Comparison between unsupervised and supervised fuzzy clustering method in interactive mode to obtain the best result for extract subtle patterns from seismic facies maps

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Abstract

Pattern recognition on seismic data is a useful technique for generating seismic facies maps that capture changes in the geological depositional setting. Seismic facies analysis can be performed using the supervised and unsupervised pattern recognition methods. Each of these methods has its own advantages and disadvantages. In this paper, we compared and evaluated the capability of two unsupervised methods Fuzzy c-means (FCM) and Gustafson Kessel (GK) and one supervised method Adaptive Neuro-Fuzzy Inference Systems (ANFIS) at revealing the presence of a channel system. The process is performed in an interactive scheme in the SeisART software to obtain the best output. The seismic facies analysis was conducted on a 3D seismic data set acquired at North Sea block F3. Based on the results, the GK method outperformed the other two methods in delineating the channel pattern.

Keywords: *Seismic Facies Analysis, Seismic Attributes, Fuzzy C-means, Gustafson Kessel, Adaptive Neuro-Fuzzy Inference Systems (ANFIS).*

Introduction

Seismic facies analysis can provide useful information about the geological properties and their variations (Thenin & Larson, 2013; Figueiredo *et al*., 2014). Seismic facies maps are obtained by analyzing multi-seismic attributes with different pattern recognition algorithms. Several algorithms have been applied to the classification of seismic facies with various degrees of success (Saggaf *et al*., 2003; Hashemi, 2010; Roy *et al*., 2013; Zhao *et al*., 2015).

Generally, a seismic facies analysis includes three main steps: i) selecting an appropriate number of attributes; ii) defining the proper number of clusters, and iii) employing an appropriate pattern recognition method (Zhao *et al*., 2015). Barnes and Laughlin (2002) showed that the proper and optimal choice of seismic attributes has a greater effect on the accuracy of the seismic facies result in comparison to the choice of pattern recognition algorithm. If relevant, physically meaningful, and independent seismic attributes are not chosen, the produced seismic facies map will lack significant geological meaning (Dorrington & Link, 2004; Chopra and Marfurt, 2005; Barnes, 2007). For selecting the correct number of attributes, several methods such as cross-plot and correlation were proposed (White, 1991; Barnes, 2007). The selected attributes can be used as inputs for the seismic facies analysis or used to generate new attributes by Principal Component Analysis (PCA) or Kernel Principal Component Analysis (KPCA) (Roweis & Saul, 2000).

Concerning a proper method to select the number of clusters, numerous methods have been presented. De Matos *et al*., (2006) used Davis Bouldin index, while Marroquin (2014) used a semi-automatic method involving the interpreter to choose the optimum number of clusters.

For the pattern recognition step, there are several supervised and unsupervised methods with their own advantages and disadvantages. For supervised methods, geological knowledge based on well data is used to train the method into obtaining the more accurate result where you have a sufficient number of well information (West *et al*., 2002; Yenugu *et al*., 2010; Guillen *et al*., 2015). On the other hand, unsupervised methods can be used when well data are not available. Additionally, these type of methods are data-driven and look for similarity and regularity in the seismic data to generate seismic facies analysis (Barnes *et al*., 2002; Zhao *et al*., 2013). In this approach, there are some methods for evaluating cluster result, based on statistical theories, indicating the nearest output to a reasonable geological model (Barnes *et al*., 2002; Coléou *et al*., 2003; de Matos *et al*., 2006, Hadiloo & Shahedani, 2016).

Seismic data, due to their inherent nature, are always associated with a degree of uncertainty and imprecision (Nikravesh & Aminzadeh, 2001). Therefore, the result of seismic facies analysis suffers from some degree of imprecision. To tackle this problem more efforts have been done by using different methods of pattern recognition algorithm (Aminzadeh & Chatterjee 1984; Tamhane *et al*., 2002; Marfurt *et al*., 2014; Marroquín, 2014; Zhao *et al*., 2015).

One of the known methods in handling the uncertainty problem is fuzzy logic (Gupta *et al*., 1979: Nikravesh & Aminzadeh, 2001; Castillo *et al*., 2011). In this paper, we use an interactive procedure that includes different methods for selecting the appropriate number of seismic attributes, choosing the desired number of clusters, and creating a facies map to reveal the presence of a channel system within the MSF4 formation with the SeisART software (Hadiloo *et al*., 2017). The 3D seismic data set used was acquired at North Sea block F3 and made available by dGB Earth Sciences B.V. and TNO companies (Song *et al*., 2017).

Methodology

The procedure of seismic facies analysis according to the workflow is shown in figure 1 is presented in the following sections.

Selecting the appropriate attributes

As mentioned in the workflow, the first step of the analysis is importing seismic data to the process. After that, it is required to select appropriate attributes according to the seismic facies analysis goal (Barnes, 2007). Therefore, it is decided to use Instantaneous amplitude, instantaneous frequency, instantaneous phase, instantaneous cosine phase, similarity, texture, and energy attributes. Many seismic attributes are unstable, unreliable, and obscure with purely mathematical quantities. Also, some others duplicate each other. Therefore, using all of them is not necessary. There are several methods to select the appropriate attributes for the analysis. The inspection process is done with the cross plots of attributes. Those attributes that have less correlation with each other can be chosen to be used in seismic facies analysis. Through this method, it is concluded that the Instantaneous amplitude, instantaneous frequency, instantaneous phase, texture attributes are the most appropriate for this analysis (Figure 2a).

Another way to reduce redundant attributes is data mining techniques such as principal component analysis (PCA) and nonlinear kernel principal component analysis (KPCA) (Roweis & Saul, 2000). These methods are statistical procedures that use an orthogonal transformation to convert the set of correlated attributes into a set of values of linearly (PCA) or nonlinear (KPCA) uncorrelated variables. This transformation is defined in such a way that the first principal component has the largest possible, and each succeeding component, in turn, has the highest variance possible under the constraint that is orthogonal to the preceding components. KPCA is considered to be a good technique to work with image data (Kuralkhanov 2010). Here KPCA for feature reduction analysis is used. Figure 2a shows cross plots of two instantaneous amplitudes and instantaneous cosine phase attributes. Also, Figure 2b shows cross plots of two attributes created by KPCA. As seen in this Figure, KPCA attributes show less correlation.

Optimum number of clusters

To obtain the appropriate number of clusters for unsupervised methods, it is needed to cluster the whole samples of horizon MSF4 of seismic attributes set with various cluster numbers and then use evaluation indices to determine the best number of clusters. In this study, six evaluation indices were used partition coefficient (PC), partition index (SC), separation index (S), Xie and Beni's index (XB), Dunn's index (DI), and Alternative Dunn index (ADI) (Dunn 1973; Balasko *et al*., 2005; Wang & Zheng, 2007).

The number of clusters, obtained by all the indices, which is most repeated, is the most probable number of clusters related to a considered sample of data. Table 1 shows the results of different evaluation indices and their selected cluster number.

Table 1. Optimum cluster number by different evaluation factors indexes

Methods of attribute selection	PC	SC	S	XB	DI	ADI
Selected seismic attributes set by cross plot method	9	7	8	4	5	4
Three attributes created by first three components of KPCA analysis	8	5	5	8	7	5

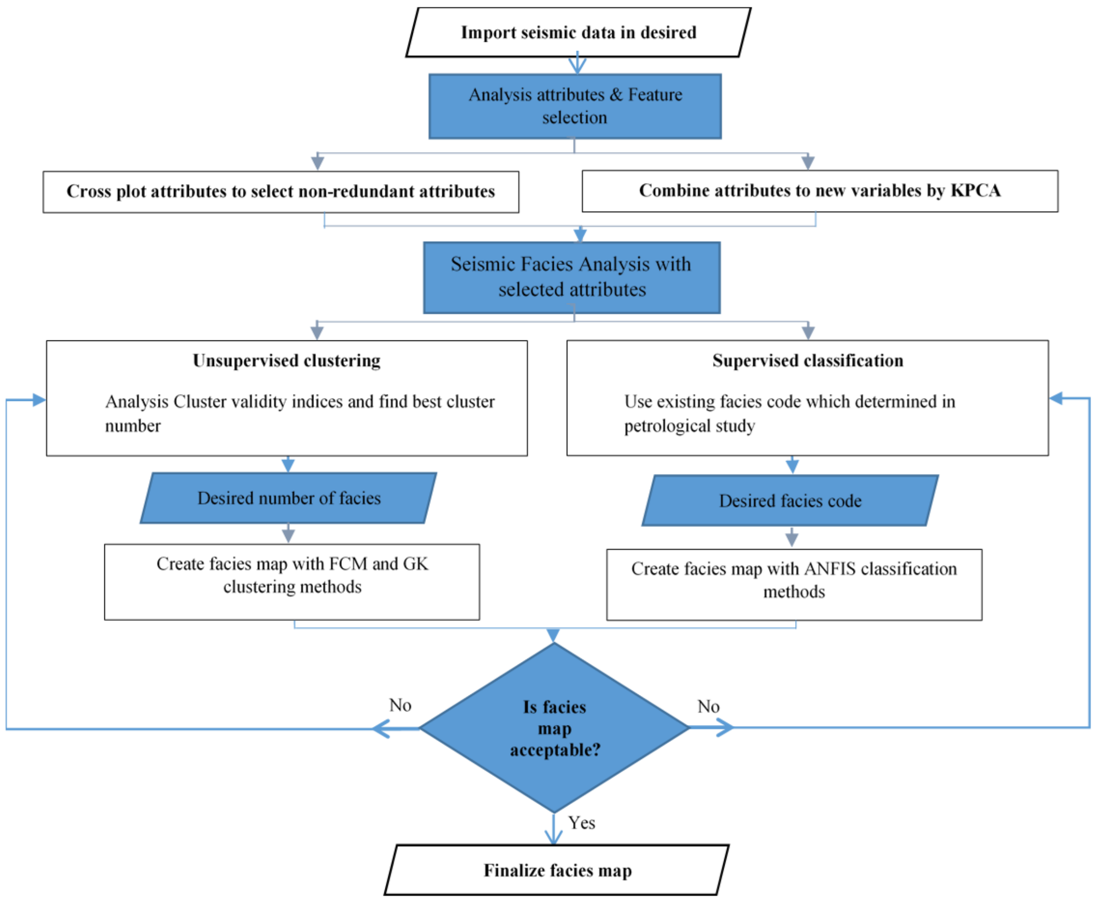

Figure 1.The workflow of seismic facies analysis

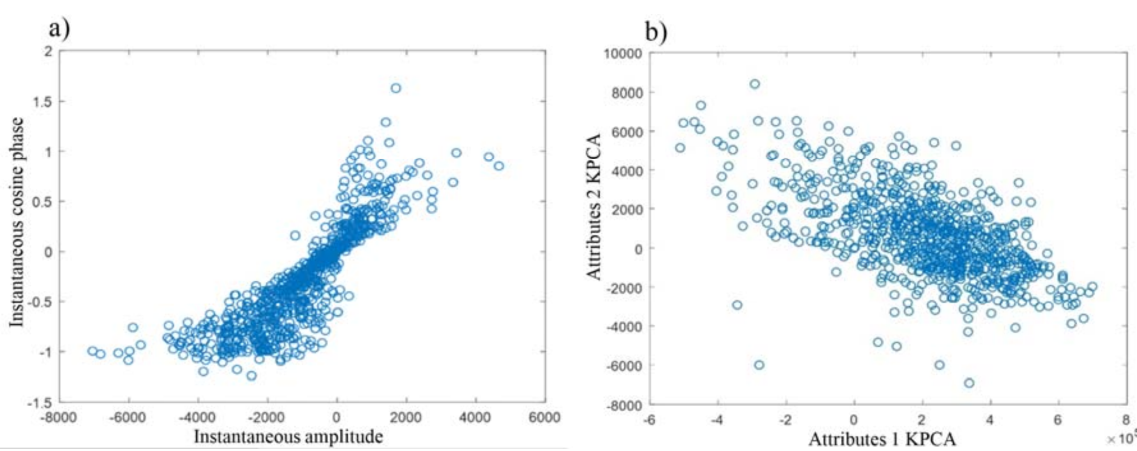

Figure 2. a) Shows cross plots of two Instantaneous amplitude and instantaneous cosine phase attributes; b) cross plots of two attributes created by KPCA.

As shown in the table, the number of four clusters is the best number for clustering the considered sample of horizon based on seismic attributes selected by cross plot method. KPCA analysis presents five clusters.

Considering the supervised clustering, the number of clusters is determined based on the analysis of petrological information of well data which here this number is four.

Clustering

We used Fuzzy c-means (FCM) (Dunn, 1973), Gustafson and Kesel (GK) (Gustafson & Kessel, 1979) fuzzy clustering methods as unsupervised clustering techniques (Bezdek, 1980; Ghosh *et al*., 2011) Adaptive Neuro-Fuzzy Inference System (ANFIS) as a supervised technique. Here we present a briefed theory of these methods.

FCM

Fuzzy c-means (FCM) is a method of clustering. This method allows one member of data to belong to two or more clusters. This method (Dunn 1973; Bezdek 2013) is based on minimization of the following objective function:

$$J_m = \sum_{i=1}^{N}\sum_{j=1}^{C} u_{ij}^m \left\| x_i - c_j \right\|^2, \qquad 1 \le m < \infty \tag{1}$$

where $\boldsymbol{m}$ is any real number greater than 1, $\boldsymbol{u_{ij}}$ is the degree of membership of $\boldsymbol{x_i}$ in the cluster $\boldsymbol{j}$, $\mathbf{x_i}$ is the $\boldsymbol{i}$th of d-dimensional measured data (here d is the number of selected attributes in sample base clustering and is the number of seismic samples in the selected horizon in trace shape clustering), $\mathbf{c_j}$ is the d-dimension center of the cluster, and ||*|| is any norm expressing the similarity between any measured data and the center. Fuzzy partitioning is carried out through an iterative optimization of the objective function shown above, with the update of membership $\boldsymbol{u_{ij}}$ and the cluster centers $\boldsymbol{c_j}$ by:

$$u_{ij} = \frac{1}{\sum_{k=1}^{C}\left(\frac{\left\| x_i - c_j \right\|}{\left\| x_i - c_k \right\|}\right)^{\frac{2}{m-1}}} \quad , \qquad c_i = \frac{\sum_{i=1}^{N} u_{ij}^m . x_i}{\sum_{i=1}^{N} u_{ij}^m} \tag{2}$$

This iteration will stop when, where ε is a termination criterion between 0 and 1, whereas k is the iteration steps. This procedure converges to a local minimum or a saddle point of $\boldsymbol{J_m}$.

Gustafson-Kessel clustering algorithm

The Gustafson-Kessel algorithm associates each group with a point and a matrix, respectively, that represent the center of the group and its covariance (Gustafson & Kessel 1979). While the original fuzzy c-means make the hypothesis that the clusters are spherical, the GK algorithm is not subject to this restriction and can identify ellipsoidal clusters. Denote f_{ir} the influence of point $\boldsymbol{i}$ on the group $\boldsymbol{r}$, the center of the group and the covariance matrix are calculated as

$$w_r = \frac{\sum_{i=1}^{n} f_{ir}^m x_i}{\sum_{i=1}^{n} f_{ir}^m} \tag{3}$$

$$A_r = \sqrt[p]{\det(S_r)} S_r^{-1} \tag{4}$$

with $S_r = \sum_{i=1}^{n} f_{ir}^m (x_i - w_r)(x_i - w_r)^T$.

where $\boldsymbol{p}$ is the feature-space dimension, $\boldsymbol{f}$ is the membership function, $\boldsymbol{m}$ is a user-defined parameter called fuzzifier. The center of the cluster is calculated as a weighted average of all data, the weights depend on the algorithm considered, as detailed in the following. The covariance matrix is defined as a fuzzy equivalent of classic covariance. Through eq. (4), a size restriction is imposed on the covariance matrix whose determinant must be 1. As a consequence, the GK algorithm is able to identify ellipsoidal groupings that are approximately the same size. This update stage of the cluster parameter is alternated with the updating of the weighting coefficients until a convergence criterion is met (Lesot & Kruse 2008).

Adaptive Neuro-Fuzzy Inference Systems (ANFIS)

ANFIS can serve as a basis for constructing a set of fuzzy if-then rules with appropriate membership functions to generate the stipulated input-output pairs with combining the topology of the neural network and fuzzy logic (Jang, 1993; Abraham, 2005). ANFIS uses the characteristics of both methods and also eliminates some disadvantages of their lonely-used case. The operation of ANFIS looks like feed-forward backpropagation network. Consequent parameters are calculated forward while premise parameters are calculated backward. In the fuzzy section, only zero or first-order Sugeno inference system or Tsukamoto inference system can be used (Tsukamoto, 1979; Takagi & Sugeno, 1985). ANFIS algorithm is composed of two membership tuning steps: coarse tuning and fine tuning. A lot of methods are proposed for these two tuning procedures (Kim *et al.,* 1997; Chen, 1999). In this paper, we used fuzzy c-means (FCM) clustering for extracting and coarse tuning of the rules that model the data behavior. For fine-tuning, which adjusts the premise and consequent parameters more precisely, we used a hybrid method which combines the gradient method and the least squares estimate (LSE) to identify parameters. Fine tuning procedures are repeated to find the appropriate parameters. Finally, output variables are obtained by applying fuzzy rules to fuzzy sets of input variables. For example:

Rule 1: If x is A_1 *and y is* B_1 *then* $f_1 = p_1x + q_1y + r_1$

Rule 2: If x is A_1 *and y is* B_2 *then* $f_2 = p_2x + q_2y + r_2$

Since ANFIS combines both neural network and fuzzy logic, it is capable of handling complex and nonlinear problems. Even if the targets are not given, ANFIS may reach the optimum result rapidly. The architecture of ANFIS consists of five layers and the number of neurons in each layer equals to the number of rules. In addition, there is no vagueness in ANFIS as opposed to neural networks (Jang *et al*., 1997; Kumar & Garg, 2004).

Implementation

The procedure of facies analysis explained above, has been implemented on a 3D seismic data set acquired at block F3 of the North Sea (Figure 3) where F3 is a block in the Dutch sector of the North Sea. The block is covered by 3D seismic in the Upper-Jurassic – Lower Cretaceous strata. The upper 1200ms of the data set consists of reflectors belonging to the Miocene, Pliocene, and Pleistocene. The large-scale sigmoidal bedding is readily apparent and consists of the deposits of a large fluviodeltaic system that drained large parts of the Baltic Sea region (Sørensen *et al*, 1997; Overeem *et al*., 2001).

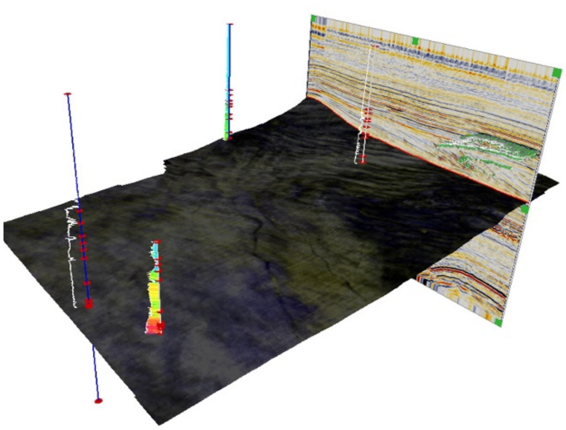

Figure 2. Seismic amplitudes on MSF4 formation.

The deltaic package consists of sand and shale, with an overall high porosity (20–33%). Some carbonate-cemented streaks are present. A number of interesting features can be observed in this package. The most striking feature is the large-scale sigmoidal bedding, with text-book quality downlap, toplap, onlap, and truncation structures. Several seismic facies can be distinguished: transparent, chaotic, linear, shingles. Well logs show the transparent facies to consist of a rather uniform lithology, which can be either sand or shale. The chaotic facies likely represent slumped deposits. The shingles at the base of the clinoforms have been shown to consist of sandy turbidites (dGB Earth Sciences B.V., 2013). We used this data to evaluate the capability of unsupervised and supervised pattern recognition methods at identifying a channel system. The log data of four wells, located at the seismic data acquisition region (Figure 3), are employed for facies analysis.

In unsupervised approach, two Fuzzy clustering methods (FCM and GK) were used and the results are shown in the figures 4 (for FCM) and 5 (for GK).

As seen in the results of FCM, the clustering algorithm applied on seismic attributes has been able to provide better results in delineating channel pattern than on KPCA attributes. Figure 5 shows results of GK clustering method and as seen they outperform those of FCM clustering method. Afterward, similar to FCM results, it can be seen that the result of the application of GK clustering method on seismic attributes is better than that of the GK clustering on KPCA attributes.

In supervised approach, ANFIS method with four clusters reported by petrological information of the existing four wells. Seismic attributes in the selected sample point in the near of wells are used to train fuzzy inference system. Figure 6 shows the results of ANFIS clustering method. The locations of the wells are remarked on the maps. As seen in the figure, ANFIS supervised method is not able to delineate the channel pattern, although as well as unsupervised methods (figures, 4 and 5), the result of clustering applied on seismic attributes (Figure 6a) has been better rather than the result of the application of clustering method on KPCA attributes (Figure 6b). The reason for the weak performance of ANFIS method can be due to the lack of wells in the location of channels.

Comparing the results of all methods could give more knowledge about seismic facies in the selected horizon. All these results help the interpreter to choose appropriate input variables, seismic attributes and analyzing a method to obtain a subtle pattern. In this proposed method, there is the ability to obtain seismic facies in the consecutive samples to follow seismic facies in thickness of horizon. This approach could reveal the shape of seismic facies analysis in a different depth. For example, in most cases, fault structure is available in all thicknesses and samples of one horizon but for the channel, this feature exists in some consecutive samples depending on the thickness of the channels, and by analyzing these samples the interpreter is able to distinguish between different features.

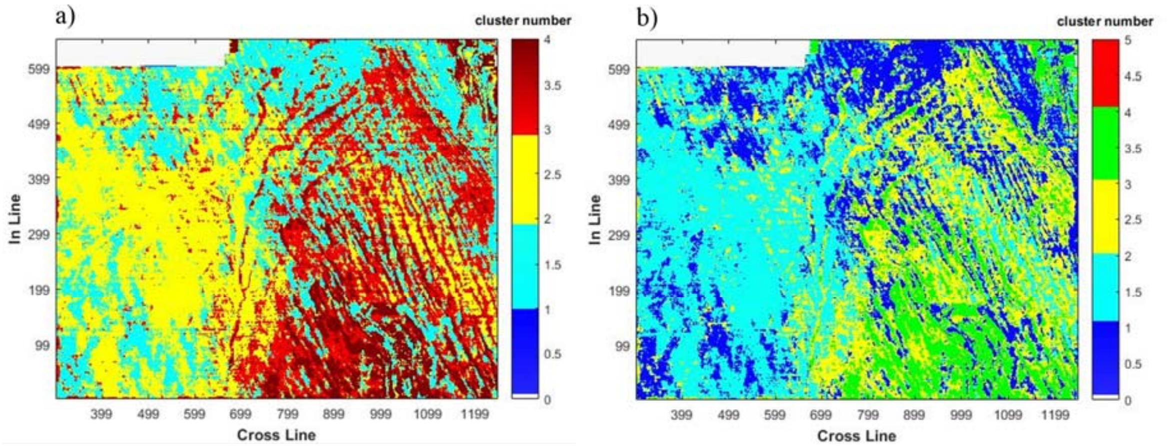

Figure 3.seismic facies analysis result obtained by FCM clustering method applied on a) seismic attributes set and b) KPCA attributes.

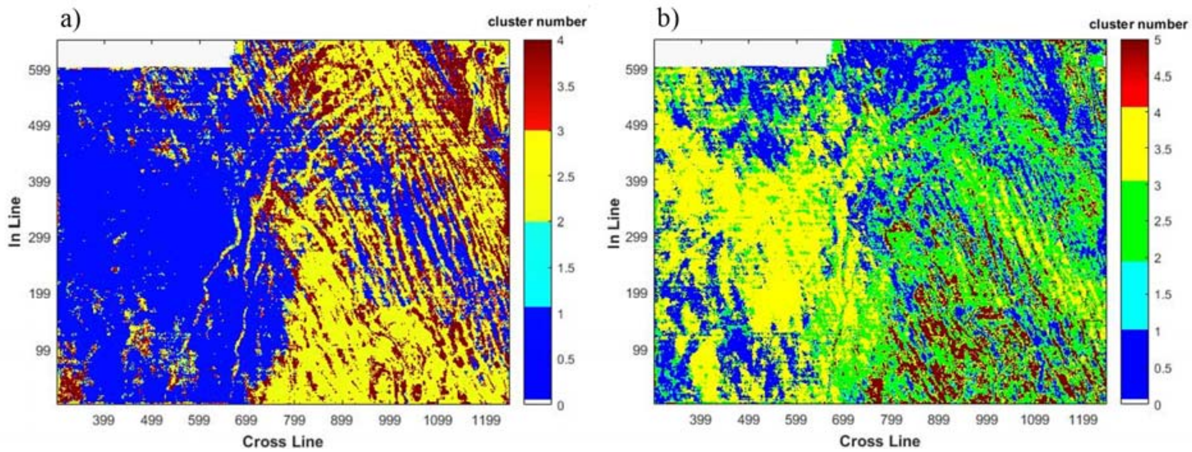

Figure 4. seismic facies analysis result obtained by GK clustering method applied on a) seismic attributes set and b) KPCA attributes.

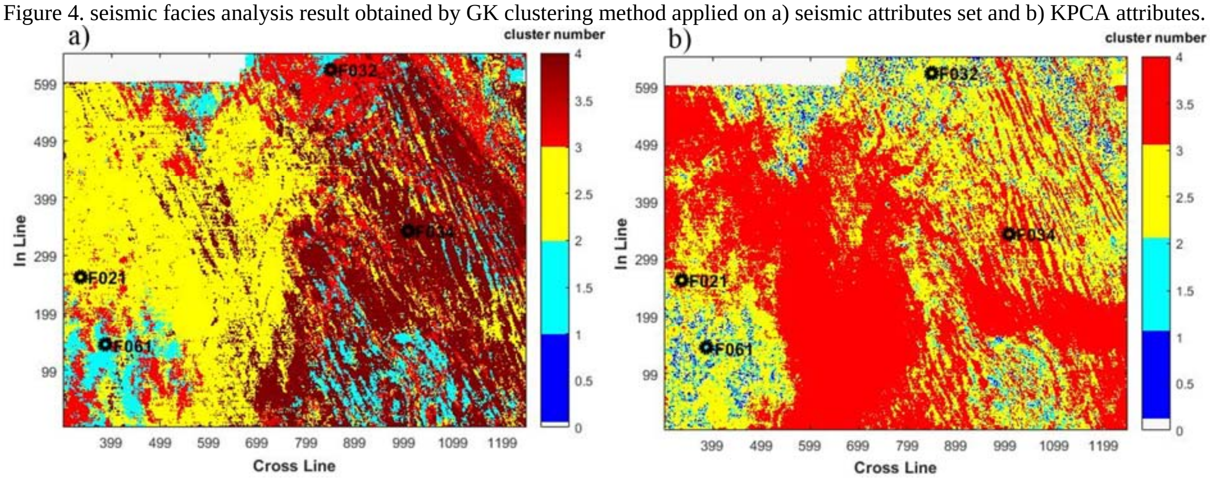

Figure 5. Seismic facies analysis result obtained by ANFIS clustering method applied on a) seismic attributes set and b) KPCA attributes. The well locations are remarked on figures.

Conclusion

In this study, seismic facies analysis was implemented using unsupervised fuzzy clustering methods of FCM and GK, and supervised ANFIS clustering method. It was shown that GK method can be able to provide a better result in delineating channel pattern than FCM method, it can be due to GK clustering formula that uses ellipsoidal clusters so that the channel pattern, as a longitudinal event, is a favorite target for ellipsoidal clusters. This result is obtained with the interaction of the interpreter to choose appropriate input and parameter to create seismic facies analysis with high accuracy. Considering the ANFIS method, it is concluded that when there are not any well data at the location of channel pattern, supervised ANFIS method cannot provide an acceptable result. Another point of the results is that the attributes created by KPCA method can deteriorate output of clustering methods when the target is channel pattern visualization.

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