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# A review of size-dependent elasticity for nanostructures

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ABSTRACT

Nanotechnology is one of the pillars of human life in the future. This technology is growing fast and many scientists work in this field. The behavior of materials in nano size varies with that in macro dimension. Therefore scientists have presented various theories for examining the behavior of materials in nano-scale. Accordingly, mechanical behavior of nano-plates, nanotubes nano-beams and nano-rodes are being investigated by Non-classical elasticity theories. This review includes the last researches on bending, buckling, and vibration of nano-plates, nano-beams, nanorods, and nanotubes which were investigated by non-local elasticity theory and nonlocal strain gradient theory. Great scholars have written valuable reviews in the field of nanomechanics. Therefore, given a large number of researches and the prevention of repetition, the articles in the past year are reviewed.

## 1. Introduction

The future of human's life will be affected by various factors. One of the most important factors is nanotechnology. The unwanted use of nanotechnology is back to several hundred years ago. Nanotechnology is the ability to produce new materials, tools and systems by controlling molecular and atomic scale, and using material properties in nano-dimension. Nanotechnology is used in many fields, including industry, medicine, agriculture, resource sustainability, aerospace, national security, electronics, and so on. It should be noted that the properties of materials, including electrical conductivity, thermal properties, mechanical properties, and other known physical and chemical properties change in the nanoscale. One of the problems in the nano scale is the change in the properties of matter. In fact, behaviors that are seen on a nanoscale scale are not predictable based on the behavior observed in macros. On the other side electrical chips and transistors will play an important role in human life. Researchers have been able to reduce the size of electrical chips by using nanotechnology. Nanotechnology is able to manufacture electrical devices in nano-scale that can detect and monitor biotic signs of the body. These sensors can be planted inside the body to report the biotic information. Nanotechnology is a new approach in all fields. Researchers at various disciplines are trying to build nano-structures that help them achieve their goals. Meanwhile, mechanical scientists play an important role in advancing nanotechnology. Experimental observations is one of the methods for modeling and studying nanoscale structures. Because of the costly nature of this method, other methods such as atomic modeling, hybrid atomic continuum mechanics are used. Modeling based on the continuum mechanics is less expensive than other methods. In the early 1900s, the Cosserat brothers proposed a theory to examine the behavior of nanomaterials. Their theory is the beginning of non-classical theories for studying the mechanical behavior of nanostructures. Some other non-classical theories for studying the behavior of nanostructures are couple stress and consistent couple stress theories [1-4], surface stress theory [5], non-local theory [6-10] and strain gradient theory [11-17]. In fact, the effect of small-scale in these theories appears as constants.

Eringen [18], by conducting reviews in the Cosserat theory, he changed its name to the micropolar [19] theory that the two names are completely equivalent to each other. There are six elastic constants for homogeneous materials, in this theory. The micropolar theory of elasticity incorporates a local rotation of points as well as the translation assumed in classical elasticity and a couple stress (a torque per unit area) as well as the force stress (force per unit area). Couple stress theory is also a special case of Cosserat elasticity theory. In this theory, the number of degrees of freedom of rotation is considered equal to the medium rotation or the main directions of rotation of strain tensor.

Nonlocal elasticity theory was introduced by Eringen [20] in the 1970s. The non-local elasticity theories are modified forms

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of classical elastic theories, in which the effect of small scale as a coefficient, expresses the relation between non-local stresses and classical stresses [21]. In this theory it is assumed that the stress at a point depends on the strain at all points, whereas in classical theories, stress at a point depends only on the strain at that point. This theory is consistent (coincident) with predictions from molecular dynamics and molecular scattering (dispersion) observations. For homogeneous and isotropic materials, linear non-local elasticity theory leads to a set of integro-partial differential equations for displacement field, which is generally difficult to solve. For special conditions, these equations are reduced to a set of partial differential equations.

The higher-order strain gradient elasticity theories, which included length-scale parameters, can show that material behavior depends on the size of the material on the micrometer scale. Mindlin [22] introduced a different elasticity model, taking into account differences in the terms related to kinetic energy and the strain energy density of nano-scale and micro-scale. In his model, in addition to the displacements and strains in the macro-scale, the additional phrases such as micro-sized deformations, as well as relative deformations, which are the differences in macro- and micro-scale deformations, and most importantly the gradient of strains have been considered. In this theory, for isentropic and homogeneous materials, there will be 18 independent parameters in elasticity relations, which would complicate the difficulty of solving elastic equations. This makes it difficult to solve the elasticity equations. After Mindlin, many researchers have tried to simplify these equations and provide models that deal with fewer parameters.

Surface effect was introduced by Gurtin and Murdoch [23]. Phenomena of surface effects appear due to the static equilibrium of atoms on the surface. These effects are present in all materials and are not related to their scale and size, but these effects are important when the surface-to-volume ratio increases dramatically, mainly due to the reduction in the size of the piece. In other words, by reducing the size of the structure to the nano-scale, the surface-to-volume ratio increases so that the surface effects cannot be ignored. In this theory it is assumed the surface of the body, like a thin layer, is ideally attached to the material, and the elasticity properties of the surface are different with the elasticity properties of the material. This theory formulated the equilibrium and structural equations of material using classical theory and the surface effects were introduced as surface tensions in the material boundary conditions.

In the last decade, there has been done a lot of valuable research on nanotechnology. In this review, we examine the researches done in a recent year in the field of nanomechanics. nonlocal and nonlocal strain gradient theories that are used to study the bending, buckling and vibration behavior of nanotubes, nano-plates and nano-beams are considered.

## 2. Nonlocal elasticity theory

### 2.1. Nanobeams and Nanorods

Apuzzo et al. [24] presented a nonlocal model for nanobeams based on a special form of the free energy depending on a participation factor which led to a nonlocal elastic structural problem governed by a sixth-order differential equation equipped with suitable kinematic and static boundary conditions and implemented a nonlocal finite element procedure. They showed the effectiveness of the approach by some examples. Bağdatli and Togun [25] employed the nonlocal Euler–Bernoulli beam theory for the vibration and stability analysis of a nanobeam conveying fluid. They considered small-scale and damping effects and assumed the beam to be traveling with a constant mean velocity along with a small harmonic fluctuation. Barati [26] investigated nonlocal and surface effects on nonlinear vibration characteristics of a flexoelectric nanobeams under magnetic field using Eringen's nonlocal elasticity and surface elasticity theories to describe size-dependency of the nanobeam. They employed Galerkin method to satisfy boundary conditions along with analytical procedures to obtain the closed-form nonlinear frequency of flexoelectric nanobeam. They showed that magnetic field intensity, flexoelectric parameter, nonlocal parameter, elastic foundation and applied voltage on the top surface of the nanobeam have great influences on nonlinear vibration frequency. Behera [27] employed Euler–Bernoulli and Timoshenko beam theories in conjunction with nonlocal elasticity theory of Eringen boundary for static analysis of nanobeams. Challamel et al. [28] studied static and dynamic behavior of an axial lattice with direct neighboring interaction loaded by some distributed forces and in interaction with an elastic medium. They constructed a nonlocal rod model by continualization scheme of the lattice difference equations and derived some exact analytical solutions for the finite lattice system under some boundary conditions. Demir and Civalek [29] by emphasizing the Eringen's nonlocal elasticity paradox for the cantilever boundary condition, developed an enhanced Eringen differential model by adding additional parameters to Eringen's nonlocal elasticity theory as an alternative solution method. They investigated bending of nano/micro beams under the concentrated and distributed loads by using Euler Bernoulli beam theory via the enhanced Eringen differential model. They also used Singularity function method and various types of boundary conditions to calculate the deflection of concentrated and distributed loaded beam. Demir et al. [30] developed the static analysis of nano-beams under the Winkler foundation and the uniform load by considering small scale effect along with Eringen's nonlocal elasticity theory using a FE approach. They used Galerkin weighted residual method to obtain the finite element equations and investigated the validity and novelty of the results for bending results. Ebrahimi and Barati [31] investigated the vibration behavior of size-dependent nano-crystalline nano-beams based on nonlocal, couple stress and Eringen's elasticity theories. They used a modified couple stress theory to capture rigid rotations of grains and incorporated Residual surface stresses into nonlocal elasticity and applied a differential transform method (DTM) satisfying various boundary conditions. Ebrahimi et al. [32] investigated the wave dispersion behavior of a rotating functionally graded material (FGMs) nanobeam by applying nonlocal elasticity theory of Eringen and assuming material properties of rotating FG nanobeam according to a power-law model. Ebrahimi and Daman [33] used nonlocal elasticity theory to investigate the free vibration of curved functionally graded piezoelectric (FGP) nanosize beam in thermal environment. They employed Analytic Navier solution to solve the governing equations obtained via the energy method for simply supported boundary conditions. The solution estimated the natural frequency for curved FGP nanobeam under the effect of a uniform temperature change and external electric voltage. Ebrahimi and Daman [34] proposed an analytical solution method for free vibration of curved functionally graded (FG) nonlocal beam supposed to different thermal loadings, by considering porosity distribution via nonlocal elasticity theory. Ebrahimi and Shaghaghi [35] proposed a non-classical beam model based on the Eringen's nonlocal elasticity theory for nonlinear vibration analysis of magneto-electro–hygro–thermal piezoelectric functionally graded (PFG) nanobeams with power-law through the thickness distribution model rested in elastic foundation. They employed Hamilton's principle to derive the equations and related boundary conditions within the framework of Euler–Bernoulli beam model with von-Karman type nonlinearity. They used Galerkin-based method to discretize the nonlinear partial differential motion equations. Ebrahimi-Nejad and Boreiry [36] performed a parametric study of bending, buckling and vibrational behavior of size-dependent piezoelectric nanobeams under thermo-magneto-mechano-electrical environment in the presence of surface effects. They employed Gurtin-Murdoch surface and Eringen's nonlocal elasticity theories in the framework of Euler–Bernoulli beam theory to obtain a new non-classical size-dependent beam model for dynamic and static analyses of piezoelectric nanobeams. They also presented numerical examples to demonstrate the effects of length, surface effects, nonlocal parameter and environmental changes (temperature, magnetic field and external voltage) on deflection, critical buckling load and natural frequency for different boundary conditions. Eltaher et al. [37] studied the effects of nonlocal elasticity and surface properties on static and vibration characteristics of piezoelectric nanobeams using thin beam theory and Gurtin–Murdoch model for mechanical and piezoelectric surface nanoscale properties. They applied nonlocal elasticity theory considering length scale effect to describe the long–range atoms interactions. Fernández-Sáez and Zaera [38] studied the problem of the in-plane free vibrations (axial and bending) of a Bernoulli–Euler nanobeam using the mixed local/nonlocal Eringen elasticity theory to analytically obtain the natural frequencies of vibration by solving two uncoupled integro-differential eigenvalue problems. They considered different kinds of end supports and analyzed the influence of both mixture parameter and length scale.

Ghaffari [39] offered a complete solution to analyze the mechanical behavior of Nano-beam under non-uniform loading considering the effects of size (nonlocal parameters), non-homogeneity constants, and different boundary conditions. They used non-local elasticity theory and kinematics of the Euler–Bernoulli beam theory for displacement field. Ghannadpour [40] investigated bending, buckling and vibration behaviors of nonlocal Timoshenko beams using a variational approach and outlining the weak form of equations. Hache et al. [41] focused on the possible justification of nonlocal beam models (at the macroscopic scale) from an asymptotic derivation based on nonlocal two-dimensional elasticity (at the material scale). They derived governing partial differential equations using Taylor series expansion, through the dimensionless depth ratio of the beam. Hosseini and Rahmani [42] analyzed the bending and vibration behavior of a curved FG nanobeam with material properties varying through the radius and using the nonlocal Timoshenko beam theory. Buckling analysis of a nano sized beam was performed by Kadıoğlu and Yaylı [43] using Timoshenko beam theory and Eringen's nonlocal elasticity theory. To obtain Fourier coefficients they assumed vertical displacement function as a Fourier sine series and rotation function as a Fourier cosine series enforced by Stokes' transformation and obtained higher order derivatives of them. Then they derived critical buckling loads by calculating determinant of resulted coefficient matrix. Kaghazian [44] performed vibration analysis of a piezoelectric nanobeam by modelling it based on Euler-Bernoulli beam theory and using the nonlocal elasticity theory to serve in several nano electromechanical systems. Kammoun [45] reported an investigation on thermo-electro-mechanical vibration of grapheme piezoelectric sandwich nanobeams and derived the governing equations based on the nonlocal elasticity theory, Timoshenko beam theory and Hamilton's principles and then solved them using generalized differential quadrature (GDQ) method. Khaniki [46] by emphasizing the inability of differential form of nonlocal elastic theory in modelling cantilever beams and inaccurate results for some type of boundaries, presented a reliable investigation on transverse vibrational behavior of rotating cantilever size-dependent beams. He used Eringen's two-phase local/nonlocal model to derive governing higher order. In order to indicate the influence of different material and scale parameters, he presented a comprehensive parametric study and discussed the results. Loghmani et al. [47] investigated the effect of discontinuities such as cracks, steps on the length, and the mass of attached buckyball on the tip of nanoresonators on natural frequencies in longitudinal vibration analysis. By using the Eringen nonlocal elasticity theory from wave viewpoint, they derived propagation, transmission and reflection functions for nanorods and obtained explicit expressions for natural frequencies. Marinca and Herisanu [48] focused on obtaining explicit analytical approximate solutions via a new procedure, namely the Optimal Auxiliary Functions Method (OAFM) for the post buckling behavior of an initially imperfect nonlocal elastic column by adopting Euler-Bernoulli hypothesis and Eringen's nonlocal elasticity. Martowicz [49] provided an overview of nonlocal formulations for models of elastic solids. He presented the physical foundations for nonlocal theories of continuum mechanics, followed by various analytical and numerical techniques. Massoud and Tahani [50] investigated the nonlinear vibration of an Euler–Bernoulli nanobeam resting on a non-linear viscoelastic foundation subjected to a harmonic electrostatic field excitation. The non-linear viscoelastic foundation is considered for both hardening and softening cases. By neglecting of the in-plane inertia and using Eringen's nonlocal elasticity theory along with Galerkin method, they derived the equation of motion. Merzouki et al. [51] developed a finite element approach for the static analysis of curved nanobeams using nonlocal elasticity beam theory based on Eringen formulation coupled with a higher-order shear deformation accounting for through-thickness stretching. Mohyeddin and Jafarizadeh [52] developed a nonlocal elastic beam model by incorporating Eringen's nonlocal constitutive equation into the large deflection beam theory for a nano-cantilever Euler–Bernoulli beam and solved the equilibrium equations in an iterative manner using the shooting method. Zamani Nejad and Hadi [53] investigated the free vibration analysis of Euler–Bernoulli nano-beams made of bi-directional functionally graded material with small scale effects. They used the non-local elasticity theory coupled with Euler–Bernoulli nano-beams to study the small scale effects on free vibration. Zamani Nejad and Hadi [9] formulated the problem of the static bending of Euler–Bernoulli nano-beams made of bi-directional functionally graded material with small scale effects. Their model was based on the Eringen's nonlocal elasticity theory applied to Euler–Bernouilli nano-beams. Zamani Nejad et al. [54] carried out buckling analysis of the nano-beams made of two-directional functionally graded materials (FGM) with small scale effects based on the nonlocal elasticity theory.

Rahimi and Rashahmadi [55] considered a novel micro-electromechanical system (MEMS) and nano-electromechanical system (NEMS) with a controllable thermo-elastic damping of axial vibration based on Eringen nonlocal theory. They presented effects of different parameters like the gradient index, nonlocal parameter, length of nanobeam and ambient temperature on the thermo-elastic damping quality factor and showed that the thermo-elastic damping can be controlled by changing different parameters. Sidhardh and Ray [56] developed a finite element model for the static analysis of smart nanobeams integrated with a flexoelectric layer on its top surface, using nonlocal elastic theory. The flexoelectric layer acted as a distributed actuator of the nanobeam. They used a layerwise displacement theory to derive the element stiffness matrices from variational principles incorporating nonlocal effects. Tufekci and Aya [57] investigated the static and dynamic behavior of a curved planar nanobeam having variable curvature and cross-section. They derived nonlocal constitutive equations by using Eringen nonlocal theory in cylindrical coordinates and then implemented into the classical beam equations. Vosoughi et al. [58] investigated the thermal buckling and post-buckling behaviors of moderately thick nanobeams subject to uniform temperature rise via employing the differential quadrature method (DQM). They derived governing equations of the nanobeams considering the von-Kármán's assumptions and used the Eringen's nonlocal elasticity theory in conjunction with the first-order shear deformation beam theory. Wang et al. [59] carried out the calibration of Eringen's small length scale coefficient $e_0$ for elastically restrained beams (in the context of buckling and axially loaded vibration) based on connection between a discrete beam model and the Eringen's nonlocal beam model. Xue et al. [60] extended classical thermoelasticity to simulate the thermoelastic responses of multilayered structures in two aspects: in mechanical sense, they used Eringen's nonlocal elasticity to depict the size-dependence; meanwhile, considered fractional order strain to describe the mechanical phenomena caused by viscoelasticity. Yaylı [61] presented longitudinal vibration analysis of FG restrained nanorods via non-local elasticity theory by assuming two axial springs attached to a FG nanorod at both ends. They then derived a coefficient matrix by considering the non-local differential relations for the FG nanorod and analyzed it via an exact eigenvalue method and finally used the results calculated from finite-element method to validate their method and discussed the influence of FG index, non-local parameter and boundary conditions on the axial frequencies of FG nanorods. Zhang et al. [62] investigated vibration characteristics of a piezoelectric nanobeam embedded in a viscoelastic medium based on nonlocal Euler–Bernoulli beam theory. They first derived the governing equations of motion and boundary conditions for vibration analysis using Hamilton's principle, where nonlocal effect, piezoelectric effect, flexoelectric effect, and viscoelastic medium were considered simultaneously. Zhang [63] by identifying three critical frequencies independent of boundary conditions together with a critical length, which determine the vibration behaviors of a nonlocal Timoshenko beam stated that unlike a local Timoshenko beam which has two frequency spectra, a nonlocal Timoshenko beam may have two frequency spectra or one frequency spectrum depending on the nonlocal effect. Zhao et al. [64] modeled a nonlocal elastic micro/nanobeam theoretically with the consideration of the surface elasticity, the residual surface stress, and the rotatory inertia in presence of nonlocal and surface effects. Zhou [65] presented a new Hamiltonian-based approach to find exact solutions for transverse vibrations of double-nanobeam-systems embedded in an elastic medium. They used the nonlocal Euler-Bernoulli and Timoshenko beam theories to model the beam within the frameworks of the symplectic methodology. After expressing the governing equations in a Hamiltonian form, they obtained exact frequency equations, vibration modes and displacement amplitudes by using symplectic eigenfunctions and end conditions.

### 2.2. Nanosheets and nanoplates

Abdollahi and Ghassemi [66], investigated surface and nonlocal effects in the analysis of buckling and vibration in rectangular single-layered graphene sheets embedded in elastic media and subjected to coupled in-plane loadings and thermal conditions. They considered the small-scale and surface effects using the Eringen's nonlocal elasticity and Gurtin-Murdoch's theory, respectively. They also used the differential quadrature method (DQM) for the solution of the relevant problems and validated the results against Navier's solutions. Ghorbanpour and Zamani [67] investigated the free vibration analysis of sandwich nanoplate with functionally graded porous core and piezoelectric face sheets. Arefi et al. [68] studied magnetic and electric buckling loads of three-layered elastic nanoplate with exponentially graded core and piezomagnetic face-sheets with material properties of the nano-graphene core obeying the exponential function along the thickness direction. They derived governing equations based on first-order shear deformation theory using variational method and investigated the influence of nanoscale by employing nonlocal piezo-magneto-elasticity theory. Arefi and Zenkour [69] presented thermo-electro-magneto-mechanical bending analysis of a sandwich nanoplate based on Kirchhoff's plate theory and nonlocal theory. The sandwich nanoplate included an elastic nano-core and two piezomagnetic face-sheets actuated by applied electric and magnetic potentials. Bachher and Sarkar [70] established a new *nonlocal theory* of generalized thermoelastic materials with voids based on *Eringen's nonlocal elasticity* and *Caputo fractional derivative* to study transient wave propagation in an infinite thermoelastic material. The material contained voids was subjected to a time-dependent continuous heat sources distributed in a plane area. Barati [71] presented new solutions to examine large amplitude vibration of a porous nanoplate resting on a nonlinear hardening elastic foundation modeled by nonlinear four-variable plate theory.

Bastami and Behjat [72] investigated buckling and free vibration of piezoelectric nano-plate on elastic foundation by employing nonlocal elasticity and classical plate theory and assuming simply supported boundary conditions and plate subjected to external electric voltage. Daneshmehr et al. [73] investigated the free vibration behaviors of the nanoplate made of functionally graded materials by applying the Eringen's nonlocal theory to study the small scale effects on natural frequencies. Dastjerdi and Akgöz [74] investigated the static and dynamic behaviors of macro and nano inhomogeneous plates made of functionally graded materials based on three-dimensional elasticity theory in combination with the nonlocal theory of Eringen. Despotovic [75] studied the problem of stability and vibration of a square single-layer graphene sheet under body force using Eringen's theory. They used classical plate theory, upgraded with nonlocal elasticity theory to formulate the differential equation of stability and vibration of the nanoplate. Ebrahimi and Barati [76] carried out damping vibration analysis of multi-phase viscoelastic nanocrystalline nanobeams on viscoelastic medium accounting for nano-grains and nano-voids sizes. They applied couple stress and surface energy effects for vibration analysis of nanocrystalline nanobeams. They assumed viscoelastic medium as infinite parallel springs as well as shear and viscous layers and employed Hamilton's principle to derive the governing equations and the related boundary conditions. They then solved the equations by applying differential transform method. Ghasemi et al. [77] studied the nonlocal buckling behavior of biaxially loaded graphene sheet with piezoelectric layers based on an orthotropic intelligent laminated nanoplate model. They used nonlocal elasticity theory in the buckling analysis to show the size scale effects on the critical buckling loads and employed third-order shear and normal deformation theory to obtain the nonlinear equilibrium equations. Ghorbanpour-Arani et al [78] investigated buckling analysis of an embedded nanoplate integrated with magnetoelectroelastic (MEE) layers based on a nonlocal magnetoelectroelasticity theory. They simulated surrounding elastic medium by the Pasternak foundation considering both shear and normal loads. The refined zigzag theory was used to model the Sandwich Nanoplate subject to both external electric and magnetic potentials. Goodarzi et al. [79] studied the free vibration behavior of rectangular FG nanoscale plates within the framework of the refined plate theory (RPT) and taking small-scale effects into account. They used the nonlocal elasticity theory to obtain the governing equations for single-layered FG nanoplate and employed the Navier's method to obtain closed-form solutions for rectangular nanoplates assuming that all edges are simply supported. Jamali and Ghassemi [80] investigated frequency analysis of rectangular piezoelectric nanoplates under in-plane forces via the surface layer and non-local small-scale hypotheses. Nazemnezhad et al. [81] investigated vibration analysis of multi-layer graphene sheets (MLGSs) by using nonlocal elasticity. They considered van der Waals interactions of every two adjacent layers in the analysis which resulted in interlayer shear effect. Their proposed formulation was according to sandwich model (SM) and Molecular Dynamic (MD) simulation was implemented to verify the model.

Norouzzadeh and Ansari [82] presented a size-dependent analysis of the surface stress and nonlocal influences on the free vibration characteristics of rectangular and circular nanoplates made of functionally graded materials with two distinct surface and bulk phases. The nonlocal and surface effects were captured by the Eringen and the Gurtin-Murdoch surface elasticity theories, respectively. Rong et al. [83] proposed an analytical Hamiltonian-based model for the dynamic analysis of rectangular nanoplates using the Kirchhoff plate theory and Eringen's nonlocal theory. By reducing the dynamic problem in a symplectic space to a unified Hamiltonian dual equation formed by a total unknown vector consisting of displacements, rotation angles, bending moments and generalized shear forces, They established exact solutions for free vibration, buckling and steady state forced vibration by the eigenvalue analysis and expansion of eigenfunction without any trial functions. Shahrbabaki [84] investigated three-dimensional free vibration of simply-supported nanoplate and wave propagation in three-dimensional infinite nonlocal solid by using suitable potential functions for Helmholtz displacement vector. They developed novel trigonometric series as approximating functions in a Galerkin based approach to deal with other boundary conditions. Shahsavari and Janghorban [85] studied the size-dependent effects on the time-dependent bending and shearing responses of single-layer graphene sheets (SLGSs) induced by displacement of the concentrated moving load along the SLGSs are. Wu et al. [86] investigated the vibration behavior of double layer graphene sheets (DLGSs) in thermal environments as energy conservation device in building materials. They used nonlocal elastic theory and classical plate theory (CLPT) to derive the governing equations and employed the element-free method to analyze the vibration behaviors of DLGSs embedded in an elastic medium. Zarei [87] studied buckling and free vibration analysis of a circular tapered nanoplate with linear variation of thickness in radial direction and subjected to in-plane forces. He employed nonlocal elasticity theory to capture size-dependent effects and Raleigh-Ritz method along with differential transform method to obtain the frequency equations for different boundary conditions. Zhang et al. [88] derived semi-analytical solutions for vibration analysis of nonlocal piezoelectric Kirchhoff plates resting on viscoelastic foundation with arbitrary boundary conditions. They first obtained the governing equations of motion and boundary conditions based on the nonlocal elasticity theory for piezoelectric materials and Hamilton's principle then developed the Galerkin strip distributed transfer function method to solve the governing equations for the semi-analytical solutions of natural frequencies. Zhang [89] studied the geometrically nonlinear vibration behavior of DLGSs using von Kármán plate model incorporated with nonlocal elasticity theory accounting for the small scale effect. Li et al. [90] introduced thermal nonlocal effect into the thermo-electro-mechanical model based on nonlocal elasticity theory to further shed light on the size-dependent coupling behavior of thermal, electric, and elastic fields. They derived coupled field equations involving size-dependent parameters and obtained the solutions using Laplace transformation methods. Karimi and Shahidi [91] investigated the influence of temperature change on the vibration, buckling, and bending of orthotropic graphene sheets embedded in elastic media including surface energy and small-scale effects is. To this aim, they used the nonlocal constitutive relations of Eringen and surface elasticity theory of Gurtin and Murdoch, respectively. They used Hamilton's principle and two-variable refined plate theory to derive the governing equations for bulk and surface of orthotropic nanoplate and employed finite difference method to solve governing equations. They verified obtained results with Navier's method and validated results reported in the literature.

### 2.3. Nanotubes and Nanoshell

Arefi and Zenkour [92] presented two-dimensional thermoelastic analysis of a functionally graded nanoshell with Material properties assumed to be mixture of ceramic and metal obeying a power law distribution based on nonlocal elasticity theory. They used first-order shear deformation theory (FSDT) for axial and radial deformations simultaneously and principle of virtual work for derivation of the governing equations. Robinson et al. [93] examined the buckling of carbon nanotube (CNT) modeled as nonlocal Euler-Bernoulli beam under self-weight and resting on elastic foundations. Belhadj [94] investigated the vibration behaviour of a nanoscale rotating shaft based single-walled carbon nanotube using Euler-Bernoulli beam model and Eringen's nonlocal theory of elasticity for the dynamic behavior of the nanorotor. They reported effects of different parameters and boundary conditions and discussed the results. Chemi et al. [95] by implementing the nonlocal Timoshenko beam theory, determined the nonlocal critical buckling loads of chiral double-walled carbon nanotubes embedded in an elastic medium using nonlocal theory. They investigated and discussed the effect of different parameters such as elastic medium, the buckling mode number, chirality, and aspect ratio on the nonlocal critical buckling loads of these structures. Farajpour [96] investigated nonlinear buckling of magneto-electro-elastic hybrid nanoshells in thermal environment using a size-dependent continuum model. The nanocomposite cylindrical shell was composed of a carbon nanotube (CNT), a microtubule (MT) and a MEE nanoscale layer coupled by polymer or filament matrix and subjected to thermo-electro-magnetic loads. They applied small nonlocal elasticity theory considering scale effect and Pasternak model to simulate the normal and shear behavior of the coupling elastic medium. He used the principle of virtual work and von Karman's strain-displacement relations to derive equations and obtained non-dimensional postbuckling loads using the Galerkin's approach. Kiani [97] explored properties of traveling transverse waves in vertically aligned jungles of single-walled carbon nanotubes (SWCNTs) in the presence of a longitudinal magnetic field using nonlocal higher-order beam theory. He established nonlocal discrete and continuous models and declared the capabilities of the continuous model in capturing the characteristics of waves by the discrete model. He investigated different effects on physical properties of the structure. Li and Hu [98] presented investigated the free torsional vibration of bi-directional FG tubes composed of two different materials with continuously varying along the radius and length directions. They employed nonlocal elasticity theory to derive the difference equation of torsional motion, which were reduced to the classical governing equation by simply setting a zero nonlocal parameter. They derived closed-form solutions of torsional frequencies and mode shapes and showed that the torsional frequencies can be significantly affected by the through-radius and through-length gradings of the bi-directional FG nanotubes.

Mu'tasim [99] studied the nonlinear free vibration and frequency veering of a single wall carbon nanotube (SWCNT) with imperfection modeled as half sine and clamped at both ends based on nonlocal elasticity theory. He used the Euler-Bernoulli Beam and Hamilton's principle to derive the nonlinear equation of motion. He considered various effects such as nonlocal elasticity, geometric initial rise/imperfection, and the effect of the axial force induced by mid-plane stretching in the derivation model of the CNT. He solved the resulting nonlinear temporal equation using the method of multiple scales (MMS) after discretizing the equations using the assumed mode method by inserting the exact linear eigenmode shape and obtained nonlinear natural frequencies of the first three modes of vibrations, for different values of rise/imperfection amplitude, and for different values of the nonlocal parameter. Rahmani et al. [100] used modified nonlocal elasticity theory to analyze the transverse forced vibration of a single-walled carbon nanotube (SWCNT) under excitation of a moving harmonic load in a parametric study and investigated the influences of different parameters on forced deflection of the nanotube in details. Size-dependent thermal buckling and post-buckling behavior of FGM nanotubes with porosities was carried out by She et al. [101] using a refined beam theory and based on Eringen nonlocal elasticity model incorporating the small scale effect. They considered two types of porosity distribution i.e. even and uneven distribution and assumed material properties to be temperature-dependent and vary in the radial direction. Tiwari and Nagar [102] Presented paper reviews on buckling analysis of nanostructure designed through the theory of nonlocal elasticity and discussed a variety of mathematical techniques to determine buckling load and applicability of nonlocal continuum models over local continuum models. They discussed impact of various parameters like size of nanostructures, nonlocal parameter and length-to-width ratio for various boundary conditions. Wen et al. [103] investigated free vibration analysis of single-walled carbon nanotubes (SWCNTs) using a higher-order theory of nonlocal elastic cylindrical beams by taking the rotary inertia, shear deformation and small scale effect into account simultaneously. Their model was capable of identically satisfying shear-free surface condition and did not need to introduce the shear correction factor. They derived characteristic equations, natural frequencies and vibration mode shapes in closed form for different boundary conditions and compared numerical results with those obtained by the molecular dynamics simulation. Zhang et al. [104] Conducted a comprehensive study of the small-scale effects on the buckling behaviors of carbon honeycombs (CHCs) by employing molecular dynamics (MD) simulations and Eringen's nonlocal elasticity theory. According to MD simulation results the small-scale effects stemming from the long-range van der Waals interaction between carbon atoms could considerably affect the buckling behaviors of CHCs. They also developed a nonlocal continuum mechanics (CM) model by employing Eringen's nonlocal elasticity theory to incorporate the small-scale effects into the theoretical analysis of the buckling of CHCs and compared nonlocal CM model with MD simulations which had good agreement under proper considerations.

## 3. Nonlocal strain gradient theory

### 3.1. Nanobeams and Nanorods

Barati [105] investigated forced vibrations analysis of nanobeams on elastic substrate and subjected to moving loads using nonlocal strain gradient theory (NSGT). He assumed the nanobeam made of functionally graded material (FGM) with even and uneven porosity distributions. After obtaining dynamic deflection of the nanobeam via Galerkin and inverse Laplace transform methods, he examined and discussed the effect of different parameters on forced vibration behavior of nanobeams. Barati [106] investigated wave dispersion in thermally affected and elastically bonded nanobeams with material imperfections or porosities evenly dispersed across the thickness by applying a general nonlocal strain-gradient elasticity model with two nonlocal and one strain-gradient parameters. He modeled nanobeam with uniform thickness and by refined shear deformation beam theory with sinusoidal transverse shear strains. Afterwards he derived the governing equations by Hamilton's rule and analytically solved to obtain wave frequencies and the velocity of wave propagation. Barretta et al. [107] investigated linear dynamics of nanobeams numerically according to a combination of Eringen's nonlocal elasticity and second-gradient strain elasticity theories. They assumed elastic properties of the beam as functionally graded in cross-section according to a power law and examined the effects of the functional grading and of the different elastic potentials on this modulation. Ebrahimi and Barati [108] used nonlocal strain gradient theory to analyze vibration characteristics of axially FG nanobeams resting on variable elastic foundation. The nonclassical nanobeam model of their study captured a length scale parameter to explore the influence of strain gradients and a nonlocal parameter to investigate the long-range interactions between the particles and was capable to be degenerated into the classical models if the material length scale and nonlocal stress field parameter are both taken to be zero. They modeled Elastic foundation consisting two layers: a Winkler layer with variable stiffness and a Pasternak layer with constant stiffness. Ebrahimi and Daman [109] proposed an analytical method to study thermo-mechanical dynamic behavior and characteristics of embedded smart shear deformable curved piezoelectric nanobeams made of porous electro-elastic FG materials using nonlocal strain gradient beam theory. They investigated the effects of pores on the mechanical and physical properties. El-Borgi et al. [110] investigated the torsional vibration of size-dependent viscoelastic nanorods modeled using Kelvin–Voigt damping model embedded in an elastic medium with different boundary conditions. They combined the nonlocal theory with the strain and velocity gradient theory to capture both softening and stiffening size-dependent behavior of the nanorods. They obtained damped eigenvalue solutions both analytically and numerically using a Locally adaptive Differential Quadrature Method (LaDQM). Faghidian [111] by using nonlocal strain gradient theory derived the governing differential and boundary conditions of dynamic equilibrium and differential constitutive equations of the classical and first-order nonlocal stress tensor in the most general form based on the Reissner stationary variational principle. They used nonlinear vibrations of size-dependent Bernoulli-Euler and Timoshenko beams to exhibit the application value of Reissner variational principle and employed the weighted residual Galerkin method, the homotopy analysis method to determine the closed form analytical solutions of the geometrically nonlinear vibration equations. Faghidian [112] employed Reissner mixed variational principle to establish the nonlinear differential and boundary conditions of dynamic equilibrium governing the flexure of beams when the effects of true shear stresses are included. They derived nonlinear size-dependent model of the Reissner nano-beam in the framework of nonlocal strain gradient elasticity theory and then obtained the closed form analytical solutions for the geometrically nonlinear flexural equations and compared to the nonlinear flexural results of the Timoshenko size-dependent beam theory and discussed different issues such as differences between the two. Fakher and Hosseini-Hashemi [113] studied the static bending and free vibration behavior of Euler nanobeams using three different approaches nonlocal strain gradient elasticity including differential, integral satisfying and integral without satisfying higher order boundary conditions. They also adopted two different types of Rayleigh–Ritz method i.e. polynomial and in the other, combination of polynomial and trigonometric as admissible functions. Then they obtained bending deflections and natural frequencies of nanobeams with different boundary. Hadi et al. [114] investigated free vibration of three-directional FG material (TDFGM) using Euler–Bernoulli nano-beam, with small scale effects and the nonlocal strain gradient elasticity theory to survey the small scale effects on natural frequencies. Jafarsadeghi-Pournaki [115] proposed a theoretical model by employing nonlocal strain gradient theory (NLSGT) and Euler-Bernoulli beam model considering nonlinear geometric effect resulting from mid-plane stretching to investigate static pull-in instability of FG electrostatic nano-bridge under the influence of electrostatic and van der Waals (vdW) forces in thermal environment. He introduced a new surface reference for eliminating the coupling between the stretching and bending due to the asymmetrical material variation along the thickness and derived the governing equation utilizing minimum energy principle, linearized by means of the step-by-step linearization method (SSLM) and solved by Galerkin based weighted residual method. Li and Hu [116] used nonlocal strain gradient theory and a size-dependent nonlinear Euler–Bernoulli beam considering the geometric nonlinearity due to the stretching effect of the mid-plane. They analytically obtained the post-buckling deflections and critical buckling forces of simply supported beams. Li and Hu [117] investigated the nonlinear bending and free vibration behaviors of the through-thickness power-law variation of two-constituent FG materials using the nonlocal strain gradient theory and size-dependent nonlinear Euler-Bernoulli and Timoshenko beam models. They considered material length scale and nonlocal parameters to account for the effects of both inter-atomic long-range force and microstructure deformation mechanism. Li et al. [118] studied the longitudinal vibration analysis of small-scaled rods model considering nonlocal parameter and material length scale parameter using nonlocal strain gradient theory. They derived analytical solutions predicting the natural frequencies and mode shapes of the rods with different boundary conditions. Li et al. [119] in another study used a size-dependent Timoshenko beam model, taking into account through-thickness power-law variation of a two-constituent FG material in the framework of the nonlocal strain gradient theory. Their model contained a material length scale parameter introduced to consider the significance of strain gradient stress field and a nonlocal parameter to consider the significance of nonlocal elastic stress field.

1. Li et al. [120] investigated the size-dependent nonlinear free vibration behavior of beam with geometric imperfections in form of even and uneven dispersion patterns and used Hamilton's principle to derive the equations and corresponding boundary conditions based on the Euler–Bernoulli beam model, the von Kármán type nonlinearity and the nonlocal strain gradient theory. They then employed Galerkin's approach to obtain approximate analytical solution free vibration of a hinged-hinged nano/micro beam. They used porous Gold for a comprehensive parametric study and determined material and scaling parameters of Au. Li et al. [121] applied the nonlocal strain gradient theory and the Euler–Bernoulli beam model to build a size-dependent inhomogeneous beam model accounting for the through-length power-law variation of a two-constituent axially FG material. They introduced a material length scale parameter and a nonlocal parameter in the axially FG beam model and solved the bending, buckling and vibration problems of axially FG beams by a generalized differential quadrature method and investigated the influences of different parameters on responses. Lv et al. [122] studied the effect of material defects on nonlinear vibration behavior of embedded FG nanobeams by employing the nonlocal strain gradient theory and considering the size-dependent governing equations accounting for the geometric nonlinearity and elastic medium. To quantify the material defects, they introduced concept of defect degree and then developed the defective FG nanobeam model. They proposed two methods, i.e., sensitivity based interval analysis method and iterative algorithm based interval analysis method to solve this model and discussed different effects at last. Rajabi et al. [123] investigated the size-dependent nonlinear vibration of Euler–Bernoulli nanobeams under moving harmonic loads traveling with variable velocities within the scope of the nonlocal strain gradient elasticity theory. They employed a multistage-linearization technique to solve the Duffing equation approximately. Sahmani et al. [124] conducted an investigation to anticipate the size-dependent nonlinear bending of FG porous micro/nano-beams with uniform distribution of porosity reinforced with graphene platelets, and subjected to the uniform distributed load together with an axial compressive load. They employed the nonlocal strain gradient elasticity theory by incorporating size effects in the third-order shear deformable beam model. Shaat [125] reduced the general nonlocal theory to the strain gradient and the couple stress theories for slowly varying acoustic waves, *i.e.,* weak nonlocal fields, and then by comparing to the general nonlocal theory, discovered the nonlocal characters of the strain gradient and couple stress theories. Moreover, by fitting the experimental dispersion curves of materials, reported the nonlocal parameters and the material coefficients and length scales of these theories for some materials including diamond, graphite, silicon, silver, gold, copper, and platinum. Tang et al. [126] investigated forced vibration of FG nanobeams resting on the nonlinear elastic foundations using the nonlocal strain gradient theory. They assumed the FG material properties to be temperature-dependent and change continuously along the thickness according to the power-law function (PFGM) or sigmoid function (SFGM). They derived the governing equations based on the Euler–Bernoulli beam theory and von-Kármán geometric nonlinearity and by considering the deviation between the geometrical and physical neutral surfaces. At last they used multiple time scale method to derive closed-form approximate solution for nonlinear forced vibration of a FG nanobeam and discussed different effects. Xu [127] investigated the size effects on the dynamic behaviors of rods within the framework of the nonlocal strain gradient elastic theory. He derived variationally consistent boundary conditions using the weighted residual method with respect to the known equation of motion of rods. Zhu and Li [128] formulated the longitudinal dynamic problem of a size-dependent elasticity rod by utilizing an integral form of nonlocal strain gradient theory. They employed convolution integral over nonlocal kernel functions nonlocal strain gradient model to account for the energies diffused from surrounding particles in a reference domain. By reducing the complicated integro-differential equations to a sixth order differential equation, they derived the nonlocal strain gradient rod under various boundary conditions and explicitly showed that the integral rod model can exert stiffness-softening and stiffness-hardening effects by considering various values of the size-dependent parameters. Zhu and Li [129] developed a size-dependent integral elasticity model for a small-scaled rod in tension based on the nonlocal strain gradient theory. To incorporate the scaling effects of nonlocal stress and microstructure-dependent strain gradient, they considered nonlocal parameter and a material length scale parameter. They stated the integral rod model is both self-consistent and well-posed.

### 3.2. Nanosheets and nanoplates

Barati [130] developed a nonlocal strain gradient plate model considering two scale parameters related to the nonlocal and strain gradient effects for vibration analysis of double-layered graded nanoplates under linearly variable in-plane mechanical loads in hygro-thermal environments. He used shear deformation plate theory needless of shear correction factors and derived governing equations via Hamilton's principle and used Galerkin's method to solve the governing equations. Barati [131] modeled a vibrating porous double-nanoplate system under in-plane periodic loads using the generalized nonlocal strain gradient theory (NSGT) to examine both stiffness-softening and stiffness-hardening effects for a more accurate analysis of nanoplates. He used a modified rule of mixture to incorporated Nanopores or nanovoids to the model. He used a refined four-variable plate theory with fewer field variables than first-order plate theory. After deriving equations, he solved them for hinged nanoplates via Galerkin's method and discussed different effects on responses. Ebrahimi and Barati [132] developed a nonlocal strain gradient plate model for vibration analysis of graphene sheets resting on elastic substrate and under nonuniform in-plane mechanical loads considering two scale parameters related to the nonlocal and strain gradient effects. They modeled graphene sheet via a two-variable shear deformation plate theory needless of shear correction factors and derived the governing equations of a nonlocal strain gradient graphene sheet via Hamilton's principle. Ebrahimi and Barati [133] studied vibration analysis of double-layered graphene sheets under biaxial in-plane loads by developing a nonlocal strain gradient plate model. For more accurate analysis, they considered two scale parameters related to the nonlocal and strain gradient effects. Ebrahimi and Barati [134] developed a nonlocal strain gradient plate model for damping vibration analysis of viscoelastic modeled via a two-variable shear deformation plate theory under hygor-thermal environments considering scale parameters and strain gradient effects. Ebrahimi and Dabbagh [135] investigated smart characteristics of waves propagating in a piezoelectric nanosize plate rested on an elastic medium including surface effects. They proposed a realistic simulation for the elastic medium by utilizing a three-parameter medium containing Winkler, Pasternak and damping coefficients. Furthermore, they examined both of the decreasing and increasing impacts of small scale influences in the framework of a nonlocal strain gradient theory (NSGT). They used Kirchhoff plate theory to derive Kinematic relations and then Hamilton's principle in order to achieve Euler-Lagrange equations of piezoelectric nanoplates. Lu et al. [136] developed size-dependent Kirchhoff and Mindlin plate models to investigate the coupling effects of nonlocal stress, strain gradient and surface energy on the dynamic response of nanoplate. They used nonlocal strain gradient theory to capture nonlocal stress and strain gradient effects and incorporated the surface energy effects by surface elasticity theory. Rajabi and Hosseini-Hashemi [137] employed the nonlocal strain gradient elasticity theory for the free vibration analysis of first-order shear-deformable orthotropic nanoplates. They used multi-term extended Kantorovich method (MTEKM) in conjunction with the generalized differential quadrature method (GDQM to solve the equations of motion. They also introduced a modified Mindlin plate model by excluding the nonlocality in the shear constitutive equations. Barati [138] presented dynamic modeling and analysis of nanoporous inhomogeneous nanoplates using generalized nonlocal strain gradient theory (NSGT) with the consideration of both stiffness-softening and stiffness-hardening effects. He modeled nanoplate porosities based on a modified rule of mixture and subjected to an in-plane harmonic load in hygro-thermal environments. They modeled the porous nanoplate according to a refined four-variable plate theory with fewer field variables than in the first-order plate theory.

Sahmani and Aghdam [139] constructed a new size-dependent inhomogeneous plate model to analyze the nonlinear buckling and postbuckling characteristics of multilayer FG composite nanoplates reinforced with graphene platelet (GPL) nanofillers under axial compressive load. To this purpose, they implemented nonlocal strain gradient elasticity theory into a refined hyperbolic shear deformation plate theory. They evaluated mechanical properties of the structure based on Halpin-Tsai micromechanical scheme. Sahmani et al. [140] introduced a functionally graded porous materials (FGPMs) reinforced with graphene platelets to improve mechanical properties and investigated the size-dependent nonlinear axial postbuckling characteristics of these structures. For this purpose, they used theory of nonlocal strain gradient elasticity incorporating both stiffness reduction and stiffness enhancement mechanisms of size effects applying to the refined exponential shear deformation plate theory. They studied three different patterns of porosity dispersion across the plate thickness. They then used an improved perturbation technique to capture the size dependencies in the nonlinear load-deflection and load-shortening responses of the reinforced FGPM micro/nano-plates and reported the results. Shahverdi and Barati [141] developed a general nonlocal strain-gradient (NSG) elasticity model for vibration analysis of porous nano-scale plates on an elastic substrate as an application for nanomechanical mass sensors. They incorporated two scale coefficients to examine the vibration characteristics more accurately. They modeled Porosity properties via a modified power-law function and Mori–Tanaka model and derived the governing equations based on Hamilton's principle under hygro-thermal loading and then solved for hinged nanoplates via Galerkin's method. Xiao et al. [142] investigated the propagation behaviors of in-plane wave in viscoelastic monolayer graphene by employing nonlocal strain gradient theory. By solving the governing equation of motion derived via Hamilton's principle, they acquired closed-form dispersion relation between phase velocity and wave number and then discussed different effects such as wave number, material length scale parameter, nonlocal parameter and damping coefficient on in-plane wave propagation behaviors by conducting numerical studies.

### 3.3. Nano-tubes, Nano-shells and nano-cone

Adeli et al. [143] investigated free torsional vibration behavior of a nonlinear nano-cone made of homogeneous and isotropic materials, based on the nonlocal strain gradient elasticity theory. The cross-sectional area of this nano-cone was assumed to vary by a nonlinear function in the longitudinal direction.

Barati [144] investigated the free vibrational behavior of porous functionally graded nanoshells in the framework of nonlocal strain gradient elasticity theory. A nonlocal parameter and a strain gradient parameter are employed to describe both stiffness reduction and stiffness enhancement of nanoshells. Porosities are evenly and unevenly distributed through the thickness of the nanoshell. First-order shear deformation theory and Galerkin's method was used to obtain vibration frequencies.

There are also other worthwhile articles that we did not scrutinize due to the large number of references [145-154].

## 4. Conclusion

Many researchers have tried to develop non-classical elasticity theories. So, many researches were done on nanobeams, nanoplates, nanoshells, nanotubes, nanorods and etc. based on nonlocal elasticity, strain gradient theory, couple stress theory, surface effect and molecular dynamics. This article pays special attention to the mechanical behavior of nanobeams, nanoplates, nanotubes and nanorods in the framework of nonlocal and nonlocal strain gradient theories. Reviewing the articles in this area shows that a lot of work has been done in a recent year. That fact reflects the pace of progress in nanotechnology.

This review shows that most of the work carried out on the basis of nonlocal and nonlocal strain gradient theories has coincided with the principle of Hamilton. Results showed that the small scale effect and material length scale parameter cannot ignored for micro/nano scale materials. Unfortunately, little experimental work has been done on the field of nanomechanics, at the same time.

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