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Non-Relativistic Limit of Neutron Beta-Decay Cross-Section in the Presence of Strong Magnetic Field

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Abstract

One of the most important reactions of the URCA that lead to the cooling of a neutron star, is neutron beta-decay ($n \rightarrow p^{+} + e^{-} + \overline{\neg}$). In this research, the energy spectra and wave functions of massive fermions taking into account the Anomalous Magnetic Moment (AMM) in the presence of a strong changed magnetic field are calculated. For this purpose, the Dirac-Pauli equation for charged and neutral fermions is solved by Perturbation and Frobenius series method, respectively. The results of the Frobenius series method are in good agreement with the results of Nikiforov-Uvarov method (NU). In continuous, using the calculated wave functions, the general relation of neutron decay cross-section in the non-relativistic limit has been obtained. This relation has been derived by the four-fermion Lagrangian within the framework of the standard model of weak interactions. These calculations from the perspective of nuclear astrophysics can be important.

Keywords: Dirac-Pauli equation; Anomalous Magnetic Moment; Four-Fermion Lagrangian.

Introduction

Study of weak processes in the presence of strong magnetic fields in neutron stars and Radio-pulsars are very important in view of astrophysical applications. The neutron stars are super-dense objects with density about of $2.8\times10^{14}\,gr/cm^{3}$ [1]. The strength of the surface magnetic field in these objects is in the range of $10^{12}\sim10^{14}G$ [2, 3]. Also, the intensity of the magnetic fields can be $10^{14}\sim10^{15}G$ for some magnetars [4, 5]. For more details about the properties of magnetars, refer to papers by Gao et al. [6] and Mereghetti et al. [7]. They showed that magnetars are neutron stars in which a strong magnetic field is the main energy source.

The neutron stars are the best laboratory for studying dense matter physics. These objects are caused by supernova explosions with an internal temperature about of $10^{11}-10^{12}K$. The processes of cooling in the neutron stars are due to emitting neutrinos. During these processes temperature decreases to $T\approx10^{10}K$ [8, 9]. The weak interactions of nucleons cause the beta-decay and the emission of neutrinos from neutron stars (the URCA process) [10, 11].

The strong magnetic fields will strongly influence on the weak interactions. The details of weak interactions related to the URCA process are presented in Ref. [12]. Even relatively weak fields can play strong role in various astrophysical problems [13].

Up to now, it is widely recognized that strong magnetic fields can be a significant factor related to

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diverse astrophysical and cosmological environments. Under the influence of strong magnetic fields in the neutron stars the direct URCA processes are as follows [14, 15]:

$$n \to p + e^- + \bar{\nu}_e \quad , p \to n + e^+ + \nu_e \quad , \quad n + \nu_e \to p + e^-$$

The URCA processes have been studied in different electromagnetic fields. The first attempts to consider the URCA processes in strong constant electromagnetic field have been investigated in Refs [16, 17]. The relativistic theory of the neutron beta-decay in strong magnetic field has been developed in [18]. Also, the probability of polarized neutron beta-decay in the presence of magnetic field was derived in Refs [19, 20]. Studenikin and Shinkevich [21] developed a relativistic approach for calculations of cross-sections of the URCA processes in strong constant magnetic fields. Many important technical details of the calculations, also useful for the later studies, are exist in their work. The decay rate for the proton inverse $\beta^{+}-$decay process accounting the AMM of nucleons in uniform constant strong magnetic field is calculated in [21, 22].

Since the real magnetic fields in neutron stars are a non-uniform [23], in the present we have considered the energy spectra, wave functions and cross-section of the beta-decay polarized neutron in a non-uniform strong magnetic field. Actually, studying such a processes can be attractive from astrophysical aspects, especially in the cooling of white dwarfs [24].

In this work, firstly, the energy spectra and the wave functions of charged fermions in an external constant magnetic field are calculated in the non-relativistic limit, and then the corrections of the energy spectra and wave functions are done in the presence of external changed magnetic field using the formal theory of perturbation. For neutral fermion the energy spectrum and the wave function has been obtained using Frobenius series method. Finally, using the four-fermion weak interaction theory [25, 26], the cross-section of neutron beta-decay has been calculated in an external non-uniform magnetic field, taking into account the AMM interaction of nucleons in the non-relativistic limit. In our calculations the incoming neutrino is supposed to be relativistic and the effects of none-zero mass of neutrino have been neglected, and we have used $\hbar = c = 1$.

The structure of this paper is as follows. In section 2, we have expressed formalism of the Dirac-Pauli equation (with AMM in external field) and the cross-section for beta- decay. Section 3 includes details of calculations and the obtained results.

Materials and Methods

For a description of particles with half spin in relativistic limit, we use the Dirac equation. In the non-relativistic limit the Dirac equation convert to the Dirac –Pauli or Pauli equation. In this section we give a brief discussion of the Dirac-Pauli equation with AMM in external field and cross-section of beta-decay.

Dirac-Pauli equation with AMM in external field

In the relativistic quantum theory, moving of a charged particle with half spin in the external electromagnetic field describes by the Dirac equation [27, 28]. In the non-relativistic limit to obtain the energy spectrum and wave function of moving fermions in an external magnetic field, it is necessary to convert the Dirac equation to the Pauli equation. The Dirac equation for fermions with AMM in external magnetic field can be written as following [28]:

$$i\partial_t \psi\left(\vec{r},t\right) = \left[\vec{\alpha}\cdot\vec{P} + \rho_3 m - \mu\rho_3\left(\vec{\Sigma}\cdot\vec{B}\right)\right]\psi\left(\vec{r},t\right), \quad (1)$$

Parameters in this equation are defined as follows:

$$\rho_3 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad , \quad \rho_1 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \vec{P} = -i\vec{\nabla} - e\vec{A}, \ \vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}, \quad \vec{\alpha} = \rho_1\vec{\sigma}, \quad (2)$$

Where $\vec{\sigma}$ - are Pauli matrices, $\vec{A}$ - is the vector potential of magnetic field I and 0 - are the 2×2 matrixes, m -is fermion mass and μ - is AMM of particle. One can easily obtain the Pauli equation with contribution of AMM of fermions, so we have:

$$i\partial_t \psi\left(\vec{r},t\right) = \left[\frac{\left(\vec{p} - e\vec{A}\right)^2}{2m} - \left(\mu + \frac{e}{2m}\right)\left(\vec{\sigma}\cdot\vec{B}\right)\right]\psi\left(\vec{r},t\right), \quad (3)$$

The stationary state of bispinor $\psi\left(\vec{r},\mathrm{t}\right)$ is defined as

$$\psi\left(\vec{r},t\right) = \exp\left(-i\varepsilon t\right)\begin{pmatrix} \psi_1\left(\vec{r}\right) \\ \psi_2\left(\vec{r}\right) \end{pmatrix}. \quad (4)$$

Where ε - is the energy of fermion in the non-relativistic limit. We are interested in solving the Eq. (3) for neutral and charged fermions in the presence of an external magnetic field. We introduced the magnetic field with cylindrical symmetry:

$$\vec{B}\left(r\right) = \left(0,\ 0,\ \frac{a}{r} + b\right) \quad (5)$$

In the Eq. (5), a and b are constant parameters. The vector potential for such magnetic field becomes:

$$A_r = A_z = 0\ ,\ \mathrm{A}_\varphi = \frac{br}{2} + a\ . \quad (6)$$

There exist many gauges which produce the magnetic field, but for convenience we introduced the gauge of mentioned in the Eq. (6).

Cross-section for beta- decay in an external field

In order to calculating the beta-decay of cross-section within the framework of the standard model of weak interactions we use the four-fermion Lagrangian as follow [21, 26]:

$$\mathcal{L} = \frac{G_F}{\sqrt{2}}\left[\bar{\psi}_p\gamma_\mu\left(1-\lambda\gamma_5\right)\psi_n\right]\left[\bar{\psi}_e\gamma^\mu\left(1-\gamma_5\right)\psi_\nu\right],\quad \gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}_{4\times4} \tag{7}$$

In above G_F is the Fermi coupled constant and $\lambda = 1.26$ is the ratio of the axial and vector constants. The total cross section of the process can be written as:

$$\sigma = \frac{L^3}{T}\sum_{\substack{phase\\space}}\left|M\right|^2 \tag{8}$$

In the Eq. (8) summation is performed over the phase space of the final particles and M is the matrix element of the process. The matrix element is related to the four-fermion Lagrangian can be written as

$$M = \int \mathcal{L}\, d^4x = \frac{G_F}{\sqrt{2}}\int\left[\bar{\psi}_p\gamma_\mu\left(1-\lambda\gamma_5\right)\psi_n\right]\left[\bar{\psi}_e\gamma^\mu\left(1-\gamma_5\right)\psi_\nu\right]d^4x \tag{9}$$

We account for the influence of the background magnetic field in the Eq. (9). In this study the massive fermions have been considered in the non-relativistic limit. Whereas the neutrino is massless, so neutrino cannot be non-relativistic. The neutrinos have not interaction with magnetic field, furthermore they have not AMM, therefore the neutrino wave function in the presence of constant magnetic field the same as changed magnetic field. According to the matter, we use the neutrino relativistic wave function in our calculations which is denoted by

$$\psi_\nu\left(\vec{x},t\right) = \frac{1}{2L^{\frac{3}{2}}}\begin{pmatrix} f_1 \\ f_2 \\ -f_1 \\ -f_2 \end{pmatrix}\exp\left(i\left(\omega_0 t - \vec{\omega}.\vec{r}\right)\right) \tag{10}$$

$$f_1 = -e^{i\theta_\nu}\left(1-\cos\varphi_\nu\right)^{\frac{1}{2}}\quad,\quad f_2 = \left(1+\cos\varphi_\nu\right)^{\frac{1}{2}} \tag{11}$$

Here ω_0, $\vec{\omega}$ are the neutrino energy and momentum respectively. We neglect effects of the neutrino mass, so $\omega_0 = \left|\vec{\omega}\right|$. The neutrino polar and azimuthally angles are denoted by θ_ν and φ_ν respectively [1, 21].

Results

To obtain the wave functions and the energy spectra of the massive fermions (electron, proton and neutron), we will solve the Dirac- Pauli equation, then we will use the wave functions for calculating the cross-section of neutron decay. This section contains three subsections which are: neutral fermion, charged fermion and the general relation of cross-section in the non-relativistic limit.

Neutral fermion

Substituting the Eq. (5) and Eq. (6) into Eq. (3) for neutral fermion we have

$$\varepsilon\psi_{1,2}\left(\vec{r}\right) = \left[\frac{\left(\vec{p}-e\vec{A}\right)^2}{2m} \mp \left(\mu + \frac{e}{2m}\right)\left(\frac{a}{r}+b\right)\right]\psi_{1,2}\left(\vec{r}\right), \tag{12}$$

Solutions of this equation in cylindrical coordinates are:

$$\psi_{1,2}\left(\vec{r}\right) = \frac{1}{L^{\frac{1}{2}}\sqrt{4\pi}}\exp\left(i\left(l\theta + k_3 z\right)\right)\cdot f\left(r\right)\begin{pmatrix} C_{1n} \\ C_{2n} \end{pmatrix}\ ,\ l=0,\ 1,\ 2,\ldots\ . \tag{13}$$

In this equation, radial function in cylindrical coordinates, $f\left(r\right)$, is calculated using the Frobenius series method as follows:

$$f\left(r\right) = e^{-\sqrt{A}r}r^l\,{}_1F_2\left(-n,\ 2l+1;\ x\right), \tag{14}$$

$$A = k_3^2 - \left[2m_n\left(\mu_n b + \varepsilon_n\right)\right]\ ,\ x = 2\sqrt{A}\ r,$$

$$k_3 = \frac{2\pi}{L}n_3\quad;\quad n_3 = \pm1,\ \pm2,\ \pm3,\ \cdots\ .$$

Where, the parameters as $m_n, \varepsilon_n, \mu_n$ are the neutron mass, energy and AMM ($\mu_n = g_n\mu_N$, $g_n = -1.91$ is Lande factor and $\mu_N = e/2m_p = 5.05\times10^{-24} erg\cdot Ga^{-1}$ is nuclear magneton), respectively. ${}_1F_2\left(\lambda,\ ;x\right)$ is Confluent Hyper-geometric function [29]. The neutron spin coefficients are

$$C_{1n} = 1 - s_n\quad,\quad C_{2n} = 1 + s_n. \tag{15}$$

The neutron spin value $s_n\left(s_n = \pm1\right)$ classifies the neutron states with respect to the spin projection to $z-$ direction ($s_n = +1$ corresponds to the spin orientation parallel to the magnetic field $\vec{B}$). From Eq. (14) and properties of Hyper-geometric function, the energy spectrum for neutron is obtained as follows:

$$\varepsilon_{n,l} = \frac{k_3^2}{2m_n} + s_n\mu_n b - \frac{m_n\mu_n^2 a^2}{2\left(n+l+0.5\right)^2}. \tag{16}$$

This relation is in good agreement with the NU method [30].

Charged fermions

The Pauli equation for charged fermion taking into account interaction of AMM particles with external constant magnetic field $\vec{B}$ is as follow:

$$£ \cdot_{1,2}(r) = \left[\frac{1}{2m_e}\left(\vec{p} + e\vec{A}\right)^2 - \left(«_e + \frac{e}{2m_e}\right)B\right]\cdot_{1,2}(r). \quad 17)$$

Solving the Eq. (17) leads to:

$$\cdot_{1,2}(\vec{r},t) = \frac{1}{L}e^{-i£^e t}e^{-i(p_{2e}y+p_{3e}z)}U_n(¦)\begin{pmatrix} C_{1e} \\ C_{2e} \end{pmatrix}; \quad (18)$$

$$¦ = \sqrt{2¥}x + \frac{p_{2e}}{\sqrt{2¥}},\quad C_{1e} = 1 - s_e,\quad C_{2e} = 1 + s_e,\quad s_e = \pm 1. \quad (19)$$

Where $U_n(¦)$ is Hermit function.

$$U_n(¦) = (-1)^n\left(\frac{\sqrt{2¥}}{2^n n!\sqrt{®}}\right)^{\frac{1}{2}} e^{\frac{¦^2}{2}}\left(\frac{d}{d¦}\right)^n\left(e^{-\frac{¦^2}{2}}\right) = e^{-\frac{¦^2}{2}}H_n(¦) \quad (20)$$

The energy spectrum of charged fermion (i.e. electron) is calculated as:

$$£_n^p = \frac{p_3^2}{2m_p} - «_p s_p B + \left(2n + 1 - s_p\right)\frac{¥}{m_p} \quad ,¥ = \frac{eB}{2}. \quad (21)$$

Here, we suppose that parameter a in Eq. (5) is small. We calculate the contribution of the perturbation term (a/r). Therefore, using the time independent perturbation theory, the electron wave function $\cdot_e^{(1)}(\vec{x},t)$ can be written as follows:

$$\cdot_e^{(1)}(\vec{x},t) = \sum_{n\neq k}\frac{m_e H_{nk}^{(e)}}{2¥ L(n-k)}\exp\left(-i£_e t\right)\exp\left(i\left(p_{2e}y + p_{3e}z\right)\right)U_n(¦)\begin{pmatrix} C_{1e} \\ C_{2e} \end{pmatrix}. \quad (22)$$

$$H' = \left(a« + \frac{ea}{2m_o}\right)\frac{1}{r} + \frac{e^2ab}{2m_o}r \rightarrow H_{nk} = \left\langle k^{(0)}\left|H'\right|n^{(0)}\right\rangle. \quad (23)$$

Where $\cdot_e^{(1)}(\vec{x},t)$, $£_e$, m_e, p_{2e}, p_{3e} and $H_{nk}^{(e)}$ are the electron wave function of one order correction , energy, mass, momentum components and energy correction under the influence magnetic field, respectively. The proton wave function $\cdot_p^{(1)}(\vec{x},t)$ can be expressed in a similar form

$$\cdot_p^{(1)}(\vec{x},t) = \sum_{m\neq q}\frac{m_p H_{mq}^{(p)}}{2¥ L(m-q)}\exp\left(-i£_p t\right)\exp\left(i\left(p_{2p}y + p_{3p}z\right)\right)U_m(¦)\begin{pmatrix} C_{1p} \\ C_{2p} \end{pmatrix}. \quad (24)$$

$$¦' = \sqrt{2¥}x + \frac{p_{2p}}{\sqrt{2¥}},\quad C_{1p} = 1 - s_p,\quad C_{2p} = 1 + s_p,\quad S_p = \pm 1. \quad (25)$$

The proton parameters are similar to electron ones. The energy spectra of ground state ($n = 0$ or $m = 0$) is:

$$\langle H\rangle_{n=0} = H_{01} = 4\sim\sqrt{®¥} + \sqrt{\frac{®}{¥}},\quad \sim = a\left(« + \frac{e}{2m_o}\right),\qquad = \frac{e^2ab}{2m_0},\quad ¥ = \frac{eB}{2}. \quad (26)$$

Also for excited state ($n = 1$ or $m = 1$), we have:

$$\langle H\rangle_{n=1} = H_{10} = \frac{13}{2}\mathsf{s}\sqrt{®¥} + \frac{47}{4}¶\sqrt{\frac{®}{¥}} \quad ,\quad ¶ = \frac{e^2ab}{2m_0} \quad ,\quad \mathsf{s} = a\left(« - \frac{e}{2m_o}\right) \quad (27)$$

Here m_0, « and e are the mass of charged fermions, AMM and charge fermions, respectively.

The general form of cross-section in the non-relativistic limit

In our calculations we have obtained the exact solution of the Dirac-Pauli equation for neutron in the presence of magnetic field. Also, the non-exact solutions of Dirac-Pauli equation for electron and proton in the non-relativistic limit are obtained using perturbation theory in the presence of non-uniform magnetic field with cylindrical symmetry. Without loss of generality, the cylindrical symmetry magnetic field $\vec{B}$ is taken in z-direction. Substituting these wave functions in Eq. (9) and after some algebra we have:

$$\begin{aligned}&\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\exp\left(i\left(£_e + £_p - ¡_0 - £_n\right)t\right)\exp\left(i\left(p_{2e} + p_{2p} - p_{2n} - ¡_2\right)y\right)\\ &\times\exp\left(i\left(p_{3e} + p_{3p} - p_{3n} - ¡_3\right)z\right)dydzdt = \left(2®\right)^3 ¢\left(£_e + £_p - ¡_0 - £_n\right)\\ &\times ¢\left(p_{2e} + p_{2p} - p_{2n} - ¡_2\right)\times ¢\left(p_{3e} + p_{3p} - p_{3n} - ¡_3\right)\end{aligned} \quad (28)$$

Integrating over the coordinate x in the matrix element of beta-decay, we get:

$$\begin{aligned}M = \frac{2G_F}{\sqrt{2}}\int\Lambda(\vec{x},t)&\Big\{\left(C_{1p}C_{1n} + C_{2p}C_{2n}\right)\left(C_{1e}f_1 + C_{2e}f_2\right)\\ &+ \sim\Big[\left(C_{1p}C_{2n} + C_{2p}C_{1n}\right)\left(C_{1e}f_1 + C_{2e}f_2\right) + \left(C_{2p}C_{1n} - C_{1p}C_{2n}\right)\\ &\times\left(C_{1e}f_2 - C_{2e}f_1\right) + \left(C_{1p}C_{1n} - C_{2p}C_{2n}\right)\left(C_{1e}f_1 - C_{2e}f_2\right)\Big]\Big\}dxdydzdt\end{aligned} \quad (29)$$

This equation can be rewritten as:

$$M = \frac{2G_F}{\sqrt{2}}\int\Lambda(\vec{x},t)C_j dxdydzdt \quad (30)$$

Where $\Lambda(\vec{x},t)$ is defined as:

$$\begin{aligned}\Lambda(\vec{x},t) = \frac{1}{2L^5}\sum_{m\neq q}\sum_{n\neq k}&\frac{m_e m_p H_{mq}^{(p)}H_{nk}^{(e)}}{4¥^2(n-k)(m-q)}F_{n'}(x)U_n(¦')U_m(¦)\exp\left(-i¡_1 x\right)\\ &\times\exp\left(i\left(£_e + £_p - ¡_0 - £_n\right)t\right)\exp\left(i\left(p_{2e} + p_{2p} - p_{2n} - ¡_2\right)y\right)\\ &\times\exp\left(i\left(p_{3e} + p_{3p} - p_{3n} - ¡_3\right)z\right)\end{aligned} \quad (31)$$

Substituting the Eq. (31) into Eq. (30), we obtain

$$M=\frac{G_F C_j}{L^5\sqrt{2}}\sum_{m\neq q}\sum_{n\neq k}\frac{m_e m_p H^{(p)}_{mq}H^{(e)}_{nk}(2®)^3}{¥^2(n-k)(m-q)}¢\left(£_e+£_p-¡_0-£_n\right)¢\left(p_{2e}+p_{2p}-p_{2n}-¡_2\right)$$
$$\times¢\left(p_{3e}+p_{3p}-p_{3n}-¡_3\right)\int_{-\infty}^{+\infty}F_{n'}(x)U_n(¦')U_m(¦)e^{(-i¡_1x)}dx$$

(32)

Using the relations of following [15]:

$$\left|¢\left(£_e+£_p-¡_0-£_n\right)\right|^2=\frac{T}{2®}¢\left(£_e+£_p-¡_0-£_n\right)$$
$$\left|¢\left(p_{2e}+p_{2p}-p_{2n}-¡_2\right)\right|^2=\frac{T}{2®}¢\left(p_{2e}+p_{2p}-p_{2n}-¡_2\right)$$
$$\left|¢\left(p_{3e}+p_{3p}-p_{3n}-¡_3\right)\right|^2=\frac{T}{2®}¢\left(p_{3e}+p_{3p}-p_{3n}-¡_3\right)$$

(33)

in which T and L are the quantization large time and distance in the y and z directions, respectively, the squared norm of the matrix element is obtained as follow:

$$|M|^2=\frac{TG_F^2}{2L^8}\left|C_j\right|^2\frac{m_e^2m_p^2}{¥^4}(2®)^3\sum_{m,q\neq0}\sum_{n,k\neq0}\frac{H^{(p)}_{q0}H^{(e)}_{n0}H^{(p)}_{m0}H^{(e)}_{k0}}{(nk)(mq)}¢\left(£_e+£_p-¡_0-£_n\right)$$
$$\times¢\left(p_{2e}+p_{2p}-p_{2n}-¡_2\right)¢\left(p_{3e}+p_{3p}-p_{3n}-¡_3\right)$$
$$\times\exp\left(-\frac{\left[2p_{2e}^2+2p_{2p}^2+¡_1^2-\left(\sqrt{A}+p_{2e}+p_{2p}\right)^2\right]}{4¥}\right)$$

(34)

in that we have:

$$\left|C_j\right|^2=2\left[(1-\cos¯_¬)\sim_1^2+(1+\cos¯_¬)\ {}_1^2-2\sim_1\ {}_1\cos¨_¬\sin¯_¬\right]$$
$$\sim_1=C_{1p}C_{1e}+\sim\left(2S_eC_{2p}+C_{1p}C_{1e}\right)$$
$${}_1=C_{1p}C_{2e}+\sim\left(2C_{2p}-C_{1p}C_{2e}\right)$$

(35)

According to the Eq. (34), we are able to calculate the total cross section of neutron decay (Eq. (8)). The phase space factor for the electron and proton in the presence of a magnetic field in the ground state is:

$$\sum_{\substack{phase\\space}}=\sum_{S_e,S_p}\int\left(\frac{L}{2®}\right)^4dp_{2e}dp_{3e}dp_{2p}dp_{3p}$$

(36)

Regard to this definition, we obtain

$$±=\frac{1}{T}\sum_{s_e,s_p}\left(\frac{L}{2®}\right)^7\int|M|^2dp_{2e}dp_{3e}dp_{2p}dp_{3p}d^3¡=\sum_{s_e,s_p}\int\frac{|M|^2}{T}\prod_i\left(\frac{L}{2®}\right)^3d^3p_i$$

(37)

$$±=\frac{\Delta}{L}\sum_{s_e,s_p}\int dp_{2e}\int dp_{3e}\int dp_{2p}\int dp_{3p}\int d^3¡\left|C_j\right|^2¢\left(p_{3e}+p_{3p}-p_{3n}+¡_3\right)¢\left(£_e+£_p+¡_0-£_n\right)$$
$$\times¢\left(p_{2e}+p_{2p}-p_{2n}+¡_2\right)\exp\left(-\frac{\left[2p_{2e}^2+2p_{2p}^2+¡_1^2-\left(\sqrt{A}+p_{2e}+p_{2p}\right)^2\right]}{4¥}\right)$$

in which

$$\Delta=\frac{G_F^2}{2(2®)^4}\frac{m_e^2m_p^2}{¥^4}\sum_{m,q\neq0}\sum_{n,k\neq0}\frac{H^{(p)}_{q0}H^{(e)}_{n0}H^{(p)}_{m0}H^{(e)}_{k0}}{(nk)(mq)}\qquad(38)$$

The integration over the electron momentum components, p_{3e} and p_{2e} are performed using the two delta functions $\{¢\left(p_{2e}+p_{2p}-p_{2n}+¡_2\right)$ and $¢\left(p_{3e}+p_{3p}-p_{3n}+¡_3\right)\}$. After these integrations we get the laws of conservation for the two momentum components, that is $p_{3e}=p_{3n}-¡_3-p_{3p}$, $p_{2e}=p_{2n}-¡_2-p_{2p}$. Finally, we have obtained the cross section of the beta decay of polarized neutron in a cylindrical symmetry external magnetic field in the non-relativistic limit as:

$$±=16(®¥)^{\frac{3}{2}}\sqrt{\frac{m_p}{6}}\sum_{k,q\neq0}\sum_{m,n\neq0}\frac{H^{(p)}_{k0}H^{(p)}_{q0}H^{(e)}_{m0}H^{(e)}_{n0}m_e^2m_p^2}{kqmn¥^4}\left(\frac{G_F^2}{®^4}\right)\frac{\left(3\sim^2+1\right)}{\sqrt{£_n-£_e-£}}$$
$$\times\exp\left(\frac{80m_n^2«_n^2a^2-21p_{2n}^2}{4¥}\right).$$

(39)

$$£=-«_ps_pB-\frac{e}{2m_p}s_pB+(2m+1)\frac{¥}{m_p}\cdot$$

This relation is the general form of cross-section in the non-relativistic limit. we can obtain cross-section of neutron decay in ground state by inserting $k=q=n=m=1$ into Eq. (39).

Discussion

In this survey, we have developed the beta-decay of polarized neutron in the presence of strong external uninformed magnetic field (variable with cylindrical symmetry). It is known a strong magnetic field can lead to beta-decay in neutron stars. In order to the description of the energy spectrum and wave functions in the URCA process, we have employed the Dirac-Pauli equation.

The Dirac-Pauli equation has been solved for neutral and charged fermions with account of AMM in the presence of a strong magnetic field with a cylindrical symmetry. Here, we have used the perturbation method for solving the Dirac-Pauli equation. The energy spectrum for massive neutral fermions in the non-relativistic limit is calculated and compared with the Nikiforov-Uvarov method (NU) [30]. The results are in good agreement together. This means that the wave functions for the non-relativistic fermions are valid. We know that the neutrinos move with the speed of light. They have not interaction with magnetic field, so we have used the relativistic wave functions of neutrinos in the presence of constant magnetic field [21] in our calculations. These wave functions have been used in calculating the matrix element (M) of the process.

The amplitude of the bound-state decay process ($|M|^2$)

has been formulated by using the four-fermion Lagrangian within the framework of the standard model of weak interactions.

These calculations from the perspective of nuclear astrophysics can be important in estimating the neutron decay cross section. We have obtained the general form for calculating the cross section of beta decay in the non-relativistic limit. This relation can be important from the perspective of nuclear astrophysics. Since, experimental astrophysics data's satisfy the existence of strong cylindrical symmetric magnetic field on the surface of forced magnetized neutron stars, our calculations and results are valid, in point of astrophysics observation.

References

1. Kauts V. L., Savochkin A.M., and Studenikin A. I. Asymmetry of Neutrino Emission from Neutron Beta Decay in Superdense Matter and a Strong Magnetic Field, *Phys. Atomic Nuclei*, **69**: 1453–1460 (2006).
2. Ghanmugan G. Magnetic Fields of Degenerate Stars, *Annu. Rev. Astron. Astrophys*, **30**: 143-184 (1992).
3. Thompson C. and Duncan R. C. The Neutron Stars. II. Quiescent Neutrino, X-Ray, and Alfven Wave Emission Soft Gamma Repeaters as Very Strongly Magnetized, *Astrophys. J*, **473**: 322 (1996).
4. Chameleon N. and Haensel P., Physics of Neutron Star Crusts, Living Rev. Rel., **11**: 10 (2008).
5. Peng Q. H. and Tong H., The physics of strong magnetic fields in neutron stars, *Mon. Not. R. Astron. Soc*., **1**: 159-162 (2007).
6. Gao Z. F., Wang N., Xu Y. and Li X. D., The effects of superhigh magnetic fields on the equations of state of neutron stars, *Astron. Nachr*., **336**: 866 (2015).
7. Mereghetti S., Pons J. and Melatos A., Magnetars: Properties, Origin and Evolution, *Space Sci. Rev*., **191**: 315-338 (2015).
8. Yan X., Guang-Zhou L., Cheng-Zhi L., Cun-Bo F., Hong-Yan W., Ming-Feng Z., En-Guang Z. The Nucleon Direct URCA Processes in a Cooling Neutron Star, *Chin. Phys. Lett*., **30:** 129501-4 (2013).
9. Yakovlev D. G., Pethick C. Neutron Star Cooling, *Ann. Rev. Astron. Astrophys*. **42**: 169-210 (2004).
10. Carlson J., Carpenter M. P., Casten R., Elster C., Fallon P., Gade A., Gross C., Hagen G., Hayes A. C., Higinbotham D. W., Howell C. R., Horowitz C. J., Jones K. L., Kondev F. G., Lapi S., Macchiavelli A., McCutchen E. A., Natowitz J., Nazarewicz W., Papenbrock T., Reddy S., Riley M. A., Savage M. J., Savard G., Sherrill B. M., Sobotka L. G., Stoyer M. A., Tsang M. B., Vetter K., Wiedenhoever I. and Wuosmaa A. H., Yennello S., White paper on nuclear astrophysics and low-energy nuclear physics, Part 2: Low-energy nuclear physics, *Prog Part Nucl Phys*., **94**: 68–124 (2017).
11. Page, D. and Applegate, J. H., The cooling of neutron stars by the direct URCA process, *Astrophys. J*, **394**: 17-20 (1992).
12. Liu, J. J., Gu W.M.: A new insight into neutrino energy loss by electron capture of iron group nuclei in magnetars surface, *Astrophys. J. Suppl. Ser*., **224**: 29 (2016).
13. Liu J. J. and Liu D. M., Influence of super-strong magnetic fields on beta decay of nuclide 59- Co in magnetar surface, *Astrophys Space Sci*., **361**:246 (2016).
14. Fujii H., Muto T., Tatsumi T., Tamagaki R., Effects of weak interaction on kaon condensation and cooling of neutron stars, *Nucl. Phys. A*, **571**: 758-783 (1994).
15. Lattimer J. M., Pethick C. J., Prakash M., Haensel P., Direct URCA Process in Neutron Stars, Phys. Rev. Lett , **66**: 2701-4 (1991).
16. Matese J., O'Connell R., Neutron beta-decay in a uniform magnetic field, *Phys. Rev.*, **180**: 1289-92 (1969).
17. Fassio-Canuto L., Neutron beta-decay in a strong magnetic field, *Phys. Rev*., **187**: 2141-6 (1969).
18. Studenikin A., Proton recoil effects in beta decay of polarized neutrons in a strong magnetic field**,** *Sov. J. Nucl. Phys.* **49**: 1031-4 (1989).
19. Ternov I., Rodionov V., Zhulego V. and Studenikin A., β-decay of Polarized Neutrons in External Electromagnetic Fields, *Sov. J. Nucl. Phys.* **28**: 747-57 (1978).
20. Baranov I., *β*-Decay of an un-polarized neutron in an intense electromagnetic field, *Sov. Phys. J.*, **17**: 533-538 (1974).
21. Shinkevich S., Studenikin A., Relativistic theory of inverse beta-decay of polarized neutron in strong magnetic field, *Pramana J. Phys.*, **65**: 215-244 (2005).
22. Bander M., Rubinstein H. R., Proton beta decay in large magnetic fields, *Phys. Lett. B*, **311**: 187-192 (1993).
23. Skobelev V. V., On the Pressure of a Neutron Gas Interacting with the Non-Uniform Magnetic Field of a Neutron Star, *Russ Phys J*, **60**: 2073 (2018).
24. Schwab J., Bildsten L. and Quataert E., The importance of Urca-process cooling in accreting one white dwarfs, *Monthly Notices of the Royal Astronomical Society*, **472**: 3390-3406 (2017).
25. Weinberg S., *Quantum theory of fields*, Cambridge University press, (2000).
26. Okun L., *Quarks and Leptons*, North-Holland (1982).
27. Landau L. M., Lifshitz E. M., *Quantum Electrodynamics*, Pergaman Press, (1979).
28. Bjorken J. B., and Drell S. D., *Relativistic Quantum Mechanics*, McGraw-Hill, (1964).
29. Abramovitz M., Stegun I. A., *handbook of Mathematical Function with Formula, Graphs and Mathematical Tables*, Denver, Newyork (1970).
30. Seidi M., Calculating of neutron energy spectrum with account of AMM in the presence of a strong magnetic field using the Nikiforov-Uvarov method , *24th Iranian Nuclear Conference*, Iran, University of Isfahan (2018).