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Comparison of Radial Consolidation Behavior of Clay under Three Types of Cyclic Loading

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ABSTRACT: Vertical drains and stone columns which have been used in infrastructure construction for highways, ports, coastal regions, etc., provide significant benefits for improving soil characteristics such as reducing the drainage length and accelerating the consolidation process. So the investigation of the radial consolidation is inevitable. Soils may be subjected to cyclic loading such as silos, tanks, etc. This paper presents semi-analytical solutions for radial consolidation and investigates the consolidation behavior under three types of cyclic loading. Consolidation under cyclic loads was calculated using the superimposition rule. Barron (1948) and Olson (1977) have presented theories for calculating radial consolidation under static and ramp load respectively. In this study, by using a set of continuous static loads or a series of infinite ramp loads, with alternatively positive and negative signs, we have extended these theories for rectangular, triangular and trapezoidal cyclic loads. The obtained analytic results demonstrate that the average degree of consolidation at the steady state depends on the integral of the load-time curve for each cycle and it increases with increase of the integral and the results indicate that change in cycle period of time does not effect on the time of getting steady state. Radial and vertical consolidation under rectangular cyclic loading have also compared and the effect of the distance between vertical drains on the time of getting steady state have investigated.

Keywords: Radial Consolidation, Rectangular Cyclic Loading, Superimposition Rule, Trapezoidal Cyclic Loading, Triangular Cyclic Loading, Vertical Drains.

INTRODUCTION

For construction of a structure on very thick saturated clay layers, the dissipation of excess pore water pressure requires a long period of time. In such cases, the damage caused by differential settlements can be prevented by allowing a portion of the settlement to take place before the beginning of construction. Also, to accelerate the consolidation process, the useful technique of building sand drains can be used.

To analyze the behavior of vertical drains, the unit cell theory was proposed by Barron

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(1948) and Richart (1957). Yoshikuni and Nakanodo (1974) theoretically treated the consolidation process by the vertical drain method, taking the well resistance into consideration. Hansbo et al. (1981) proposed a theory that incorporates important parameters such as vertical discharge capacity, remoulding effects during installation, and filter resistance. Many more studies have been published (Lu and Xie, 2011; Walker, 2011; Deng et al., 2013; Covo-Torres et al., 2015; Lu et al., 2015).

In practical geotechnical engineering, soils beneath many structures, such as oil and water tanks, highway embankments, ocean banks, etc., undergo cyclic loading (Ying-Chun and Kang-He, 2005). The discrepancies between the settlements obtained from theoretical predictions based on the theory of consolidation under static loads and the settlements obtained from field measurements have required consideration of consolidation under cyclic loadings (Wilson and Elgohary, 1974). Since the pioneering work of Shiffman (1958), the challenge of deriving an analytical solution for investigation consolidation settlement of soils under time-dependent loading has captured the attention of researchers. Shiffman studied consolidation settlements due to the time-dependent loading by dividing the linear loading into a series of step loads and used the rule of superimposing. Wilson and Elgohary (1974) proposed an analytical solution for the progress of consolidation of elastic clays subjected to rectangular cyclic loading. Terzaghi's conventional theory extended by Olson (1977) to cover the one dimensional consolidation under ramp loading and many various solutions have been proposed for consolidation of soils under cyclic loading based on different assumptions and considerations. Razouki and Schanz (2011) presented a study of the one-dimensional consolidation process under haversine repeated loading with and without rest period. The analysis was carried out using a hybrid coupled, analytical and numerical implicit finite difference technique. Also Razouki et al (2013) have presented an exact analytical solution of the nonhomogeneous partial differential equation governing the conventional one-dimensional consolidation under haversine repeated loading. Lo et al (2016) have presented a closed form solution for one-dimensional consolidation in unsaturated soils under cyclic loading. The pore water and air pressures along with the total settlement was derived by employing a Fourier series representation in the spatial domain and a Laplace transformation in the time domain. Consolidation with vertical drains under time-dependent loads have also studied before. Lei et al. (2016) have derived an analytical solution to predict consolidation with vertical drains under impeded drainage boundary conditions and multi-ramp surcharge loading. And many other theories have studied consolidation behavior of clay under cyclic loading (Ouria et al., 2013; Ni et al., 2013; Abbaspour, 2014; Speirs et al., 2014).

To make clear the possible engineering application of each loading type, we can consider a silo that is full in odd months of a year and it is empty in even months, in this situation the soil under the silo has been subjected to rectangular cyclic loading. The soil under a silo that is filled in one month constantly and is getting emptied in other month, is subjected to triangular cyclic loading. If there is a gap in time between the end of filling the silo and the beginning of emptying, the type of applied cyclic load is trapezoidal.

This paper presents a semi-analytical solution for radial consolidation under cyclic loading. This method has also been used by Toufigh and Ouria (2009). In that research, each full cycle of loading was replaced by a pair of static loads with different signs. Based on Terzaghi's theory, the degree of consolidation was calculated for each static load and the results were superimposed.

MATERIALS AND METHODS

In this research, a method based on a superimposition rule is employed to calculate the pore water pressure and the degree of consolidation for radial consolidation of clay under rectangular, triangular and trapezoidal cyclic loading. It is assumed that the coefficient of consolidation c_{vr} is constant during the consolidation and this study is based on elastic consolidation theory. Also, excess pore water pressure can dissipate only horizontally. Based on the previous section, radial consolidation of clay under constant and ramp loading have been studied before and in the presented method, by using a set of continuous static loads or a series of infinite ramp loads, with alternatively positive and negative signs, we can extend these theories for rectangular, triangular and trapezoidal cyclic loading as shown in Figure 1.

It is assumed that the full cycle period for the three types of cyclic loading mentioned above, are equal (t_c). Other properties of these load cases are shown in Figure 1.

Radial Consolidation under Rectangular Cyclic Loading

Figure 1a shows a rectangular cyclic loading system which is adapted in figure 1b to a set of continuous static loads. As can be seen, each full cycle of rectangular cyclic load was replaced by a pair of static loads with positive and negative signs.

Finally, based on Barron's equation (Barron, 1948) and superimposition rule, the average degree of consolidation at the end of n half cycles of the over consolidated clay layer, can be calculated by the following equation:

$$U_{hcn} = (-1)^n \sum_{i=1}^{n} (-1)^i U(T_i) \quad (1)$$

where $T_i = i\,T_c/2$. For calculating U_{hcn}, $U(T_i)$ is required. Based on Barron's equation, $U(T_i)$ can be expressed as:

$$U(T_i) = 1 - \exp\left[\frac{-8T_i}{F(n)}\right] \quad (2)$$

where

$$F(n) = \frac{n^2}{n^2-1}\ln(n) - \frac{3n^2-1}{4n^2} \quad (3)$$

$$n = \frac{r_e}{r_w} \quad (4)$$

Radial Consolidation under Triangular Cyclic Loading

Figure 1c shows a triangular cyclic loading system which is adapted in Figure 1d to a series of ramp loads. Each full cycle of triangular cyclic load was replaced by a pair of ramp loads with positive and negative signs. The degree of consolidation can be defined based on Olson's equations.

As in the previous section, based on Olson's equations and superimposition rule, the average degree of consolidation at the end of n half cycles of the over consolidated clay layer, can be calculated by Eq. (1).

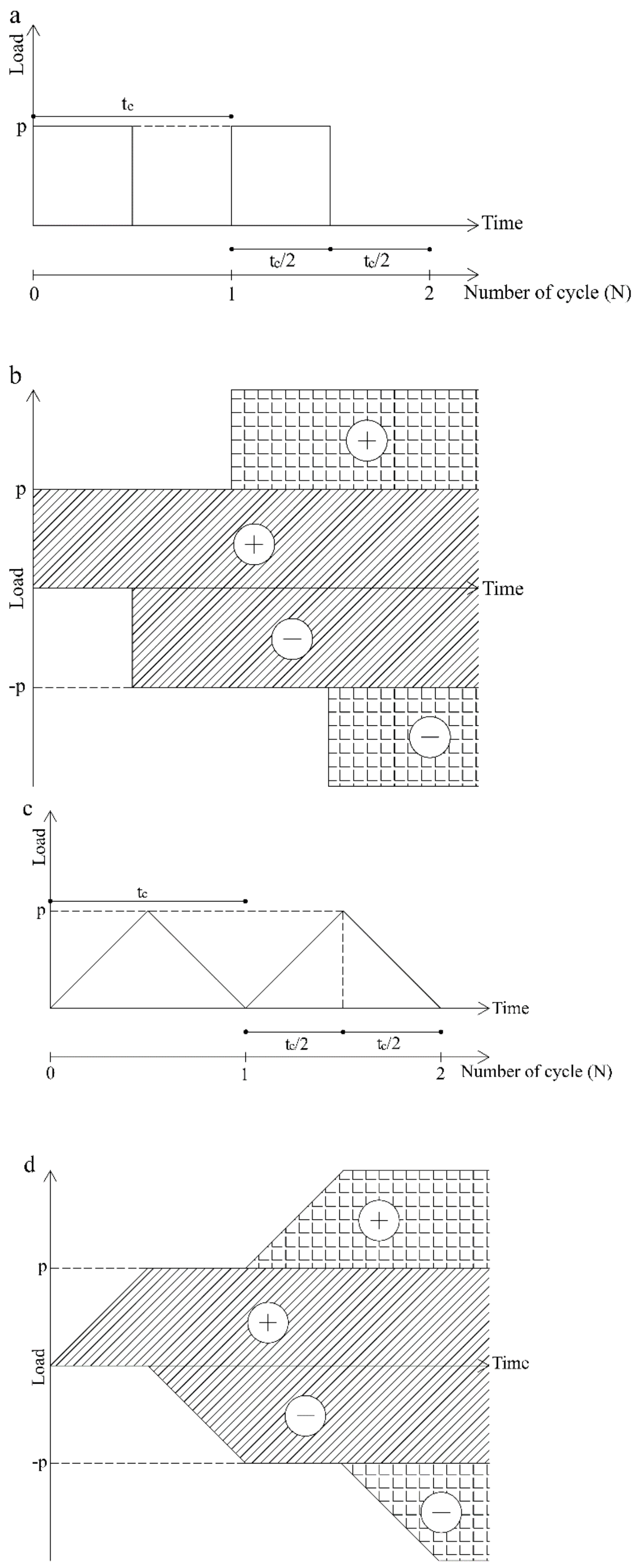
Based on Olson's equation in radial consolidation, $U(Ti)$ can be expressed as:
For $T_i < 0.5\, T_c$:

$$U_r = \frac{2T_i - \frac{2}{A}\left[1 - \exp\left(A.T_i\right)\right]}{T_c} \quad (5)$$

For $T_i \geq 0.5\, T_c$:

$$U_r = 1 - \frac{2}{A.T_c}\left[\exp\left(\frac{A.T_c}{2}\right) - 1\right]\exp\left(-A.T_i\right) \quad (6)$$

where


a
Load
t_c
p
Time
$t_c/2$
$t_c/2$
0
1
2
Number of cycle (N)
b
p
Load
Time
-p
c
Load
t_c
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Number of cycle (N)
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Load
Time
-p

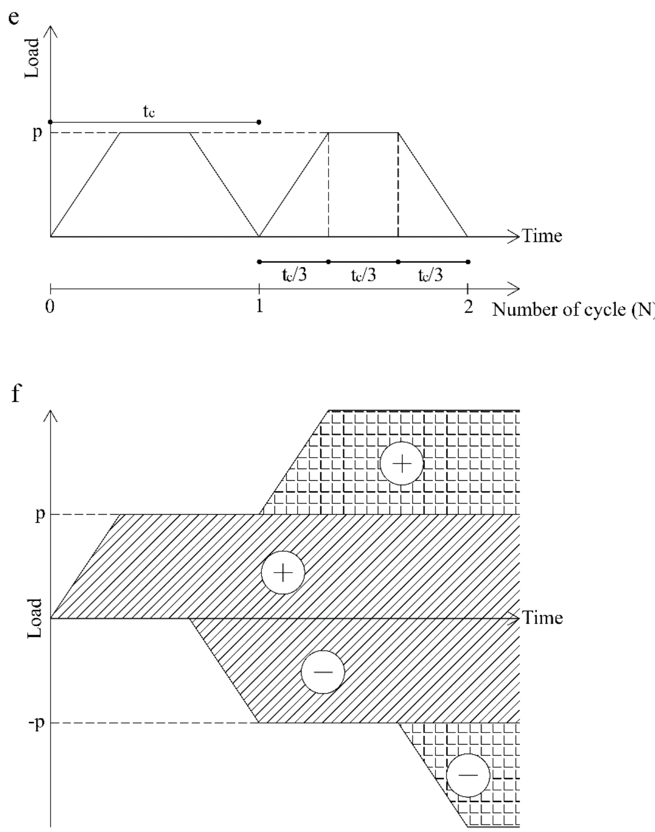


Fig. 1. The three types of cyclic loading adapted by the superimposition rule

$$T_i = \frac{C_{vr} t_i}{r_e^2} \tag{7}$$

$$T_c = \frac{C_{vr} t_c}{r_e^2} \tag{8}$$

$$A = \frac{2}{F(n)} \tag{9}$$

Radial Consolidation under Trapezoidal Cyclic Loading

This case is similar to the previous section and we use Olson's equation and superimposition rule to calculate trapezoidal cyclic loading (Figures 1e and 1f). Average degree of consolidation at the end of *N* cycles of the over consolidated clay layer, can be calculated by the following equation:

$$U_{cN} = \sum_{i=1}^{N} \left[\mathrm{U}((N-i+1)\mathrm{T}_c) - \mathrm{U}((\frac{3(N-i)+1}{3})T_c) \right] \tag{10}$$

For this case Olson's equation (Olson 1977) for radial consolidation, *U(Ti)* has been modified into the following equations:

For $T_i < \dfrac{T_c}{3}$:

$$U_r = \frac{3T_i - \frac{3}{A}\left[1 - \exp\left(A T_i\right)\right]}{T_c} \tag{11}$$

For $T_i \geq \dfrac{T_c}{3}$:

$$U_r = 1 - \frac{3}{A T_c}\left[\exp\left(\frac{A T_c}{3}\right) - 1\right]\exp\left(-A T_i\right) \tag{12}$$

APPLICATION OF THE PRESENTED METHODS

The procedure of using the presented methods for calculating radial consolidation of clay is described in the following example.

Radial consolidation calculation of OC clay layer with c_{vr} = 0.00462 m^2/day, r_w = 0.38 m, r_e = 1.52 m and t_c = 30 days is examined. All three types of cyclic loading applied to the clay layer are shown in figure1.

In order to calculate the average degree of consolidation, the time factor for a full cycle and other required parameters are calculated:

$$T_c = \frac{c_{vr} t_c}{{r_e}^2} = \frac{0.00462 \times 30}{(1.52)^2} = 0.06$$

$$n = \frac{r_e}{r_w} = \frac{1.52}{0.38} = 4$$

$$F(n) = \frac{n^2}{n^2 - 1}\ln(n) - \frac{3n^2 - 1}{4n^2} =$$

$$\frac{4^2}{4^2 - 1}\ln(4) - \frac{3 \times (4^2) - 1}{4 \times (4^2)} = 0.744$$

$$A = \frac{2}{F(n)} = 2.6869$$

Average degree of consolidation at the end of the first half cycle:

a) For rectangular cyclic loading (by using Eq. (1)):

$$U_{hc1} = U(T_1) = U(0.5T_c)$$

$U(0.5T_c)$ would be obtained from Eq. (2):

$$U_{hc1} = U(0.5T_c) = 1 - \exp\left[\frac{-4T_c}{F(n)}\right] = 0.2757$$

b) For triangular cyclic loading (by using Eq. (1)):

$$U_{hc1} = U(T_1) = U(0.5T_c)$$

$U\ (0.5T_c)$ would be obtained from Eq. (6):

$$U_{hc1} = 1 -$$

$$\frac{2}{AT_c}\left[\exp\left(\frac{AT_c}{2}\right) - 1\right]\exp\left(\frac{-AT_c}{2}\right) = 0.03924$$

Average degree of consolidation at the end of the first full cycle:

a) For rectangular cyclic loading (by using Eq. (1)):

$$U_{c1} = U_{hc2} = U(T_c) - U(0.5T_c)$$

As presented previously, the value of $U\ (0.5T_c)$ is calculated. $U\ (T_c)$ can be calculated by using Eq. (2):

$$U(T_c) = 1 - \exp\left[\frac{-8T_c}{F(n)}\right] = 0.4753$$

So, U_{c1} can be calculated:

$$U_{c1} = 0.4753 - 0.2757 = 0.1996$$

b) For triangular cyclic loading (by using Eq. (1)):

$$U_{c1} = U_{hc2} = U(T_c) - U(0.5T_c)$$

The value of $U(0.5T_c)$ as previous section is calculated. $U(T_c)$ can be calculated by using Eq. (6):

$$U(T_c) = 1 -$$

$$\frac{2}{AT_c}\left[\exp\left(\frac{AT_c}{2}\right) - 1\right]\exp\left(-AT_c\right) = 0.11365$$

So, U_{c1} can be calculated:

$$U_{c1} = 0.11365 - 0.03924 = 0.0744$$

c) For trapezoidal cyclic loading (by using Eq. (10)):

$$U_{c1} = U(T_c) - U(T_c / 3)$$

From Eq. 12:

$$U(T_c) = 1 -$$

$$\frac{3}{AT_c}\left[\exp\left(\frac{AT_c}{3}\right) - 1\right]\exp\left(-AT_c\right) = 0.1256$$

$$U(T_c / 3) = 1 -$$

$$\frac{3}{AT_c}\left[\exp\left(\frac{AT_c}{3}\right) - 1\right]\exp\left(\frac{-AT_c}{3}\right) = 0.0264$$

So, U_{c1} can be calculated:

$$U_{c1} = 0.1256 - 0.0264 = 0.0992$$

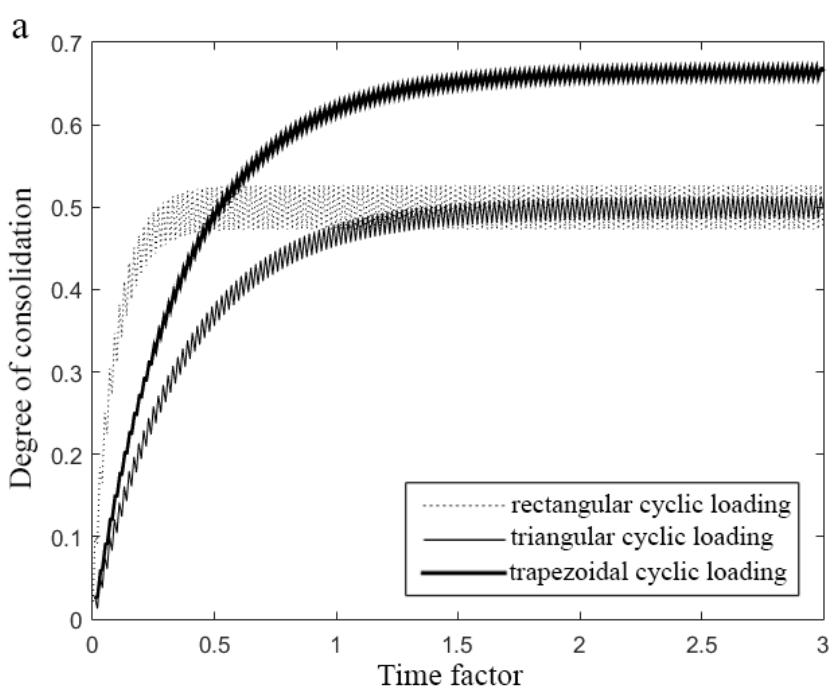
Above procedure can be repeated for the next cycles for calculating the degree of consolidation. The results of the calculations are shown in Table 1.

Table 1. The results of presented example

N	Time (day)	U_{cN} Rectangular Cyclic Loading	Triangular Cyclic Loading	Trapezoidal Cyclic Loading
1	30	0.1996337	0.0743941	0.0991981
2	60	0.3043996	0.1377133	0.1836287
3	90	0.3593798	0.1916062	0.2554903
4	120	0.3882329	0.2374761	0.3166539
5	150	0.4033747	0.2765175	0.3687121
6	180	0.4113210	0.3097468	0.4130205
7	210	0.4154911	0.3380293	0.4507328
8	240	0.4176796	0.3621014	0.4828309
9	270	0.4188280	0.3825899	0.5101506
10	300	0.4194307	0.4000284	0.5334032
20	600	0.4200953	0.4798381	0.6398226
50	1500	0.4200963	0.4995715	0.6661354
100	3000	0.4200963	0.4997295	0.6663460

RESULTS AND DISCUSSION

Figure 2 illustrates average degree of consolidation for the above mentioned cyclic loadings. The soil properties and geometric characteristics are the same as the previous section. Load characteristics are also the same as application except the full cycle period. From Figure 2 it may be seen that for radial consolidation under rectangular and triangular cyclic loading (with equal integral of load-time curve for each cycle) with equal cycle period, the average of degree of consolidation are equal at the steady state. For radial consolidation under trapezoidal cyclic loading and with cycle period equal to above states, the average of degree of consolidation is more than them at the steady state (because the integral of load-time curve for trapezoidal cyclic loading for each cycle is more than rectangular and triangular cyclic loading).



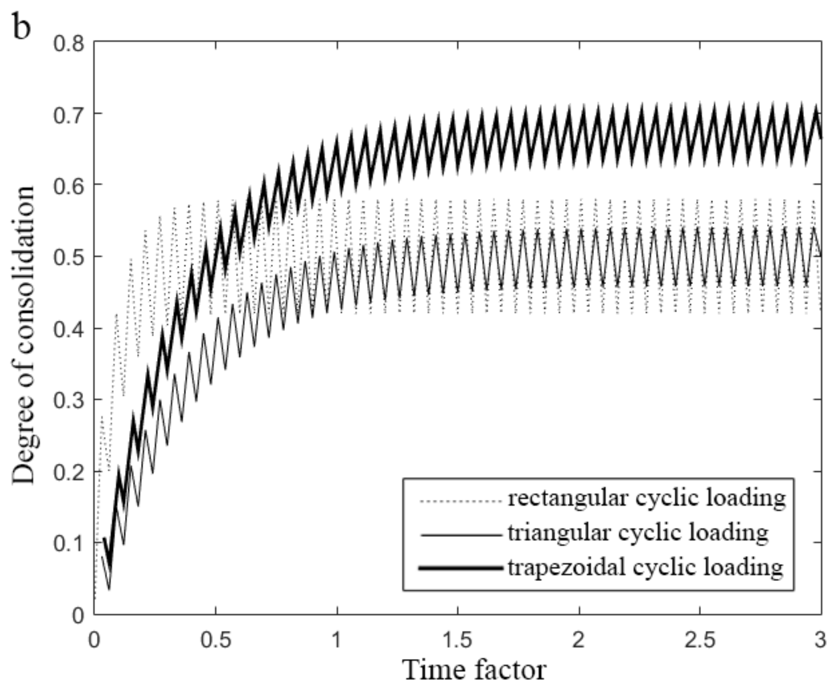


Fig. 2. The average degree of consolidation: a) T_c = 10 days, b) T_c = 30 days

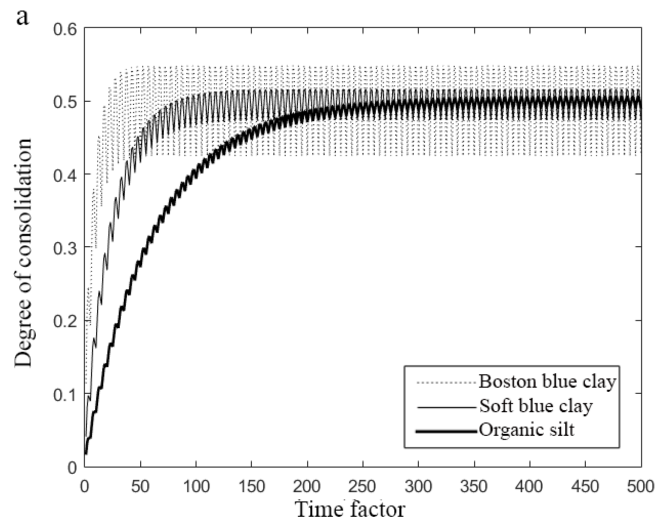
The degree of consolidation of soil under trapezoidal cyclic loading in radial consolidation is less than the rectangular cyclic loading from the beginning and becomes more than it after a period of time. The degree of consolidation under triangular cyclic loading in radial consolidation is less than the loading states mentioned above.

The time of getting to the steady state in radial consolidation of soil under triangular and trapezoidal cyclic loading are approximately equal and more than if under rectangular cyclic loading.

Figures 3-5 were plotted to investigate the influence of type of soil on radial consolidation under cyclic loading. Geometric characteristics are the same mentioned before and soil properties are shown in Table 2.

Table 2. Soil properties

	c_v		k_h/k_v		Adopted Value of c_v	
Soil type	**Typical value (cm^2/s)×10^{-4}**	**Reference**	**Experimental Value**	**Reference**	**Vertical Consolidation (m^2/day)×10^{-4}**	**Radial Consolidation (m^2/day)×10^{-4}**
Soft blue clay	1.6-26	Wallace and Otto (1964)	1.5	Basett and Brodie (1961)	90	135
Boston blue clay	40±20	Ladd and Luscher (1965)	0.7-3.3	Haley and Aldrich (1969)	260	430
Organic silt	2-10	Lowe, Zaccheo, and Feldman (1964)	1.2-1.7	Tsien (1955)	35	53



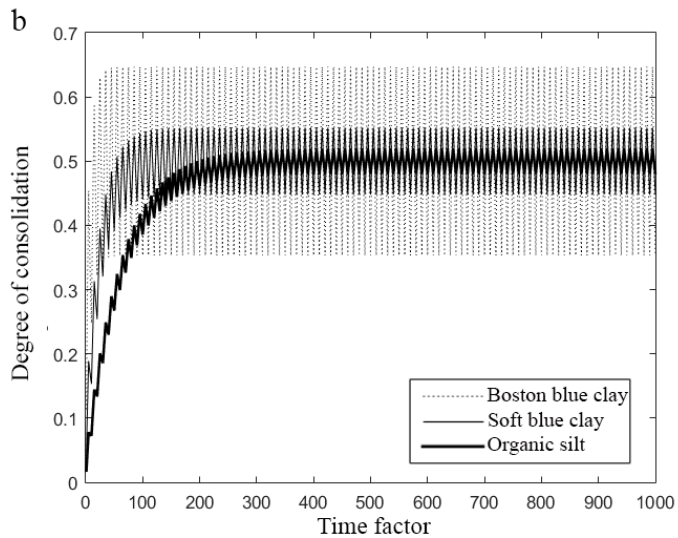
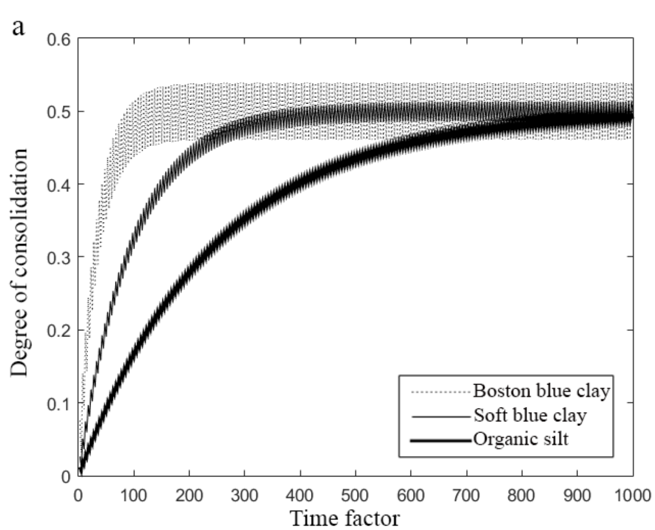


Fig. 3. The average degree of consolidation under rectangular cyclic loading: a) T_c = 5 days, b) T_c = 10 days



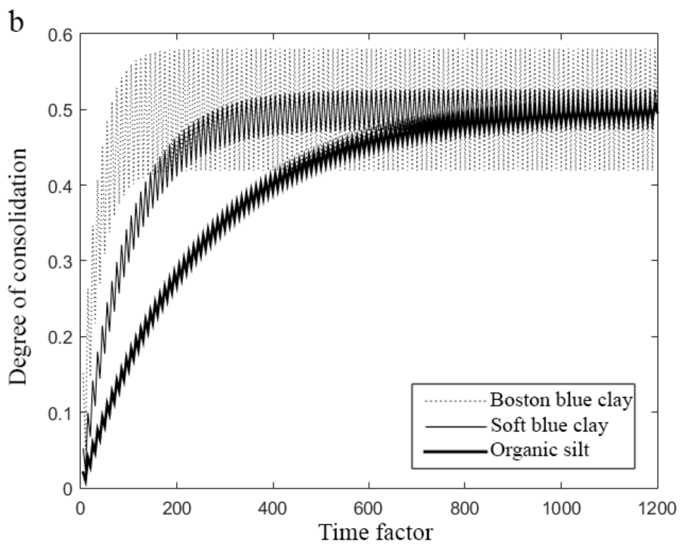


Fig. 4. The average degree of consolidation under triangular cyclic loading: a) T_c = 5 days, b) T_c = 10 days

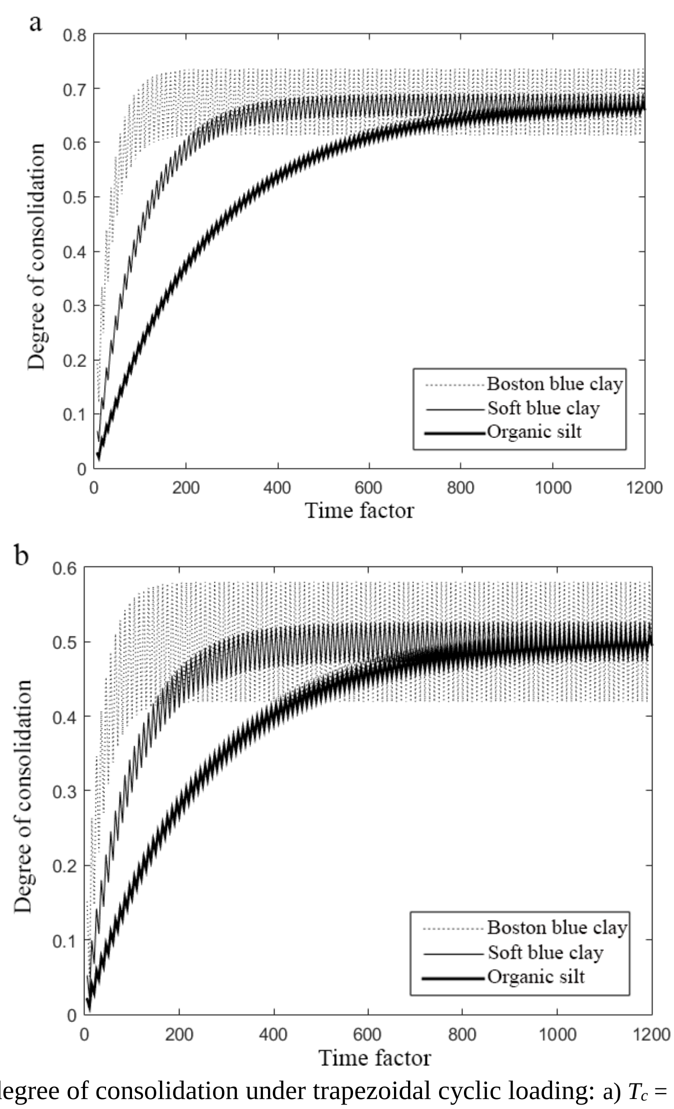
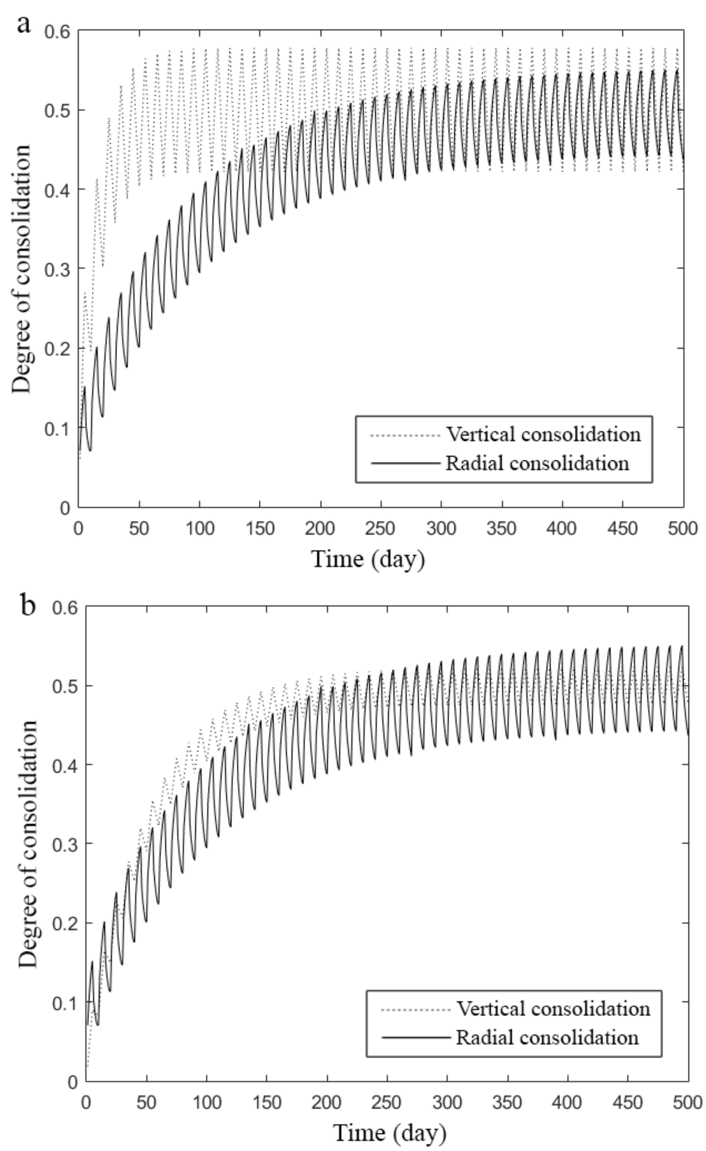
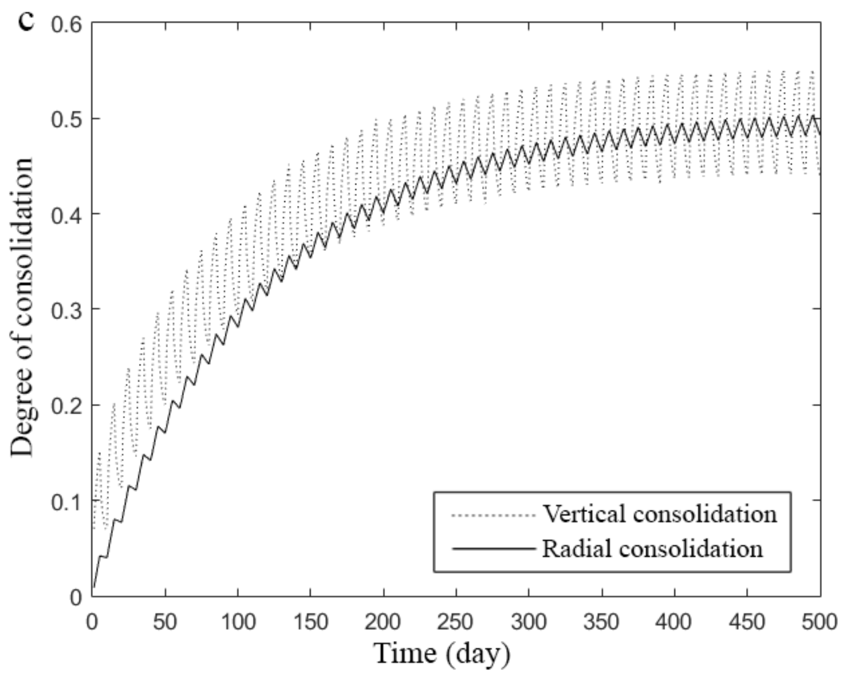


Fig. 5. The average degree of consolidation under trapezoidal cyclic loading: a) T_c = 10 days, b) T_c = 20 days

The results indicate that the time of reaching steady state decreases with increase of consolidation coefficient and change in cycle period of time does not affect the time of getting steady state.

Figures 6-8 illustrates the degree of consolidation of the three types of soil in radial and vertical consolidation under rectangular cyclic loading to investigate the effect of using vertical prefabricated drains or stone columns to accelerate the consolidation of soft soils. The calculation of vertical consolidation under rectangular cyclic loading is based on the study of Toufigh and Ouria (2009). The coefficient of consolidation of each soil is shown in Table 2 and the soil properties and geometric characteristics for each case are given in Table 3. As it can be seen from Table 2, for organic silt and soft blue clay, the ratio of radial coefficient of consolidation to vertical coefficient of consolidation is 1.5 and it can be seen from Figures 3 and 5 that when the ratio of drainage length of radial consolidation to vertical becomes 2, the average degree of consolidation for radial consolidation is less than for vertical consolidation.





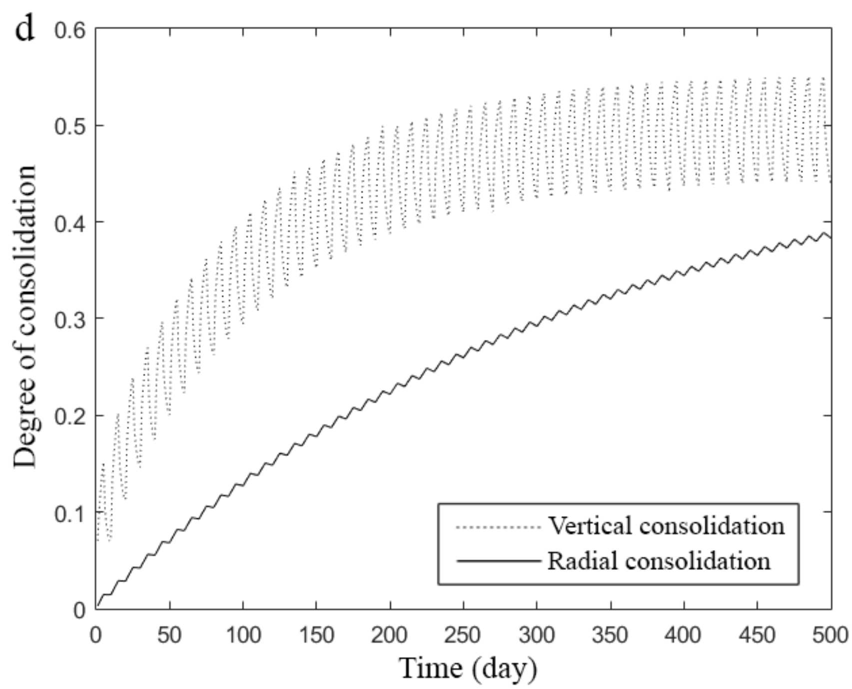
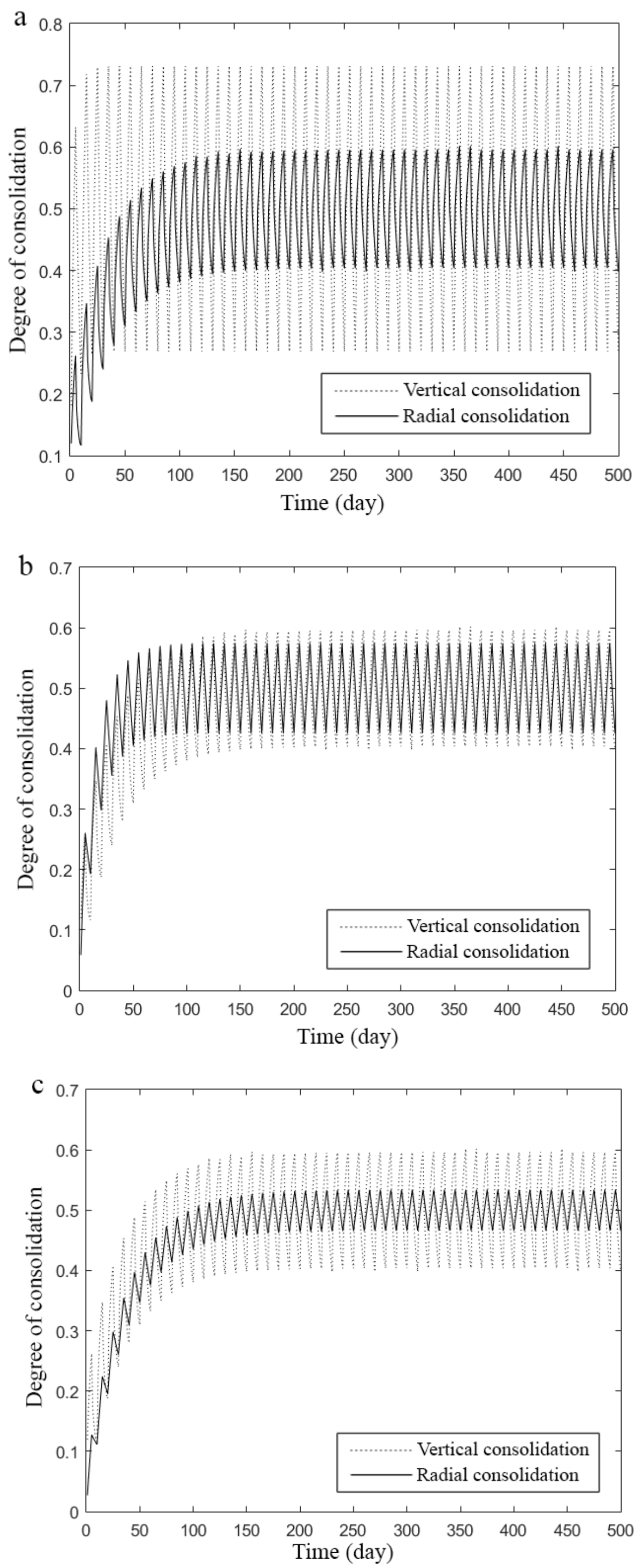


Fig. 6. The average degree of consolidation under rectangular cyclic loading for soft blue clay: a) case 1, b) case 2, c) case 3, d) case 4

Table 3. Load and geometric characteristics

Case	Cycle Period (day)	Radial Consolidation r_w (m)	Radial Consolidation r_e (m)	Vertical Consolidation H_{dr} (m)
1	10	0.38	1.52	1.52
2	10	0.38	2.28	1.52
3	10	0.38	3.04	1.52
4	10	0.38	4.56	1.52


a
Degree of consolidation
Time (day)
Vertical consolidation
Radial consolidation
b
Degree of consolidation
Time (day)
Vertical consolidation
Radial consolidation
c
Degree of consolidation
Time (day)
Vertical consolidation
Radial consolidation

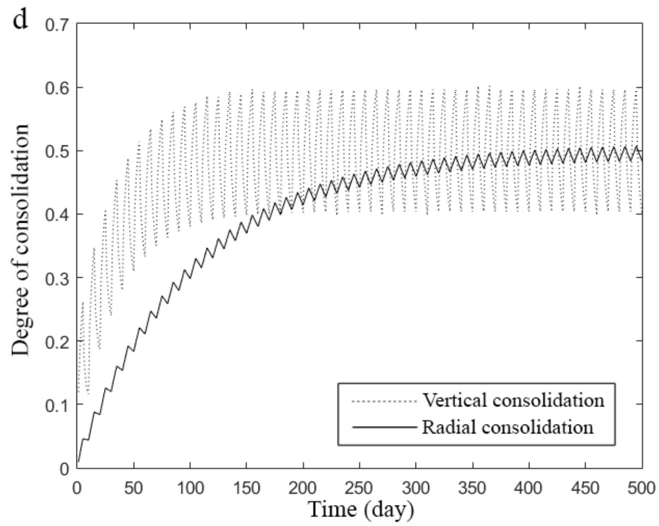
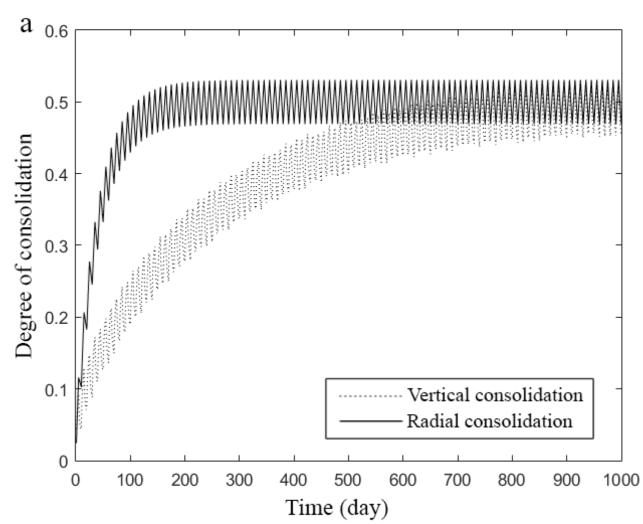
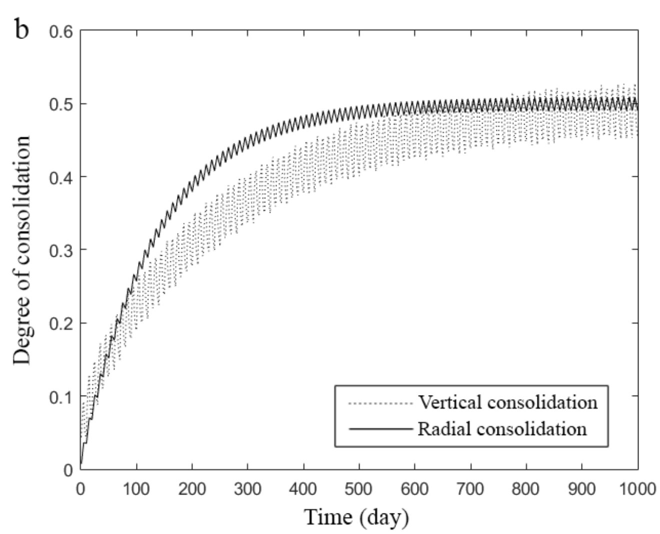


Fig. 7. The average degree of consolidation under rectangular cyclic loading for Boston blue clay: a) case 1, b) case 2, c) case 3, d) case 4





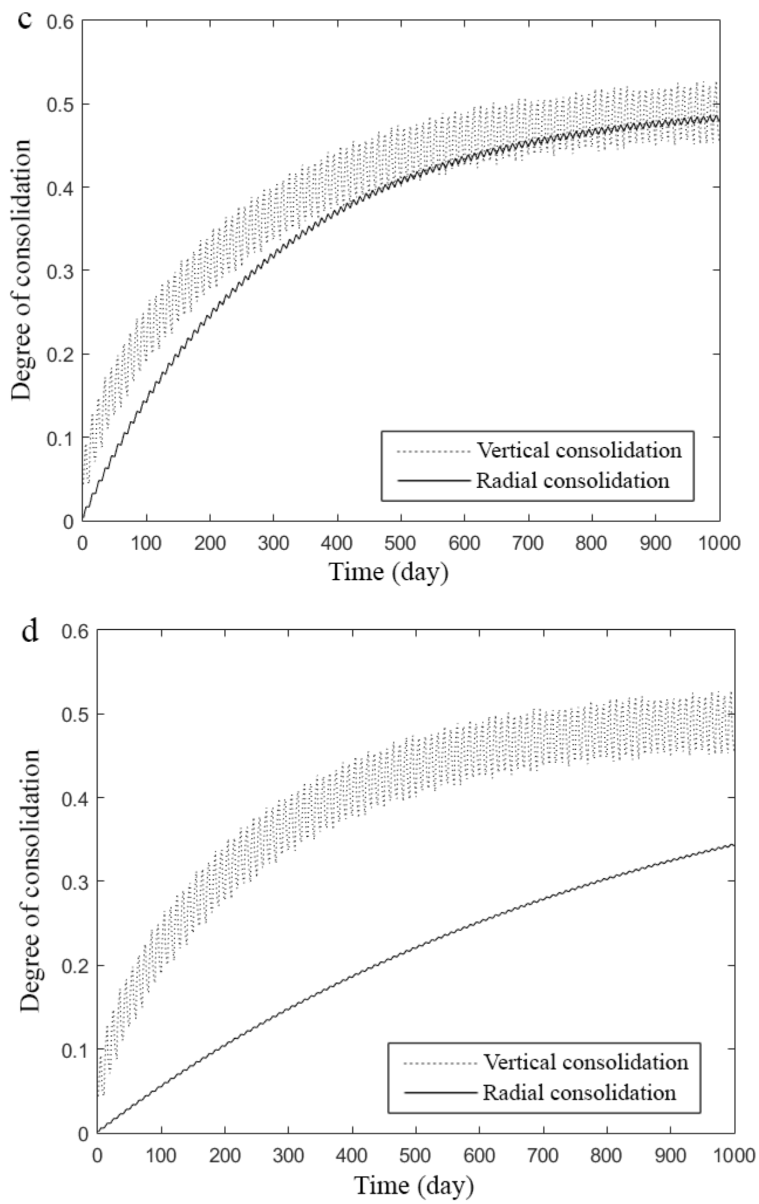


Fig. 8. The average degree of consolidation under rectangular cyclic loading for organic silt: a) case 1, b) case 2, c) case 3, d) case 4

For Boston blue clay, the ratio of radial coefficient of consolidation in to vertical coefficient of consolidation is 1.7 and when the ratio of drainage length of radial consolidation in to vertical becomes 2, the average of degree of consolidation for radial consolidation is equal to vertical from beginning to steady state (case 3). Therefore radial consolidation and vertical consolidation under rectangular cyclic loading become equal when:

$$1 < \frac{r_e / H_{dr}}{k_h / k_v} < 1.5 \tag{13}$$

where r_e : is radius of the equivalent circle (= $d_e/2$), H_{dr} : is the length of vertical drainage, k_h: is the horizontal coefficient of permeability and k_v: is the vertical coefficient of permeability.

CONCLUSIONS

The following conclusions may be drawn from this study:

1. The average of degree of consolidation in steady state depends on the integral of the load-time curve for each cycle and it increases with increase of the integral.
2. The degree of consolidation of soil under trapezoidal cyclic loading is less than for the rectangular cyclic load from the beginning and becomes more than it after a period of time. Degree of consolidation is less than the loading states mentioned above for triangular cyclic loading.
3. The time of reaching steady state, is lower for rectangular cyclic loading than the other types of cyclic loading that have been investigated in this research.
4. Change in cycle period of time does not effect on the time of getting steady state.
5. Radial consolidation and vertical consolidation under rectangular cyclic loading become equal when the ratio of drainage length of radial consolidation to vertical is about 1-1.5 times the ratio of radial coefficient of consolidation to vertical.

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