance of LCA can be further tested by dividing the feasible solutions to some leagues (e.g. league one, two etc) set based on the quality of solutions, where the qualifiers of each league move to a so-called premier league. This would help reducing the exploration time.

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