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Modelling of Suddenly Expanded Flow Process in Supersonic Mach Regime using Design of Experiments and Response Surface Methodology

Jaimon D. Quadros [a,], S. A. Khan [b], and Antony A. J. [c]*

[a] *Department of Mechanical Engineering, Birla Institute of Technology, RAK campus, Ras-Al-Khaimah, UAE*
[b] *Department of Mechanical Engineering, International Islamic University Malaysia (IIUM), Kuala Lampur, Malaysia*
[c] *Department of Mechanical Engineering, Bearys Institute of Technology, Mangalore, India*

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ABSTRACT

The present work is an attempt to model, analyze, and control the flow at the base of an abruptly expanded circular duct by using design of experiments (DOE) and response surface methodology (RSM). Tiny-jets in the form of orifice were positioned at an interval of 90^0, 6.5 mm from the primary axis of the main jet of the nozzle. Experiments were conducted to measure two responses namely, base pressure without the use of micro jets or active control (WoC) and base pressure with the use of micro jets or active control (WC). Mach number (M), nozzle pressure ratio (NPR), area ratio (AR) and length to diameter ratio (L/D) were considered as the input variables (parameters), which control the outputs (i.e. base pressure). Non-linear regression models based on central composite design (CCD) and Box-Behnken design (BBD) have been developed in order to facilitate the input-output relationships. Moreover, the significance of main, square and interaction terms of the developed models have been tested by performing analysis of variance (ANOVA). The ANOVA and significance test results and their respective correlation coefficient values indicate that both the CCD and BBD regression models are statistically adequate for both the base pressure responses of without control and with control respectively. The performances of the nonlinear models have been validated for accuracy prediction by use of 15 test cases. The performance of BBD model is found to be better in forecasting base pressure for both cases of without control and with control when compared to the CCD model.

1. Introduction

In the past few decades, the concept of base flows at high speed has been emerging as an important area of research due to its relevance in the field of external aerodynamics (Bansal et al. [1]). Such research mainly includes the study of flow field at the base of space shuttle, launch vehicles, and missiles which develop a low pressure recirculation region at its base corner. The pressure at the base is comparatively less when compared to the free stream atmospheric pressure. This difference in pressure leads to the formation of low pressure at the base leading to the formation of base drag which is approximately equal to 67 % of the net drag on a 3-D body travelling at inertia levels where the Mach number is approximately equal to 1 (i. e. $0.9 < M > 1.3$). A number of passive as well as the active control methods have been implemented for modifying the geometry of the base region or energizing the base region and are commonly called as boat-tailing and base bleed respectively. These control methods are much committed towards one particular aim of reducing the base drag. Therefore, from the above discussions it is imperative for one to distinguish the possible blend of parameters in order to accomplish base pressure control bringing about increment or decrement of it, based upon the application desired. For instance, in order to reduce the base drag, there is a need for increasing the base pressure to the level of free stream atmospheric pressure whereas in case of combustion chamber, it is required to decrease the base pressure to a very low value for improved mixing of fuel and air.

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[*] Corresponding author. e-mail: jaimonq@gmail.com

Wick [2] experimentally studied effects of inertia level, thickness and type of boundary layer on suddenly expanded flows at sonic Mach number. It was concluded that the boundary layer thickness and type of the corner largely contributed towards development of base pressure at the expansion corner. Also, that boundary layer can be considered as one of the preliminary sources of fluid for corner flow. Korst [3] designed a physical flow model for base pressure concerned with flow cases that were transonic and supersonic in nature wherein base flow beyond the wake is either sonic or supersonic. The model was developed based on interaction concepts between the viscous flow and atmospheric flow and the law of mass conservation in the wake. Experimental studies were carried out by Rathakrishnan [4] to study the consequences of employing ribs on abruptly expanded axisymmetric passage, emphasizing on reducing the pressure in the base region in the expanded duct. The optimum aspect ratio of the annular ribs that caused minimum disturbance in the pressure field of the expanded duct was found to be 3:1. Also in the case of a plain duct, it was concluded that an L/D range of 3 to 5 developed minimum base pressure for passive control. Moreover, annular ribs having aspect ratios of 3:2 and 3:3 increased pressure at the base corner as they induced oscillations into the duct pressure field. Rathakrishnan et al. [5] examined the flow of air streams in circular pipe with abrupt augmentation in area. They presumed that the pressure in the base corner is highly dependent on nozzle pressure ratio, area ratio and length to diameter ratio. The reattachment length also was dependent on L/D ratio and the exit nozzle area. Viswanath et al. [6] studied the effect of passive devices for the purpose of controlling base drag at Mach 2.0. The devices primarily involved base cavities and ventilated cavities. The results observed significant reduction in base drag by use of ventilated cavities. As high as 50 percent increment in base pressure and 3 to 5 percent reduction in base drag were obtained at Mach 2.0 for a revolutionary body. Khan et al. [7] conducted experiments to investigate the effect of microjets on base pressure in a suddenly expanded duct. The Mach numbers studied were 2.0, 2.5 and 3.0. The experiments were conducted for an overexpansion level of (P_e/P_a=0.277). It was found that micro jets served as active controllers for base pressure. The work further concluded that for a given a given Mach number and nozzle pressure ratio, one can identify the L/D ratio of the duct that will result in maximum increase/decrease of base pressure. Khan et al. [8] carried out experimental investigations in order to study active control of base pressure with microjets for Mach numbers 1.87, 2.2 and 2.58. The experiments were conducted for nozzle pressure ratios of 3, 5, 7, 9 and 11 respectively. Increase in base pressure upto 95 percent was observed for certain combination of parameters of the study. Khan et al. [9] studied the effect of microjets on suddenly expanded flows for Mach numbers 1.25, 1.3, 1.48, 1.6, 1.8 and 2.0. The experiments were conducted for an under expansion level of (Pe/Pa=1.5). Their studies found that micro jets were effective and influence the flow field in the dead region when the jets are under expanded. Khan et al. [10] examined the effect of microjets for suddenly expanded flows for nozzles for a perfectly expanded case. It was found that the microjets were not effective for Mach numbers 1.25, 1.3, 1.48, 1.6, 1.8 and 2.0. At these Mach numbers the base pressure magnitude experiences a marginal increase. The reason for this trend is the presence of a weak wave at the nozzle lip. At this point when the micro jets are activated, the Mach numbers were unable to bring about an appreciable change in the base pressure. Another important observation made was that the correctly expanded flows were dominated by waves. Control effectiveness and effect of expansion level in a suddenly expanded flow for Mach numbers 1.25, 1.3, 1.48, 1.6, 1.8, 2.0, 2.5 and 3.0 was studied by Khan et al. [11]. The experiments were conducted for nozzle pressure ratios of 3, 5, 7, 9 and 11 respectively. It was concluded that the values of base pressure increased with increase in area ratio for a given Mach number, L/D ratio and nozzle pressure ratio. This increase in base pressure was due to the increase in the relief available for the flow due to the increase in the back ward facing step height ratio. Badrinarayan [12] studied base flows at supersonic speeds experimentally. The measurements were done in the wake at the trailing edge of the blunt base of 2-D and 3-D bodies at M = 2.0. The results demonstrate the behaviour of separated flows and also point out the significance of flow reversal. Baig et al. [13] conducted experimental investigations for manipulating the base pressure through a suddenly expanded passage. To achieve this objective, micro-jets were employed as active controllers for controlling the base pressure. The Mach numbers they studied were 1.87, 2.2 and 2.58. The area ratio studied was 2.56 and L/D ratios tested were from 10 to 1. The experiments were conducted at NPRs from 3 to 11 in steps of two. The tests indicated an increase in base pressure upto 65 percent for certain combinations of flow parameters. From the literature cited above, most conventional engineering approaches have been used to identify selection of most influencing process variables on base pressure without and with active control. This has led the researchers to search for an alternative tool to study, identify, control, analyze and establish the complex input output relationships for better understanding in of controlling the suddenly expanded flows. Statistical design of experiments (DOE) incorporates the technique for outlining the set of experiments, collecting the appropriate set of data from the planned experiments, and analyzing the data using regression analysis to reach important inference on the created input–output relationship of the system Patel et al. [14]. Considerable research has been carried out by the distinguished researchers using design of experiments and statistical Taguchi method to tackle problems related to different process designs which involve fluid related problems. However, this approach has not been considered yet in analyzing flow control problems or problems related to suddenly expanded flows. This is the reason for the current study to be conducted. Hence the statistical Taguchi method can be used to tackle various problems related to suddenly expanded flows.

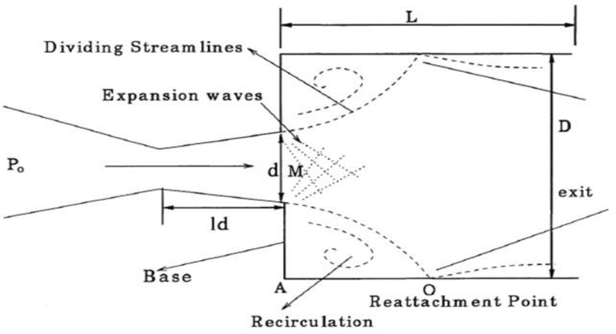


Figure 1. Suddenly expanded flow field.

RSM is a compilation of two approaches enforced for developing relationships between input and output and also identifies the curvature effects of the response function. Central composite design (CCD) and Box Behnken Design (BBD) are the two dominant classes of response surface methodologies, where in the input variables are needed to be set at three or more number of levels [15]. The CCD is rotatable provided the output prediction accuracy is same around the design center. However, these types of designs need input which are to be set at 5 levels. On the other hand, the input requires three levels for the design to be non-rotatable. It is of prime importance to decide whether the use of rotatable or non-rotatable design would be convenient. Hence the present study employs a non-rotatable CCD with a three level set input variables. Box-Behnken design is a rotatable design formed by the combination of 2k factorial designs with insufficient block designs, with k being the total count of input parameters [14].

Suddenly expanded flows without and with control can be an interesting study owing to many applications. One of such applications include the space program wherein behavior of base pressure is of prime importance for devising a mechanism that can control the base pressure in order to facilitate either its increase or decrease and has been reported in [16, 17]. This mechanism can eventually be used for high end applications such as minimizing base pressure in the ignition chamber for augmenting the blending; increase the base pressure in the case of rockets and missiles to bring about a decrease in the base drag. On the forefront, this study will help in deciding the most influential factors affecting suddenly expanded flows thus owing to reduced cost and energy consumption. Thus the present work focusses on

- Developing nonlinear input output relationships by use of statistical tools like DOE and RSM.
- Perform statistical analysis to develop nonlinear models based on non-rotatable CCD and check for adequacy. Fifteen test cases have been used to validate the developed models for practical suitability.
- One more set of nonlinear models were developed by use of BBD and effectiveness of these models were analyzed by testing them for random 15 test cases.

The performance of the two implemented models i.e. CCD and BBD were compared as per the above said validation criteria and amongst the best of these was chosen as per the least rate variation in predictability, for single of the responses. Additionally, attempts have been made to analyze the complex relationships of input parameters on base pressure by means of surface plots.

2. Nozzle Design

The Nozzle design for Mach number 3.0 as shown in Figure 2 consists of a certain set of parameters that are used as standards for its design. The flow generally occurring from supersonic Mach numbers is Blowdown flows. The parameters involved are stated below:

1. Exit diameter i.e. Ae of Nozzle (10 mm fixed).
2. Throat area (A*) is obtained from Genick [18].
3. The angle between the exit diameter and throat of the nozzle is maintained at an angle of approximately 5^0 to 8^0.
4. The angle between the inlet diameter and throat is maintained at an angle of approximately 18^0 to 30^0.
5. Thus the angel between throat and exit is maintained at 6^0 and angel between inlet and throat is maintained at 18^0 for all nozzles viz. Mach 2.0, Mach 2.5 and Mach 3.0 respectively.

3. Methodology

3.1 Selection of variables and their corresponding levels

The selection of parameters and their operating ranges are extremely important for this study for development of consistent control over the process. The present work has opted for process variables and their corresponding levels predominantly on the basis of available literature and some trial experiments conducted. From the previous studies, it has already been concluded that Mach number, nozzle pressure ratio and area ratio have largely influenced base pressure. From Baig et al. [13], an optimum L/D ratio resulting in highest value of the stagnation pressure at the exit plane was identified for a given area ratio, Inertia level and NPR. This value of L/D ratio was found to be the minimum value of duct length for the flow to circulate, separate and reattach and this was also found to be function of the area ratio, Mach number and NPR. Thus the present work emphasizes four parameters viz. Mach number, nozzle pressure ratio, area ratio and length to diameter ratio as highly influential variables for conducting the experiments. The Mach number (inertia level) and NPR have a direct influence on base pressure that is measured at the exit of the nozzle. The influence of micro-jets affecting base pressure is also primarily dependent on the level of expansion which is again dictated by NPR and Mach number. It is observed that an increase in the value of NPR, yields micro-jets to be more effective thereby causing increment in the base pressure for Mach 1.25 to Mach 2.0. However, the control becomes ineffective for Mach numbers of beyond 2.0 resulting in the decrement of the base pressure when compared to the ones without the use of control. This case explicitly takes place for the Mach numbers ranging from 2.5 to 3.0 and is reported by [19]. Thus Mach number and NPR have been chosen as preliminary factors for the present study. In the case of area ratio, it has been already reported that, base pressure is affected by area ratio due to its dependence on Mach number [11, 20, 21 and 22]. Furthermore, base pressure as a dependent variable of L/D becomes imperative in order to derive the minimum duct length that would be required for flow attachment. This L/D ratio is different for different area ratios and hence it can be thoroughly construed that L/D for a particular Mach number and NPR is largely a function of the relief to the flow preferred. Thus on the basis of above conclusions, the process parameters selected along with their corresponding levels implemented in the present study are as shown in Table 1.

3.2 Conduction of experiments

The base pressure for suddenly expanded flows has been measured for different combinations of process parameters. The experiments are conducted in a view to establish nonlinear regression models according to the design matrices and also test the developed models (test cases). The design matrix for CCD and BBD implemented for experimental data collection has been presented in Tables 2 and 3 respectively. Experimental work and data collection is explained in the subsequent section of the paper. The nonlinear regression models based on the design matrices have been developed for the responses – base pressure without control and with active control respectively. Analysis of variance (ANOVA) is performed to check the statistical adequacy of each response individually. The prediction accuracy of the developed models has been examined by assistance of fifteen random experiments and the best model is selected for each response.

4. Experimentation

The complete experimental setup is shown in Figure 3 as discussed in [7, 8, 9, 10, 11, 13 and 23]. One of the nozzles of Mach number 3.0 designed for experimentation, shown in Figure

Table 1. Factors and their respective levels.

Process Parameters			Levels		
Description	Uncoded	Coded	Low (-1)	Middle (0)	High (1)
Mach number	A	X_1	2	2.5	3
NPR	B	X_2	5	7	9
Area Ratio (AR)	C	X_3	3.25	4.75	6.25
L/D Ratio	D	X_4	4	6	8

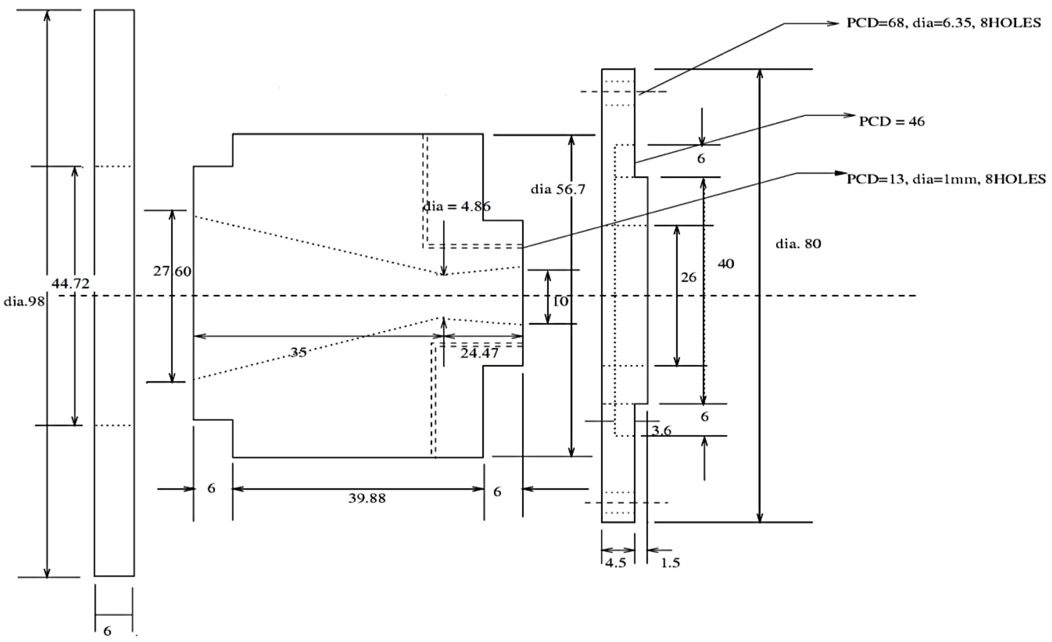


Figure 2. Nozzle design for Mach 3.0.

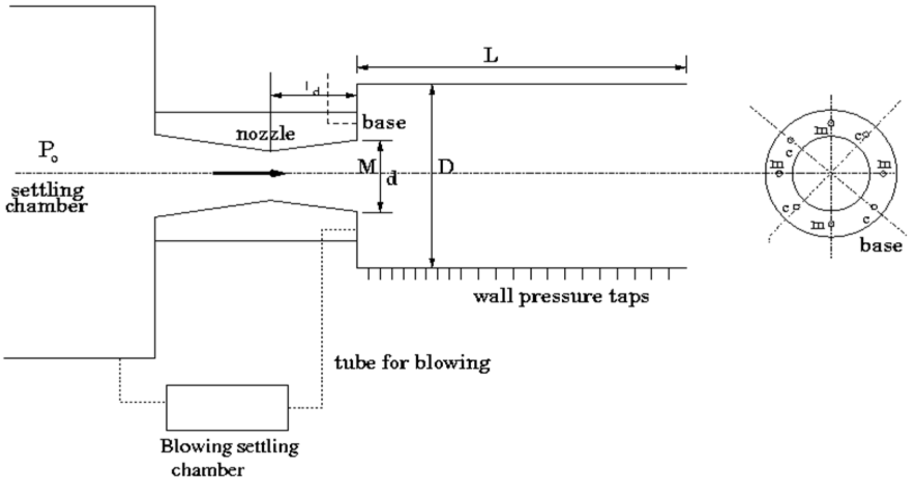


Figure 3. Experimental set up.

4. At the exit periphery of the nozzle, there are eight holes (refer Figure 3 and Figure 4), four of which (marked c) were used for blowing and the remaining four (marked m) were used for base pressure (P_b) measurement. Axisymmetric ducts having L/D ratios of 4, 6 and 8 as shown in Figure 5 have been incorporated in order to facilitate the CCD and BBD designs. The required level of expansion i.e. NPR was maintained by use of a settling chamber and air compressor and is shown in Figures 6 and 7 respectively. A PSI system 2000 make pressure transducer was implemented for measuring base pressure. It consisted of 16 channels with pressure ranging from 0-300psi. The readings were displayed for averaged 250 samples per second. The measured information comprises of base pressure (P_b) dispersion along the expanded duct along with nozzle pressure ratio (NPR) characterized as the ratio of pressure in the settling chamber (stagnation pressure) to the atmospheric pressure (P_a). This NPR also happens to be the condition of micro-jets implemented in the present study. The values measured were non-dimensionalized by dividing them by ambient atmospheric pressure (P_a). The responses involve non-dimensional base pressure measurements without the use of active control i.e. P_b/P_a (WoC) and with the use of active control i.e. P_b/P_a (WC). CCD and BBD matrices were employed for conducting experiments and two replicates have been considered for base pressure measurement. All the non-dimensional base pressures are presented within an uncertainty band of ±2.6 percent. All the results are repeatable within ±3 percent.

5. Results and Discussions

This section details about the statistical analysis of the data collected experimentally by use of two nonlinear regression models. Licensed Minitab 17 software has been used for this purpose. Tests like ANOVA, significance tests and prediction accuracy tests have been performed for determining their practical significance on the experiments conducted. The developed models were validated by fifteen random experimental test cases. Finally, the model performances are also compared among themselves.

5.1 Model Developments and Statistical Analysis

Nonlinear models for the two base pressure responses have been developed by utilizing the available experimental data. The responses along with the results of statistical analysis are given below

5.1.1. Response- Base pressure (WoC)

The following nonlinear models have been developed based on CCD and BBD for the response –base pressure without the use of active control (WoC). The input output relationships have been derived using the collected experimental data which was later implemented into the commercially licensed MINITAB software. The Eqs. for CCD and BBD are shown below

Figure 4. Nozzle.

Figure 5. Axisymmetric ducts.

Figure 6. Settling chamber.

Figure 7. Air compressor.

Table 2. CCD design matrix.

Central Composite Design (CCD)						
Process Parameters					Reponses	
S. I. No.	A	B	C	D	P_b/Pa (WoC)	P_b/P_a (WC)
1	-1	-1	-1	1	-	-
2	0	1	0	0	-	-
3	1	-1	-1	-1	-	-
4	1	0	0	0	-	-
5	-1	1	1	1	-	-
6	1	1	1	-1	-	-
7	0	0	0	1	-	-
8	-1	0	0	0	-	-
9	0	0	1	0	-	-
10	0	1	0	0	-	-
11	0	0	0	0	-	-
12	0	0	-1	0	-	-
13	-1	-1	-1	-1	-	-
14	0	0	0	1	-	-
15	0	0	1	0	-	-
16	0	0	1	0	-	-
17	1	-1	1	1	-	-
18	-1	1	-1	-1	-	-
19	-1	1	1	1	-	-
20	1	-1	1	-1	-	-
21	1	0	0	0	-	-
22	1	1	-1	1	-	-
23	0	-1	0	0	-	-
24	-1	-1	1	-1	-	-
25	-1	1	-1	1	-	-
26	-1	-1	-1	1	-	-
27	-1	1	-1	1	-	-

Table 3. BBD design matrix**.**

Box Behnken Design (CCD)						
Process Parameters					Reponses	
S. I. No.	A	B	C	D	P_b/P_a (WoC)	P_b/P_a (WC)
1	-1	-1	0	0	-	-
2	1	-1	0	0	-	-
3	-1	1	0	0	-	-
4	1	1	0	0	-	-
5	0	0	-1	-1	-	-
6	0	0	1	-1	-	-
7	0	0	-1	1	-	-
8	0	0	1	1	-	-
9	-1	0	0	-1	-	-
10	1	0	0	-1	-	-
11	-1	0	0	1	-	-
12	1	0	0	1	-	-
13	0	-1	-1	0	-	-
14	0	1	-1	0	-	-
15	0	-1	1	0	-	-
16	0	1	1	0	-	-
17	-1	0	-1	0	-	-
18	1	0	-1	0	-	-
19	-1	0	1	0	-	-
20	1	0	1	0	-	-
21	0	-1	0	-1	-	-
22	0	1	0	-1	-	-
23	0	-1	0	1	-	-
24	0	1	0	1	-	-
25	0	0	0	0	-	-
26	0	0	0	0	-	-
27	0	0	0	0	-	-

$$\begin{aligned}(\mathbf{P_b/P_a})_{\mathbf{CCD}} &= 0.41505 + 0.19774X_1 - 0.08465\,X_2 \\ &+ 0.08351\,X_3 - 0.02625\,X_4 + 0.0030\,X_1^2 \\ &+ 0.0215\,X_2^2 - 0.0275\,X_3^2 + 0.0741\,X_4^2 \\ &+ 0.01541\,X_1X_2 - 0.01684\,X_1X_3 - 0.00891\,X_1X_4 - \\ &0.04809\,X_2X_3 + 0.02097\,X_2X_4 - 0.00478\,X_3\,X_4\end{aligned} \tag{1}$$

$$\begin{aligned}(\mathbf{P_b/P_a})_{\mathbf{BBD}} &= 0.4270 + 0.20015\,X_1 - 0.12050\,X_2 \\ &+ 0.12010\,X_3 - 0.00012\,X_4 - 0.07696\,X_1^2 \\ &+ 0.02922\,X_2^2 - 0.01875\,X_3^2 + 0.03672\,X_4^2 \\ &+ 0.01756\,X_1X_2 - 0.02125\,X_1X_3 - 0.0100\,X_1X_4 \\ &+ 0.00313X_2X_3 - 0.04756X_2X_4 + 0.05056\,X_3X_4\end{aligned} \tag{2}$$

The significance test results for response base pressure (WoC) using CCD and BBD have been presented in Table 4. These tests provide significance of the process parameters, their square terms and two parameter interactions. The significance test is conducted for the nonlinear base pressure model using CCD and presented in Eq. (1) at a confidence level of 95% (Patel et al. 2014). The terms X_1, X_2, X_3, X_4, square terms- X_4^2, interaction terms- X_1X_2, X_1X_3, X_2X_3, X_2X_4 were found to be the significant factors that affect the base pressure without control as their corresponding P values were less than 0.05. It is also noticed that, the square terms are X_1^2, X_2^2 and X_3^2 are found to be insignificant as their P values are more than 0.05 (Table 4). This indicates that the relation of Mach number, nozzle pressure ratio and area ratio with base pressure might be linear in nature. However, the square term of X_4^2 is found to be significant as its P value is less than 0.05 thereby indicating nonlinear relationship of base pressure with L/D ratio. On the other hand, for the BBD model, the terms X_1, X_2, X_3, X_4, square terms- X_2^2, X_3^2, X_4^2, and interaction terms- X_1X_2, X_1X_3, X_2X_4, X_3X_4 have been found to be significant factors affecting base pressure according to their corresponding P values as they are less than 0.05. Here, the square term X_1^2 is having P value of more than 0.05 indicating that the relation between base pressure and Mach number might be linear in nature, whereas square terms X_2^2, X_3^2 and X_4^2 are found to be significant indicating a nonlinear relationship of base pressure with nozzle pressure ratio, area ratio and length to diameter ratio. The process parameters can be coded using the following relationships

X_1= [(A-2.5)/0.5], X_2= [(B-7)/2],

X_3= [(C-4.75)/1.5], X_4= [(D-6)/2]

The response Equations for base pressure (WoC) in uncoded

$$\begin{aligned}(\mathbf{P_b/P_a})_{\mathbf{CCD}} &= -0.222 + 0.387\,A - 0.1114\,B + 0.3497\,C- \\ &0.2424\,D + 0.0121\,A^2 + 0.00538\,B^2 - 0.01222\,C^2 \\ &+ 0.01853\,D^2 + 0.01541\,AB - 0.02246\,AC \\ &-0.00891\,AD - 0.01603\,BC + 0.00524\,BD - 0.00159\,CD\end{aligned} \tag{3}$$

Table 4. Significance rest results, coefficients, standard error coefficients, T Statistics and P- values obtained using CCD and BBD model for response base pressure (WoC).

Design	Central Composite Design				Box Behnken Design			
Terms	Coefficient	SE Coefficient	T	P	Coefficient	SE Coefficient	T	P
Constant	0.41505	0.00947	43.82	0.000	0.42700	0.00663	64.36	0.000
X_1	0.19774	0.00607	32.64	0.000	0.20015	0.00332	60.34	0.000
X_2	-0.08465	0.00606	-13.9	0.000	-0.12050	0.00332	-36.33	0.000
X_3	0.08351	-0.00606	13.79	0.000	0.12010	0.00332	36.21	0.000
X_4	-0.02625	0.00606	-4.33	0.000	0.00012	0.00498	-23.2	0.000
$X_1{*}X_1$	0.0030	0.01600	0.17	0.851	-0.07696	0.00332	0.03	0.980
$X_2{*}X_2$	0.0215	0.01600	1.34	0.183	0.02922	0.00498	5.87	0.000
$X_3{*}X_3$	-0.0275	0.01600	-1.71	0.090	-0.01875	0.00498	-3.77	0.000
$X_4{*}X_4$	0.0741	0.01600	4.63	0.000	0.03672	0.00498	7.38	0.000
$X_1{*}X_2$	0.01541	0.00643	2.40	0.018	0.01756	0.00575	3.06	0.003
$X_1{*}X_3$	-0.01684	0.00643	-2.62	0.010	-0.02125	0.00575	-3.7	0.000
$X_1{*}X_4$	-0.00891	0.00643	-1.39	0.169	0.01000	0.00575	1.74	0.085
$X_2{*}X_3$	-0.04809	0.00643	-7.48	0.000	-0.00313	0.00575	-0.54	0.588
$X_2{*}X_4$	0.02097	0.00643	3.26	0.002	-0.04756	0.00575	-8.28	0.000
$X_3{*}X_4$	-0.00478	0.00643	-0.73	0.459	0.05056	0.00575	8.80	0.000

Table 5. Coefficient of multiple correlation and insignificant terms of nonlinear models for response base pressure (WoC).

Model	Correlation coefficient with all R terms	Correlation coefficient without insignificant terms	Insignificant terms
CCD	0.9451	0.9419	AA, BB, CC, AD, CD
BBD	0.9871	0.9832	AA, AD, BC

$$
\begin{aligned}
(\mathbf{P_b/P_a})_{\mathbf{BBD}} &= 0.4270 + 0.20015\,X_1 - 0.12050\,X_2 \\
&+ 0.12010\,X_3 - 0.00012\,X_4 - 0.07696\,X_1^2 \\
&+ 0.02922\,X_2^2 - 0.01875\,X_3^2 + 0.03672\,X_4^2 \\
&+ 0.01756\,X_1X_2 - 0.02125\,X_1X_3 - 0.0100\,X_1X_4 \\
&+ 0.00313X_2X_3 - 0.04756X_2X_4 + 0.05056\,X_3X_4
\end{aligned}
\tag{4}
$$

Here X_1, X_2, X_3 and X_4 indicates the process parameters namely Mach number (A), nozzle pressure ratio (B), area ratio (C) and length to diameter ratio (D) in coded form respectively. Similarly A, B, C and D represent the process parameters namely Mach number, nozzle pressure ratio, area ratio and length to diameter ratio respectively in uncoded (real) form. The significance test has been carried out for both CCD and BBD based regression models separately and the insignificant terms have been identified for the base pressure (WoC) (Refer Table 5). Additionally, multiple correlation coefficients have been determined statistically in order to evaluate the precision of the models developed. The multiple correlation coefficients indicate that the model fit the assumed response equations and are found to be 0.9451 for CCD and 0.9871 for BBD (both values close to 1.0) thereby proving the accurate predictability of the equations. The significance of main, square and interaction terms of the developed models have been tested by performing ANOVA. The results of ANOVA tests for both nonlinear based CCD and BBD regression models are presented in Table 6 for the response base pressure (WoC). It is important to note that all linear, square and interaction parameters are found to be significant as their P values are found to be less than 0.05 for both models. However, even lack of fit exists for both the nonlinear models and was found to be significant as its P-values are lesser than 0.05. It is imperative to note that, by removal of insignificant terms the lack of fit becomes significant. It is to be noted that, a simpler regression equation can be produced by removal of insignificant terms, however reduces the prediction accuracy of the model.

The ANOVA and significance test results and their respective correlation coefficient values indicate both CCD and BBD regression models are statistically adequate for the response base pressure (WoC).

5.1.2. Response- Base pressure (WC)

The Eqs. 5 and 6 represent the non-linear regression equations for CCD and BBD obtained for the response- base pressure (WC). The significance test results for base pressure (WC) using CCD and BBD have been conducted at a confidence level of 95%. The coefficients of multiple correlation and the insignificant terms identified for the pair of nonlinear models are presented in Table 7.

$$
\begin{aligned}
(\mathbf{P_b/P_a})_{\mathbf{CCD}} &= -0.129 + 0.275\,A - 0.1115\,B \\
&+ 0.3470\,C - 0.2219\,D + 0.0324\,A^2 + \\
&0.00593\,B^2 - 0.01107\,C^2 + 0.01762\,D^2 \\
&+ 0.01498\,AB - 0.01760\,AC - 0.01170\,AD \\
&- 0.01803\,BC + 0.00570\,BD - 0.00288\,CD
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
(\mathbf{P_b/P_a})_{\mathbf{BBD}} &= 0.130 + 0.284\,A - 0.0909\,B \\
&+ 0.1359\,C - 0.1490\,D + 0.0345\,A^2 + \\
&0.00851\,B^2 - 0.00859\,C^2 + 0.00941\,D^2 \\
&- 0.00012\,AB - 0.01717\,AC + 0.00394\,AD \\
&- 0.00275\,BC - 0.01247\,BD + 0.01556\,CD
\end{aligned}
\tag{6}
$$

It has been observed that for the CCD model, the square terms- AA, BB, CC are found to be insignificant as their corresponding P values are found to be greater than 0.05. Thus from the significance tests, it is clearly evident that the square terms AA, BB and CC which are termed as insignificant indicating that a linear relationship exists between base pressure (WC) and Mach number, nozzle pressure ratio and area ratio. However, the square term DD was found to be significant indicating that, a nonlinear relationship exists between base pressure (WC) and length to diameter ratio. Moreover, it is also observed that, the interaction term AC is found to be insignificant as its P value was greater than 0.05. This case is contrasting when compared to the significance test for CCD model of base pressure (WoC). This delineates the fact that, interaction term AC plays a vital role in differentiating base pressure results with and without the use of active control. Here, interaction term AC represents the combination of Mach number and area ratio. At this point, when active control is employed or micro jets are activated, base pressure assumes lower values for area ratio 4.75 when compared to that for without control. This is mainly due to the combined effect of relief enjoyed by the flow, Mach number and reattachment length which is responsible for this behavior [10]. The significance test for BBD model identifies the only square term i.e. AA to be insignificant as its corresponding P value was greater than 0.05. This indicates that the relationship of base pressure with Mach number may be linear in nature. Here again the interaction terms AD and BC are found to be insignificant. The results obtained for BBD for the response base pressure (WC) is similar to those observed earlier, for the case of base pressure (WoC) (refer Table 5). The results of ANOVA show that all the linear, square and interaction terms for both the CCD and BBD models are found to be significant at the confidence levels of 95% as their values have been found to be less than 0.05 (refer Table 8). However, even lack of fit exists for both the nonlinear models and is found to be significant as its P-value are lesser than 0.05. The results from ANOVA and significance test indicate that, the non-linear regression models based upon CCD and BBD are statistically adequate for the response base pressure (WC).

5.2 Testing and Comparison of Models

It is imperative that the nonlinear CCD and BBD models developed be tested for their statistical adequacy. For this purpose, test cases were performed randomly and real experiments were conducted to record the different responses of the above test cases. The experiments were conducted for selected values of Mach number, nozzle pressure ratio, area ratio and length to diameter ratio falling in the respective range of their levels (refer Table 1), however different from those conducted as per CCD and BBD design matrix. The values of process parameters for performing random experiments are shown in Table 9. The response wise performance of the model has been presented below.

5.2.1 Response- Base pressure (WoC)

The base pressure values obtained from the randomly generated 15 test have been compared with the base pressure values obtained from the developed nonlinear regression models for the response of base pressure (WoC). The line of best fit is used to make the comparison. Here, the experimental values obtained are compared with those of corresponding model predicted values. It has been observed that, the best fit line obtained for the CCD model as shown in Figure 8a shows deviation of data points from the ideal, y=x line. However majority data points have seen to lie closer to the ideal line for the BBD model (Figure 8b) therefore indicating better predictability of BBD model when compared to the CCD model for the response (base pressure without control).

Table 6. Results of ANOVA- Base Pressure (WoC).

Design		Central Composite Design					Box Behnken Design				
Source	DF	Seq. SS	Adj. SS	Adj. MS	F	P	Seq. SS	Adj. SS	Adj. MS	F	P
Regression	14	4.23383	4.23383	0.30242	114.45	0.000	3.75725	3.75725	0.26838	508.15	0.000
Linear	4	3.88291	3.88291	0.97073	367.37	0.000	3.59646	3.59646	0.89911	1702.42	0.000
Square	4	0.13486	0.13486	0.03371	12.76	0.000	0.06978	0.06978	0.01744	33.03	0.000
Interaction	6	0.21606	0.21606	0.03601	13.63	0.000	0.09102	0.09102	0.01517	28.72	0.000
Res. Error	93	0.24574	0.24574	0.00265	-	-	0.04912	0.04912	0.00053	-	-
Lack of Fit	10	0.24568	0.24568	0.02457	34176.02	0.000	0.04936	0.04936	0.00494	768.07	0.000
Pure Error	83	0.00006	0.00006	0.000	-	-	0.00005	0.00005	0.000	-	-
Total	107	4.47957	4.47957	-	-	-	3.80637	3.80637	-	-	-

Table 7. Coefficient of multiple correlation and insignificant terms of nonlinear models for response base Pressure (WC).

Model	Correlation coefficient with all R terms	Correlation coefficient without insignificant terms	Insignificant terms
CCD	0.9401	0.9371	AA, BB, CC, AC, AD, CD
BBD	0.9872	0.9838	AA, AD, BC

Table 8. Results of ANOVA- Base Pressure (WC).

Design		Central Composite Design					Box Behnken Design				
Source	DF	Seq. SS	Adj. SS	Adj. MS	F	P	Seq. SS	Adj. SS	Adj. MS	F	P
Regression	14	4.15686	4.15686	0.29692	104.15	0.000	3.83737	3.83737	0.27410	510.89	0.000
Linear	4	3.75226	3.75226	0.93806	329.05	0.000	3.68181	3.68181	0.92045	1715.64	0.000
Square	4	0.14602	0.14602	0.03650	12.80	0.000	0.07689	0.07689	0.01922	35.83	0.000
Interaction	6	0.25858	0.25858	0.04310	15.12	0.000	0.07867	0.07867	0.01311	24.44	0.000
Res. Error	93	0.26512	0.26512	0.00285	-	-	0.04990	0.04990	0.00054	-	-
Lack of Fit	10	0.26509	0.26509	0.02651	105098.5	0.000	0.04936	0.04936	0.00494	766.16	0.000
Pure Error	83	0.000	0.000	0.000	-	-	0.00053	0.00053	0.00001	-	-
Total	107	4.42198	4.42198	-	-	-	3.88727	3.88727	-	-	-

The estimations of percentage deviation for prediction are found to lie in the range of –12.92% to +15.88% for the CCD model and -23.56% to +7.37% for the BBD models respectively (Figures 9a & 9b). It has been clearly observed that, for the CCD model most of the data points lie on the positive side, whereas, the data points are distributed on either side of the reference line for the BBD model. It is also essential to note that, BBD model has demonstrated far better prediction with regard to average absolute percent deviation for the response of base pressure without control (see Figure 10).

5.2.2 Response- Base pressure (WC)

Comparison of the predicted model values alongside their individual genuine values through the best fit line are as shown in Figure 11. It can be observed that the distribution of data points is closely distributed on both sides of the best fit line for the BBD model (Figure 11b). Hence it can be thoroughly construed that the BBD model performs better than the CCD model (Figure 11a & 11b) for nonlinear regression model of base pressure (WC). Furthermore, the percentage deviation in prediction was found to

Table 9. Input output data of test cases.

Test No.	M	NPR	AR	L/D	P_b/P_a (WoC)	P_b/P_a (WC)
1.	2.5	5	3.25	5	0.476	0.460
2.	3	7	3.25	5	0.595	0.573
3.	3	7	4.75	5	0.699	0.671
4.	2.5	7	3.25	5	0.347	0.330
5.	2	7	6.25	8	0.376	0.368
6.	2.5	9	6.25	8	0.374	0.369
7.	3	9	6.25	5	0.651	0.641
8.	2	7	6.25	5	0.327	0.360
9.	2.5	5	3.25	8	0.388	0.397
10.	2.5	5	3.25	4	0.557	0.539
11.	3	7	6.25	8	0.692	0.689
12.	3	9	4.75	4	0.619	0.700
13.	3	5	6.25	6	0.809	0.810
14.	2.5	7	4.75	5	0.489	0.471
15.	2	9	6.25	6	0.191	0.200

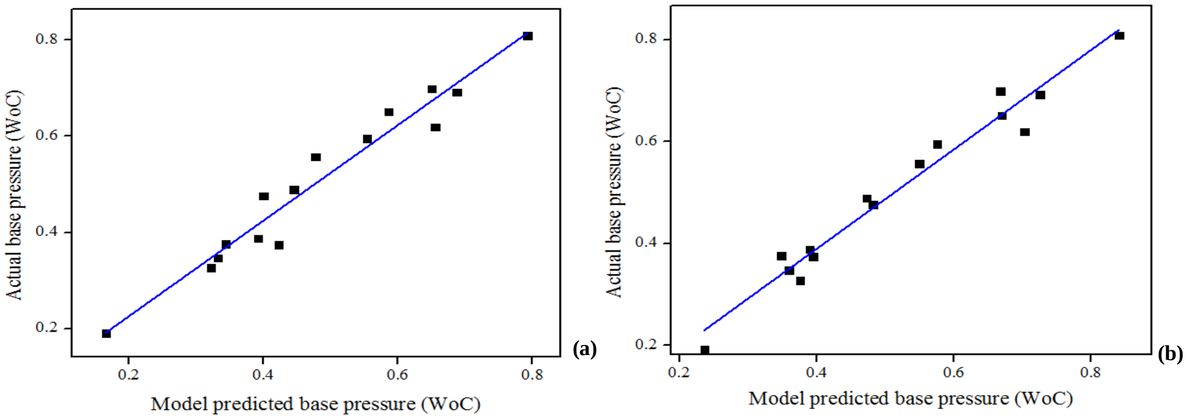


Figure 8. Comparison of model predicted base pressure (WoC) with actual base pressure (WoC) for (a) CCD; (b) BBD.

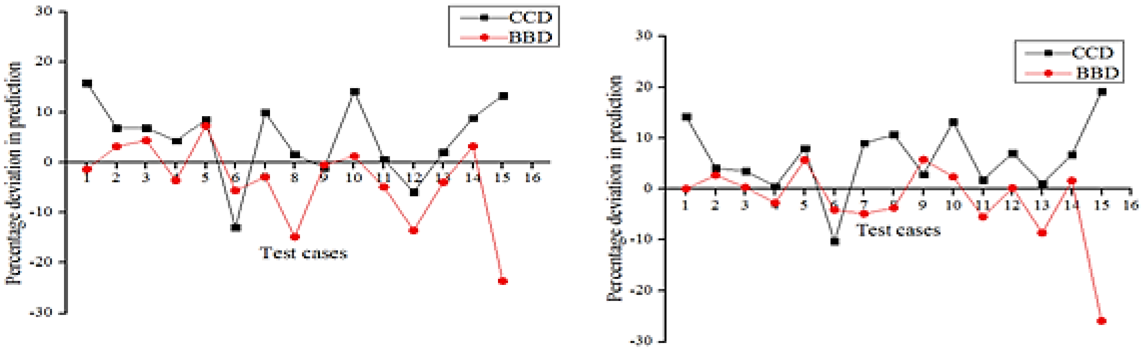


Figure 9. Standard deviation in prediction of 15 test cases for (a) base pressure (WoC); (b) base pressure (WC).

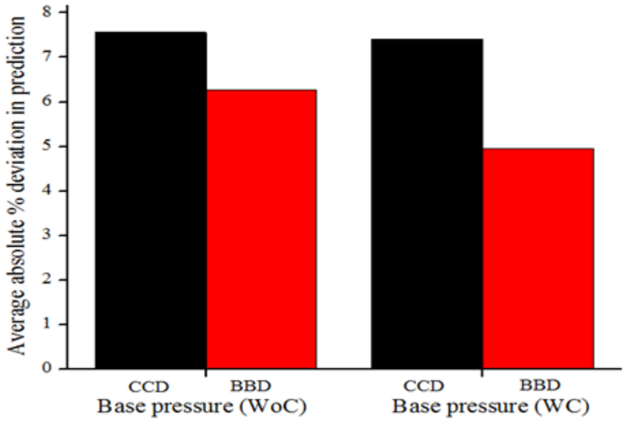


Figure 10. Comparison in terms of average absolute percent deviation in prediction of test cases for different responses.

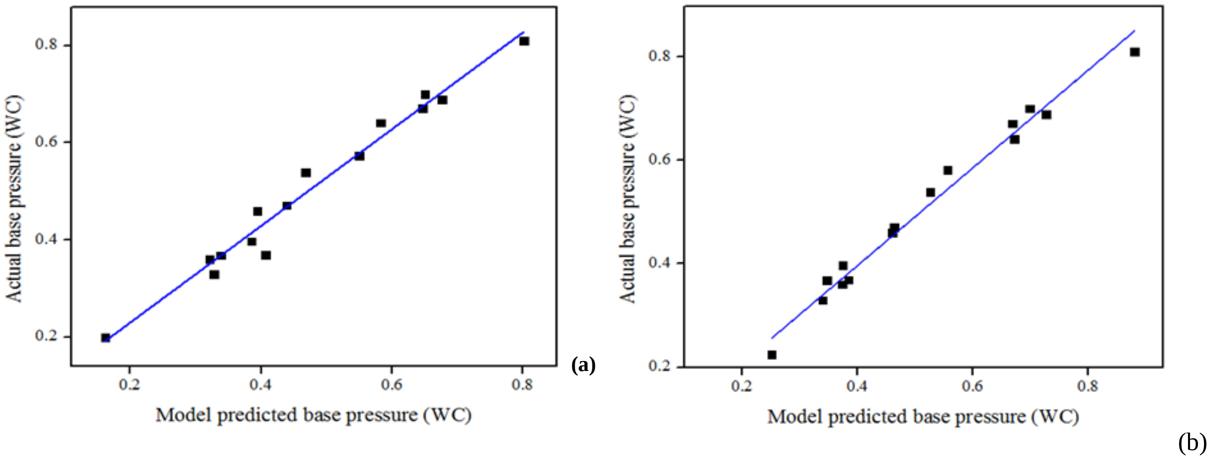


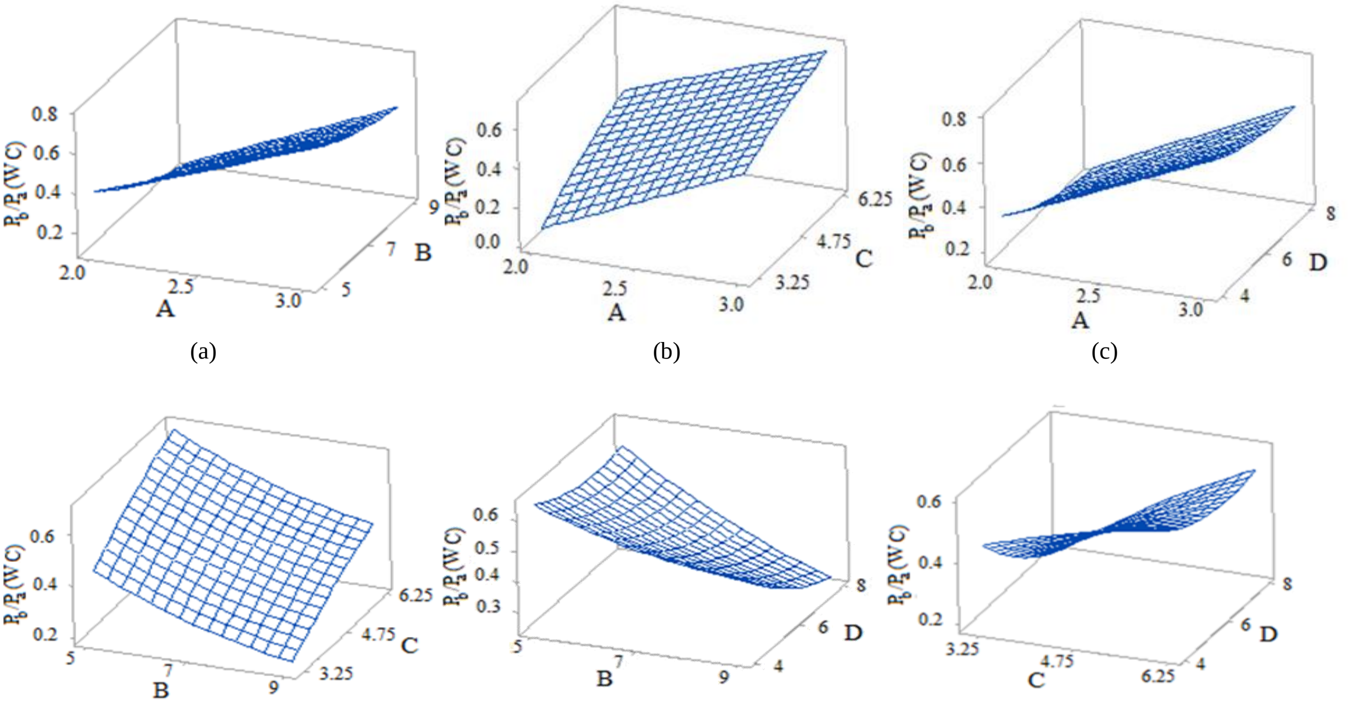
Figure 11. Comparison of model predicted base pressure (WC) with actual base pressure (WC) (a) CCD; (b) BBD.

remain in the range of -10.27% to +19.23% for CCD and -25.86% to +5.74% for the BBD model as shown in Figure 8b. Here, it is also important to note that majority of the data points for the CCD model lie above the reference line. Moreover, the BBD model shows better performance in terms of average absolute percentage deviation in predicting the response for base pressure (WC) (refer Figure 10). The better performance might be due to the ability to accurately capture the nonlinearity of the process.

5.3 Surface Plots

Based on the above results obtained, the BBD model has been found to be more convenient in prediction of base pressure. Based on the BBD model, surface plots for non-dimensional base pressure with respect to the process parameters have been analyzed. The results obtained from the significance tests are found to match well with the ones' obtained in surface plots. Surface plots for base pressure with control have been presented in Figure 12. It is important to note that, the results for base pressure with control showed the best value for average absolute percentage deviation for the BBD model (Figure 10). Hence the surface plots for base pressure with control have been plotted. Following observations were made in the study of surface plots for base pressure based on BBD approach.

- Increase in Mach number increases the base pressure linearly and an increase in NPR reduces the base pressure with area ratio and length to diameter ratio kept constant (Figure 12a). This is mainly due to the high over expansion of the jets at Mach 2.5 and 3.0 which experiences a stronger effect at the nozzle exit thereby indicating a strong linear relationship of Mach number with base pressure. However these shocks have larger shock angles and hence flow deflections are smaller. Therefore, even though flows behind these shocks experience increase in pressure, they will not be able to influence the base region. Hence, the base pressure is dictated by the re-circulating flow rather than the flow behind the shock [13]. It is also observed that contribution of Mach number towards this response is more when compared to that of NPR.
- The Figure 12(b) delineates the influence of Mach number and area ratio on base pressure. As the area ratio increases, the base pressure also tends to increase. This increase in values for base pressure for higher area ratios at high Mach numbers is due to the relief enjoyed by the flow and the vortex at the base which is not able to create enough suction which otherwise will be able to do so for the lower area ratios [24, 25].
- The Figure 12(c) shows a marginal change in the base pressure for increasing L/D ratios. The reason for this may be that, at a particular area ratio, suction at the base decreases and hence base pressure experiences an infinitesimal change with increasing L/D [13, 23]. The resulting surface plot is found to be almost flat, indicating a strong linear relationship with base pressure and is in good agreement with the significance test conducted for BBD model as shown in Table 8.



(a) (b) (c)

(d) (e) (f)

Figure 12. Surface plots of non-dimensional base pressure with (a) Mach number and NPR, (b) Mach number and AR, (c) Mach number and L/D ratio, (d) NPR and AR, (e) NPR and L/D ratio and (f) AR and L/D ratio.

- Figure 12(d) shows decrease in base pressure for increased levels of NPR. A closer look at the base region of the enlarged duct will give a possible explanation to this case. For a given area ratio, the expansion level at the nozzle exit determines the base pressure level. The prime area of concern at this point is the level of over expansion. This over expansion causes shock at the nozzle exit which is very powerful for lower NPRs, resulting in very high level of base pressure [13]. As the NPR increases, the shock at the nozzle exit becomes weaker and the level of over expansion comes down [19, 23 and 25]. It is also observed that contribution of area ratio towards this response is more (due to its steep increase) when compared to that of NPR. Hence the variation of base pressure with respect to NPR seems to be linear, whereas area ratio seems to be slightly nonlinear.
- In Figure 12 (e & f), it is clearly noticed that the base pressure observes curvature and a transition at L/D = 6 upto which it decreases and then increases with increase in L/D from 6 to 8 (Figure 12e), and decreases marginally from L/D 6 to 8 for (Figure 12f). This non linearity in base pressure variation is due to the influence of atmospheric pressure which plays a significant role owing to flow development at lower L/D ratios when compared to higher L/D ratios where after the base pressure value again increases [10, 23]. This duct value of L/D=6 is in good agreement the findings obtained by Rehman et al. [19]. Here again base pressure shows nonlinear variation with respect to (NPR and L/D ratio); (area ratio and L/D ratio).

6. Conclusions

Modelling has been carried out to determine the input– output relationships of the base pressure without and with control utilizing the two nonlinear regression models namely CCD and BBD of experiments.

- It has been observed that base pressure without control and with control for the CCD model has a more or less a linear relationship with Mach number, nozzle pressure ratio and area ratio whereas, a nonlinear relationship with L/D ratio. However for the BBD base models, base pressure exerts a nonlinear relationship with nozzle pressure ratio, area ratio and L/D ratio whereas, a linear relationship with Mach number.

- Nonlinear regression models in light of CCD and BBD are being tried and verified for their statistical adequacy and forecasting capability with the assistance of ANOVA. It has been observed that all models are found to be statistically adequate.

- The performances of CCD and BBD models have been compared response-wise by utilizing test cases. The average absolute percent deviation has been used as the criterion in order to choose the optimum model for each observation. It is important to note that, performance of BBD is found to be better in forecasting base pressure for both cases of without and with control when compared to the CCD model.

- The present work presents a methodology to model and analyze base pressure process utilizing statistical tools. Further, it will help for reducing base drag, when one needs to consider expanding base pressure to the more extreme and similarly for improvement of mixing when one needs to go for diminishing base pressure to the lowest possible value. The regression models can be used to predict the response values without conducting experiments for the set of process parameters.

- Surface plots for base pressure with respect to process parameters were plotted based on BBD approach and the results were found to agree well with the significance test results. The surface plots conveyed that Mach number, nozzle pressure ratio and area ratio showed significant contribution towards base pressure variation.

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