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Longitudinal magnetic field effect on torsional vibration of carbon nanotubes

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ABSTRACT

Torsional dynamic analysis of carbon nanotubes under the effect of longitudinal magnetic field is carried out in the present study. Torque effect of an axial magnetic field on a carbon nanotube has been defined using Maxwell's relation. Nonlocal governing equation and boundary conditions for carbon nanotubes are obtained by using Hamilton's minimum energy principle. Eringen's nonlocal stress gradient elasticity theory is used in the formulation. Fourth order nonlocal equation of motion is solved by utilizing differential quadrature method. Clamped-clamped and clamped-free nonlocal boundary conditions are considered. Nonlocal and axial magnetic field effects on torsional vibration of carbon nanotubes are investigated. The magnetic field has significant effects on the dynamics of carbon nanotubes and may lead to torsional buckling. Critical torsional buckling load reduces with nonlocal effects. Nonlocality shows softening effect on carbon nanotube's lattice structure. Present results can be used in the design and analysis of nanoelectromechanical products like nano-motors.

1. Introduction

After the discovery by Iijima [1], carbon nanotubes (CNTs) have been used in many applications due to their superior physical, thermal and mechanical properties like material strength, low density, electrical and heat conductivity. Researchers have developed applications of CNTs in nanoelectromechanical products [2,3] like nano-bearing [4] or nano-gearbox [5] and pharmaceutical industry [6].

Static and dynamic analysis of CNTs are very important at the design stage. Magnetic field, temperature, density or concentration changes can affect the dynamics of the CNT, especially in torsional deformation. CNTs can be modeled using continuum and discrete models. In the discrete models, force and moment resultants on each node are solved by using static and dynamic equilibrium conditions. Lattice dynamics and molecular dynamic (MD) simulation are the discrete models. In these models numerical calculations take a lot of time because of the number of nodes. For example, the dynamic analysis of long or wide CNTs can take several days in some analysis.

In continuum models, a CNT is assumed as a continuous elastic body. Elasticity theory can be used in the modeling of their static and dynamic behaviors. It is shown in previous studies that classical elasticity theory is inadequate in the mechanical modeling of CNTs due to its size independent modeling characteristics. Most materials exhibit size dependent elastic behavior in length scale range 1-50nm. Eringen [7,8] dealt with this problem and proposed length scale dependent nonlocal elasticity. Eringen calibrated his model by fitting nonlocal theory results to the lattice dynamics results.

The magnetic field effect on CNTs has been recently investigated by researchers. Many studies have been published in the last years. Possible applications of magnetically sensitive CNTs have been developed by researchers. Ajiki and Ando [9] studied the electronic states of CNTs in a magnetic field. Bellucci et al. [10] investigated the vertical magnetic field effect on the transport properties of CNTs. Nanotube based field-effect transistors were studied by Fedorov et al. [11]. Magnetically driven torsional actuation of a multi-walled carbon nanotube (MWCNT) yarn artificial muscle was modeled by Lee et al. [12].

Wang et al. [13] dealt with the wave propagation problem of CNTs embedded in an elastic medium under the effect the longitudinal magnetic field. Li et al. [14] investigated the dynamic characteristics of MWCNTs under the effect of a transverse magnetic field. Li and Wang [15] achieved the dynamic analysis magneto-elastic CNTs with axial magnetic field effect. Murmu et al. investigated the longitudinal magnetic field effect on the vibration of a CNT system [16], transverse vibration of magnetically sensitive double-walled carbon nanotube (DWCNT) [17] and the influence of a transverse magnetic field on the axial vibration of CNTs [18]. Narendar et al. [19] studied the

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longitudinal magnetic field effect on wave dispersion characteristics SWCNTs. Wang et al. [20] investigated the van der Waals interaction and transverse magnetic field effect on vibration characteristics of MWCNTs. Ghorbanpour Arani et al. investigated the longitudinal magnetic field effect on wave propagation in an embedded DWCNT conveying fluid [21] and nonlocal nonlinear vibration instability of a double-CNT system [22]. Li et al. [23] studied the buckling, bending and free vibration of magnetoelectroelastic nonlocal Timoshenko nanobeam. Wang et al. [24] investigated the wave propagation in a fluid-conveying SWCNT with temperature and magnetic field effects. Chang [25] studied the statistical nonlinear dynamic behaviors of a SWCNT subjected to a longitudinal magnetic field.

Kiani has published remarkable papers about magnetic field effect on CNTs. Free transverse vibrations of elastically supported DWCNTs [26], free dynamic deflection of an elastically supported DWCNTs [27], the vibration and instability of a SWCNT [28] and free in-plane and out-of-plane vibrations of SWCNTs [29] with magnetic field effect were studied by Kiani. Also the axial buckling behavior [30], the axial load-bearing capacity [31] and the vibrations and instability of pre-tensioned current-carrying nanowires [32] were investigated by Kiani accounting the surface energy effect.

In recent years, carbon nanotubes have been modeled as functionally graded (FG) material with size dependent theories in studies. FG nanoplates [33], bi-directional FG nanobeams [34,35], stress analysis of rotating FG nano-disks [36], FG nano-disks under thermoelastic loading [37], vibrations of three-dimensionally graded nanobeams [38], pull-in behavior of FG nanobeams [39] and thermoelastic analysis of rotating FG nano-disks have been investigated by researchers. Size dependent analysis for nanostructures were carried out in [40,41]. Adeli et al. studied the torsional vibration of nano-cone using nonlocal strain gradient theory. Farajpour et al. [42] investigated the nonlinear vibration, electrical and magnetic instabilities of nanofilms. Hadi et al. [43] modeled numerically of a cell membrane under pressure.

Carbon nanotubes can be used as a nanomotors which is excited by magnetic field. Some recent studies presented the possible application of magnetically actuated nanomotors [44–47]. In literature search, magnetic field effect on torsional vibration of CNTs has not been investigated previously according to the author's knowledge. Longitudinal and transverse vibrations of CNTs under the effect of magnetic field were studied in previous studies. But in these works, nonlocal magnetic field effect was not taken into consideration. In the present study, torque effect of the longitudinal magnetic field is defined by using Lorentz magnetic force. The nonlocal governing equation and boundary conditions of CNT are obtained using Hamilton Principle and Eringen's nonlocal elasticity theory. The effects of some parameters like magnetic field, nonlocal parameter and nanotube length are investigated in detail. In addition to the previous studies, fourth order differential equation of motion for CNT, which includes the nonlocal magnetic field effect, is solved by using Differential Quadrature Method (DQM).

2. Analysis

The present study is based on the longitudinal magnetic field effect on carbon nanotube's torsional dynamics. It is assumed that, the longitudinal magnetic field acts at opposite directions on the upper and lower surface of the CNT (illustrated in Figs. (1) and (2)). In this section, firstly the longitudinal magnetic field effect and the circumferential Lorentz Force on CNT are denoted. Governing equation and boundary conditions, including the magnetic field force are obtained by using Hamilton's Principle and the nonlocal elasticity theory.

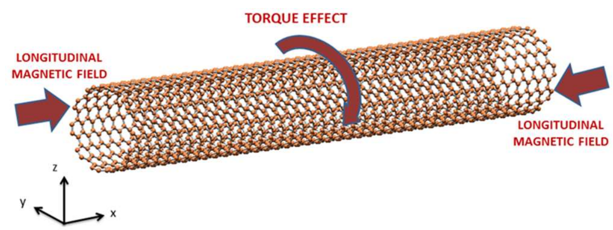

Figure 1. CNT Atomic Lattice Structure Model

2.1. Magnetic field effect

Longitudinal magnetic field equations may be obtained from the Maxwell's Relations [48]. The Lorentz force resultants due to the longitudinal magnetic field along the x, y, and z directions has been defined in previous studies [16]:

$$F_x = 0 \tag{1}$$

$$F_y = \eta H_x{}^2 \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y \partial z}\right) \tag{2}$$

$$F_z = \eta H_x{}^2 \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 v}{\partial y \partial z}\right) \tag{3}$$

For the present torsional vibration analysis of a CNT, only the circumferential displacement, $v = v(x,t)$ is taken into account. The Lorentz Force in the y direction is (Eq. (2)):

$$F_y = \eta H_x{}^2 \left(\frac{\partial^2 v}{\partial x^2}\right) \tag{4}$$

The angular deformation can be written in circumferential direction as $v = \theta R$. Where θ is the angular displacement and R is the radius of the CNT. Eq. (4) can be written as:

$$F_y = \eta H_x{}^2 R^2 \left(\frac{\partial^2 \theta}{\partial x^2}\right) \tag{5}$$

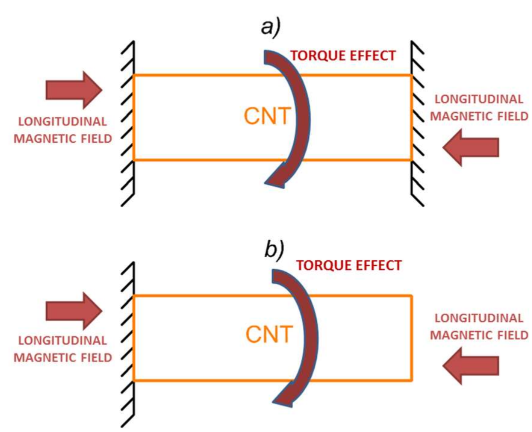

Figure 2. Continuum Model of the Present Problem a) Clamped-Clamped b) Clamped-Free

Magnetic torque of the circumferential Lorentz force in circumferential direction is:

$$T_\theta = F_y R = \eta {H_x}^2 R^3 \left(\frac{\partial^2\theta}{\partial x^2}\right) \tag{6}$$

Eq. (6) is the effect of the longitudinal magnetic field in circumferential direction of CNT.

2.2. Nonlocal Elasticity Theory for CNTs

The differential form of nonlocal relation can be written as [7,8,49]:

$$(1-\mu\nabla^2)\tau_{kl} = \lambda\varepsilon_{rr}\delta_{kl} + 2G\varepsilon_{kl} \tag{7}$$

where τ_{kl} is the nonlocal stress tensor, ε_{kl} is the strain tensor, λ and G are the Lame constants, $\mu=(e_0a)^2$ is called the nonlocal parameter, a is an internal characteristic length and e_0 is a constant. Eringen determined the e_0 parameter as 0.39 with matching the dispersion curves based on the atomic models. Eq. (7) can be used in order to write stress in an elasticity problem for the shear deformation in one dimensional form [50]:

$$\left(1-\mu\frac{\partial^2}{\partial x^2}\right)\tau = G\gamma \tag{8}$$

where γ and τ are the shear strain and the shear stress of the CNT, respectively. The torque relation can be expressed as:

$$T = \int_A \tau z \, dA \tag{9}$$

where A is the cross sectional area of the CNT. Multiplying the both sides of Eq. (9) by z and integrating them with respect to the cross sectional area of CNT lead to the following constitutive equation:

$$T - (e_0a)^2\frac{\partial^2 T}{\partial x^2} = GI_P\frac{\partial\theta}{\partial x} \tag{10}$$

where I_P is polar moment of inertia for CNT and defined as:

$$I_P = \pi\frac{({R_2}^4 - {R_1}^4)}{2} \tag{11}$$

2.3. Equation of Motion

The equation of motion and boundary conditions are obtained using the Hamilton Principle and Nonlocal Elasticity. The Hamilton Principle can be written as:

$$\int_{t_1}^{t_2}[\delta W + \delta E_K - \delta E_P] = 0 \tag{12}$$

where W denotes the work done by the magnetic torque, E_K denotes the kinetic energy and E_P denotes the potential energy of the CNT. They are defined as [51,52]:

$$W = \int_0^L \eta {H_x}^2 R^3 \left(\frac{\partial^2\theta}{\partial x^2}\right)\theta dx \tag{13}$$

$$E_K = \int_0^L \rho I_P \left(\frac{\partial\theta}{\partial t}\right)^2 dx \tag{14}$$

$$E_P = \int_0^L GI_P \left(\frac{\partial\theta}{\partial x}\right)^2 dx \tag{15}$$

If W, E_K and E_P are defined according to the nonlocal elasticity theory and variational principle, following equations are obtained:

$$\delta W = \int_{t_1}^{t_2}\int_0^L \frac{\partial}{\partial x}\left[\eta {H_x}^2 R^3 \frac{\partial\theta}{\partial x}\right]\delta\theta dxdt + \int_{t_1}^{t_2}\int_0^L \frac{\partial^2}{\partial x^2}\left[\mu\eta {H_x}^2 R^3 \frac{\partial^2\theta}{\partial x^2}\right]\delta\theta dxdt \tag{16}$$

$$\delta E_K = \int_0^L\int_{t_1}^{t_2} \frac{\partial}{\partial t}\left[\rho I_P\left(\frac{\partial\theta}{\partial t}\right)\right]\delta\theta dtdx + \int_{t_1}^{t_2}\int_0^L \frac{\partial}{\partial x}\left[\mu\rho I_P\left(\frac{\partial^3\theta}{\partial x\partial t^2}\right)\right]\delta\theta dxdt \tag{17}$$

$$\delta E_P = \int_{t_1}^{t_2}\int_0^L \frac{\partial}{\partial x}\left[GI_P\left(\frac{\partial\theta}{\partial x}\right)\right]\delta\theta dxdt \tag{18}$$

If Eq. (12) is rearranged according to Eq. (16)-(18), Eq. (19) is obtained:

$$\left\{-\int_{t_1}^{t_2}\int_0^L \frac{\partial}{\partial x}\left[\eta {H_x}^2 R^3\frac{\partial\theta}{\partial x}\right]\delta\theta dxdt + \int_{t_1}^{t_2}\left[\eta {H_x}^2 R^3\frac{\partial\theta}{\partial x}\right][\delta\theta(L) - \delta\theta(0)]dt\right\} + \left\{\int_{t_1}^{t_2}\int_0^L\frac{\partial^2}{\partial x^2}\left[\mu\eta {H_x}^2R^3\frac{\partial^2\theta}{\partial x^2}\right]\delta\theta dxdt + \int_{t_1}^{t_2}\left[\mu\eta {H_x}^2R^3\left(\frac{\partial^2\theta}{\partial x^2}\right)\right]\left[\frac{\partial\delta\theta(L)}{\partial x} - \frac{\partial\delta\theta(0)}{\partial x}\right]dt - \int_{t_1}^{t_2}\left[\mu\eta {H_x}^2R^3\left(\frac{\partial^3\theta}{\partial x^3}\right)\right][\delta\theta(L)-\delta\theta(0)]dt\right\} + \left\{-\int_0^L\int_{t_1}^{t_2}\frac{\partial}{\partial t}\left[\rho I_P\left(\frac{\partial\theta}{\partial t}\right)\right]\delta\theta dtdx + \int_0^L\left[\rho I_P\left(\frac{\partial\theta}{\partial t}\right)\right][\delta\theta(t_2) - \delta\theta(t_1)]dx\right\} - \left\{\int_{t_1}^{t_2}\int_0^L\frac{\partial^2}{\partial x\partial t}\left[\mu\rho I_P\left(\frac{\partial^2\theta}{\partial x\partial t}\right)\right]\delta\theta dtdx - \int_{t_1}^{t_2}\left[\mu\rho I_P\left(\frac{\partial^3\theta}{\partial x\partial t^2}\right)\right][\delta\theta(L)-\delta\theta(0)]dt\right\} - \left\{-\int_{t_1}^{t_2}\int_0^L\frac{\partial}{\partial x}\left[GI_P\left(\frac{\partial\theta}{\partial x}\right)\right]\delta\theta dtdx + \int_{t_1}^{t_2}\left[GI_P\left(\frac{\partial\theta}{\partial x}\right)\right][\delta\theta(L) - \delta\theta(0)]dt\right\} = 0 \tag{19}$$

If Eq. (19) is reorganized, following equation is obtained:

$$\int_{t_1}^{t_2}\int_0^L\left\{-\left[\eta {H_x}^2R^3\left(\frac{\partial^2\theta}{\partial x^2}\right)\right] + \left[\mu\eta {H_x}^2R^3\left(\frac{\partial^4\theta}{\partial x^4}\right)\right] - \left[\rho I_P\left(\frac{\partial^2\theta}{\partial t^2}\right)\right] + \left[\mu\rho I_P\left(\frac{\partial^4\theta}{\partial x^2\partial t^2}\right)\right] + \left[GI_P\left(\frac{\partial^2\theta}{\partial x^2}\right)\right]\right\}\delta\theta dtdx + \int_{t_1}^{t_2}\left\{\left[\eta {H_x}^2R^3\frac{\partial\theta}{\partial x}\right] - \left[\mu\eta {H_x}^2R^3\left(\frac{\partial^3\theta}{\partial x^3}\right)\right] - \left[\mu\rho I_P\left(\frac{\partial^3\theta}{\partial x\partial t^2}\right)\right] - \left[GI_P\left(\frac{\partial\theta}{\partial x}\right)\right]\right\}[\delta\theta(L) - \delta\theta(0)]dt - \int_{t_1}^{t_2}\left[\mu\eta {H_x}^2R^3\left(\frac{\partial^2\theta}{\partial x^2}\right)\right]\left[\frac{\partial\delta\theta(L)}{\partial x} - \frac{\partial\delta\theta(0)}{\partial x}\right]dt = 0 \tag{20}$$

According to Eq. (19), the governing equation of motion of a nanotube under the effect of the magnetic field can be written as:

$$GI_P\left(\frac{\partial^2\theta}{\partial x^2}\right) = \rho I_P\left(\frac{\partial^2\theta}{\partial t^2}\right) - \mu\rho I_P\left(\frac{\partial^4\theta}{\partial x^2\partial t^2}\right) + \eta {H_x}^2R^3\left(\frac{\partial^2\theta}{\partial x^2}\right) - \mu\eta {H_x}^2R^3\left(\frac{\partial^4\theta}{\partial x^4}\right) \tag{21}$$

Inserting μ=0 leads to the classical equations of motion for torsional vibration of a CNT including a magnetic field. The boundary conditions are obtained as:

$$\left[\eta {H_x}^2R^3\frac{\partial\theta}{\partial x} - \mu\eta {H_x}^2R^3\left(\frac{\partial^3\theta}{\partial x^3}\right) - \mu\rho I_P\left(\frac{\partial^3\theta}{\partial x\partial t^2}\right) - GI_P\left(\frac{\partial\theta}{\partial x}\right)\right][\delta\theta] = 0 \tag{22}$$

$$\left[-\mu\eta {H_x}^2R^3\left(\frac{\partial^2\theta}{\partial x^2}\right)\right]\left[\frac{\partial\delta\theta}{\partial x}\right] = 0 \tag{23}$$

Similarly, inserting μ=0 in Eqs. (22) and (23) leads to the boundary conditions of classical elasticity theory.

2.4. DQM Solution

A CNT with diameter d and length L is considered. Eq. (21) can be written in the following form with a negative magnetic field effect:

$$GI_P\left(\frac{\partial^2\theta}{\partial x^2}\right) = \left(1-\mu\frac{\partial^2}{\partial x^2}\right)\rho I_P\left(\frac{\partial^2\theta}{\partial t^2}\right) - \left(1-\mu\frac{\partial^2}{\partial x^2}\right)\eta {H_x}^2R^3\left(\frac{\partial^2\theta}{\partial x^2}\right) \tag{24}$$

Eq. (24) is reorganized with dimensionless nanotube length parameter $\left(\bar{x} = \frac{x}{L}\right)$ and the harmonic vibration assumption $(\theta(\bar{x},t) = \psi(\bar{x})e^{j\omega t})$, where ψ is the angular deformation amplitude of the nanotube, j^2=-1, ω is the torsional angular frequency and t is the time.

$$\frac{\partial^4\theta}{\partial\bar{x}^4}\left[-\frac{\mu}{L^2}\frac{\eta {H_x}^2R^3}{GI_P}\right] + \frac{\partial^2\theta}{\partial\bar{x}^2}\left[1 - \frac{\mu}{L^2}\frac{\rho I_P\omega^2L^2}{GI_P} + \frac{\eta {H_x}^2{R_i}^3}{GI_P}\right] + \theta\left[\frac{\rho I_P\omega^2L^2}{GI_P}\right] = 0 \tag{25}$$

Eq. (24) can be written in the following dimensionless form:

$$\frac{\partial^4\theta}{\partial\bar{x}^4}\left[-\frac{\mu}{L^2}MT\right]+\frac{\partial^2\theta}{\partial\bar{x}^2}\left[1-\frac{\mu}{L^2}\Omega^2+MT\right]+\theta[\Omega^2]=0 \tag{26}$$

where MT denotes the dimensionless magnetic torque parameter and Ω denotes the non-dimensional frequency parameter (NDFP) and they are defined as:

$$MT=\frac{\eta H_x{}^2R^3}{GI_P} \quad , \quad \Omega^2=\frac{\rho\omega^2L^2}{G} \tag{27}$$

Eq. (26) is a fourth order linear differential equation. It can be solved analytically, but root searching process may take too much computation time. As a time-saving approach, the Differential Quadrature Method (DQM) is an effective method for the solution of linear or nonlinear differential equations with boundary and initial values. DQM was proposed firstly by Bellman and Casti [53] and its application to solid mechanics problems may be found in literature [54]. Also, DQM has been used recently in vibration of nanotube problems by scientists [55–60].

Non-dimensional frequencies of the nanostructures stacks in a limit value at higher modes with nonlocal elasticity approach [61]. To determine the mode number of a nonlocal frequency, analytical solution results may jumped between modes in a small interval. As a discrete approach, DQM can give exact mode frequencies in a sequential order.

According to the DQM, the n^{th} order derivative of a single variable function ($f(x)$) can be defined approximately over the interval $[0\ L]$ at x_i as below [54]:

$$\left.\frac{d^nf(x)}{dx^n}\right|_{x=x_i}\cong\sum_{j=1}^{N}\beta_{ij}^{(n)}f(x_j) \tag{28}$$

where $\beta_{ij}^{(n)}$ is the n^{th} order weighting coefficient at the i^{th} point calculated for the j^{th} sampling point of the domain. In Eq. (28), N is the total number of sampling points of the grid distribution and $f(x_j)$ is the value of the function at the j^{th} point. The weighting coefficients for the first order derivative are:

$$\beta_{ij}^{(1)}=\frac{L^{(1)}(x_i)}{(x_i-x_j)L^{(1)}(x_j)} \quad , \quad i,j=1,2,\dots,N \quad , \quad i\neq j \tag{29}$$

$$\beta_{ii}^{(1)}=-\sum_{j=1,j\neq i}^{N}\beta_{ij}^{(1)} \quad , \quad i,j=1,2,\dots,N \quad , \quad i=j \tag{30}$$

In Eq. (29), the first derivative of Lagrange interpolating polynomials at each point x_k $(k=1,\dots\dots,N)$ is:

$$L^{(1)}(x_k)=\prod_{l=1,l\neq k}^{N}(x_k-x_l) \quad , \quad k=1,\dots\dots,N \quad , \quad i\neq j \tag{31}$$

Higher order derivatives $(n=2,3,4,\dots\dots,N-1)$ may be obtained iteratively:

$$\beta_{ij}^{(n)}=n\left(\beta_{ii}^{(n-1)}\beta_{ij}^{(1)}-\frac{\beta_{ij}^{(n-1)}}{(x_i-x_j)}\right) \quad , \quad i,j=1,2,\dots\dots,N \quad , \quad i\neq j \tag{32}$$

$$\beta_{ii}^{(n)}=-\sum_{j=1,j\neq i}^{N}\beta_{ij}^{(n)} \quad , \quad i,j=1,2,\dots,N \quad , \quad i=j \tag{33}$$

The grid point distribution is assumed as well-known Gauss-Chebyshev-Lobatto point distribution [62]:

$$x_i=\frac{L}{2}\left(1-\cos\left(\frac{i-1}{N-1}\pi\right)\right) \quad , \quad i=1,2,\dots\dots,N \tag{34}$$

where L is the length of the nanotube. Governing equation of motion for the magnetically affected nanotube according to DQM can be written as:

$$\left(\sum_{j=1}^{N}\beta_{ij}^{(4)}\psi_j\right)\left[-\frac{\mu}{L^2}MT\right]+\left(\sum_{j=1}^{N}\beta_{ij}^{(2)}\psi_j\right)\left[1-\frac{\mu}{L^2}\Omega^2+MT\right]=-(\psi_j)[\Omega^2] \quad , \quad i=2,3,\dots\dots,(N-2) \tag{35}$$

where ψ_j is the angular deformation amplitude of the j^{th} point.

Boundary conditions are applied using DQM as considered in the literature [54]. The Clamped-Clamped (C-C) boundary conditions are:

$$\bar{x}=0 \;\rightarrow\; \begin{matrix}\psi_i=0\\ \left(\mu\eta H_x{}^2R^3\right)\sum_{j=1}^{N}\beta_{ij}^{(2)}\psi_j=0\end{matrix} \quad , \quad i=1$$

$$\bar{x}=1 \;\rightarrow\; \begin{matrix}\psi_i=0\\ \left(\mu\eta H_x{}^2R^3\right)\sum_{j=1}^{N}\beta_{ij}^{(2)}\psi_j=0\end{matrix} \quad , \quad i=N \tag{36}$$

and Clamped-Free (C-F) boundary conditions for DQM:

$$\bar{x}=0\rightarrow\begin{cases}\psi_i=0\\ \left(\mu\eta H_x{}^2R^3\right)\displaystyle\sum_{j=1}^{N}\beta_{ij}^{(2)}\psi_j=0\end{cases} \quad , \quad i=1$$

$$\bar{x}=1\rightarrow$$
$$\begin{cases}\sum_{j=1}^{N}\beta_{ij}^{(1)}\psi_j\\ \left(\eta H_x{}^2R^3\right)\sum_{j=1}^{N}\beta_{ij}^{(1)}\psi_j-\left(\mu\eta H_x{}^2R^3\right)\sum_{j=1}^{N}\beta_{ij}^{(3)}\psi_j \quad , \quad i=N\\ -(\mu\rho I_P\omega^2)\sum_{j=1}^{N}\beta_{ij}^{(1)}\psi_j-(GI_P)\sum_{j=1}^{N}\beta_{ij}^{(1)}\psi_j=0\end{cases} \tag{37}$$

N quadrature analog equations are needed for solving the differential governing equation of motion. First two and last two equation can be obtained from boundary conditions in Eqs. (36)-(37) and the remaining (N-4) equations can be obtained from Eq. (35). Governing equation of motion may be defined in matrix form as below:

$$\begin{bmatrix}[S_{bb}] & [S_{bd}]\\ [S_{db}] & [S_{dd}]\end{bmatrix}\begin{bmatrix}\{\psi_b\}\\ \{\psi_d\}\end{bmatrix}=-\Omega^2\begin{bmatrix}\{0\}\\ \{\psi_d\}\end{bmatrix} \tag{38}$$

where the subscripts b and d defines the grid points used in quadrature analog of the boundary conditions and the governing differential equation. $\{\psi_b\}$ is a (4x1) column vector and contains the angular deformation amplitude values on boundary conditions. If the $\{\psi_b\}$ vector is eliminated, following characteristic eigen-value equation will be obtained:

$$[S]\{\psi_d\}+\Omega^2[I]\{\psi_d\}=0 \tag{39}$$

where $[I]$ is the identity matrix, Ω is the eigen-value and $\{\psi_d\}$ is the eigen-vector which describes the mode shape of nanotube. S matrix is an order of $(N-4)x(N-4)$ and can be defined as below:

$$[S]=[S_{dd}]-[S_{db}][S_{bb}]^{-1}[S_{bd}] \tag{40}$$

3. Numerical Results and Discussion

In this section, the nonlocal and longitudinal magnetic field effects on the torsional vibration of the CNT are investigated. Many studies can be found about physical properties of CNTs. Nanotube radius has an essential role on the shear modulus (G). Shear modulus(G) and density(ρ) of the CNT is determined from previous studies [63,64]. Material properties of CNT are given in the Table (1).

3.1. Validation of present model

Validation of the present nonlocal CNT model has been carried out in previous study [65]. A discrete model (Lattice Dynamic) results have been used in order to compare the stress gradient nonlocal model. The nonlocal theory results are in good agreement with the discrete model results.

Accuracy of the DQM solution depends on the number of terms used in the analysis. In Table (2), the convergence of non-dimensional DQM frequencies can be seen. Nonlocal and magnetic field effects are assumed nearly zero. Results are identically same for the N≥10.

Table 1. Material Properties for CNT

CNT	Inner Radius (nm)	Density (kg/m^3)	Shear Modulus (TPa) [64]
Armchair (6,6)	0.41	4962	0.40

Table 2. Comparison of the Analytical and DQM Solution Results (MT=0.01 , μ=0.01nm^2)

C-C				C-F			
Analytical	DQM			Analytical	DQM		
	N = 5	N = 10	N = 15		N = 5	N = 10	N = 15
3.1511	3.1079	3.1511	3.1511	1.5778	1.5966	1.5779	1.5779

3.2. Results

In this section, the variation of non-dimensional frequency parameter (NDFP) of the magnetically affected CNT with the nonlocal parameter (μ) and dimensionless magnetic torque (MT) are investigated. In figures (3) and (4), variation of NDFP with nonlocal parameter (μ) and dimensionless magnetic torque (MT) is given for Clamped-Clamped (C-C) and Clamped-Free (C-F) boundary conditions, respectively. It is well known that the nonlocal elasticity predicts softening at the nano-length scale when there is no magnetic field [65,66]. It is seen that the nonlocal effect decreases the NDFP for both boundary conditions. Magnetic field moment increases the NDFP contrary to the nonlocality. Nanotube length has direct effect on the nonlocality. In longer nanotubes, nonlocal effect vanishes because of the size dependency. So, only magnetic field effect on NDFP is seen at L=20nm case. Same observations can be interpreted form the figures (5) and (6). Magnetic torque effect increases and nonlocality decreases the NDFP.

In figures (7) and (8) it was observed that, for some values of magnetic torque, the vibration frequency becomes zero that means the rod loses its stability. This may be called torsional buckling. NDFP is zero when MT=-1 with very low influence of nonlocality. It seems that torsional buckling occurs with static deformation at this point. With increasing effect of nonlocality, critical buckling load reduces as a result of softening effect of nonlocal theory. That means CNT can buckle easily.

Variation of critical buckling torque with nonlocal parameter in 1st, 2nd and 3rd mode frequency are shown in Figs. (9) and (10). It is interesting to note that, nonlocal approach shows softening effect on CNT structure and as a result it can buckle on small torques. With increasing nanotube length, nonlocality lost its effectiveness and CNT is stiffer than before. Critical buckling torque increases with the loss of nonlocality. This situation is reverse for the classical continuum approach and only explained with nonlocal theories.

4. Conclusion

In the present study, longitudinal magnetic field effect on torsional vibration of CNT is investigated. Hamilton's principle and Eringen's nonlocal elasticity theory are used in order to obtain the governing equation of motion and boundary conditions. DQM is used in solution of differential governing equation. Effects of parameters (magnetic torque and nonlocal parameter) are depicted in the figures.

Nonlocality and magnetic field have opposite effects on CNT's dynamics. Magnetic field increases and nonlocal parameter decreases the NDFP. Nanotube length (L≥20nm) diminishes the nonlocality. Torsional buckling in CNT occurs under the effect of magnetic field. Nonlocality facilitate the buckling with softening effect on CNT lattice structure. Nonlocal effect decreases the critical buckling load. In contrary to the classical continuum mechanics, critical buckling load affected by nonlocality. Torsional buckling case can be studied in more detail in future studies.

Present results may be useful in the design and analysis of nanomotors.

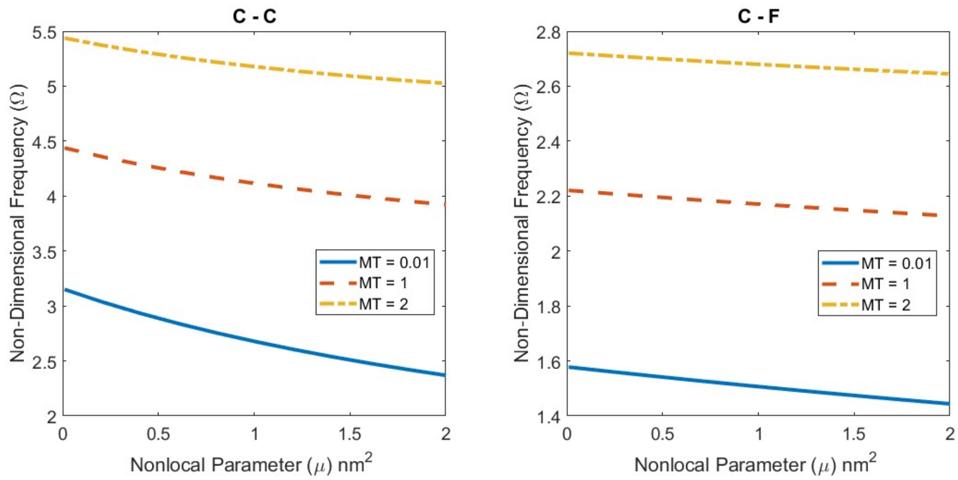

Figure 3. Variation of NDFP with Nonlocal Parameter (L=5nm)

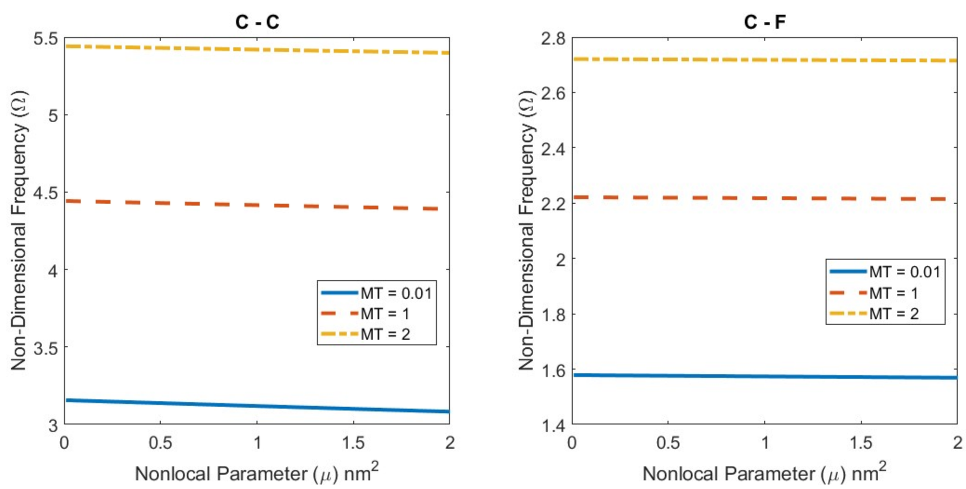

Figure 4. Variation of NDFP with Nonlocal Parameter (L=20nm)

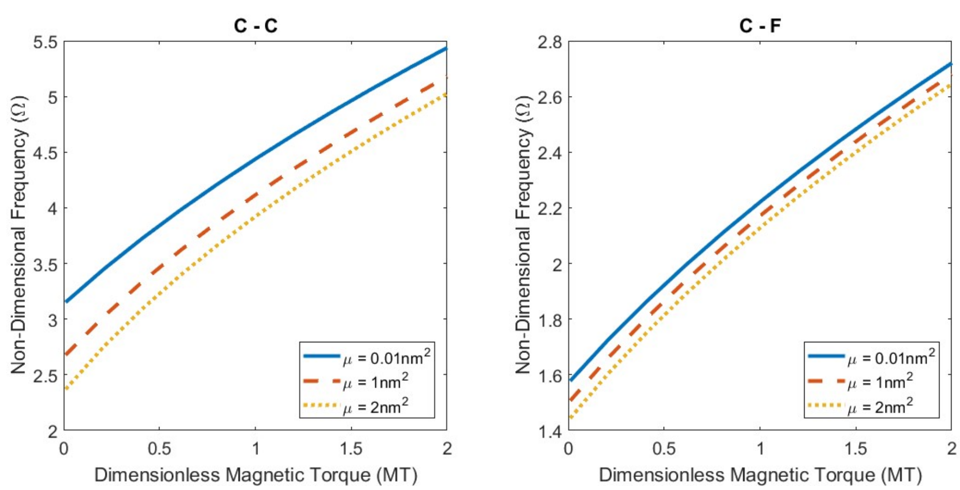

Figure 5. Variation of NDFP with Dimensionless Magnetic Torque (MT) (L=5nm)

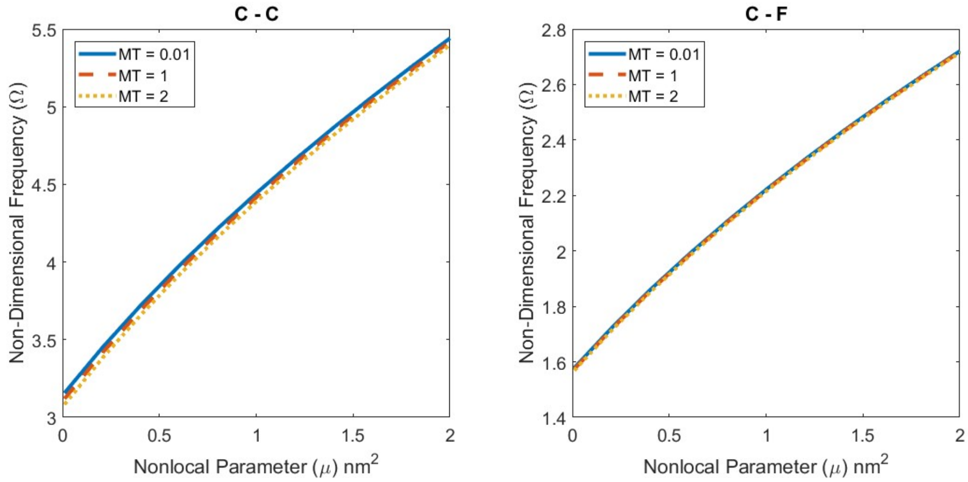

Figure 6. Variation of NDFP with Dimensionless Magnetic Torque (MT) (L=20nm)

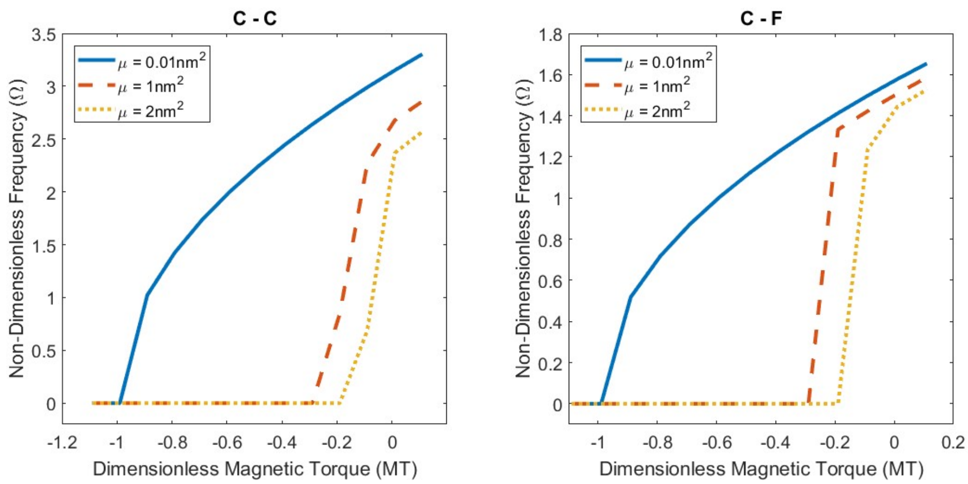

Figure 7. Torsional Buckling Case (L=5nm)

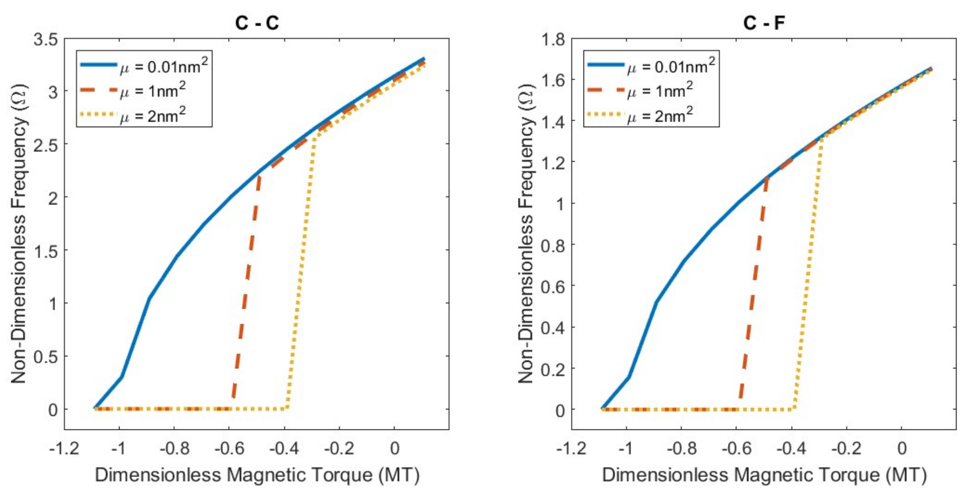

Figure 8. Torsional Buckling Case (MT) (L=20nm)

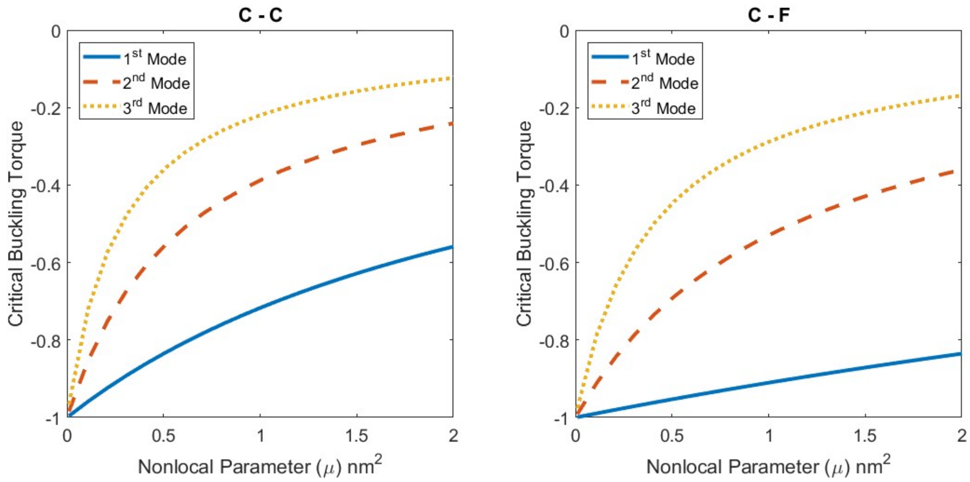

Figure 9. Variation of Critical Buckling Torque with Nonlocal Parameter (L=5nm)

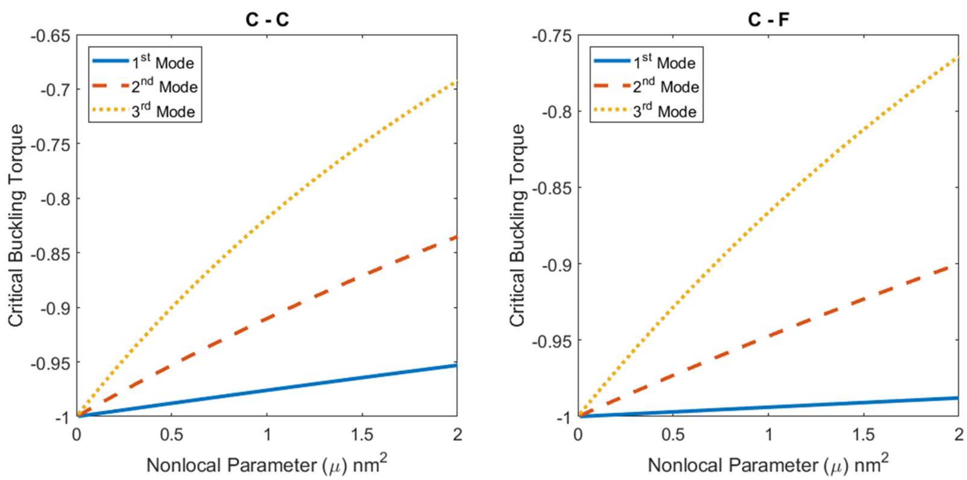

Figure 10. Variation of Critical Buckling Torque with Nonlocal Parameter (L=20nm)

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