A Semi-Analytical Method for History Matching and Improving Geological Models of Layered Reservoirs: CGM Analytical Method

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Abstract

History matching is used to constrain flow simulations and reduce uncertainty in forecasts. In this work, we revisited some fundamental engineering tools for predicting waterflooding behavior to better understand the flaws in our simulation and thus find some models which are more accurate with better matching. The Craig-Geffen-Morse (CGM) analytical method was used to predict recovery performance calculations and it was simple enough to be applied in a spreadsheet. In this study, the analytical approach of history matching was applied to a layered reservoir from a shallow marine deposit which was composed of different facies includes lower shoreface facies (LSF), middle shoreface facies (MSF) and upper shoreface facies (USF). Truncated Gaussian Simulation (TGS) is often used to stochastically distribute the facies in the geological model around a deterministic mean representation. The actual distribution is often hard to determine. Starting with the deterministic element of the facies distributions, corrections were made by matching the CGM method predictions to historical data. These corrections were amalgamated in the model and produced a much better history match. Further, the modifications were used to condition the stochastic simulator to provide a geologically more robust model that also matched history. The results showed that the variation of the total field production rate (FPR) between the deterministic model and history data was reduced about 19.8% (from 21.52% to 1.73%) after applying history match analytically.

Keywords

Craig-Geffen-Morse analytical method;
History matching, Improving geological models;
Waterflood performance;
Uncertainty reduction.

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1. Introduction

Waterflooding is a common and most economical method for the fluid injections to support the reservoir pressure and increase the oil recovery. It is widely used to displace oil due to its availability and high efficiency. In 1865, this technique was introduced accidently by flowing water from a shallow water-bearing layer into the lower oil pay zone which was happened in Pothole, Pennsylvania. John (1880) stated that this approach was continued for some decades [1]. In 1924, the water was first injected by a five-spot pattern in Bradford field [2]. There are many methods for predicting and forecasting the waterflooding performance [3-11]. From these all methods, there are three well known and widely used approaches such as Dykstra-Parsons, Buckley-Leverett, and Craig-Geffen-Morse (CGM). Because it is the combination of other methods with some more corrections and improvement, CGM approach is predicting the waterflood performance more realistically [12].

Kruger (1961) was the first person who developed a numerical technique to quantify the distributions of areal permeability in the reservoir by matching the measured and predicted production data. The results of that study showed that this numerical technique is applicable for matching data at different reservoir conditions and for 2D flow calculations and analysis [13]. At the same time, the concept of the layered reservoir was analyzed using the simple reservoir engineering techniques by Hutchinson et al. [14]. Afterwards, Bennion et al. (1966) applied a stochastic model for predicting the reservoir stratification. In that work, they established a method that can be used to determine different parameters of the layered system include continuity of permeable zones and lateral extending of the shale [15]. On the other hand, Craig et al. (1970) investigated the effect of reservoir description on waterflood performance predictions; they identified that the effect of permeability stratification on oil recovery was greater than the effect of gravity forces [16]. The size and location of the layers with different values of permeability are significant. In order to select the proper number of layers with optimum thicknesses two factors must be considered; flow capacity and/or equal thickness [16].

In 1988, Wu proposed a semi-empirical approach to predict the waterflood process by using classical waterflooding models. Through adjusting the effect of some dominant parameters include displacement mechanisms, vertical variation of permeability, sweep efficiency and mobility ratio, the technique was found to be practical and effective for evaluating and managing the initial waterflood project and matching the performance of the mature waterflood project. However, that approach was limited just for providing the performance of the total field, not each individual well [12]. Spivy et al. (1994) introduced an analytical procedure for predicting and history matching the production data such as calculating production rate and cumulative production. This procedure was applicable for a well with linearly varying the bottom hole pressure with time at a contestant production pressure [17]. In order to identify the vertical and areal distributions of the injection water, the CGM method was applied to match the actual production data of the field in the Palogande-Cebu oil field in Colombia. Analyzing the well injectivity and injection-production curve of this approach resulted in a good match between the historical and calculated data of the fluid distributions [18]. Lerma (2003) studied the capability of an analytical method for waterflood performance and history match in a layered reservoir. By applying that approach, some important parameters include expected schedule of production, oil recovery, and the duration of the production from each layer were identified [19]. In addition, Gomez et al. (2009) illustrated an analytical methodology, composed of CGM technique, for estimating and simulating the vertical and areal efficiencies of the waterflooding process. From applying this approach in La Cira-Fantas field in Colombia, they found out that 38% of the injected water had been lost during flowing due to lack in hydraulic connectivity between the injection and production wells [20].

History match, as an important tool, is always applied for estimating the real reservoir parameters and reducing their uncertainties [21]. Hence, Samandari et al. (2011) applied a semi-analytical method to hydraulically fractured shale gas wells in Barnett play in West Texas. After obtaining an acceptable match of the field production data, some reservoir parameters include effective fracture and matrix permeability were determined [22]. Olalotiti-Lawal et al. (2015) presented a semi-analytical method for estimating the performance of the unconventional reservoir and indicating the fluid flow through single or multiple fractured wells. The results of their investigation showed that this method can be applied to both vertical and horizontal wells of oil or gas reservoirs with the permeability ranging from nanoDarcy to one millidarcy [23]. Moreover, Tostado et al. (2016) introduced a new analytical technique for history matching the production performance of the cyclic steam simulation in a heavy oil reservoir; an inconsistent pattern found for the production by the steam cycle in a single well and across the field [24]. In the same way, Young et al. (2017) studied using some analytical models to indicate the waterflooding performance and history matching in a large sandstone reservoir in the Middle East. The results of this study represented a high reduction in uncertainties of reservoir parameters and better history match prediction [25].

In this paper, we applied the CGM analytical method to predict and history match the waterflood performance. In this way, the uncertainty of some reservoir parameters can be reduced and the exact thicknesses of the various layers in the multi-layered reservoir were identified.

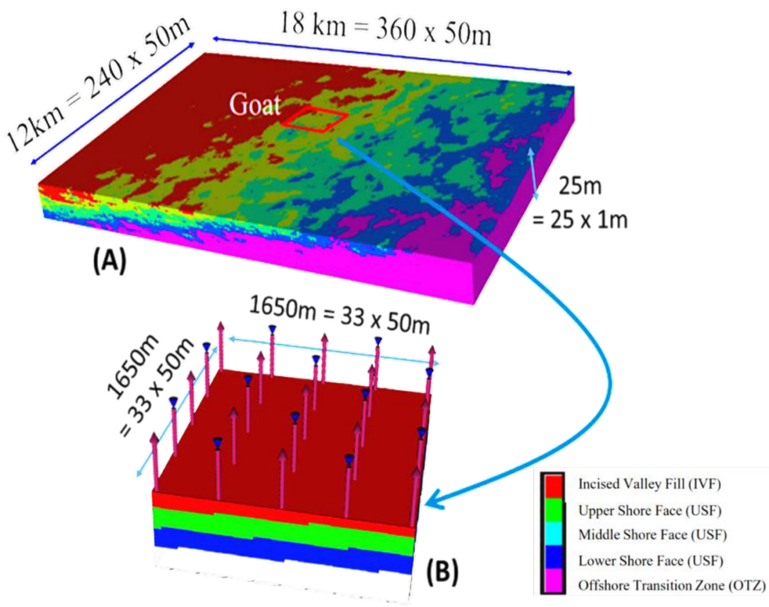


Figure 1. The stochastic realization model of the shoreface deposit and simple deterministic model. **(A)** the stochastic model of the total shoreface deposit created by Petrel modeling software which is composed of 5 layers including Incised Valley Fill, USF, MSF, LSF and Offshore Transition Zone. **(B)** a simple deterministic model created by Eclipse simulation software which is composed of three main layers include USF, MSF and LSF.

2. Data and Simulation Model Overview

As far as the analytical approach is concerned, a number of basic introductory parameters were needed to perform the CGM procedure such as water injection rates, fluid and rock properties. The values of cumulative water injection were presumed but the properties of reservoir fluid and rock provided for the shoreface deposit, which are referred to the previous works, are shown in Table 1 and 2. Additionally, other parameters including relative permeability ratio curve, fractional flow profile, mobility ratio, average and frontal water saturation and secondary water saturation (Sw2) for each layer had been calculated from the known values of the relative permeability table.

From the given data and based on the dimensions of the area of interest, which is named Goat, a simple deterministic model was created as shown in Fig. 1. The dimensions of the model are 1650 m, 1650 m and 25 m in X, Y and Z, respectively; and the total number of cells are 27,225 cells (dx: 50 m, dy: 50 m and dz: 1 m). This model is essentially composed of three main layers, namely, Lower Shoreface (LSF), Middle Shoreface (MSF) and Upper Shoreface (USF). Each of these layers has its own different petrophysical properties such as porosity, permeability, and net to gross. The total number of wells in this model are 13 producers and 12 injectors which were arranged in a five-spot pattern with 400m spacing as shown in Fig. 2. The producers were controlled with the fixed bottom hole pressure (at least 1400 psi) and injectors by the field voidage.

Table 1. Reservoir fluid properties - PVT data

Properties	Units	Values
Water viscosity	cP	0.44
Oil viscosity	cP	2
Density of oil/water	Ratio	0.8 (45° API)
Oil Formation Volume Factor (B_o)	bbl/stb	1.2
Water Formation Volume Factor (B_w)	bbl/stb	1

The distributions of the facies of a geological model corresponding to the approved semi-analytical model need to be modified. In this way, the model of the area of interest with 100 realizations was created by Petrel software as shown in Fig. 3. These models were generated by applying the 'Truncated Gaussian with Trends' (TGT) technique of facies modeling.

Table 2. **Reservoir rock properties of all layers**

Layer	NTG	Porosity	Permeability, mD		
			K_x	K_y	K_z
LSF	0.25	0.15	5	5	0.001
MSF	0.60	0.17	20	20	0.01
USF	0.95	0.25	250	250	10

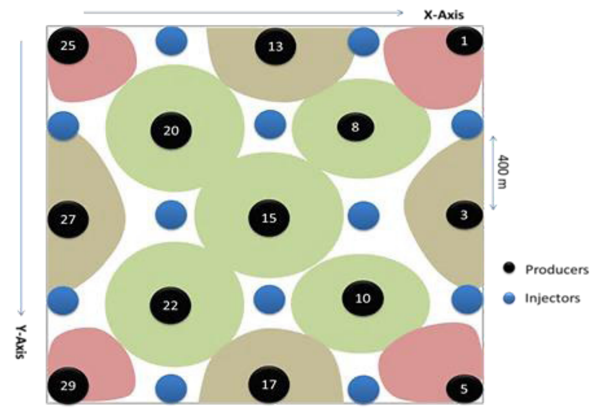


Figure 2. Positions of all production and injection wells in the simulation and geological models.

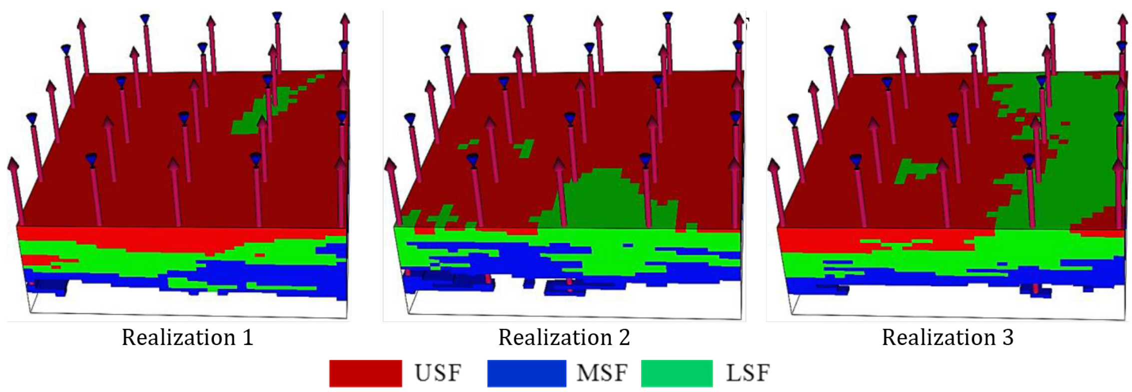


Figure 3. Examples of different stochastic realization models which are made by petrel for the area of interest (Goat sector).

3. Methodology

The methodology used in this paper is basically divided into three main parts (Fig. 4); recovery performance calculations, semi-analytical history match and updating the geological model. First, for recovery performance calculations, the CGM analytical method was used to build a spreadsheet. Second, once the analytical model was verified, the analytical and simulation results were compared and matched with the historical data by changing the thickness of the layers in both the analytical and deterministic models. Finally, based on the best match gained from the semi-analytical history matching, the geological model was updated by modifying the facies proportions around production wells.

3.1 Waterflooding recovery performance

The CGM method combines the displacement mechanism, areal sweep efficiency, injectivity and stratification to predict the performance of the waterflooding process. The CGM procedure for one specific layer in well production 3 was prepared in a spreadsheet. In this study, the upper shoreface was selected as a base layer because it is the most effective one in the shoreface system. The same procedure was then applied for all three layers and production wells. The procedure of the CGM method to predict recovery performance in a layered reservoir is as follows [1, 2, 15, 21]:

1. Splitting the reservoir into the applicable number of layers.
2. Determining the performance of the particular layer (base layer).
3. Plotting liquid rates include oil production rate (Q_o), water production rate (Q_w) and water injection rate (i_w), and cumulative liquid volumes, such as cumulative oil production (N_p), cumulative water production (W_p), and cumulative water injection (W_{inj}) versus time for the base layer.
4. Estimating the values of permeability with thickness (Kh), permeability with porosity (K∅) and permeability by porosity (K/∅) for each layer.
5. To estimate the performance of any other layer i.e. layer (i), pick the sequence of time (t) and find N_p^*, W_p^*, W_{inj}^*, Q_o^* and Q_w^* by reading the plotted values in step 3 at time (t^*) which can be determined from using Eq. 1.
   And then calculate the performance of layer (i) at any time (t) by using following equations (Eq. 2 - 6).
6. Estimating the total performance at any given time (t) by summing the values of each layer.

$$t_i^* = t\,\frac{(k/\emptyset)_i}{(k/\emptyset)_b} \quad (1)$$

$$\boldsymbol{N_p} = \boldsymbol{N_p^*}\,\frac{(\boldsymbol{\emptyset h})_{\boldsymbol{i}}}{(\boldsymbol{\emptyset h})_{\boldsymbol{b}}} \quad (2)$$

$$W_p = W_p^*\,\frac{(\emptyset h)_i}{(\emptyset h)_b} \quad (3)$$

$$\boldsymbol{W_{inj}} = \boldsymbol{W_{inj}^*}\,\frac{(\boldsymbol{\emptyset h})_{\boldsymbol{i}}}{(\boldsymbol{\emptyset h})_{\boldsymbol{b}}} \quad (4)$$

$$\boldsymbol{Q_o} = \boldsymbol{Q_o^*}\,\frac{(\boldsymbol{k/h})_{\boldsymbol{i}}}{(\boldsymbol{k/h})_{\boldsymbol{b}}}\,\frac{(\boldsymbol{\emptyset h})_{\boldsymbol{i}}}{(\boldsymbol{\emptyset h})_{\boldsymbol{b}}} \quad (5)$$

$$\boldsymbol{Q_w} = \boldsymbol{Q_w^*}\,\frac{(\boldsymbol{k/h})_{\boldsymbol{i}}}{(\boldsymbol{k/h})_{\boldsymbol{b}}}\,\frac{(\boldsymbol{\emptyset h})_{\boldsymbol{i}}}{(\boldsymbol{\emptyset h})_{\boldsymbol{b}}} \quad (6)$$

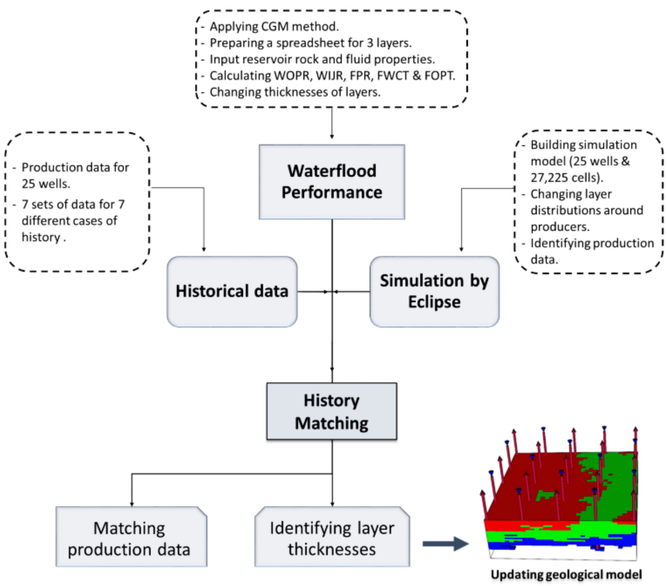


Figure 4. Steps and procedure of the semi-analytical history match to update geological model

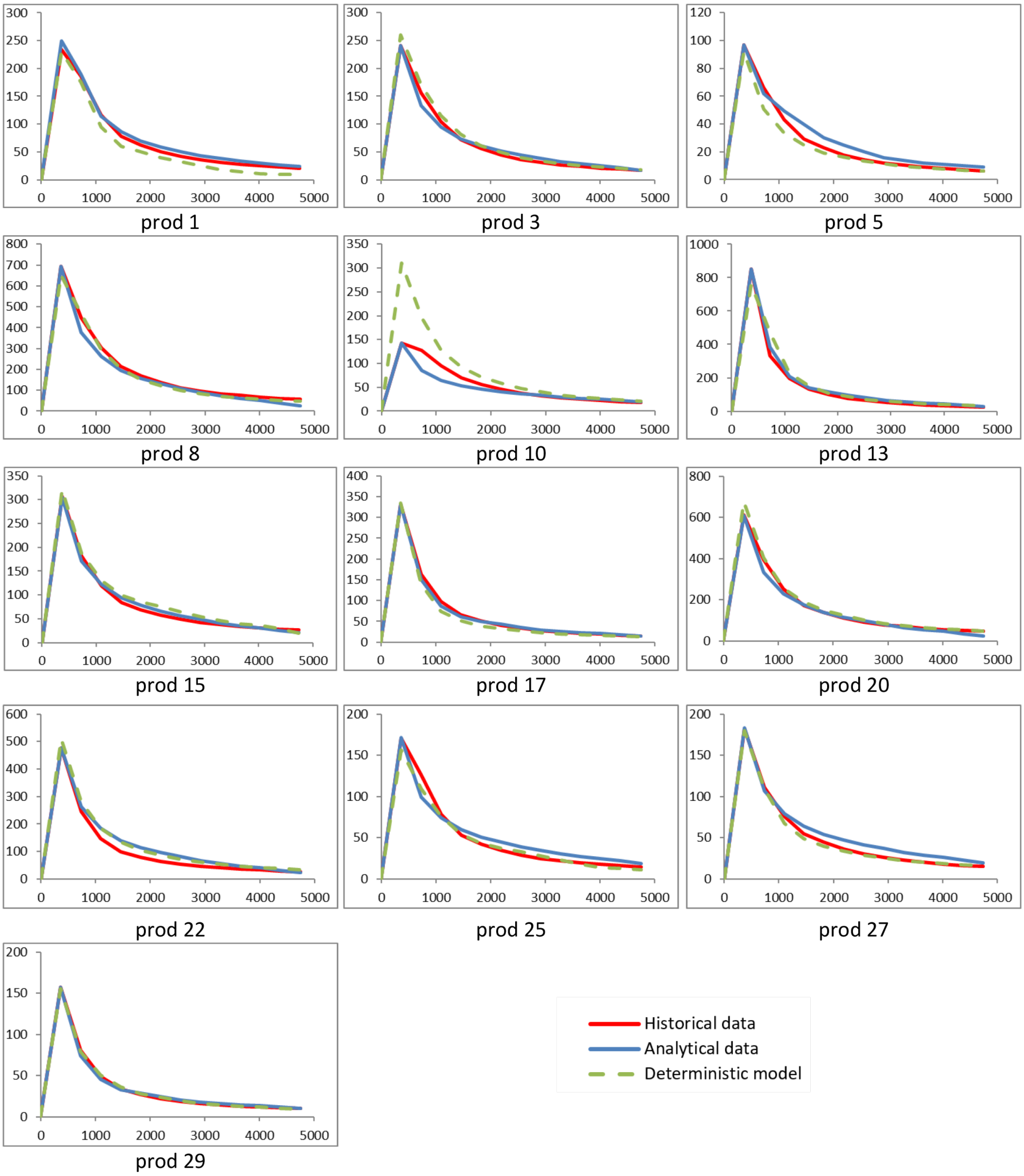


Figure 5. WOPR values from history, analytic and deterministic model for 13 production wells in Case 7.

3.2 History match

In this study, the history match was performed by applying a semi-analytical technique. In this way, the analytical results of the CGM procedure form the spreadsheet for each production well were plotted with the historical data to observe the mismatch between production data (Fig. 5). In order to history match the production data, the uncertain parameters must be changed in the spreadsheet. In this method, only modifying some parameters would be effective including porosity, permeability, and thicknesses of the layers because the cumulative oil production and oil production rate depend on these parameters. The porosity and permeability of each layer were accurately known, but the only inaccurate parameter is the thickness of layers because in this kind of reservoir there are many discontinuities of the facies between different production wells. Hence, this study focused on the thicknesses of the three main layers around each producer. Therefore, the thicknesses of these layers around each single production well in the spreadsheet were modified to get the real values. These accurate values of layer heights added to the deterministic model of simulation. Finally, by comparing analytical, simulation and historical production data, the effectiveness of this matching process was evaluated graphically.

3.3 Accuracy of the semi-analytical method

In order to evaluate the accuracy of the semi-analytical method of history matching, the Root Mean Square Error (RMSE) and Normalized Root Mean Square (NRMSE) approaches were applied. These methods were used to estimate the difference between the predicted values ($X_{predict,i}$) and actual historical values of production data ($X_{actual,i}$). In this way, the difference, RMSE, and NRMSE, between the CGM analytical results and historical data was determined using Eq. 7 and 8, respectively.

$$\mathbf{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}(\mathbf{X_{predict,i}}-\mathbf{X_{actual,i}})^2}{\mathbf{n}}} \quad (7)$$

$$\mathbf{NRMSE} = \frac{\mathbf{RMSE}}{\mathbf{X_{predict,max}}-\mathbf{X_{predict,min}}} \quad (8)$$

3.4 Updating geological model

When the best match of the production data was achieved between the deterministic model (from Eclipse simulation software) and the historical data by applying the semi-analytical history match, the geological model could be updated. This was done by comparing the best deterministic model with the different stochastic realization models (geological models), which were created by Petrel modeling software. When the perfect match of the filed production data between the best deterministic model and one of the realization models was achieved, the matched stochastic realization model (geological model) needs to be updated. This could be done by changing the local layer proportions around each production well based on the production data. In this work, main focusing was on well oil production rate (WOPR)well water cut (WWCT) of each producer, field oil production rate (FOPR), total field oil production rate (FOPT), total field water cut (FWCT) and total field production rate (FPR) of the total field.

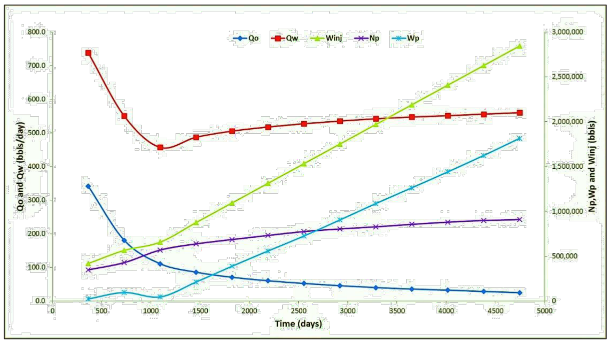


Figure 6. The CGM results of the fluid production and injection from the three layers in production well 3.

4. Results and Discussion

The calculations of oil recovery performed in a spreadsheet for producer 3, which is surrounded by injection wells 2, 4 and 7 (Fig. 2). The production and injection results, from CGM spreadsheet, for the three USF, MSF and LSF layers from the producer 3 are shown in Fig. 6. In this step, it was assumed that each layer has its own uniform thickness i.e. USF=16.4 ft, MSF=32.8 ft, and LSF=16.4 ft. But, whenever any of these values is being changed, the results will also vary and the most effective layer in the system is the UFS because it has better reservoir rock properties. The accuracy of the spreadsheet procedure is verified by comparing the results of producer 3 with the historical data. as shown in Fig. 7, it is clear that the results are getting closer to the historical data after changing the height of the layers from the uniform to the random values i.e. USF=20.8 ft, MSF=15.24 ft, and LSF=22.96 ft.

4.1 History Match

The simulation was first done for the simple model with the uniform thicknesses of each layer by using Eclipse 100. Then, it was applied for the modified deterministic model after changing the proportion of the facies to those values gained from the spreadsheet. WOPR from the simple model, deterministic model, analytical and historical data were plotted together as shown in Fig. 8. It is obvious that the results of the simple model do not match with the historical data, but a good match between the analytical, the modified deterministic model and historical data were achieved because of having fewer uncertainty thicknesses in both analytical and modified deterministic models. After confirming the results of the well production 3, the same procedure was applied for the remaining 12 wells. For each production well, the particular thickness of each of the three layers was identified by comparing and matching the values of WOPR with historical data in the spreadsheet. Then, the height of each layer around each single production well was changed in the deterministic model based on their effective drainage area. The results of WOPR from the analytical spreadsheet and modified deterministic model with the historical data were plotted together versus time in days as shown in Fig. 5. A good match can be seen between the production data for almost all the production wells in this case (case 7).

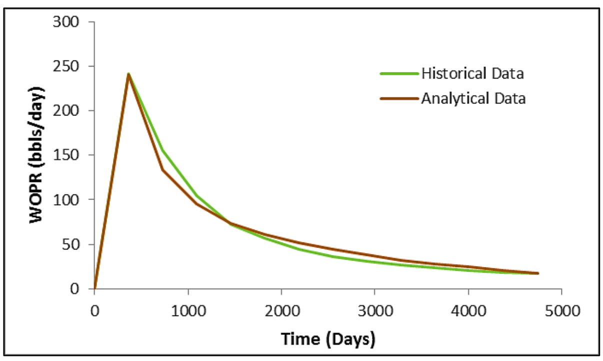


Figure 7. Analytical and historical data of WOPR from producer 3.

In order to obtain closer distributions of the three layers to the real values, the same procedure was applied to 6 more sets of historical data (6 cases). The different values of thicknesses around each production well in various cases of the historical data were identified, analytically. This is because of having different distributions of layers around production wells in each case f history data and this confirms the accuracy of this analytical method of history match. The production data of the all producers were also different in the 7 cases. WOPR of producer 3 from various cases was taken as an example to see these differences as shown in Fig. 9. The value of WOPR was very low in case 2, which was about 100 bbl/day, but it was the highest in both cases 3 and 4 which was about 700 bbl/day. While in case 1, producer 3 was able to produce 480 bbl of oil per day.

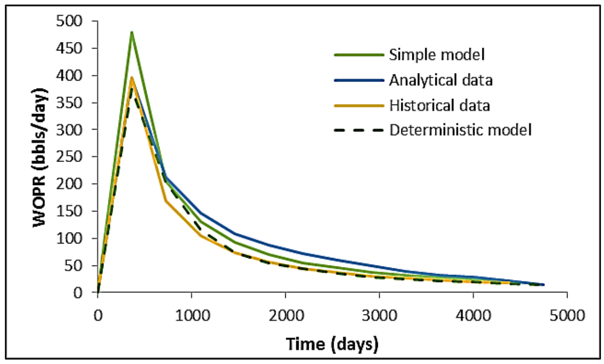


Figure 8. WOPR of producer 3 from analytical data, historical data, simple and deterministic models.

4.2 Accuracy of the semi-analytical method

In order to check the accuracy of this analytical method of history matching, the RMSE and NRMSE approaches were used to identify the ratio of error of the FOPR, FOPT, FWCT and WOPR data. By looking into the results of these methods in all the 7 cases, it is possible to state that the attribution error of the analytical approach varied from 1% to 10.2%. Amongst all the seven cases, case 7 has the lowest ratio of errors for all production variables including FOPT, FWCT, and average WOPR and FOPR as shown in Fig. 10. In this history match case, the value of errors between actual (historical data) and predicted simulation data are varied from 1% to 8.6%. FOPT gives the lowest error ratio which is about 1%, but in term of the daily production rate, there is the highest ratio of error which is around 8.6%. This mismatch could be due to the difference of the production rate at the late stage of prediction. The concern to the water production, a good match between the historical and simulated FWCT was achieved with the error ratio about 2%. This could be due to the assumptions of the analytical method such as piston-like displacement and 100% of vertical sweep efficiency.

From the comparison between calculated values of error measurements such as RMSE and NRMSE for all cases, case 7 gives the smallest values of error. Hence, the best deterministic model related to the case 7 was selected as the most accurate model from semi-analytical history match. From the best deterministic model, the most realistic heights of layers (Table 3) around each production well were identified which would be used for upgrading the geological model. The heights of layers around almost all of the production wells are varied and now close to the real heights in the reservoir with no or less uncertainty. For instance, wells 1, 3 and 5 are on the same line, but they have different heights of layers i.e. the height of USF is about 6 m in producer 1 and reduced to 3 m and 1 m in producer 3 and 5, respectively.

Table 3. The heights of USF, MSF and LSF around 13 production wells in best (final) deterministic model in case 7

Layer	Thickness (meter)												
	prod 1	**prod 3**	**prod 5**	**prod 8**	**prod 10**	**prod 13**	**prod 15**	**prod 17**	**prod 20**	**prod 22**	**prod 25**	**prod 27**	**prod 29**
USF	6	3	1	5	1	10	2	4	5	4	4	2	3
MSF	6	7	8	8	2	4	7	4	9	2	7	7	5
LSF	3	4	6	3	10	2	6	7	2	7	5	5	6

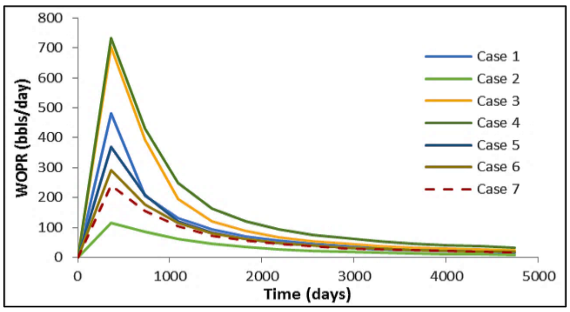


Figure 9. WOPR of producer 3 from Eclipse for different cases

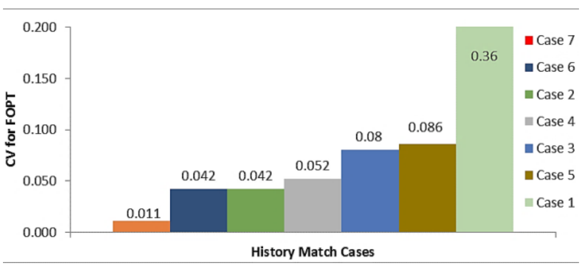


Figure 10. The ratio of mismatch between simulated and historical FOPT for various history match cases (1, 2, 3, 4, 5, 6, and 7).

4.3 Updating the Geological Model

From semi-analytical history match, the best deterministic model with the best-matched data was identified to be case 7 and it is named as the Final Deterministic Model. The shoreface deposit was modeled by petrel with 100 stochastic realizations. The field production data and pressure of the final deterministic model were compared with the results of the all realization models, separately. Amongst all realizations, only the perfect match of the FOPR, FOPT, FPR, FWCT, and FWPT was achieved between the final deterministic model and realization model 51 as shown in Fig. 11.

Even though a good match between the final deterministic model and realization model 51 was achieved in terms of the field production variables, the local facies distribution around each producer might be different. Thus, the local proportions of the three layers around each producer in the realization model 51 decided to be altered and updated to those values identified in the final deterministic model by history matching. For this purpose, the values of WOPR for each well from both models were compared. The results from some wells were matched such as producers 1, 8 and 20. The facies proportions around these wells don't need any modifications. However, the results of the remaining producers in the realization model 51 weren't matched properly with the final deterministic model and can be divided into two following groups; wells with higher production rate than the final deterministic model and wells with lower production rate than the final deterministic model. Those wells which have the larger layer thicknesses in the realization model 51 were giving a higher production rate as well, include producer 3, 5, 10, 15, 25 and 27. The wells having a lower production rate are surrounded by the smaller layer heights such as producer 13, 17, 22 and 29. Based on these differences in thicknesses of layers and oil production rate, the local layer distributions around each production well in the realization model 51 should be modified to those values gained by the history matching.

When the realization model 51 was updated, analytically, by modifying the heights of layers around all the production wells, the values of WOPR and WWCT of the updated model were compared with the historical data as shown in Fig. 12. It can be seen that the values of WOPR from producer 15, as an example for all the other producers, is well-matched between the updated model and the historical data.

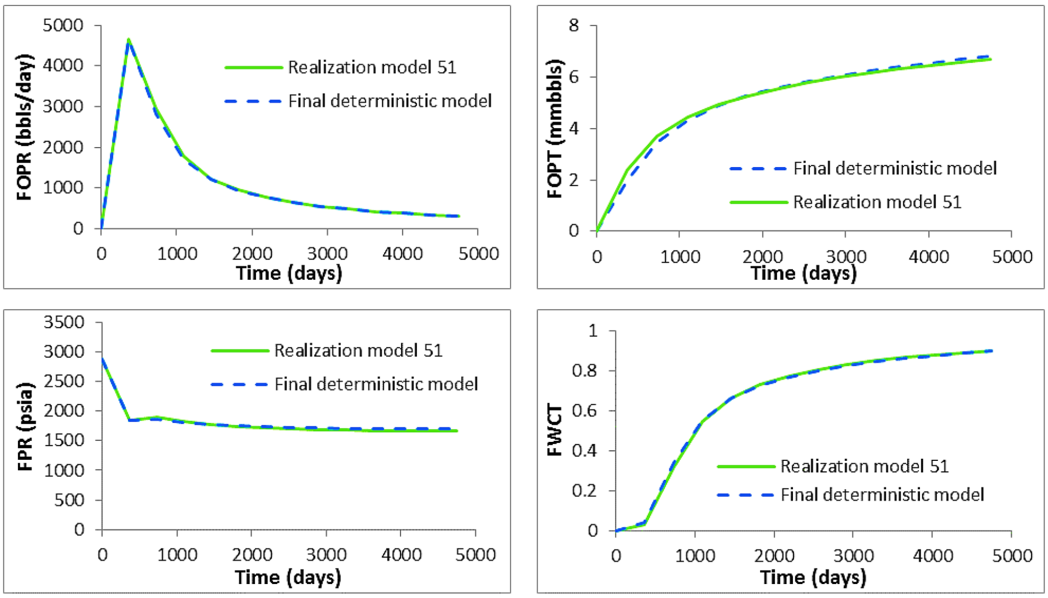


Figure 11. Field production variables and field pressure data from the Final Deterministic Model and Stochastic Realization Model 51.

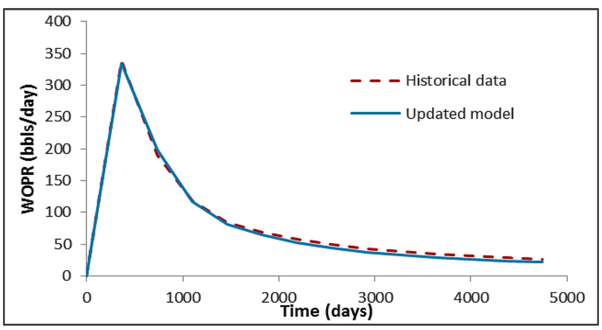
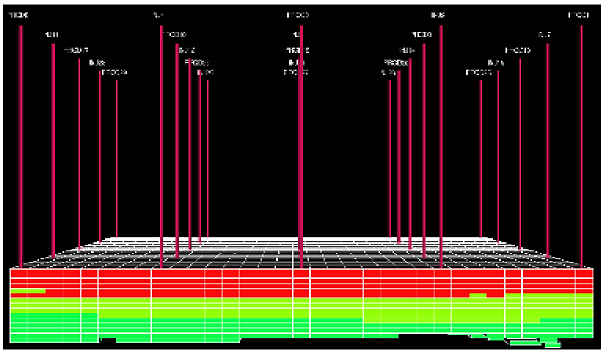


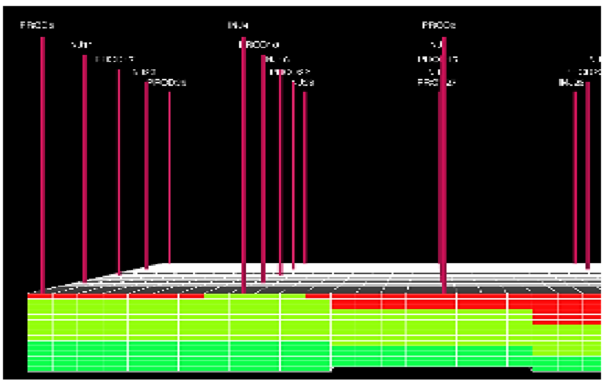
Figure 12. WOPR of producer 15 from the updated model and historical data.

The distributions of the three layers around production well 1, 3 and 5 in the realization model 51 and updated model are shown in Fig. 13. The proportion of the three layers are different in both models. The thickness of USF around producer 5 in the realization model 51 is about 5 m but it was reduced to about 1 m in the updated model. However, the thickness of the MSF was increased from 4 m to 8 m around the same well; LSF has the same height in which is about 6 m in both models. The thicknesses of the USF, MSF, and LSF around well 3 were also changed from 6 m, 4 m and 4 m to 3 m, 7 m and 6 m in the updated model, respectively. While, in well 1 the proportions of the three layers were slightly changed in the updated model.

In order to observe the variations between the simple deterministic model, updated model, and historical data after completing the semi-analytical history match, the FOPT results from the three models were compared together as shown in the Fig. 14. Before doing a history match, the production data of the simple deterministic model were varied with the historical data with high uncertainty which was about 21.52%. But this uncertainty was reduced about 19.8 % (from 21.52% to 1.73%) in the updated geological model and a good match with historical data was obtained. This indicates that the method applied in this work for history match was accurate and can be used for updating the geological model.



Realization model 51 (before history match and upgrading)



Updated geological model (after history match and upgrading)

Figure 13. Facies distributions around producers 1, 3 and 5 of the updated model and realization 51 model.

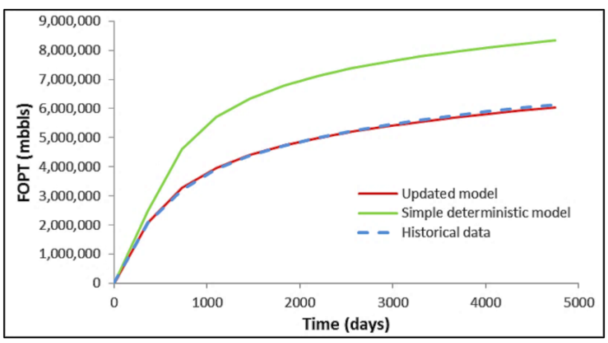


Figure 14. Variation of the FOPT between deterministic, updated model and historical data.

5. Conclusion

The main objective of this paper was to describe a semi-analytical method to history match the production data of the layered reservoir. In this study, CGM method was used for predicting the waterflooding performance for the three layers of the shoreface deposit include USF, MSF, and LSF. The production data of the reservoir, with the uniform heights of layers around production wells, weren't matched with historical production data because of having uncertainty in some parameters, such as the heights of layers. In order to get a good match between production data and the model, it was needed to identify the real heights of these three layers around all production wells. Early results showed that this analytical method of history match worked quite well and could be applied in the field scale. The followings are the main points drawn from this study:

- From semi-analytical history matching, the uncertainty in layer thicknesses around all production wells was reduced.
- The real thicknesses of the three layers around all production wells were identified, analytically.
- The geological model was improved by changing the local layer distributions regardless of their real thicknesses.
- A good match of the production data between the updated model and historical data was obtained.

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